

Interference Pricing for SINR-based Random Access Game

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Abstract—In this paper, we study the problem of random access with interference pricing in wireless ad hoc networks using non-cooperative game theory. While most of the previous works in random access games are based on the protocol model, we analyze the game under the more accurate signal-to-interference-plus-noise-ratio (SINR) model. First, under the setting with fixed interference linear pricing, we characterize the existence of the Nash equilibrium (NE) in the random access game. In particular, when the utility functions of all the players satisfy a risk aversion condition, we show that the game is a *S-modular* game and characterize the convergence of the strategy profile to the NE. Then, under the setting with adaptive interference linear pricing, we propose an iterative algorithm that aims to solve the network utility maximization (NUM) problem. Convergence of the solution to a Karush-Kuhn-Tucker (KKT) point of the NUM problem is studied. It can be shown that the solution obtained under the protocol model may result in starvation for some users due to the inaccurate interference pricing. Simulation results show that our proposed algorithm based on the SINR model achieves a higher average utility than the algorithm based on the protocol model and a carrier sense multiple access (CSMA) scheme implemented in a slotted time system.

Index Terms—Medium access control (MAC), random access, SINR model, non-cooperative game theory, S-modular game, network utility maximization, interference pricing.

I. INTRODUCTION

IN a wireless ad hoc network, a medium access control (MAC) protocol is used to coordinate the access of the users to the shared wireless medium. In general, there are two main classes of MAC protocols: *scheduling* and *random access*. In a scheduling-based MAC, the transmissions of the users are scheduled orderly in an attempt to prevent packet collisions among the users. In a random access MAC, the users need to contend for the channel for transmission. In this paper, we focus on random access MAC protocols because of their scalability and flexibility. Due to the contention nature of the random access protocols, it is important to study the interaction of multiple rational users with *strategic interdependence* [1], where the utility of a user depends on both its own action and the actions of the other users.

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Game theory is a useful tool in analyzing the strategic behavior of users with self-interest in various random access problems. In [2], Cagalj *et al.* modeled carrier sense multiple access with collision avoidance (CSMA/CA) using game theory. Both normal-form and repeated-form CSMA/CA games were formulated and the existence of a Nash equilibrium (NE) was shown. In [3], game theory was applied to analyze the behaviour of selfish nodes in a one-shot random access game. Necessary and sufficient conditions for the NE were proposed, and the asymptotic properties of the system were studied. In [4], Cui *et al.* proposed a comprehensive framework to study non-cooperative random access games. The Nash equilibria were characterized for various settings and distributed algorithms were proposed. In [5], a cartel maintenance repeated game framework was proposed. A trigger-punishment rule was designed so that it is always in each user's best interest to cooperate. Yang *et al.* in [6] and [7] studied distributed random access for wireless ad hoc network with pricing based on non-cooperative game theory. The convergence and uniqueness of the NE were analyzed. Long *et al.* in [8] formulated a non-cooperative game for random access and power control in wireless ad hoc networks. An asynchronous distributed algorithm was proposed that converges to the NE globally. Lee *et al.* in [9] reverse-engineered an exponential backoff random access protocol. They showed that each user is implicitly participating in a non-cooperative game by adjusting its transmission probability to maximize its utility function. Chen *et al.* in [10] proposed an analytical framework for random access using game theory. Distributed algorithms were proposed to achieve the NE.

In the analysis of the MAC protocols using game theory in [4], [6], [7], [9], [10], the *protocol model* [11] is used to account for the effect of multi-user interference. Under the protocol model, a transmission is successful if the receiver is within the transmission range of its intended transmitter and outside the interference range of other transmitters. However, in reality, the interference at the receiver is the *cumulative* power received from other nodes that are concurrently transmitting. As a result, the *signal-to-interference-plus-noise-ratio (SINR) model* [11] characterizes the effect of interference more accurately than the protocol model, which is based on a simplifying assumption on the multi-user interference. Under the SINR model, a transmission is successful if and only if the SINR at the intended receiver is above a predefined threshold that depends on the transmit power, adopted modulation and coding schemes. Despite its higher complexity, some recent works in contention-based *random access* [12]–[14] have adopted the SINR model due to its higher practicality and

accuracy in modeling.

In this work, we consider the random access problem in a wireless ad hoc network with linear *interference pricing*, which is a form of *congestion pricing* that encourages the efficient use of the network resources [15]–[17]. In a network with a very low level of congestion, the bandwidth is often under-utilized. However, on the other extreme when the level of congestion is high, serious service degradation will result. By incurring an appropriate interference price on each user which is proportional to its transmission probability, we aim to maximize the aggregate utility of all the users and maintain the level of congestion at a satisfactory level. It should be noted that the interference pricing is a linear term that is similar to the energy cost studied in the previous literature [18]–[20].

In the first part of the paper, we consider the setting with *fixed* linear interference pricing. We study the interactions of the rational users, who have perfect information on some parameters of the other users, using non-cooperative game theory and prove the existence of the NE. In particular, if all the users satisfy a risk aversion condition, then we can prove that the random access game is a *S-modular* game [21] (i.e., a supermodular game or a submodular game [22]) under the SINR model. In the second part of the paper, we consider the setting with *adaptive* linear interference pricing. We propose an iterative interference pricing scheme that aims to solve the network utility maximization (NUM) problem based on the Karush-Kuhn-Tucker (KKT) conditions.

In summary, the contributions of our work are as follows:

- We characterize the existence of the NE in the random access game with fixed interference pricing under the SINR model.
- We extend the previous results in [4], [6], [7] on supermodular random access game under the protocol model to the S-modular game under the more accurate SINR model.
- We present an adaptive interference pricing scheme for the random access game, where the solution converges to a KKT point of the NUM problem.
- Simulation results show that our proposed algorithm based on the SINR model achieves a higher average utility than that based on the protocol model and a CSMA scheme.

The framework that we present in this paper is unique compared to the prior work in random access game. Although some previous works have shown that the random access game is either a supermodular game in [4], [6], [7] or a submodular game [9] under the protocol model, we extend their results to a more general model, which includes both the SINR and protocol models, and consider the S-modular game. For the interference pricing scheme, it should be noted that a similar approach has been applied in [23] to study the power control problem, and in [6] to study the random access problem under the protocol model. In contrast, we consider the random access problem under the SINR model. Moreover, it was proved in [23] that the game is still a supermodular game when the price is adaptive, so that the convergence of the best response update is guaranteed. However, in the random access setting in this paper, it can be shown that the game is *not* a supermodular

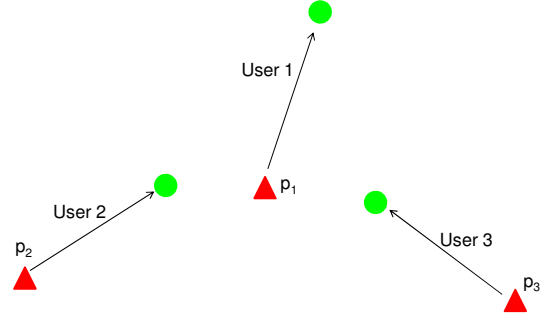


Fig. 1. A wireless ad hoc network with a set of users $\mathcal{N} = \{1, 2, 3\}$, where the triangles and circles represent the transmitters and receivers, respectively. p_i denotes the transmission probability of user i . The protocol model suggests that users 2 and 3 do not interfere with user 1. However, the SINR model reveals that the *simultaneous* transmissions by users 2 and 3 will interfere with user 1, which is not taken into account in the protocol model.

game when the adaptive pricing is used. Thus, the convergence of the solution to a fixed point is not obvious. For the NUM problem, although a similar problem has been considered in [13], [14], these works obtained the near-optimal solution of the NUM problem by successive convex optimization. In contrast, we consider a different solution approach in this paper by applying the iterative interference price update in the random access game.

The rest of the paper is organized as follows. The system model is described in Section II. We study the interactions of the rational users with fixed interference pricing in Section III. In Section IV, we propose an adaptive interference pricing scheme that aims to maximize the aggregate utility. Simulation results are presented in Section V. Conclusions and future work are given in Section VI.

II. SYSTEM MODEL

Consider a wireless ad hoc network with several nodes located in a neighbourhood, the transmission between the transmitter (source) and the receiver (destination) is within one-hop. We define $\mathcal{N}$ as the set of one-hop transmitter/receiver pairs or links in the wireless network, and we refer to each transmitter/receiver pair as a *user*. The total number of users is $N = |\mathcal{N}|$. However, it should be noted that *not* all the nodes are one-hop neighbours to each other as shown in Fig. 1. We adopt a slotted MAC protocol, where time is divided into equal time slots. The users attempt to access the shared channel at the beginning of each time slot according to their *transmission probabilities* in each channel. That is, each user $i \in \mathcal{N}$ can access the wireless channel with a certain transmission probability $p_i \in \mathcal{P}_i = \{p | p_i^{min} \leq p \leq p_i^{max}\}$. We define a transmission probability vector $\mathbf{p} = (p_i, \forall i \in \mathcal{N})$.

For the *SINR model*, if user $i \in \mathcal{N}$ chooses to transmit, then the SINR at receiver i is given by

$$\theta_i = \frac{P_i G_{ii}}{I_i + n_i}, \quad (1)$$

where P_i is the transmit power of user i , and we assume that it is fixed. G_{ij} is the channel gain from the transmitter of user i to the receiver of user j . I_i and n_i are the interference and

noise powers received by user i , respectively. The communication of user i is successful if

$$\theta_i \geq \theta_i^{th} \Leftrightarrow I_i \leq \frac{P_i G_{ii}}{\theta_i^{th}} - n_i, \quad (2)$$

where θ_i^{th} is the SINR threshold.

Let $\mathbb{N}_i$ be the power set (i.e., the set of all subsets) of $\mathcal{N} \setminus \{i\}$. As an example, for $\mathcal{N} = \{1, 2, 3\}$, $\mathbb{N}_2 = \{\{\}, \{1\}, \{3\}, \{1, 3\}\}$. Assuming that the transmit powers $(P_i, \forall i \in \mathcal{N})$ are fixed, we define $\mathbb{M}_i$ as a set where each element is a set of users that can transmit simultaneously with user i without affecting the reception of receiver i (i.e., θ_i^{th} can be achieved). The set $\mathbb{M}_i$ obtained with the SINR model is given by

$$\mathbb{M}_{i, SINR} = \left\{ \mathcal{M} \in \mathbb{N}_i \mid \sum_{m \in \mathcal{M}} P_m G_{mi} \leq \frac{P_i G_{ii}}{\theta_i^{th}} - n_i \right\}. \quad (3)$$

Under the protocol model, user m is an *interferer* or *one-hop neighbour* to user i if the SINR due to the interference from user m only is below the SINR threshold. That is,

$$\frac{P_i G_{ii}}{P_m G_{mi} + n_i} < \theta_i^{th}. \quad (4)$$

Let $\mathcal{I}_i = \left\{ m \in \mathcal{N} \setminus \{i\} \mid \frac{P_i G_{ii}}{P_m G_{mi} + n_i} < \theta_i^{th} \right\}$ be the set of interferers of user i . Notice that any subset of users in $\mathcal{N} \setminus (\mathcal{I}_i \cup \{i\})$ is an element of $\mathbb{M}_i$ obtained with the protocol model. Thus, we have

$$\mathbb{M}_{i, PTC} = \left\{ \mathcal{M} \in \mathbb{N}_i \mid P_m G_{mi} \leq \frac{P_i G_{ii}}{\theta_i^{th}} - n_i, \forall m \in \mathcal{M} \right\}. \quad (5)$$

It should be noted that the protocol model we use in this paper is slightly different from that in [11], where a transmission is successful if the receiver is within the transmission range of its intended transmitter and outside the interference range of the other transmitters.

Let $\mathbf{p}_{-i} = (p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_N) \in \mathcal{P}_{-i} = \mathcal{P}_1 \times \dots \times \mathcal{P}_{i-1} \times \mathcal{P}_{i+1} \times \dots \times \mathcal{P}_N$. The average data rate of user i is given by

$$r_i(\mathbf{p}, \mathbb{M}_i) = \mu_i p_i S_i(\mathbf{p}_{-i}), \quad (6)$$

where

$$S_i(\mathbf{p}_{-i}) = \sum_{\mathcal{M} \in \mathbb{M}_i} \left(\prod_{m \in \mathcal{M}} p_m \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq i} (1 - p_k) \right). \quad (7)$$

$S_i(\mathbf{p}_{-i})$ represents the conditional probability of successful transmission of user i given that user i transmits. We assume that user i is transmitting with the fixed data rate $\mu_i = W \log_2(1 + \theta_i^{th})$, where W is the channel bandwidth. That is, the packet reception at receiver i is successful if and only if the achieved SINR is no less than θ_i^{th} . For the rest of the paper, we assume that sets $\mathbb{M}_i$ in (3) and (5) are given, so we write $r_i(\mathbf{p}, \mathbb{M}_i)$ in (6) as $r_i(\mathbf{p})$ for simplicity.

Let $U_i(r_i(\mathbf{p}))$ be the utility function of user $i \in \mathcal{N}$, which is an increasing, strictly concave, and twice continuously differentiable function in $r_i(\mathbf{p})$. We further assume that the utility function $U_i(r_i(\mathbf{p}))$ is bounded for $\forall \mathbf{p} \in \mathcal{P}, \forall i \in \mathcal{N}$,

where $\mathcal{P} = \mathcal{P}_1 \times \mathcal{P}_2 \times \dots \times \mathcal{P}_N$. An example of the utility function is the α -fair function [24] defined as

$$U_i(r_i) = \begin{cases} (1 - \alpha_i)^{-1} r_i^{1 - \alpha_i}, & \text{if } \alpha_i \in [0, 1) \cup (1, \infty), \\ \ln r_i, & \text{if } \alpha_i = 1, \end{cases} \quad \forall i \in \mathcal{N}. \quad (8)$$

Without proper coordination, the *dominant strategy* of a *rational* user $i \in \mathcal{N}$ is to set $p_i = p_i^{max}$ in order to maximize its utility $U_i(r_i(\mathbf{p}))$. If every rational user is following this strategy, then the average data rate of every user may be zero in some cases. Thus, the system may need to coordinate the transmission. In this paper, we consider the use of the *interference pricing* on the transmission probabilities of the users to manage the level of interference. Specifically, we assume that the system charges user i a linear price $c_i \geq 0$, which is proportional to its transmission probability p_i , but is independent of the transmission probabilities $\mathbf{p}_{-i}$ of the other users. In fact, it is possible for the system to estimate $\mathbf{p}$ by listening to the shared wireless medium and learning from the contention history of the users [25] in order to implement the pricing scheme. Thus, the surplus (i.e., utility minus payment) of user i is given by

$$\sigma_i(\mathbf{p}, c_i) = U_i(r_i(\mathbf{p})) - c_i p_i. \quad (9)$$

In this paper, we assume that user i needs to obtain perfect information on some parameters, which include μ_i , c_i , $\mathbb{M}_i$, and $\mathbf{p}_{-i}$, to update its transmission probability p_i . To obtain this information, we assume that receiver i and the system send the values of data rate μ_i and price c_i to transmitter i , respectively. Moreover, each user m broadcasts its transmission probability p_m , transmit power P_m , received noise power n_m , and position to the other users using broadcast protocols, such as limited-scope message flooding [26], such that user i can determine $\mathbf{p}_{-i}$ and $\mathbb{M}_i$. Since this information has to be broadcast by each user, the signalling overhead grows linearly with the number of users N .

In the following section, we consider the setting where the prices $\mathbf{c} = (c_i, \forall i \in \mathcal{N})$ are fixed. We model the interactions of the users as a random access game, and characterize the existence of the NE. In Section IV, in order to achieve the maximum aggregate utility of the users, we propose an adaptive pricing scheme for the system to dynamically adjust its prices for the random access game.

III. NON-COOPERATIVE RANDOM ACCESS GAME WITH FIXED INTERFERENCE PRICING

In this section, we assume that the prices $\mathbf{c}$ of all the users are fixed. We study the interactions of the rational users in a random access game using non-cooperative game theory. In particular, we study the *existence*, *uniqueness*, *equilibrium selection*, and *efficiency* of the NE [22]. We also consider a special case of the random access game that can be modeled as a *S-modular game* [21], [27], [28]. Notice that we use the terms *users* and *players* interchangeably.

Specifically, we consider a random access game $\mathcal{G} = (\mathcal{N}, \mathcal{P}, \boldsymbol{\sigma})$ in normal-form, where $\mathcal{N}$ is the set of users. $\mathcal{P} = \mathcal{P}_1 \times \mathcal{P}_2 \times \dots \times \mathcal{P}_N$ is the Cartesian product of the action sets of all the users, where $\mathcal{P}_i$ is the action set of user i .

$\sigma(\mathbf{p}, \mathbf{c}) = (\sigma_1(\mathbf{p}, c_1), \sigma_2(\mathbf{p}, c_2), \dots, \sigma_N(\mathbf{p}, c_N))$ is the vector of surplus functions of all the users.

A. General Case

We define the *best response* of user $i \in \mathcal{N}$ as the transmission probability p_i that maximizes its surplus when the transmission probabilities $\mathbf{p}_{-i}$ and price c_i are given. That is,

$$\begin{aligned} BR_i(\mathbf{p}_{-i}, c_i) &= \arg \max_{p_i \in \mathcal{P}_i} \sigma_i(\mathbf{p}, c_i) \\ &= \arg \max_{p_i \in \mathcal{P}_i} U_i(r_i(p_i, \mathbf{p}_{-i})) - c_i p_i. \end{aligned} \quad (10)$$

Then, we define the NE¹ as the intersection of the best response correspondence of all the players.

Definition 1: A strategy $\mathbf{p}^*$ is a NE of game $\mathcal{G}$ if

$$\sigma_i(p_i^*, \mathbf{p}_{-i}^*, c_i) \geq \sigma_i(p_i, \mathbf{p}_{-i}^*, c_i), \forall p_i \in \mathcal{P}_i, \forall i \in \mathcal{N}. \quad (11)$$

In the general case with increasing and strictly concave utility functions, we can prove the *existence* of the NE in random access game $\mathcal{G}$.

Theorem 1: Game $\mathcal{G}$ has at least one NE.

Proof: First, notice that $\mathcal{P}_i$ is a non-empty, convex, and compact subset of a finite-dimensional Euclidean space for all $i \in \mathcal{N}$. Moreover, the surplus function $\sigma_i(\mathbf{p}, c_i)$ is continuous in $\mathbf{p}$ for all $i \in \mathcal{N}$, and it is concave (and thus quasi-concave) in p_i for all $i \in \mathcal{N}$. From the theorem due to Debreu, Glicksburg, and Fan [29, pp. 39], game $\mathcal{G}$ has at least one NE. ■

Theorem 2: The NE $\mathbf{p}^*$ of game $\mathcal{G}$ is characterized by

$$p_i^* = \left[\frac{1}{\mu_i S_i(\mathbf{p}_{-i}^*)} U_i'^{-1} \left(\frac{c_i}{\mu_i S_i(\mathbf{p}_{-i}^*)} \right) \right]_{p_i^{min}}^{p_i^{max}}, \quad \forall i \in \mathcal{N} \quad (12)$$

where $[z]_w^y = \min\{\max\{z, w\}, y\}$ and $U_i'(r_i)$ is the first derivative of $U_i(r_i)$.

Proof: Because problem (10) is a convex optimization problem, when $\mathbf{p}_{-i} = \mathbf{p}_{-i}^*$ is given, the optimal solution of problem (10) is given by (12). From Definition 1, $\mathbf{p}^*$ is the NE of game $\mathcal{G}$. ■

The following result is immediate from Theorem 2.

Corollary 1: With α -fair utility functions defined in (8), the NE $\mathbf{p}^*$ of game $\mathcal{G}$ is given by

$$p_i^* = \left[\sqrt[\alpha_i]{\frac{1}{c_i (\mu_i S_i(\mathbf{p}_{-i}^*))^{\alpha_i - 1}}} \right]_{p_i^{min}}^{p_i^{max}}, \quad \forall i \in \mathcal{N}. \quad (13)$$

In particular, if $\alpha_i = 1, \forall i \in \mathcal{N}$, then the *unique* NE is given by $p_i^* = [1/c_i]_{p_i^{min}}^{p_i^{max}}, \forall i \in \mathcal{N}$.

Although we can prove the existence of the NE in the general case, the convergence of the strategy profile to a NE is not guaranteed in general [10]. In the next section, we will focus on a special case that be modeled as a S-modular game with nice convergence properties.

B. Special Case: S-modular Game

In this section, we consider a special case of the random access game that can be modeled either as a *supermodular game* or as a *submodular game*. Although some of the results have been proved under the protocol model, we show that they still hold under a more general model that includes both the SINR and protocol models.

First, we define the *coefficient of relative risk aversion* (CRRA) [1] of utility function $U_i(r_i)$ as

$$\chi_i = -\frac{r_i U_i''(r_i)}{U_i'(r_i)}, \quad (14)$$

where $U_i''(r_i)$ is the second derivative of $U_i(r_i)$. χ_i measures the relative concavity of the utility function. Since $\mathcal{P}_i = [p_i^{min}, p_i^{max}]$ is a single dimensional compact subset of $\mathbb{R}$ and $\sigma_i(\mathbf{p}, c_i)$ is continuous in $\mathbf{p}$, a supermodular game and a submodular game are defined as follows [27], [28]:

Definition 2: Random access game $\mathcal{G}$ is a *supermodular game* if $\sigma_i(\mathbf{p}, c_i)$ has an *increasing difference* in $(p_i, \mathbf{p}_{-i})$ on $\mathcal{P}_i \times \mathcal{P}_{-i}, \forall i \in \mathcal{N}$. That is,

$$\frac{\partial^2 \sigma_i(\mathbf{p}, c_i)}{\partial p_i \partial p_j} \geq 0, \forall j \in \mathcal{N} \setminus \{i\}, i \in \mathcal{N}. \quad (15)$$

Conversely, random access game $\mathcal{G}$ is a *submodular game* if $\sigma_i(\mathbf{p}, c_i)$ has a *decreasing difference* in $(p_i, \mathbf{p}_{-i})$ on $\mathcal{P}_i \times \mathcal{P}_{-i}, \forall i \in \mathcal{N}$, where the inverse inequality in (15) holds.

1) *Supermodular Game:* The following results show that game $\mathcal{G}$ is a supermodular game if the CRRA of the utility functions of all the players are larger than or equal to one. It includes the case when all the users have α -fair functions in (8) with parameters $\alpha_i \geq 1$. Previous works in [4], [6], [7] have shown that the random access game considered under the protocol model is a supermodular game. By representing the data rate of user i in the form of (6) and capturing the contention relationships of user i using set $\mathbb{M}_i$, the generalization of the previous result to the general model, which includes both the protocol and SINR models, is greatly simplified.

Theorem 3: If the CRRA of user i is greater than or equal to one $\forall i \in \mathcal{N}$, then game $\mathcal{G}$ is a supermodular game.

Proof: First, we will show that if $\chi_i \geq 1$, then the utility function $U_i(r_i(\mathbf{p}))$ has an increasing difference in $(p_i, \mathbf{p}_{-i})$ on $\mathcal{P}_i \times \mathcal{P}_{-i}$. That is, $\frac{\partial^2 U_i(r_i(\mathbf{p}))}{\partial p_i \partial p_j} \geq 0, \forall j \in \mathcal{N} \setminus \{i\}$. Let $i \in \mathcal{N}$ and $j \in \mathcal{N} \setminus \{i\}$ be given. From (6), since $r_i(\mathbf{p}) = \mu_i p_i S_i(\mathbf{p}_{-i})$, we have

$$\frac{\partial U_i(r_i(\mathbf{p}))}{\partial p_i} = \mu_i U_i'(r_i(\mathbf{p})) S_i(\mathbf{p}_{-i}), \quad (16)$$

and

$$\frac{\partial^2 U_i(r_i(\mathbf{p}))}{\partial p_i \partial p_j} = \mu_i \frac{\partial S_i(\mathbf{p}_{-i})}{\partial p_j} \left[r_i(\mathbf{p}) U_i''(r_i(\mathbf{p})) + U_i'(r_i(\mathbf{p})) \right]. \quad (17)$$

In the random access setting, since $S_i(\mathbf{p}_{-i})$ is inversely proportional to p_j , we have $\frac{\partial S_i(\mathbf{p}_{-i})}{\partial p_j} \leq 0$. (Please refer to (28) and (29) in the proof of Lemma 2 for the detailed mathematical proof.) Also, $\chi_i \geq 1$ implies that $r_i(\mathbf{p}) U_i''(r_i(\mathbf{p})) + U_i'(r_i(\mathbf{p})) \leq 0$. Thus, we have $\frac{\partial^2 U_i(r_i(\mathbf{p}))}{\partial p_i \partial p_j} \geq 0$. Since

¹For conciseness, pure strategy NE is simply referred to as NE in this paper.

$\frac{\partial^2 \sigma_i(\mathbf{p}, c_i)}{\partial p_i \partial p_j} = \frac{\partial^2 U_i(r_i(\mathbf{p}))}{\partial p_i \partial p_j}$, $\forall j \in \mathcal{N} \setminus \{i\}$, $\forall i \in \mathcal{N}$, by Definition 2, the result follows. ■

Next, we study the convergence of the supermodular game. Let $\mathcal{T}$ be the set of all the time slots when random access game $\mathcal{G}$ is played. Let $\mathcal{T}_i \subseteq \mathcal{T}$ be the time slots that player $i \in \mathcal{N}$ updates its transmission probability. In other words, player i plays game $\mathcal{G}$ at time $t \in \mathcal{T}_i$. The *best response update* of player i is the update of transmission probability p_i that *maximizes* the surplus function $\sigma_i(\mathbf{p}, c_i)$ in (9) given the transmission probabilities $\mathbf{p}_{-i}(t)$ of the other players at time t . Specifically, it is given by

$$p_i(t+1) = \begin{cases} \arg \max_{p_i^{min} \leq p_i \leq p_i^{max}} [U_i(r_i(p_i, \mathbf{p}_{-i}(t))) - c_i p_i], & \text{if } t \in \mathcal{T}_i, \\ p_i(t), & \text{otherwise.} \end{cases} \quad (18)$$

For a *sequential* best response update, the players update one by one in a round-robin manner. In this way, we have $\mathcal{T}_i \cap \mathcal{T}_j = \emptyset$, $\forall i, j \in \mathcal{N}$, $i \neq j$. In contrast, for a *simultaneous* best response update, the players update at the same time such that $\mathcal{T}_i = \mathcal{T}_j$, $\forall i, j \in \mathcal{N}$, $i \neq j$. It should be noted that transmitter i needs to obtain the value of data rate μ_i from receiver i , price c_i from the system, and $\mathbf{p}_{-i}(t)$ from the other users to update its transmission probability $p_i(t+1)$. Let $\mathcal{P}_{NE}$ be the set of NE of game $\mathcal{G}$. The following theorem characterizes the NE and the *monotone* convergence properties of a supermodular game in general.

Theorem 4: In a supermodular game $\mathcal{G}$, (a) The set of NE $\mathcal{P}_{NE}$ is non-empty. (b) Set $\mathcal{P}_{NE}$ has the smallest $\underline{\mathbf{p}}_{NE}$ and the largest element $\bar{\mathbf{p}}_{NE}$, where $\underline{\mathbf{p}}_{NE}, \bar{\mathbf{p}}_{NE} \in \mathcal{P}_{NE}$. That is, if $\mathbf{p} \in \mathcal{P}_{NE}$, then $\underline{\mathbf{p}}_{NE} \preceq \mathbf{p} \preceq \bar{\mathbf{p}}_{NE}$. (c) With the sequential or simultaneous best response update, at time $t = 1$, if the starting points $\mathbf{p}(1) = \mathbf{p}^{min}$ or $\mathbf{p}(1) = \mathbf{p}^{max}$ are chosen, then the strategy profiles converge monotonically to $\underline{\mathbf{p}}_{NE}$ or $\bar{\mathbf{p}}_{NE}$, respectively.

Proof: The proofs of (a) and (b) are due to [27]. For (c), since we assume that the utility functions are *strictly* concave, the best response of each user (i.e., the optimal solution of problem (10)) is a singleton, and the result thus follows from [27, Theorems 4.3.2 and 4.3.4] and [30]. ■

For (b), it should be noted that the converse is not necessarily true. That is, if $\underline{\mathbf{p}}_{NE} \preceq \mathbf{p} \preceq \bar{\mathbf{p}}_{NE}$, it is possible that $\mathbf{p} \notin \mathcal{P}_{NE}$.

The following result establishes that $\underline{\mathbf{p}}_{NE}$ is the *Pareto-dominant* equilibrium that each user achieves the highest surplus in set $\mathcal{P}_{NE}$. It can be shown that the previous results in [4], [7] under the protocol model are also applicable to our more general model that includes both the SINR and protocol models, and we omit the proof.

Theorem 5: In a supermodular random access game $\mathcal{G}$, $\underline{\mathbf{p}}_{NE}$ is the Pareto-dominant equilibrium in $\mathcal{P}_{NE}$. That is, $\sigma_i(\underline{\mathbf{p}}_{NE}, c_i) \geq \sigma_i(\mathbf{p}, c_i)$, $\forall \mathbf{p} \in \mathcal{P}_{NE}$, $\forall i \in \mathcal{N}$.

2) *Submodular Game:* Similar to Theorem 3, game $\mathcal{G}$ is a submodular game if the CRRA of the utility functions of all the players are between zero and one. It includes the α -fair functions in (8) with parameters $0 < \alpha_i < 1$ and the utility function $\theta_i \log(1 + r_i)$ with parameter $\theta_i \geq 0$ [23].

Theorem 6: If $0 < \chi_i < 1$, $\forall i \in \mathcal{N}$, then game $\mathcal{G}$ is a submodular game.

In general, the submodular games do not possess the monotone convergence results as in the supermodular games. Nevertheless, we can characterize the convergence of the strategy profile in the submodular games with the simultaneous best response update.

Theorem 7: In a submodular game, with the simultaneous best response update, at time $t = 1$, if $\mathbf{p}(1) = \mathbf{p}^{min}$, then (a) The strategy profile converges to two limits, $\hat{\mathbf{p}}$ and $\check{\mathbf{p}}$. That is, as $t \rightarrow \infty$, we have $\mathbf{p}(2t+1) \rightarrow \hat{\mathbf{p}}$ and $\mathbf{p}(2t) \rightarrow \check{\mathbf{p}}$. (b) When the two limits coincide such that $\hat{\mathbf{p}} = \check{\mathbf{p}}$, then $\hat{\mathbf{p}}$ is the NE.

Proof: Notice that the strategy space $\mathcal{P}_i$ of player i is independent of $\mathbf{p}_{-i}$. Using similar arguments as in the proof of [21, Theorem 5.1], two monotone sequences will be generated. ■

It should be noted that Theorem 4 is related to the properties of a supermodular game, where we leverage on the results in [27], [30]. For Theorem 7, although a similar technique has been applied in [31] to analyze a submodular game, we consider a different system model and utility function.

IV. NETWORK UTILITY MAXIMIZATION WITH ADAPTIVE INTERFERENCE PRICING

In this section, we study how the interference prices $\mathbf{c}$ should be adjusted adaptively for game $\mathcal{G}$ to achieve maximum network aggregate utility. Specifically, we consider that the system aims to solve the NUM problem

$$\begin{aligned} & \underset{\mathbf{p}}{\text{maximize}} \quad \sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p})) \\ & \text{subject to} \quad p_i^{min} \leq p_i \leq p_i^{max}, \quad \forall i \in \mathcal{N}. \end{aligned} \quad (19)$$

Due to the product form of the variables in (6), problem (19) is *non-convex*, even if the utility functions are concave. As a result, problem (19) is difficult to solve in general.

Let ν_i and λ_i be the Lagrange multipliers associated with the constraints $p_i^{min} \leq p_i$ and $p_i \leq p_i^{max}$, respectively. We have the following KKT conditions on the local optimal solution of problem (19).

Lemma 1: For a local optimal solution $\mathbf{p}^*$ of problem (19), there exist unique Lagrange multipliers λ_i^*, ν_i^* , $\forall i \in \mathcal{N}$ such that

$$p_i^{min} \leq p_i^* \leq p_i^{max} \quad \text{and} \quad \lambda_i^*, \nu_i^* \geq 0, \quad \forall i \in \mathcal{N}, \quad (20)$$

$$\nu_i^*(p_i^* - p_i^{min}) = 0 \quad \text{and} \quad \lambda_i^*(p_i^{max} - p_i^*) = 0, \quad \forall i \in \mathcal{N}, \quad (21)$$

$$\frac{\partial U_i(r_i(\mathbf{p}^*))}{\partial p_i} + \sum_{j \in \mathcal{N} \setminus \{i\}} \frac{\partial U_j(r_j(\mathbf{p}^*))}{\partial p_i} + \nu_i^* - \lambda_i^* = 0, \quad \forall i \in \mathcal{N}. \quad (22)$$

Proof: The result follows directly from [32, pp.316]. ■

Motivated by the above KKT conditions, we propose the following pricing scheme for the users to solve problem (19). Let $\theta_{i,j}$ be the price that user $i \in \mathcal{N}$ has to pay for generating interference to user $j \in \mathcal{N}$, which is defined as

$$\theta_{i,j}(\mathbf{p}) \triangleq -\frac{\partial U_j(r_j(\mathbf{p}))}{\partial p_i}. \quad (23)$$

In fact, price $\theta_{i,j}(\mathbf{p})$ represents the marginal increase in utility of user j per unit decrease in the transmission probability of user i . As a result, user i aims to maximize its surplus, which is equal to its utility minus the total payments. Let $\boldsymbol{\theta}_i = (\theta_{i,j}, \forall j \in \mathcal{N} \setminus \{i\})$. User i aims to solve the following optimization problem when $\mathbf{p}_{-i}$ and $\boldsymbol{\theta}_i$ are fixed.

$$\begin{aligned} & \underset{p_i}{\text{maximize}} \quad U_i(r_i(\mathbf{p})) - p_i \sum_{j \in \mathcal{N} \setminus \{i\}} \theta_{i,j} \\ & \text{subject to} \quad p_i^{\min} \leq p_i \leq p_i^{\max}. \end{aligned} \quad (24)$$

Since the objective function is concave and the constraint is linear, problem (24) is a *convex* optimization problem. Moreover, notice that when $c_i = \sum_{j \in \mathcal{N} \setminus \{i\}} \theta_{i,j}$, $\forall i \in \mathcal{N}$ are chosen, problems (10) and (24) are identical. Thus, we can use this pricing scheme for game $\mathcal{G}$ in an attempt to solve the NUM problem (19).

Let $\hat{\mathcal{S}}_{j,i} = \left\{ \mathcal{S} \mid \mathcal{S} \in \mathcal{N} \setminus \{j, i\}, \mathcal{S} \in \mathbb{M}_j, \mathcal{S} \cup \{i\} \in \mathbb{M}_j \right\}$ and $\hat{\mathcal{S}}_{j,i} = \left\{ \mathcal{S} \mid \mathcal{S} \in \mathcal{N} \setminus \{j, i\}, \mathcal{S} \in \mathbb{M}_j, \mathcal{S} \cup \{i\} \notin \mathbb{M}_j \right\}$ be two sets of users that exclude users i and j . We have the following *closed-form* expression on the price $\theta_{i,j}(\mathbf{p})$.

Lemma 2: The price $\theta_{i,j}(\mathbf{p})$ is non-negative and is non-decreasing in p_i . It is given by

$$\theta_{i,j}(\mathbf{p}) = \mu_j p_j U_j'(r_j(\mathbf{p})) \sum_{\mathcal{S} \in \hat{\mathcal{S}}_{j,i}} \left(\prod_{s \in \mathcal{S}} p_s \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{S}, k \neq i, j} (1 - p_k) \right), \quad \forall i, j \in \mathcal{N}, i \neq j, \forall \mathbf{p} \in \mathcal{P}. \quad (25)$$

Proof: First, notice that

$$\theta_{i,j}(\mathbf{p}) = - \frac{\partial U_j(r_j(\mathbf{p}))}{\partial S_i(\mathbf{p}_{-i})} \frac{\partial S_i(\mathbf{p}_{-i})}{\partial p_i}. \quad (26)$$

For the first term on the right hand side of (26), we have

$$\frac{\partial U_j(r_j(\mathbf{p}))}{\partial S_i(\mathbf{p}_{-i})} = \mu_j p_j U_j'(r_j(\mathbf{p})) \geq 0. \quad (27)$$

Moreover, we can write $S_j(\mathbf{p}_{-j})$ in (7) as

$$\begin{aligned} S_j(\mathbf{p}_{-j}) &= \sum_{\mathcal{S} \in \hat{\mathcal{S}}_{j,i}} \left(\prod_{s \in \mathcal{S}} p_s \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{S}, k \neq i, j} (1 - p_k) \right) \\ &+ \sum_{\mathcal{S} \in \hat{\mathcal{S}}_{j,i}} \left(\prod_{s \in \mathcal{S}} p_s \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{S}, k \neq i, j} (1 - p_k) \right) (1 - p_i). \end{aligned} \quad (28)$$

Thus, we have

$$\frac{\partial S_j(\mathbf{p}_{-j})}{\partial p_i} = - \sum_{\mathcal{S} \in \hat{\mathcal{S}}_{j,i}} \left(\prod_{s \in \mathcal{S}} p_s \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{S}, k \neq i, j} (1 - p_k) \right) \leq 0. \quad (29)$$

Combining the results from (26), (27), and (29), we obtain $\theta_{i,j}(\mathbf{p}) \geq 0$ in (25). Moreover, since $U_j(r_j(\mathbf{p}))$ is a non-decreasing function in p_i and the term $\sum_{\mathcal{S} \in \hat{\mathcal{S}}_{j,i}} \left(\prod_{s \in \mathcal{S}} p_s \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{S}, k \neq i, j} (1 - p_k) \right)$ is independent of p_i , $\theta_{i,j}(\mathbf{p})$ is a non-decreasing function in p_i . ■

Under the protocol model, the interference pricing can be simplified as follows.

Lemma 3: Under the protocol model, the price $\theta_{i,j}(\mathbf{p})$ is given by

$$\theta_{i,j}(\mathbf{p}) = \begin{cases} \mu_j p_j U_j'(r_j(\mathbf{p})) \prod_{k \in \mathcal{I}_j \setminus \{i\}} (1 - p_k), & \text{if } i \in \mathcal{I}_j, \\ 0, & \text{otherwise,} \end{cases} \quad (30)$$

where $\mathcal{I}_j = \left\{ m \in \mathcal{N} \setminus \{i\} \mid \frac{P_j G_{jj}}{P_m G_{mj} + n_j} < \theta_j^{\text{th}} \right\}$ is the set of interferers of user j .

Proof: Under the protocol model, we have $S_j(\mathbf{p}_{-j}) = \prod_{k \in \mathcal{I}_j} (1 - p_k)$. From (26) and (27), the result follows. ■

However, it should be noted that although the MAC protocols are designed using either the protocol model or SINR model, the packet receptions are indeed performed under the SINR model in reality. Thus, the simplification in interference pricing under the protocol model in Lemma 3 may result in *starvation* for some users with zero data rate, as illustrated by the following example.

Example 1: We consider a wireless ad hoc network shown in Fig. 1, where the transmit powers of all users are the same. At a certain transmit power level P , we can observe the following: Since transmitter 1 is close to receivers 2 and 3, user 1 interferes with users 2 and 3. However, since transmitters 2 and 3 are far away from receiver 1, users 2 and 3 do not interfere with user 1 as long as they do not transmit simultaneously. Users 2 and 3 are far from each other and do not interfere with each other. Under the protocol model, since $\theta_{2,1}(\mathbf{p}) = \theta_{3,1}(\mathbf{p}) = 0$ from Lemma 3, we have $c_2 = 0$. Similarly, we have $\theta_{3,1}(\mathbf{p}) = \theta_{3,2}(\mathbf{p}) = 0$, which implies that $c_3 = 0$. With a zero price, users 2 and 3 will transmit with the maximum allowed transmission probabilities $p_2^{\max}$ and $p_3^{\max}$, respectively. If $p_2^{\max} = p_3^{\max} = 1$, then user 1 is starved with zero data rate $r_1(\mathbf{p}) = 0$ by evaluating under the more accurate SINR model.

We then propose the iterative algorithm for random access game $\mathcal{G}$ with adaptive interference pricing that aims to solve the NUM problem (19) in Algorithm 1. The random access game and the interference price update are performed iteratively for a total of Ψ iterations (line 4). In each iteration l , game $\mathcal{G}$ is played for T time slots (lines 5-17). However, different from the power control setting in [23], it can be shown that our random access game with adaptive pricing is *not* a supermodular game. Thus, the strategy profile may not converge by using the best response update. In order to facilitate the convergence, we introduce the *gradient play* [9], [10], where player i updates its transmission probability p_i that *improves* its surplus function $\sigma_i(\mathbf{p}(t), c_i(l))$ in (9) along the *ascent direction* $d(\mathbf{p}(t), c_i(l))$ with a step size $\beta(l) \geq 0$ when the transmission probabilities $\mathbf{p}(t)$ at time t are given. For example, for the steepest ascent method [32, pp. 25], we choose

$$\begin{aligned} d(\mathbf{p}(t), c_i(l)) &= \frac{\partial \sigma_i(\mathbf{p}(t), c_i(l))}{\partial p_i} \bigg/ \left| \frac{\partial \sigma_i(\mathbf{p}(t), c_i(l))}{\partial p_i} \right| \\ &= \frac{\mu_i U_i'(r_i(\mathbf{p}(t))) S_i(\mathbf{p}_{-i}(t)) - c_i(l)}{\left| \mu_i U_i'(r_i(\mathbf{p}(t))) S_i(\mathbf{p}_{-i}(t)) - c_i(l) \right|}. \end{aligned} \quad (31)$$

Algorithm 1 *Iterative Random Access Game with Adaptive Interference Price Update to solve problem (19).*

```

1: Initialize the starting point  $\mathbf{p}(1) \in \mathcal{P}$  and  $\mathbf{c}(1) \succeq \mathbf{0}$  and
   the iteration counter  $l := 1$ 
2: Initialization for game  $\mathcal{G}$ : Number of game iterations  $T$ 
   and the set of time slots  $\mathcal{T}_i, \forall i \in \mathcal{N}$ 
3: Initialization for the interference price update: Step size
    $\beta(1) > 0$ , step size adjustment factor  $0 < \delta < 1$ , total
   number of iterations  $\Psi$ , state information  $flag := 0$ , set
   of price update iterations  $\mathcal{L}_{price}$ 
4: while  $l \leq \Psi$ 
5:   Random Access Game  $\mathcal{G}$  with interference prices  $\mathbf{c}(l)$ 
6:   Set  $t := 1$ 
7:   while  $t \leq T$ 
8:     for all  $i \in \mathcal{N}$ 
9:       if  $t \in \mathcal{T}_i$  then
10:        Set  $d(\mathbf{p}(t), c_i(l)) := \frac{\mu_i U'_i(r_i(\mathbf{p}(t))) S_i(\mathbf{p}_{-i}(t)) - c_i(l)}{\mu_i U'_i(r_i(\mathbf{p}(t))) S_i(\mathbf{p}_{-i}(t)) - c_i(l)}$ 
11:        Set  $p_i(t+1) := \left[ p_i(t) + \beta(l) d(\mathbf{p}(t), c_i(l)) \right]_{p_i^{min}}^{p_i^{max}}$ 
12:       else
13:         Set  $p_i(t+1) := p_i(t)$ 
14:       end if
15:     end for
16:     Set  $t := t + 1$ 
17:   end while
18:   Set  $\mathbf{p}^*(l) := \mathbf{p}(t)$ 
19:   Interference Price Update
20:   if  $l \in \mathcal{L}_{price}$  then
21:     Set  $\theta_{i,j}(\mathbf{p}^*(l))$  using (25) with  $\mathbf{p} = \mathbf{p}^*(l), \forall j \in \mathcal{N} \setminus \{i\}, \forall i \in \mathcal{N}$ 
22:     Set  $c_i(l+1) := \sum_{j \in \mathcal{N} \setminus \{i\}} \theta_{i,j}(\mathbf{p}^*(l)), \forall i \in \mathcal{N}$ 
23:   else
24:     Set  $c_i(l+1) := c_i(l), \forall i \in \mathcal{N}$ 
25:   end if
26:   Step Size Update
27:   if (oscillation is detected) and ( $flag = 0$ ) then
28:     Set  $flag := 1$ 
29:   end if
30:   if  $flag = 1$  then
31:     Set  $\beta(l+1) := \delta \beta(l)$ 
32:   else
33:     Set  $\beta(l+1) := \beta(l)$ 
34:   end if
35:   Set  $l := l + 1$ 
36: end while

```

Thus, for $t \in \mathcal{T}_i$, the gradient play is given by

$$p_i(t+1) := \left[p_i(t) + \beta(l) d(\mathbf{p}(t), c_i(l)) \right]_{p_i^{min}}^{p_i^{max}} \quad (32)$$

Compared with the best response update, the gradient play can be viewed as a *better response* [10] update. The solution of game $\mathcal{G}$ in iteration l is recorded in $\mathbf{p}^*(l)$ (line 18). Let $\mathcal{L}_{price}$ be the set of iterations when the interference price update is performed. The price is updated in iteration $l \in \mathcal{L}_{price}$

based on (25) (lines 19-25). However, it should be noted that when the steepest ascent method is used, we will have $d(\mathbf{p}(t), c_i(l)) \in \{-1, 1\}$ numerically. The use of gradient play in (32) for the update of $\mathbf{p}(t)$ may result in oscillation with a small amplitude when a small step size $\beta(l) > 0$ is chosen. To facilitate the convergence of the algorithm, once an oscillation is detected, we set $flag$ to be equal to one (lines 27-29). Then, the step size will be diminished by an adjustment factor $0 < \delta < 1$ in the following iterations (lines 30-34). After running Algorithm 1, the fixed point may be reached, which is defined as follows:

Definition 3: $(\mathbf{p}^*, \mathbf{c}^*)$ is a *fixed point* if $p_i^* = BR_i(\mathbf{p}_{-i}^*, c_i^*), \forall i \in \mathcal{N}$ and $c_i^* = \sum_{j \in \mathcal{N} \setminus \{i\}} \theta_{i,j}(\mathbf{p}^*), \forall i \in \mathcal{N}$, where $BR_i(\mathbf{p}_{-i}, c_i)$ is the best response function defined in (10).

We can show that set of fixed points is in fact the set of KKT points of problem (19). That is, the local optimal points or saddle points of problem (19) can be obtained by Algorithm 1. In the special case where problem (19) has only one KKT point, Algorithm 1 results in the *globally* optimal solution of problem (19). However, in general, there are multiple KKT points, and the final solution of Algorithm 1 depends on the starting point $\mathbf{p}(1) \in \mathcal{P}$ and $\mathbf{c}(1) \succeq \mathbf{0}$.

Lemma 4: The fixed point $\mathbf{p}^*$ satisfies the KKT conditions of problem (19).

Proof: Since problem (24) is *convex*, the following KKT conditions for each $i \in \mathcal{N}$ are both *necessary* and *sufficient* [33].

$$p_i^{min} \leq p_i^* \leq p_i^{max} \quad \text{and} \quad \lambda_i^*, \nu_i^* \geq 0, \quad (33)$$

$$\nu_i^*(p_i^* - p_i^{min}) = 0 \quad \text{and} \quad \lambda_i^*(p_i^{max} - p_i^*) = 0, \quad (34)$$

$$\frac{\partial U_i(r_i(p_i^*, \mathbf{p}_{-i}^*))}{\partial p_i} + \sum_{j \in \mathcal{N} \setminus \{i\}} \frac{\partial U_j(r_j(\mathbf{p}^*))}{\partial p_i} + \nu_i^* - \lambda_i^* = 0. \quad (35)$$

Notice that (33)-(35) are almost the same as (20)-(22), except that the latter are satisfied $\forall i \in \mathcal{N}$. Since the union of the above KKT conditions $\forall i \in \mathcal{N}$ is equal to the KKT conditions of problem (19) in Lemma 1, the set of fixed points coincides with the set of KKT points of problem (19). ■

V. PERFORMANCE EVALUATIONS

In this section, we evaluate the performance of Algorithm 1 under both the SINR and protocol models, and compare it with a CSMA scheme. Unless specified otherwise, we assume that the nodes are randomly placed in a 50 m $\times$ 50 m area. The communication distance of a user is randomly selected to be between 5 m and 25 m. We assume that the data rate of all the users is the same such that $\mu_i = 10$ Mbps, $\forall i \in \mathcal{N}$. For simplicity, we model the channel gain as $G_{i,j} = 1/d_{i,j}^\gamma$, where $d_{i,j}$ is the distance between the transmitter of user i and the receiver of user j , and γ is the path loss exponent. We adopt γ to be equal to 2. The transmit powers of all the users are equal and set to a value which yields a minimum signal-to-noise-ratio (SNR) of 10 dB at the receivers. The SINR threshold is $\theta_i^{th} = \theta^{th}, \forall i \in \mathcal{N}$, and is set to 0 dB. We assume that all the users have the same α -fair utility functions with $\alpha_i = \alpha, \forall i \in \mathcal{N}$. We consider that $p_i^{min} = p^{min} = 0.01$

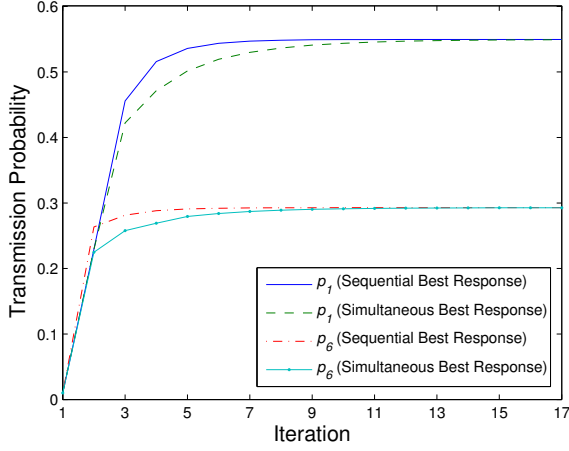


Fig. 2. The convergence of transmission probabilities p_1 and p_6 using sequential and simultaneous best response updates in a supermodular game with $N = 6$ and $\alpha = 2$ under the SINR model. The interference prices are fixed in this case, where $c_i = 2, \forall i \in \mathcal{N}$.

and $p_i^{max} = p^{max} = 0.999, \forall i \in \mathcal{N}$. Notice that if we choose $p^{min} = 0$ or $p^{max} = 1$, the utility functions of some users may be unbounded, which violates our assumption in Section II. In Algorithm 1, at time $t = 1$, we choose the starting point $\mathbf{p}(1) = \mathbf{p}^{min}$ and $c_i(1) = \sum_{j \in \mathcal{N} \setminus \{i\}} \theta_{i,j}(\mathbf{p}(1)), \forall i \in \mathcal{N}$. We choose the initial step size $\beta(1) = 0.01$ and the step size adjustment factor $\delta = 0.9$. For the random access game, we consider that sequential gradient play is used, where the users update their transmission probabilities one by one in a round-robin manner. Notice that whenever the system parameters are changed, the users should exchange this information and rerun Algorithm 1 again.

First, we evaluate the random access game $\mathcal{G}$ with fixed interference pricing under the SINR model by plotting the trajectories of p_1 and p_6 in Fig. 2 in a network with six players. We assume that the interference prices are fixed such that $c_i = 2, \forall i \in \mathcal{N}$. By using the α -fair functions with $\alpha = 2$, we know from Theorem 3 that $\mathcal{G}$ is a supermodular game. In this way, from Theorem 4(c), by choosing the starting point $\mathbf{p}(1) = \mathbf{p}^{min}$, the strategy profiles converge *monotonically* to the same solution when the sequential or simultaneous best response updates are used. In fact, the solution corresponds to the smallest NE $\mathbf{p}_{NE}$. In each iteration, we consider that each user updates its transmission probability once. From Fig. 2, we can observe that the sequential best response update results in a faster convergence than the simultaneous best response update.

Next, we illustrate the interference pricing and transmission probability assignment obtained from Algorithm 1 for the network topology shown in Fig. 3 with $N = 6$ and $\alpha = 1.5$. For the interference price update in Algorithm 1, we consider that it is performed in every iteration such that $\mathcal{L}_{price} = \{1, 2, \dots, \Psi\}$. We set $\Psi = 500$. Under the SINR model, from Fig. 4, we can see that the transmission probabilities of all the users converge. In fact, we can verify that the solution $(\mathbf{p}^*, \mathbf{c}^*)$ obtained from Algorithm 1 is indeed a fixed point as defined in Definition 3. In Fig. 5, we plot the interference

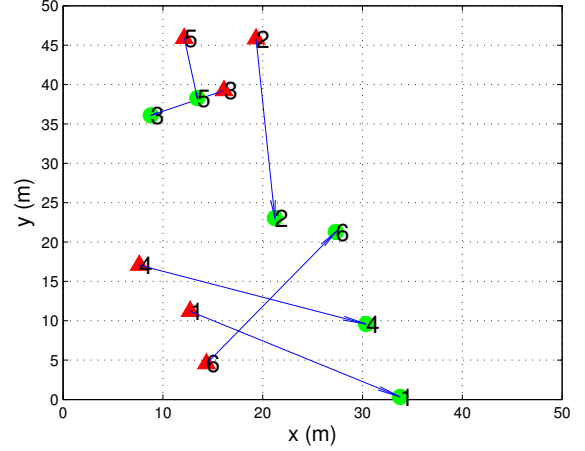


Fig. 3. An example of a wireless ad hoc network with six users.

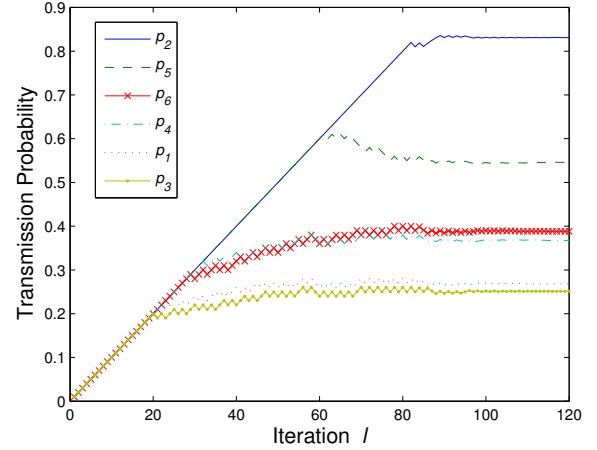


Fig. 4. The convergence of transmission probabilities $\mathbf{p}^*(l)$ of the six users in the wireless ad hoc network shown in Fig. 3 under the SINR model using Algorithm 1 with $\alpha = 1.5$.

price c_2 of user 2 against its transmission probability p_2 when $p_j = 0.7, \forall j \in \mathcal{N} \setminus \{2\}$ under both the SINR and protocol models. For the protocol model, we observe that $c_2 = 0$ for all the cases. Since user 2 does not interfere with other users under the protocol model, we have $\theta_{2,j}(\mathbf{p}) = 0, \forall j \in \mathcal{N} \setminus \{2\}$ from Lemma 3. Thus, $c_2 = \sum_{j \in \mathcal{N} \setminus \{2\}} \theta_{2,j}(\mathbf{p}) = 0$. On the other hand, under the SINR model, user 2 is in fact generating interference to the other users in the system. From Fig. 5, we can observe that the interference price c_2 increases with the transmission probability p_2 in order to control user 2's interference to the other users. Moreover, the simulation results reveal that the design based on the protocol model leads to the starvation of user 6. The reason is that both users 2 and 5 are not interferers to all the other users in the system under the protocol model. As a result, we have $c_2 = 0$ and $c_5 = 0$, which implies that $p_2^* = BR_2(\mathbf{p}_{-2}, c_2) = p_2^{max}$ and $p_5^* = BR_5(\mathbf{p}_{-5}, c_5) = p_5^{max}$. However, the SINR model suggests that users 2 and 5 in fact interfere with the transmission of user 6 when they transmit simultaneously. Thus, in reality, user 6 receives a close-to-zero data rate $r_6 \approx 0$.

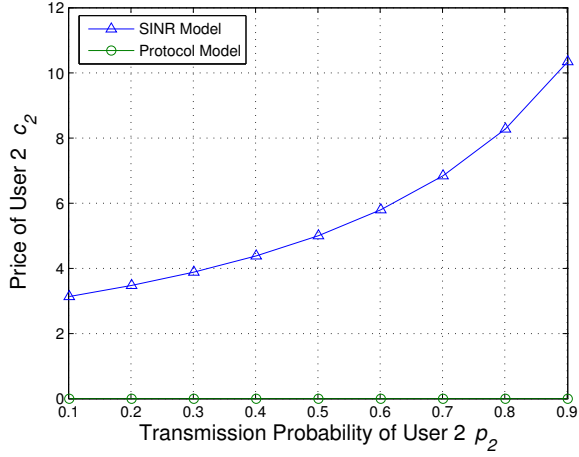


Fig. 5. The interference price c_2 of user 2 against its transmission probability p_2 when $p_j = 0.7, \forall j \in \mathcal{N} \setminus \{2\}$ and $\alpha = 1.5$. Notice that under the SINR model, the interference price c_2 is directly proportional to the transmission probability p_2 . However, it is inferred from the less accurate protocol model that user 2 does not interfere with any other users, and it is thus required to pay nothing.

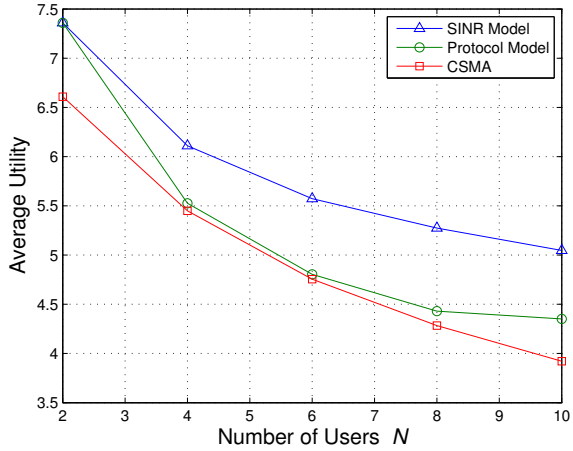
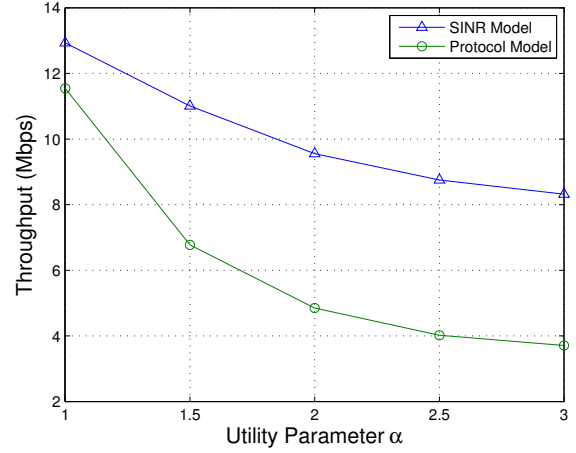
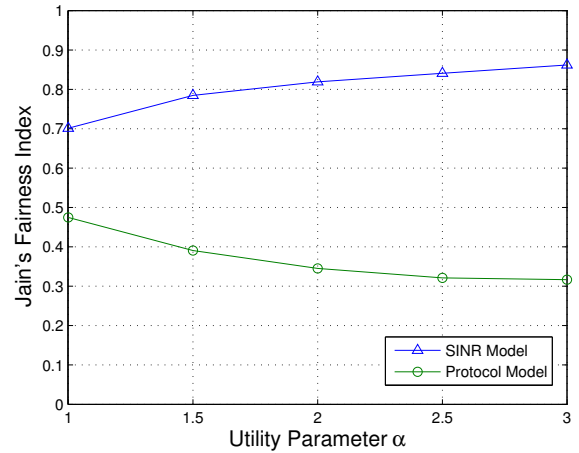


Fig. 6. Average utility versus the number of users N for Algorithm 1 using the SINR model, the protocol model, and a CSMA scheme with $\alpha = 0.8$. Notice that the design based on the SINR model achieves the highest average utility.

Then, we compare the average utility achieved with Algorithm 1 for the SINR model (using $\mathbb{M}_i = \mathbb{M}_{i,SINR}, \forall i \in \mathcal{N}$) and the protocol model (using $\mathbb{M}_i = \mathbb{M}_{i,PTC}, \forall i \in \mathcal{N}$), and a CSMA scheme implemented in a slotted time system. Notice that the packet receptions of all the schemes are indeed performed under the SINR model. The average utility is defined as the aggregate utility in the system divided by the total number of users N . The result is obtained for $\alpha = 0.8$ by averaging over 1000 different random topologies when N varies. For the interference price update in Algorithm 1, we consider that it is performed in every ten iterations. That is, $\mathcal{L}_{price} = \{10, 20, \dots, \Psi\}$. We set $\Psi = 200$. The operation of the CSMA scheme is similar to the one used in the IEEE 802.11 standard, except that the interframe spacing is not implemented. We assume that the sensing is performed based



(a) Throughput



(b) Jain's fairness index

Fig. 7. (a) The throughput and (b) Jain's fairness index against the utility parameter α when $N = 8$. As we can see, the design based on the SINR model results in a more efficient and fairer allocation than that based on the protocol model. Moreover, different degrees of tradeoff between the throughput and fairness can be achieved by tuning the parameter α under the SINR model.

on the protocol model, where the channel is sensed busy if any one-hop neighbour of the user transmits. We choose the minimum and maximum contention window sizes $aCW_{min} = 13$ and $aCW_{max} = 1023$ [34, pp. 536], respectively. As shown in Fig. 6, we can see that the design based on the SINR model always achieve a higher average utility than the design based on the protocol model and a CSMA scheme. Moreover, we can observe that the average utility decreases with N . It is because when N increases, the contention for transmission in a random access system increases, which reduces the average utility of each user. It should be noted that when N is equal to 2, the protocol model is identical to the SINR model, which results in the same achieved average utility.

In Fig. 7, we evaluate the throughput (i.e., the network efficiency) and fairness of the design based on the SINR and protocol models under different values of the utility parameter α for $N = 8$ in 1000 different scenarios. The degree of fairness is measured by the Jain's fairness index [35] defined

as $(\sum_{i \in \mathcal{N}} r_i(\mathbf{p}))^2 / (N \sum_{i \in \mathcal{N}} r_i(\mathbf{p})^2)$. As shown in Fig. 7, the design based on the SINR model results in a higher throughput and Jain's fairness index than that based on the protocol model. That is, a more efficient and fairer resource allocation is achieved under the SINR model. Moreover, by increasing the value of α , we can see that the throughput decreases and the Jain's fairness index increases under the SINR model as shown in Figs. 7 (a) and (b), respectively. So the parameter α acts as a knob to control the *tradeoff* between the network efficiency and fairness [31].

VI. CONCLUSIONS

In this paper, we studied the random access games with linear interference pricing under the SINR model in wireless ad hoc networks. In the first part of the paper, we assumed that the prices are fixed. We studied the interactions of the rational users and characterize the existence of the NE. In particular, we considered a special case of a S-modular random access game, and analyzed the efficiency and convergence of the NE. In the second part of the paper, we proposed an iterative interference pricing algorithm for the random access game that aims to maximize the aggregate utility of the users. It was shown that the fixed point of the algorithm satisfies the KKT conditions of the NUM problem. Simulation results showed that Algorithm 1 based on the SINR model achieves a higher average utility than the algorithm based on the protocol model and a CSMA scheme implemented in a slotted time system. Interesting topics for future work include the extension of the analysis of the random access game to a multi-channel setting, and the study of the revenue maximization pricing scheme.

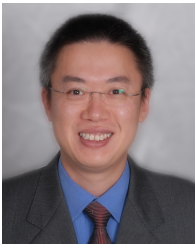
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