

SINR-based Random Access for Cognitive Radio: Distributed Algorithm and Coalitional Game

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Abstract—In this paper, we study the problem of multi-channel medium access control (MAC) in cognitive radio (CR) networks. While most of the previously proposed MAC protocols for CR networks are heuristic and are based on the simplistic protocol model, we design a distributed MAC protocol using the more accurate signal-to-interference-plus-noise-ratio (SINR) model. First, we assume that the secondary users are cooperative and formulate the problem of assigning transmission and listening probabilities for random access as a non-convex network utility maximization problem. We propose a three-phase algorithm that converges to a near-optimal solution after solving a number of convex optimization problems distributively. Simulation results show that our proposed algorithm based on the SINR model achieves a higher aggregate throughput than other schemes which are based on the protocol model. Then, we consider the case that the secondary users are rational. We use coalitional game theory to study the incentive issues of user cooperation in a given channel for the SINR model. In particular, we use the solution concept of the core to analyze the stability of the grand coalition, and the solution concept of the Shapley value to fairly divide the payoff among the users. We show that the Shapley value lies in the core when all the users are one-hop neighbours of each other. We illustrate the Shapley value and the core with a numerical example.

Index Terms—Cognitive radio networks, multi-channel medium access control (MAC), random access, SINR model, network utility maximization, non-convex optimization, coalitional game theory.

I. INTRODUCTION

WITH the licensed radio spectrum being under-utilized [1], cognitive radio (CR) [2] has emerged as the solution to the spectrum scarcity problem. To address the problem of spectrum sharing between the primary (licensed) users and secondary (unlicensed) users, the *commons model* and the *property model* [3] have been proposed. In the commons model, the secondary users access the spectrum holes *opportunistically* so that they do not cause interference to the primary users. In the property model, the primary users are allowed to *trade* some of their temporarily unused spectrum to the secondary users in exchange for monetary return. In this paper, we consider the setting where a set of channels from the primary network are available to the secondary CR

network, e.g., in the form of *dynamic spectrum leasing* [4]–[6] in the property model or by *spectrum sensing* [7], [8] in the commons model. In order to implement a scalable system that is adaptive to the dynamic network changes in a CR network, *distributed* medium access control (MAC) protocols should be employed by the secondary users.

By exploiting multiple orthogonal channels, the overall system performance of a CR network can be improved, since transmissions can take place simultaneously without causing multi-user interference. Different multi-channel MAC protocols have been proposed for CR networks. In [9], Cordeiro *et al.* proposed a distributed cognitive MAC protocol that includes a slotted beaconing period for nodes to negotiate on the channel usage. Su *et al.* proposed in [10] two sensing policies for the physical layer and a packet scheduling algorithm for the MAC layer of a distributed CR network. Jia *et al.* proposed in [11] a hardware-constrained cognitive MAC protocol that coordinates the contention and spectrum usage among the secondary users. Timmers *et al.* proposed in [12] an energy-efficient distributed multi-channel MAC protocol for a multi-hop CR network, which is based on the timing structure of the power-saving mode used in the IEEE 802.11 standard.

In the throughput analysis of multi-channel MAC protocols, such as [10], [12], the *protocol model* or *unit disk model* [13] is widely used to account for the effect of multi-user interference due to its simplicity in characterizing the physical layer. Under the protocol model, a transmission is successful if the receiver is within the transmission range of its intended transmitter and outside the interference range of other transmitters. However, in reality, the interference at the receiver is the *cumulative* power received from other nodes that are concurrently transmitting. As a result, the *signal-to-interference-plus-noise-ratio (SINR) model* or *physical model* [13] characterizes the effect of interference more accurately. Under the SINR model, a transmission is successful if and only if the SINR at the intended receiver is above a predefined threshold that depends on the adopted modulation and coding schemes. Despite its higher complexity, the SINR model is getting more attention in recent years due to its higher practicality and accuracy in modeling. Some recent works have investigated contention-based *random access* protocols [14], [15] and collision-free *scheduling* protocols [16], [17] using the SINR model.

Since most of the proposed multi-channel MAC protocols for CR networks are *heuristic* in nature and apply the simplistic protocol model, in this paper, we propose a *distributed* random access protocol for CR networks, which is based on the *multi-channel SINR model* and the mathematical frame-

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work of network utility maximization (NUM) [15], [18]–[20]. In particular, we extend the mathematical models in [15] and [20], where the former focused on the single-channel SINR model, while the latter focused on the multi-channel protocol model. The resulting *non-convex* optimization problem is more difficult to solve than the problems in [15] and [20]. In particular, our problem involves the dimension of channel selection which was absent in [15], and entails a more accurate and complex interaction among the users due to the SINR model which was absent in [20].

In the formulation of the NUM problem, we assume that all the secondary users are *cooperative*. Thus, an interesting question is what happens if the users are *rational* and they aim to maximize their own utilities? Previous works, such as [21], use *non-cooperative game theory* to analyze the behaviour of rational users in CR networks. However, this approach is more appropriate for analyzing the behaviour of *individual* rational users. To analyze what a *group* of rational users can achieve under the SINR model, where the effect of interference is cumulative, *coalitional game theory* [22] is a more suitable tool. Coalitional game theory has found many applications in communication networks [8], [23], [24]. In [24], it was applied to study the behaviour of users under the SINR model. However, [24] investigated cooperative communications, whereas we consider random access in CR networks. To the best of our knowledge, this work is the first paper that applies coalitional game theory to study random access under the SINR model.

In summary, the contributions of our work are as follows:

- We first assume that the secondary users are cooperative. We formulate the problem of random access with multiple channels as a NUM problem using the SINR model, where the optimization variables are the transmission and listening probabilities of the users.
- We propose a distributed three-phase algorithm using convex optimization and the coordinate ascent method to obtain a near-optimal solution for the non-convex NUM problem.
- We then study the case where the secondary users are rational. We formulate the problem as a coalitional game to analyze the interactions among the users under the SINR model. We apply the solution concepts of the *core* and the *Shapley value* [22] to characterize the stability and fair allocation of the aggregate utility among the rational users, respectively. We show that the Shapley value lies in the core when all users are one-hop neighbours.
- Simulation results show that the proposed scheme based on the SINR model achieves a higher aggregate throughput than other schemes which are based on the protocol model. A numerical example is given to illustrate both the Shapley value and the core.

The rest of the paper is organized as follows. The system model is described in Section II. We formulate the random access problem in Section III and present our distributed algorithm in Section IV. The coalitional game is discussed in Section V and simulation results are presented in Section VI. Conclusions and future work are given in Section VII.

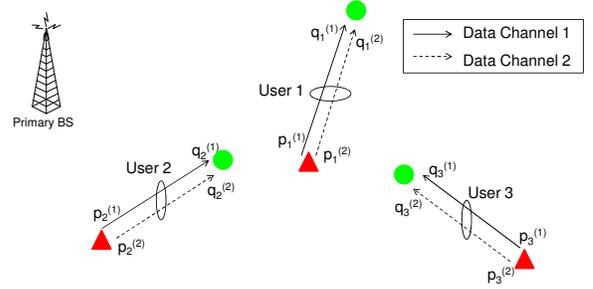


Fig. 1. A CR network with set of users $\mathcal{N} = \{1, 2, 3\}$, where the triangles and circles represent the transmitters and receivers, respectively. The set of available orthogonal data channels $\mathcal{C} = \{1, 2\}$ is provided by the primary BS. $p_i^{(c)}$ and $q_i^{(c)}$ denote the transmission and listening probabilities for user i in channel c , respectively.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a CR network with several secondary nodes located in a neighbourhood, where a set of orthogonal data channels \mathcal{C} and one control channel are obtained from the primary base station (BS), e.g. in the form of spectrum leasing. The data channels are used for data transmissions, and the control channel is used for the exchange of control messages. The total number of data channels is $C = |\mathcal{C}|$. We consider only single-hop transmissions between the secondary nodes. We define \mathcal{N} as the set of one-hop transmitter/receiver pairs or links in the CR network, and we refer to each transmitter/receiver pair as a *user*. The total number of users is $N = |\mathcal{N}|$. We adopt a slotted MAC protocol, where time is divided into equal time slots. The users attempt to access the shared channel at the beginning of each time slot according to their *transmission probabilities* in each channel. That is, each user $i \in \mathcal{N}$ can access a channel c with a certain transmission probability $p_i^{(c)}$, and we define a vector $\mathbf{p} = (p_i^{(c)}, \forall i \in \mathcal{N}, c \in \mathcal{C})$. Also, we introduce a vector $\mathbf{q} = (q_i^{(c)}, \forall i \in \mathcal{N}, c \in \mathcal{C})$, where $q_i^{(c)}$ is the *listening probability* of receiver i in channel c . We have the following constraints:

$$\sum_{c \in \mathcal{C}} p_i^{(c)} \leq 1 \quad \text{and} \quad \sum_{c \in \mathcal{C}} q_i^{(c)} \leq 1, \quad \forall i \in \mathcal{N}. \quad (1)$$

For the *SINR model*, if user $i \in \mathcal{N}$ chooses to transmit in channel $c \in \mathcal{C}$, then the SINR at receiver i is given by

$$\theta_i^{(c)} = \frac{P_i G_{ii}^{(c)}}{\iota_i^{(c)} + n_i^{(c)}}, \quad (2)$$

where P_i is the transmit power of user i . $G_{ij}^{(c)}$ is the channel gain from the transmitter of user i to the receiver of user j in channel c . $\iota_i^{(c)}$ and $n_i^{(c)}$ are the interference and noise powers received by user i in channel c , respectively. Given that receiver i has tuned to channel c for reception, the communication of user i is successful if

$$\theta_i^{(c)} \geq \theta_i^{th} \Leftrightarrow \iota_i^{(c)} \leq \frac{P_i G_{ii}^{(c)}}{\theta_i^{th}} - n_i^{(c)}, \quad (3)$$

where θ_i^{th} is the SINR threshold. Let \mathbb{N}_i be the power set (i.e., the set of all subsets) of $\mathcal{N} \setminus \{i\}$. As an example, for

$\mathcal{N} = \{1, 2, 3\}$, $\mathbb{N}_2 = \{\{\}, \{1\}, \{3\}, \{1, 3\}\}$. Assuming that the transmit powers ($P_i, \forall i \in \mathcal{N}$) are fixed, we define $\mathbb{M}_i^{(c)}$ as a set where each element is a set of users that can transmit simultaneously with user i without affecting the reception of receiver i in channel c (i.e., θ_i^{th} can be achieved). The set $\mathbb{M}_i^{(c)}$ obtained with the SINR model is given by

$$\mathbb{M}_{i,SINR}^{(c)} = \left\{ \mathcal{M} \in \mathbb{N}_i : \sum_{m \in \mathcal{M}} P_m G_{mi}^{(c)} \leq \frac{P_i G_{ii}^{(c)}}{\theta_i^{th}} - n_i^{(c)} \right\}. \quad (4)$$

If the *protocol model* is used, only pairwise interference is considered. User m is an *interferer* or *one-hop neighbour* to user i if the SINR due to the interference from user m only is below the SINR threshold. That is,

$$\frac{P_i G_{ii}^{(c)}}{P_m G_{mi}^{(c)} + n_i^{(c)}} < \theta_i^{th}. \quad (5)$$

The set $\mathbb{M}_i^{(c)}$ obtained with the protocol model is given by

$$\mathbb{M}_{i,PTC}^{(c)} = \left\{ \mathcal{M} \in \mathbb{N}_i : P_m G_{mi}^{(c)} \leq \frac{P_i G_{ii}^{(c)}}{\theta_i^{th}} - n_i^{(c)}, \forall m \in \mathcal{M} \right\}. \quad (6)$$

Intuitively, more users are allowed to transmit simultaneously in the protocol model than in the SINR model. This is confirmed by the following lemma.

Lemma 1: $\mathbb{M}_{i,SINR}^{(c)} \subseteq \mathbb{M}_{i,PTC}^{(c)}$.

Proof: Observing the fact that $\sum_{m \in \mathcal{M}} P_m G_{mi}^{(c)} \leq \frac{P_i G_{ii}^{(c)}}{\theta_i^{th}} - n_i^{(c)}$ in (4) implies $P_m G_{mi}^{(c)} \leq \frac{P_i G_{ii}^{(c)}}{\theta_i^{th}} - n_i^{(c)}, \forall m \in \mathcal{M}$ in (6), it follows directly that $\mathbb{M}_{i,SINR}^{(c)} \subseteq \mathbb{M}_{i,PTC}^{(c)}$. ■

Example 1: We consider Fig. 1 where the transmit powers of all users are the same. Assuming that all users have selected channel 1, at a certain transmit power level P , we can observe the following: Since transmitter 1 is close to receivers 2 and 3, user 1 interferes with users 2 and 3. However, since transmitters 2 and 3 are far away from receiver 1, users 2 and 3 do not interfere with user 1 as long as they do not transmit simultaneously. Users 2 and 3 are far from each other and do not interfere with each other. For the protocol model, we have $\mathbb{M}_{1,PTC}^{(1)} = \{\{\}, \{2\}, \{3\}, \{2, 3\}\}$, $\mathbb{M}_{2,PTC}^{(1)} = \{\{\}, \{3\}\}$, and $\mathbb{M}_{3,PTC}^{(1)} = \{\{\}, \{2\}\}$. However, the protocol model does not take into account that user 1 may be interfered when both users 2 and 3 transmit *simultaneously*. In this case, we have $\mathbb{M}_{1,SINR}^{(1)} = \{\{\}, \{2\}, \{3\}\} \subset \mathbb{M}_{1,PTC}^{(1)}$, $\mathbb{M}_{2,SINR}^{(1)} = \mathbb{M}_{2,PTC}^{(1)}$, and $\mathbb{M}_{3,SINR}^{(1)} = \mathbb{M}_{3,PTC}^{(1)}$.

The probability of successful transmission of user i in channel c is given by

$$p_i^{succ,(c)} = p_i^{(c)} q_i^{(c)} \sum_{\mathcal{M} \in \mathbb{M}_i^{(c)}} \left(\prod_{m \in \mathcal{M}} p_m^{(c)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq i} (1 - p_k^{(c)}) \right). \quad (7)$$

We define $\mathbb{M}_i = \{\mathbb{M}_i^{(c)}, \forall c \in \mathcal{C}\}$. The average data rate of user i is given by

$$r_i(\mathbf{p}, \mathbf{q}_i, \mathbb{M}_i) = \sum_{c \in \mathcal{C}} \mu_i^{(c)} p_i^{succ,(c)}, \quad (8)$$

where $\mu_i^{(c)}$ is the peak data rate for user i in channel c , and vector $\mathbf{q}_i = (q_i^{(c)}, \forall c \in \mathcal{C})$ contains the listening probabilities of receiver i in all the channels. Given \mathbf{p} and \mathbf{q}_i , we have the following lemma, which states that the average data rate r_i is over-estimated when the protocol model is used.

Lemma 2: $r_i(\mathbf{p}, \mathbf{q}_i, \mathbb{M}_{i,PTC}) \geq r_i(\mathbf{p}, \mathbf{q}_i, \mathbb{M}_{i,SINR})$.

Proof: From (8) and Lemma 1, we have

$$\begin{aligned} r_i(\mathbf{p}, \mathbf{q}_i, \mathbb{M}_{i,PTC}) &= r_i(\mathbf{p}, \mathbf{q}_i, \mathbb{M}_{i,SINR}) + \sum_{c \in \mathcal{C}} \mu_i^{(c)} p_i^{(c)} q_i^{(c)} \\ &\quad \times \sum_{\mathcal{M} \in \mathbb{M}_{i,PTC}^{(c)} \setminus \mathbb{M}_{i,SINR}^{(c)}} \left(\prod_{m \in \mathcal{M}} p_m^{(c)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq i} (1 - p_k^{(c)}) \right) \\ &\geq r_i(\mathbf{p}, \mathbf{q}_i, \mathbb{M}_{i,SINR}). \quad \blacksquare \end{aligned}$$

For the rest of the paper, we assume that sets $\mathbb{M}_i^{(c)}$ in (4) and (6) are given, so we write $r_i(\mathbf{p}, \mathbf{q}_i, \mathbb{M}_i)$ as $r_i(\mathbf{p}, \mathbf{q}_i)$ for simplicity.

III. NETWORK UTILITY MAXIMIZATION

We now formulate the multi-channel random access problem as a NUM problem with vectors \mathbf{p} and \mathbf{q} as the optimization variables. The NUM problem is given by

$$\begin{aligned} &\text{maximize}_{\mathbf{p}, \mathbf{q}} \quad \sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}, \mathbf{q}_i)) \\ &\text{subject to} \quad \sum_{c \in \mathcal{C}} p_i^{(c)} \leq 1, \sum_{c \in \mathcal{C}} q_i^{(c)} \leq 1, \quad \forall i \in \mathcal{N}, \\ &\quad \quad \quad 0 \leq p_i^{(c)}, q_i^{(c)} \leq 1, \quad \forall i \in \mathcal{N}, c \in \mathcal{C}, \end{aligned} \quad (9)$$

where $U_i(r_i(\mathbf{p}, \mathbf{q}_i))$ is a concave and non-decreasing function in $r_i(\mathbf{p}, \mathbf{q}_i)$. However, due to the product form of the variables in (7), problem (9) is *non-convex*, even if the utility functions are concave. As a result, the problem is difficult to solve in general. An example of a concave utility function useful for resource allocation is the α -fair function [25] defined as

$$U_i(r_i) = \begin{cases} (1 - \alpha_i)^{-1} r_i^{1 - \alpha_i}, & \text{if } \alpha_i \in [0, 1) \cup (1, \infty), \forall i \in \mathcal{N}. \\ \ln r_i, & \text{if } \alpha_i = 1, \end{cases} \quad (10)$$

Intuitively, r_i increases when $p_i^{(c)}$ increases or when $p_j^{(c)}$ decreases, $j \neq i$. This is confirmed by the following lemma:

Lemma 3: For $i \in \mathcal{N}$, we have: (a) $r_i(\mathbf{p}, \mathbf{q}_i)$ is a non-decreasing function of $p_i^{(c)}, \forall c \in \mathcal{C}$.

(b) $r_i(\mathbf{p}, \mathbf{q}_i)$ is a non-increasing function of $p_j^{(c)}, \forall j \in \mathcal{N} \setminus \{i\}, c \in \mathcal{C}$.

Proof: (a) From (8), r_i can be written in the form $r_i(\mathbf{p}, \mathbf{q}_i) = \sum_{c \in \mathcal{C}} x_i^{(c)} p_i^{(c)}$, where

$$x_i^{(c)} = \mu_i^{(c)} q_i^{(c)} \sum_{\mathcal{M} \in \mathbb{M}_i^{(c)}} \left(\prod_{m \in \mathcal{M}} p_m^{(c)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq i} (1 - p_k^{(c)}) \right).$$

Since $x_i^{(c)} \geq 0$ and it is independent of $p_i^{(c)}$, $r_i(\mathbf{p}, \mathbf{q}_i)$ is a non-decreasing function of $p_i^{(c)}, \forall c \in \mathcal{C}$.

(b) Let $j \in \mathcal{N} \setminus \{i\}$ be given. We first define two sets of users that exclude users i and j :

$$\tilde{\mathcal{S}}_{i,j}^{(c)} = \left\{ \mathcal{S} : \mathcal{S} \in \mathcal{N} \setminus \{i, j\}, \mathcal{S} \in \mathbb{M}_i^{(c)}, \mathcal{S} \cup \{j\} \in \mathbb{M}_i^{(c)} \right\}$$

and $\hat{\mathcal{S}}_{i,j}^{(c)} = \left\{ \mathcal{S} : \mathcal{S} \in \mathcal{N} \setminus \{i, j\}, \mathcal{S} \in \mathbb{M}_i^{(c)}, \mathcal{S} \cup \{j\} \notin \mathbb{M}_i^{(c)} \right\}$.

From (8), we can write r_i as

$$r_i(\mathbf{p}, \mathbf{q}_i) = \sum_{c \in \mathcal{C}} \mu_i^{(c)} p_i^{(c)} q_i^{(c)} \times \left[\sum_{\mathcal{S} \in \hat{\mathcal{S}}_{i,j}^{(c)}} \left(\prod_{s \in \mathcal{S}} p_s^{(c)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{S}, k \neq i, j} (1 - p_k^{(c)}) \right) + \sum_{\mathcal{S} \in \hat{\mathcal{S}}_{i,j}^{(c)}} \left(\prod_{s \in \mathcal{S}} p_s^{(c)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{S}, k \neq i, j} (1 - p_k^{(c)}) \right) (1 - p_j^{(c)}) \right],$$

which is a non-increasing function of $p_j^{(c)}$, $\forall c \in \mathcal{C}$. \blacksquare

Although it is possible that the users may occupy more than one channel at an optimal solution, we can show based on Lemma 3 that we can always find another optimal solution where each user occupies only one channel.

Theorem 1: A global optimal solution of problem (9), $(\mathbf{p}^*, \mathbf{q}^*)$, is in the form:

$$p_i^{(c)*} \begin{cases} \in [0, 1], & \text{if } c = c_i, \\ = 0, & \text{otherwise,} \end{cases} \text{ and } q_i^{(c)*} = \begin{cases} 1, & \text{if } c = c_i, \\ 0, & \text{otherwise,} \end{cases} \quad (11)$$

where c_i is the channel chosen by user i .

Proof: Assume that (\mathbf{p}, \mathbf{q}) is feasible in problem (9), but \mathbf{p} and \mathbf{q} are not in the form of (11). From (8), we have

$$r_i(\mathbf{p}, \mathbf{q}_i) = \sum_{c \in \mathcal{C}} s_i^{(c)}(\mathbf{p}) q_i^{(c)}, \quad (12)$$

where

$$s_i^{(c)}(\mathbf{p}) = \mu_i^{(c)} p_i^{(c)} \sum_{\mathcal{M} \in \mathbb{M}_i^{(c)}} \left(\prod_{m \in \mathcal{M}} p_m^{(c)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq i} (1 - p_k^{(c)}) \right).$$

We define

$$c_i = \arg \max_{c \in \mathcal{C}} s_i^{(c)}(\mathbf{p}), \quad \forall i \in \mathcal{N}, \quad (13)$$

and $\mathbf{q}_i^* = (q_i^{(c)*}, \forall i \in \mathcal{N}, c \in \mathcal{C})$, where

$$q_i^{(c)*} = \begin{cases} 1, & \text{if } c = c_i, \\ 0, & \text{otherwise.} \end{cases} \quad (14)$$

We have

$$r_i(\mathbf{p}, \mathbf{q}_i) = \sum_{c \in \mathcal{C}} s_i^{(c)}(\mathbf{p}) q_i^{(c)} \leq s_i^{(c_i)}(\mathbf{p}) = r_i(\mathbf{p}, \mathbf{q}_i^*), \quad \forall i \in \mathcal{N},$$

where the inequality in the middle is due to the definition of c_i in (13) and the fact that $\sum_{c \in \mathcal{C}} q_i^{(c)*} \leq 1$. Since $U_i(r_i)$ is a non-decreasing function in r_i , $\forall i \in \mathcal{N}$, we have

$$\sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}, \mathbf{q}_i)) \leq \sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}, \mathbf{q}_i^*)). \quad (15)$$

Given \mathbf{q}^* , we have

$$r_i(\mathbf{p}, \mathbf{q}_i^*) = \mu_i^{(c_i)} p_i^{(c_i)} \times \sum_{\mathcal{M} \in \mathbb{M}_i^{(c_i)}} \left(\prod_{m \in \mathcal{M}} p_m^{(c_i)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq i} (1 - p_k^{(c_i)}) \right). \quad (16)$$

Since \mathbf{p} is not in the form as shown on the left hand side of (11), there exists $c \neq c_i$ such that $p_i^{(c)} > 0$. We define $\mathbf{p}^* = (p_i^{(c)*}, \forall i \in \mathcal{N}, c \in \mathcal{C})$, where

$$p_i^{(c)*} = \begin{cases} p_i^{(c)}, & \text{if } c = c_i, \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

Notice that r_i in (16) is independent of $p_i^{(c)}$ for $c \neq c_i$, and it is a non-increasing function of $p_j^{(c)}$, $\forall j \in \mathcal{N} \setminus \{i\}, c \in \mathcal{C}$ as shown in Lemma 3(b). Thus, we have

$$r_i(\mathbf{p}, \mathbf{q}_i^*) \leq r_i(\mathbf{p}^*, \mathbf{q}_i^*), \quad \forall i \in \mathcal{N}. \quad (18)$$

Since $U_i(r_i)$ is a non-decreasing function in r_i , $\forall i \in \mathcal{N}$, we have

$$\sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}, \mathbf{q}_i^*)) \leq \sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}^*, \mathbf{q}_i^*)). \quad (19)$$

Combining (15) and (19), we have

$$\sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}, \mathbf{q}_i)) \leq \sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}, \mathbf{q}_i^*)) \leq \sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}^*, \mathbf{q}_i^*)). \quad (20)$$

To sum up, given any feasible point (\mathbf{p}, \mathbf{q}) , we can always find another feasible point $(\mathbf{p}^*, \mathbf{q}^*)$ in the form of (17) and (14), which yields an objective value that is not smaller than that for (\mathbf{p}, \mathbf{q}) and each user occupies only one channel. The result thus follows. \blacksquare

IV. THREE-PHASE DISTRIBUTED ALGORITHM USING SEQUENTIAL CONVEX OPTIMIZATION

In this section, our goal is to solve non-convex NUM problem (9). We propose a low-complexity three-phase algorithm where the transmitters and receivers have to solve a number of convex optimization problems distributively. Convergence and local optimality of the solution are guaranteed.

A. Transmission Probability Optimization

We define the vector $\mathbf{p}_i = (p_i^{(c)}, \forall c \in \mathcal{C})$. Transmitter $i \in \mathcal{N}$ needs to solve the following local optimization problem, which has the same objective function as problem (9):

$$\begin{aligned} & \underset{\mathbf{p}_i}{\text{maximize}} && U_i \left(\sum_{c \in \mathcal{C}} o_i^{(c)} p_i^{(c)} \right) \\ & && + \sum_{j \in \mathcal{N} \setminus \{i\}} U_j \left(\sum_{c \in \mathcal{C}} \left(v_{ji}^{(c)} p_i^{(c)} + w_{ji}^{(c)} (1 - p_i^{(c)}) \right) \right) \\ & \text{subject to} && \sum_{c \in \mathcal{C}} p_i^{(c)} \leq 1, \quad 0 \leq p_i^{(c)} \leq 1, \quad \forall c \in \mathcal{C}, \end{aligned} \quad (21)$$

where

$$o_i^{(c)} = \mu_i^{(c)} q_i^{(c)} \sum_{\mathcal{M} \in \mathbb{M}_i^{(c)}} \left(\prod_{m \in \mathcal{M}} p_m^{(c)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq i} (1 - p_k^{(c)}) \right), \quad (22)$$

$$v_{ji}^{(c)} = \mu_j^{(c)} p_j^{(c)} q_j^{(c)} \times \sum_{\mathcal{M} \in \mathbb{M}_j^{(c)}: i \in \mathcal{M}} \left(\prod_{m \in \mathcal{M} \setminus \{i\}} p_m^{(c)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq j} (1 - p_k^{(c)}) \right), \quad (23)$$

and

$$w_{ji}^{(c)} = \mu_j^{(c)} p_j^{(c)} q_j^{(c)} \times \sum_{\mathcal{M} \in \mathbb{M}_j^{(c)}: i \notin \mathcal{M}} \left(\prod_{m \in \mathcal{M}} p_m^{(c)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq j, i} (1 - p_k^{(c)}) \right). \quad (24)$$

The coefficients $o_i^{(c)}$, $v_{ji}^{(c)}$, and $w_{ji}^{(c)}$ should be computed by transmitter i based on the broadcast messages from other transmitters and receivers.

Theorem 2: Problem (21) is a *convex* optimization problem in \mathbf{p}_i .

Proof: First, the constraints in problem (21) are linear. Also, as $o_i^{(c)}$, $v_{ji}^{(c)}$, and $w_{ji}^{(c)}$ are independent of $p_i^{(c)}$, and since the arguments within the utility functions are linear in \mathbf{p}_i , the objective function is concave in \mathbf{p}_i [26, pp. 79]. Thus, problem (21) is a convex optimization problem. ■

Hence, we can solve problem (21) by using the *interior point method* [26].

B. Listening Probability Optimization

Receiver $i \in \mathcal{N}$ needs to solve the following *local* optimization problem with the same objective function as problem (9).

$$\begin{aligned} & \underset{\mathbf{q}_i}{\text{maximize}} && U_i \left(\sum_{c \in \mathcal{C}} a_i^{(c)} q_i^{(c)} \right) + \sum_{j \in \mathcal{N} \setminus \{i\}} U_j (r_j(\mathbf{p}, \mathbf{q}_j)) \\ & \text{subject to} && \sum_{c \in \mathcal{C}} q_i^{(c)} \leq 1, \quad 0 \leq q_i^{(c)} \leq 1, \quad \forall c \in \mathcal{C}, \end{aligned} \quad (25)$$

where

$$a_i^{(c)} = \mu_i^{(c)} p_i^{(c)} \sum_{\mathcal{M} \in \mathbb{M}_i^{(c)}} \left(\prod_{m \in \mathcal{M}} p_m^{(c)} \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq i} (1 - p_k^{(c)}) \right). \quad (26)$$

Theorem 3: Let $c_i = \arg \max_{c \in \mathcal{C}} a_i^{(c)}$. A *closed-form* solution of problem (25) is

$$q_i^{(c)*} = \begin{cases} 1, & \text{if } c = c_i, \\ 0, & \text{otherwise.} \end{cases} \quad (27)$$

Proof: First, notice that $a_i^{(c)}$ and $\sum_{j \in \mathcal{N} \setminus \{i\}} U_j (r_j(\mathbf{p}, \mathbf{q}_j))$ are independent of $q_i^{(c)}$. Since U_i is a non-decreasing function, problem (25) is equivalent to the following linear programming problem

$$\begin{aligned} & \underset{\mathbf{q}_i}{\text{maximize}} && \sum_{c \in \mathcal{C}} a_i^{(c)} q_i^{(c)} \\ & \text{subject to} && \sum_{c \in \mathcal{C}} q_i^{(c)} \leq 1, \\ & && 0 \leq q_i^{(c)} \leq 1, \quad \forall c \in \mathcal{C}, \end{aligned} \quad (28)$$

the solution of which is given by (27). ■

C. Three-Phase Distributed Algorithm

Having introduced the local optimization problems for the transmitter and receiver of user $i \in \mathcal{N}$, we are now ready to present Algorithm 1 for obtaining a near-optimal solution of problem (9) based on the *coordinate ascent method* [27, pp. 207]. Let $\mathbf{p}_{-i} = (\mathbf{p}_1, \dots, \mathbf{p}_{i-1}, \mathbf{p}_{i+1}, \dots, \mathbf{p}_N)$ and

Algorithm 1 *Three-Phase Distributed Algorithm to Obtain a Near-optimal Solution for Problem (9).*

```

1: Initialize  $\mathbf{p}^*$  such that  $\sum_{c \in \mathcal{C}} p_i^{(c)*} \leq 1, \forall i \in \mathcal{N}$ , and  $0 \leq p_i^{(c)*} \leq 1, \forall i \in \mathcal{N}, c \in \mathcal{C}$ 
2: Initialize  $\mathbf{q}^*$  such that  $\sum_{c \in \mathcal{C}} q_i^{(c)*} \leq 1, \forall i \in \mathcal{N}$ , and  $0 \leq q_i^{(c)*} \leq 1, \forall i \in \mathcal{N}, c \in \mathcal{C}$ 
3: Set the convergence threshold  $\epsilon > 0$ 
4: Set the iteration counter  $t := 1$ 
5: Set  $u := -\infty$  and  $\Delta := \infty$ 
6: Phase I: Channel Probing
7: while  $\Delta > \epsilon$ 
8:   for each transmitter  $i \in \mathcal{N}$ 
9:     if  $t \in \mathcal{T}_i$  then
10:       Calculate  $o_i^{(c)}, \forall c \in \mathcal{C}$  using (22)
11:       Calculate  $v_{ji}^{(c)}, \forall j \in \mathcal{N} \setminus \{i\}, c \in \mathcal{C}$  using (23)
12:       Calculate  $w_{ji}^{(c)}, \forall j \in \mathcal{N} \setminus \{i\}, c \in \mathcal{C}$  using (24)
13:       Solve problem (21) to obtain the solution  $\mathbf{p}_i^*$ 
14:       Broadcast  $\mathbf{p}_i^*$  to other users using the control channel
15:       Set  $u^*(t) := \sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}^*, \mathbf{q}_i^*))$ 
16:       Set  $t := t + 1$ 
17:     end if
18:   end for
19:   Set  $\Delta := u^*(t) - u$  and  $u := u^*(t)$ 
20: end while
21: Phase II: Channel Selection
22: for each receiver  $i \in \mathcal{N}$ 
23:   if  $t \in \mathcal{T}_i$  then
24:     Calculate  $a_i^{(c)}, \forall c \in \mathcal{C}$  using (26)
25:     Set  $c_i := \arg \max_{c \in \mathcal{C}} a_i^{(c)}$ 
26:     Set  $q_i^{(c)*}, \forall c \in \mathcal{C}$  using (27)
27:     Broadcast  $\mathbf{q}_i^*$  to other users using the control channel
28:     Set  $p_i^{(c)*} := 0$ , if  $c \neq c_i, \forall c \in \mathcal{C}$ 
29:     Set  $u^*(t) := \sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}^*, \mathbf{q}_i^*))$ 
30:     Set  $t := t + 1$ 
31:   end if
32: end for
33: Phase III: Transmission Probability Allocation
34: Set  $\Delta := \infty$ 
35: Repeat Lines 7 to 20 once

```

$\mathbf{q}_{-i} = (\mathbf{q}_1, \dots, \mathbf{q}_{i-1}, \mathbf{q}_{i+1}, \dots, \mathbf{q}_N)$. Considering transmitter i , the basic idea of this method is that we fix \mathbf{p}_{-i} and \mathbf{q} , and maximize the aggregate utility $\sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}, \mathbf{q}_i))$ with respect to \mathbf{p}_i (i.e., problem (21)). Similarly, for receiver i , we fix \mathbf{p} and \mathbf{q}_{-i} , and maximize the aggregate utility $\sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}, \mathbf{q}_i))$ with respect to \mathbf{q}_i (i.e., problem (25)). The updates of the solutions are carried out *successively*. Notice that the solution of problem (25) as stated in Theorem 3 represents a channel selection. Once the channel is selected by the receiver, the transmitter will not attempt to transmit in other channels, to which the receiver is not listening. As a result, the receivers should defer their decisions of selecting a channel until after the transmitters have coordinated their transmission probabilities.

With this idea, we propose our Algorithm 1 with three phases. In phase I, the receivers are initialized to listen to each channel with a certain probability. The transmitters then probe the channels by adjusting their transmission probabilities until the aggregate utility converges. In phase II, each transmitter/receiver pair selects the channel that results in the highest average data rate. The reason for choosing only one

channel is given by Theorem 1. In phase III, based on this channel selection, the transmitters adjust their transmission probabilities again until the aggregate utility converges.

In Algorithm 1, \mathcal{T}_i is the set of time slots in which user $i \in \mathcal{N}$ solves the local optimization problem. Also, we use variable u to keep track of the aggregate utility achieved in the previous iteration, and we let $u^*(t)$ be the aggregate utility achieved in iteration t . The algorithm transitions from phase I to phase II and from phase III to the exit if the difference $\Delta = u^*(t) - u$ is less than the predefined convergence threshold ϵ . The complexity of Algorithm 1 is relatively low because it involves solving only convex problem (21) and evaluating closed-form equation (27). Transmitter i and receiver i need only to broadcast the solutions \mathbf{p}_i^* and \mathbf{q}_i^* in (21) and (27) using the control channel, respectively. Thus, the signalling overhead grows linearly with the number of users N .

We have the following theorems that show the convergence of Algorithm 1 and the local optimality of $(\mathbf{p}^*, \mathbf{q}^*)$ obtained by Algorithm 1. Notice that even in a centralized setting, there is no guarantee that we can obtain the globally optimal solution of problem (9) due to its non-convexity.

Theorem 4: The aggregate utility $u^*(t)$ converges to a fixed point u^* . That is, $\lim_{t \rightarrow \infty} u^*(t) = u^*$. Moreover, $u^*(t)$ is a non-decreasing sequence in t . That is, $u^*(t) \leq u^*(t+1)$ for all $t \geq 0$.

Proof: In both phases I and III, because we fix \mathbf{p}_{-i}^* and \mathbf{q}^* to solve problem (21) for \mathbf{p}_i^* , and update the solution of transmission probabilities \mathbf{p}^* in the Gauss-Seidel manner [27, pp. 185], we can show by [27, Proposition 3.9, pp. 219] that $u^*(t)$ converges to a fixed point. In each iteration t , since we are maximizing the objective function $\sum_{i \in \mathcal{N}} U_i(r_i(\mathbf{p}^*, \mathbf{q}_i^*))$ over some variables, while the other variables are fixed, we must have $u^*(t) \leq u^*(t+1)$ for all $t \geq 0$. ■

Theorem 5: The solution $(\mathbf{p}^*, \mathbf{q}^*)$ is at least a *local* optimal solution of problem (9).

Proof: Let $(\mathbf{p}^*, \mathbf{q}^*)$ be a fixed point obtained using Algorithm 1. That is, \mathbf{p}_i^* is the optimal solution of problem (21) given $\mathbf{p}_{-i} = \mathbf{p}_{-i}^*$ and $\mathbf{q} = \mathbf{q}^*$. Also, \mathbf{q}_i^* is the optimal solution of problem (25) given $\mathbf{q}_{-i} = \mathbf{q}_{-i}^*$ and $\mathbf{p} = \mathbf{p}^*$. (Notice that from (17)-(19) in the proof of Theorem 1 and (27) in Theorem 3, we can show that \mathbf{q}^* will not change even if we involve problem (25) for listening probability optimization in phase III.) Since problems (21) and (25) are *convex*, the Karush-Kuhn-Tucker (KKT) conditions are both *necessary* and *sufficient* [26]. $(\mathbf{p}^*, \mathbf{q}^*)$ must satisfy the union of all the KKT conditions of problems (21) and (25), for all $i \in \mathcal{N}$, which are equal to the KKT conditions of problem (9). That is, $(\mathbf{p}^*, \mathbf{q}^*)$ is a stationary point [28] in problem (9). Because $(\mathbf{p}^*, \mathbf{q}^*)$ is obtained by solving a number of minimization problems (21) and (25), which have the same objective function as problem (9), it is clear that $(\mathbf{p}^*, \mathbf{q}^*)$ is at least a local optimal solution of problem (9). ■

V. COALITIONAL GAME THEORY FOR SINR MODEL

In the previous section, we have assumed that all the secondary nodes cooperate to maximize the aggregate utility. This gives rise to the question of what happens if the users

are *rational* and aim to maximize their own utilities. In fact, if user i is rational and there is no coordination among the users, user i may choose to transmit in a particular channel c_i by setting $p_i^{(c_i)} = q_i^{(c_i)} = 1$ and $p_i^{(c)} = q_i^{(c)} = 0$ for $c \neq c_i$ in order to maximize U_i as suggested by the proof of Theorem 1. Hence, a significant amount of interference will be generated. In the worst case (e.g., when the number of channels C is small), it is possible that the utilities of all the users will be zero. To prevent this problem, the users may coordinate among themselves in the form of a *coalition*. The users belonging to the same coalition coordinate their transmission and listening probabilities to maximize the aggregate utility, which is then divided among themselves.

Example 2: We continue with Example 1, and assume that the three users have selected channel 1 and transmit with power P . We assume that their peak data rates are $\mu_1 = 5$, $\mu_2 = 2$, and $\mu_3 = 1$. If all the users are willing to coordinate their transmission probabilities \mathbf{p} , the optimal transmission probabilities based on throughput maximization (i.e., $\alpha = 0$ in problem (9)) for both the SINR and protocol models are given by $p_1^* = 1$, $p_2^* = 0$, and $p_3^* = 0$. From (10), the corresponding utilities are $U_1 = 5$ and $U_2 = U_3 = 0$. However, if users 2 and 3 are rational, they may not be satisfied with zero utility. For the protocol model, users 2 and 3 have no *bargaining power* with user 1 to increase their utilities. On the other hand, the SINR model reveals that users 2 and 3 can threaten user 1 to transmit simultaneously and jam user 1's transmission. This effect is not captured by the protocol model. In the following, we apply coalitional game theory to study the incentives of rational user cooperation and the payoff distribution among the users. We note that coalitions can also be formed in the protocol model. However, in this case, the significance of the formation of coalitions may be undermined by the fact that the protocol model does not capture the cumulative effect of interference.

A. Coalitional Game

Since the channels are orthogonal, we focus on one particular channel, and refer to the set of users that have selected that channel by \mathcal{N} for notational simplicity. In this case, the average data rate of user i in (8) can be simplified to

$$r_i(\mathbf{p}) = \mu_i p_i \sum_{\mathcal{M} \in \mathbb{M}_i} \left(\prod_{m \in \mathcal{M}} p_m \right) \left(\prod_{k \in \mathcal{N} \setminus \mathcal{M}, k \neq i} (1 - p_k) \right), \quad (29)$$

where we drop the superscript for channel c and the term for the listening probability. We further restrict our attention to non-decreasing concave utility functions $U_i(r_i)$, where $U_i(0) = 0$.

We define the coalitional game \mathcal{G} with *transferable utility* [22] as a pair (\mathcal{N}, v) , where \mathcal{N} is the set of players or the *grand coalition*, and $v : 2^{\mathcal{N}} \rightarrow \mathbb{R}$ is the *value* of a coalition $\mathcal{S} \subseteq \mathcal{N}$ that the members of the coalition can distribute among themselves. In our problem, this value is defined as

$$\begin{aligned} v(\mathcal{S}) = \underset{\mathbf{p}}{\text{maximize}} \quad & \sum_{i \in \mathcal{S}} U_i(r_i(\mathbf{p})) \\ \text{subject to} \quad & 0 \leq p_i \leq 1, \quad \forall i \in \mathcal{S}, \\ & p_j = 1, \quad \forall j \in \mathcal{N} \setminus \mathcal{S}. \end{aligned} \quad (30)$$

That is, $v(\mathcal{S}) = \sum_{i \in \mathcal{S}} U_i(r_i(\mathbf{p}^*(\mathcal{S})))$, where $\mathbf{p}^*(\mathcal{S})$ is the optimal solution of problem (30). The users within coalition \mathcal{S} coordinate among themselves to maximize the aggregate utility, subject to the *worst-case interference* from users $j \in \mathcal{N} \setminus \mathcal{S}$ when they choose transmission probabilities $p_j = 1$. All users in set $\mathcal{N} \setminus \mathcal{S}$ are not coordinating with the users within coalition \mathcal{S} . Instead, each user $j \in \mathcal{N} \setminus \mathcal{S}$ transmits with $p_j = 1$ in order to maximize its own utility, because $U_j(r_j(\mathbf{p}))$ is a non-decreasing function in p_j from Lemma 3(a). $v(\mathcal{S})$ can be obtained from Algorithm 1 with a few minor changes: Choose $C = 1$. Run line 13 if $i \in \mathcal{S}$, but replace it with “Set $p_i^* := 1$ ” if $i \in \mathcal{N} \setminus \mathcal{S}$.

The property of *superadditivity* [22] is often observed in coalitional games, including game \mathcal{G} . It is defined as follows:

Definition 1: A game is superadditive if $v(\mathcal{S}_1 \cup \mathcal{S}_2) \geq v(\mathcal{S}_1) + v(\mathcal{S}_2), \forall \mathcal{S}_1, \mathcal{S}_2 \subset \mathcal{N}$ with $\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset$.

Theorem 6: Game \mathcal{G} is superadditive.

Proof: Let $\mathbf{p}^*(\mathcal{S}_1)$, $\mathbf{p}^*(\mathcal{S}_2)$, and $\mathbf{p}^*(\mathcal{S}_1 \cup \mathcal{S}_2)$ be the optimal probabilities maximizing $v(\mathcal{S}_1)$, $v(\mathcal{S}_2)$, and $v(\mathcal{S}_1 \cup \mathcal{S}_2)$, respectively, as defined in (30). For $\mathcal{S}_1 \cap \mathcal{S}_2 = \emptyset$, we construct a vector $\mathbf{p}(\mathcal{S}_1 \cup \mathcal{S}_2)$, where the i^{th} element is

$$p_i(\mathcal{S}_1 \cup \mathcal{S}_2) \triangleq \begin{cases} p_i^*(\mathcal{S}_1), & \text{if } i \in \mathcal{S}_1, \\ p_i^*(\mathcal{S}_2), & \text{if } i \in \mathcal{S}_2, \\ 1, & \text{otherwise.} \end{cases}$$

So $\mathbf{p}(\mathcal{S}_1 \cup \mathcal{S}_2)$ is feasible in problem (30) with $\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2$. From (30), we have $p_i^*(\mathcal{S}_1) = 1$ if $i \in \mathcal{N} \setminus \mathcal{S}_1$. Thus, we have

$$p_i^*(\mathcal{S}_1) \begin{cases} = p_i(\mathcal{S}_1 \cup \mathcal{S}_2), & \text{if } i \in \mathcal{S}_1, \\ \geq p_i(\mathcal{S}_1 \cup \mathcal{S}_2), & \text{if } i \in \mathcal{N} \setminus \mathcal{S}_1. \end{cases}$$

From Lemma 3(b), we have

$$r_i(\mathbf{p}^*(\mathcal{S}_1)) \leq r_i(\mathbf{p}(\mathcal{S}_1 \cup \mathcal{S}_2)), \forall i \in \mathcal{S}_1.$$

Since U_i is a non-decreasing function of r_i , we have

$$U_i(r_i(\mathbf{p}^*(\mathcal{S}_1))) \leq U_i(r_i(\mathbf{p}(\mathcal{S}_1 \cup \mathcal{S}_2))), \forall i \in \mathcal{S}_1,$$

which implies that

$$\sum_{i \in \mathcal{S}_1} U_i(r_i(\mathbf{p}^*(\mathcal{S}_1))) \leq \sum_{i \in \mathcal{S}_1} U_i(r_i(\mathbf{p}(\mathcal{S}_1 \cup \mathcal{S}_2))).$$

Similarly, we have

$$\sum_{i \in \mathcal{S}_2} U_i(r_i(\mathbf{p}^*(\mathcal{S}_2))) \leq \sum_{i \in \mathcal{S}_2} U_i(r_i(\mathbf{p}(\mathcal{S}_1 \cup \mathcal{S}_2))).$$

Overall, we have

$$\begin{aligned} & \sum_{i \in \mathcal{S}_1} U_i(r_i(\mathbf{p}^*(\mathcal{S}_1))) + \sum_{i \in \mathcal{S}_2} U_i(r_i(\mathbf{p}^*(\mathcal{S}_2))) \\ & \leq \sum_{i \in \mathcal{S}_1 \cup \mathcal{S}_2} U_i(r_i(\mathbf{p}(\mathcal{S}_1 \cup \mathcal{S}_2))) \leq \sum_{i \in \mathcal{S}_1 \cup \mathcal{S}_2} U_i(r_i(\mathbf{p}^*(\mathcal{S}_1 \cup \mathcal{S}_2))), \end{aligned}$$

which concludes the proof. \blacksquare

B. The Core

To determine the *stability* of the grand coalition, we use the solution concept of the *core* [22]. It is possible that a subset of users may opt out of the grand coalition to form a smaller coalition, if the users in the smaller coalition receive higher utilities than when they participate in the grand coalition. In that case, the core is *empty*. The core is formally defined as follows:

Definition 2: The core is the set of feasible utility allocation vectors $\mathbf{U} = (U_i, \forall i \in \mathcal{N})$ where

$$\mathcal{U}_{\text{core}} = \left\{ \mathbf{U} : \sum_{i \in \mathcal{N}} U_i = v(\mathcal{N}), \sum_{i \in \mathcal{S}} U_i \geq v(\mathcal{S}), \forall \mathcal{S} \subset \mathcal{N} \right\}. \quad (31)$$

In some special cases, it can be shown that the core is non-empty. One such special case is when all the users are one-hop neighbours to each other (i.e., user i is a one-hop neighbour to user $j, \forall i, j \in \mathcal{N}, i \neq j$, where *one-hop neighbour* is defined in (5)). In this case, since $\mathbb{M}_{i, \text{SINR}}$ and $\mathbb{M}_{i, \text{PTC}}$ are null sets $\forall i \in \mathcal{N}$ from (4) and (6), the SINR model is identical to the protocol model. So, the average data rate of user i in (29) can further be simplified as

$$r_i(\mathbf{p}) = \mu_i p_i \prod_{j \in \mathcal{N} \setminus \{i\}} (1 - p_j). \quad (32)$$

Theorem 7: If all the users are one-hop neighbours to each other, then the core is non-empty.

Proof: Since $p_j = 1, \forall j \in \mathcal{N} \setminus \mathcal{S}$ from (30), we have $r_i = 0, \forall i \in \mathcal{S} \subset \mathcal{N}$ from (32) if all the users are one-hop neighbours to each other, which implies that $v(\mathcal{S}) = 0, \forall \mathcal{S} \subset \mathcal{N}$. Notice that any vector $\mathbf{U} = (U_i, \forall i \in \mathcal{N} : \sum_{i \in \mathcal{N}} U_i = v(\mathcal{N}), U_i \geq 0, \forall i \in \mathcal{N})$ satisfies $\sum_{i \in \mathcal{S}} U_i \geq v(\mathcal{S}) = 0, \forall \mathcal{S} \subset \mathcal{N}$. So $\mathbf{U} \in \mathcal{U}_{\text{core}}$, and the core is thus non-empty. \blacksquare

C. Shapley Value

As a solution concept, the core has a few drawbacks. It can be *empty* and the allocation of payoff according to the core may be *unfair*. In Example 2, with the use of the SINR model, we can show that $v(\{1, 2\}) = v(\{1, 3\}) = v(\{1, 2, 3\}) = 5$ using (30). The only allocation of utilities that lies in the core is $U_1 = 5, U_2 = U_3 = 0$. This allocation is stable since no smaller coalitions can be formed where the members can receive a higher payoff than when they are in the grand coalition. However, it is unfair in the division of the payoff among the users as it does not take into account the contribution of each user to a coalition. In the following, we propose to use the *Shapley value* [22] to *fairly* divide the payoff among the players. Let the total number of users in coalition \mathcal{S} be $S = |\mathcal{S}|$.

Definition 3: The Shapley value is the payoff allocation vector $\phi(v) = (\phi_1(v), \dots, \phi_N(v))$, where

$$\phi_i(v) = \sum_{\mathcal{S} \subseteq \mathcal{N} \setminus \{i\}} \frac{S!(N-S-1)!}{N!} [v(\mathcal{S} \cup \{i\}) - v(\mathcal{S})]. \quad (33)$$

In fact, $\phi_i(v)$ represents the expected *marginal contribution* of user i to different coalitions \mathcal{S} without user i . The Shapley value has a number of nice properties. First, we have

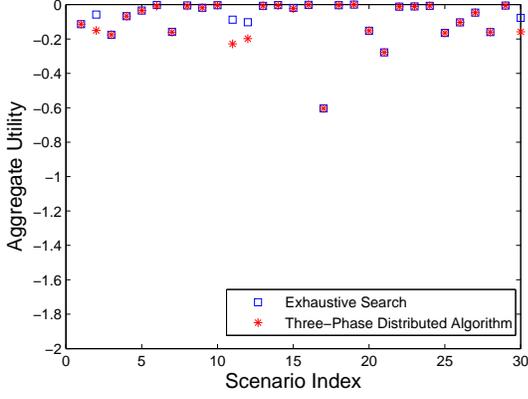


Fig. 2. Aggregate utility obtained using an exhaustive search and the three-phase distributed algorithm (i.e., Algorithm 1) based on the multi-channel SINR model. We can see that Algorithm 1 achieves a near-optimal solution.

$\sum_{i \in \mathcal{N}} \phi_i(v) = v(\mathcal{N})$. Moreover, it is fair in the sense that users who make the same contribution to different coalitions receive the same payoff. Mathematically, if $v(\mathcal{S} \cup \{i\}) = v(\mathcal{S} \cup \{j\})$, $\forall \mathcal{S} \in \mathcal{N} \setminus \{i, j\}$, then $\phi_i(v) = \phi_j(v)$. As we have discussed in Example 2, with the use of the SINR model, users 2 and 3 can threaten to leave the coalition to jointly jam user 1's transmission. The Shapley value in this case is $\phi(v) = (3.33, 0.83, 0.83)$ and both users 2 and 3 receive positive utilities. It is worth mentioning that since users 2 and 3 have no bargaining power in the protocol model, we can show that the Shapley value in this case is $\phi(v) = (5, 0, 0)$ and both users 2 and 3 receive zero utility.

In general, the Shapley value is not directly related to the core. However, the Shapley value lies in the core in some special cases, including the case where all the users are one-hop neighbours to each other for our problem.

Theorem 8: If all the users are one-hop neighbours to each other, then (a) $\phi_i(v) = \frac{v(\mathcal{N})}{N}$, $\forall i \in \mathcal{N}$, and (b) $\phi(v) \in \mathcal{U}_{core}$.

Proof: (a) If all the users are one-hop neighbours to each other, we have $v(\mathcal{S}) = 0$, $\forall \mathcal{S} \subset \mathcal{N}$, from the proof of Theorem 7. From (33), notice that the only non-zero term in the summation is given by $\mathcal{S} = \mathcal{N} \setminus \{i\}$. Therefore, we have $\phi_i(v) = \frac{(N-1)!(N-(N-1)-1)!}{N!} [v(\mathcal{N}) - v(\mathcal{N} \setminus \{i\})] = \frac{v(\mathcal{N})}{N}$.

(b) From part (a), we have $\sum_{i \in \mathcal{N}} \phi_i(v) = v(\mathcal{N})$. Also, $\sum_{i \in \mathcal{S}} \phi_i(v) = Sv(\mathcal{N})/N > 0 = v(\mathcal{S})$, $\forall \mathcal{S} \subset \mathcal{N}$. From (31), we know that $\phi(v) \in \mathcal{U}_{core}$. ■

Thus, in this case, the payoff allocation vector $\phi(v)$, which distributes the total payoff equally among the users, is in the core. Empirical investigations regarding the core and the Shapley value in the general setting, where not all the users are one-hop neighbours, are provided in the next section.

VI. PERFORMANCE EVALUATIONS

In this section, we evaluate the performances of Algorithm 1 for the SINR and protocol models, and compare with that of a heuristic scheme. We also illustrate the significance of the core and the Shapley value. Unless specified otherwise, we assume that the secondary nodes are randomly placed in a $50 \text{ m} \times 50 \text{ m}$ area. The peak data rate of a user is randomly

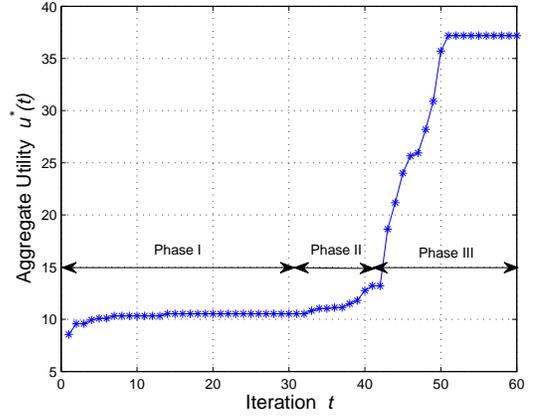


Fig. 3. Convergence of the aggregate utility $u^*(t)$ using the three-phase distributed algorithm (i.e., Algorithm 1). Notice that the aggregate utility obtained in each iteration is non-decreasing. The users probe the channels in phase I and select the best channel in phase II. In phase III, the transmission probabilities are adjusted based on the channels selected in phase II.

selected to be between 1 Mbps and 10 Mbps. For simplicity, we do not take into account the effect of fading and model the channel gain as $G_{i,j}^{(c)} = 1/d_{i,j}^\gamma$, where $d_{i,j}$ is the distance between the transmitter of user i and the receiver of user j , and γ is the path loss exponent. We adopt $\gamma = 2$. When the effect of channel fading is considered, Algorithm 1 is still applicable. In this case, after estimating the channel gain $G_{i,j}^{(c)}$ in every coherence interval, we rerun Algorithm 1 to obtain an updated solution. The transmit powers of all the users are equal and set to a value which yields a minimum signal-to-noise-ratio (SNR) of 10 dB at the receivers. The SINR threshold is $\theta_i^{th} = \theta^{th}$, $\forall i \in \mathcal{N}$, and is set to 0 dB. The convergence threshold ϵ is set to 10^{-4} . All the users have the same α -fair utility functions with $\alpha_i = \alpha$, $\forall i \in \mathcal{N}$. For initialization, we use $p_i^{(c)*} = q_i^{(c)*} = 1/C$, $\forall i \in \mathcal{N}, c \in \mathcal{C}$ in lines 1 and 2 in Algorithm 1.

We first evaluate the optimality of the solution obtained with Algorithm 1. We consider the case of five users, two orthogonal channels with identical channel conditions, and $\alpha = 5$. The optimal solution under the SINR model is obtained with an exhaustive search. As shown in Fig. 2, the solution obtained with Algorithm 1 is near-optimal. In Fig. 3, we evaluate the convergence of Algorithm 1 for $N = 10$, $C = 3$, and $\alpha = 0$. From Theorem 4, the algorithm converges to a fixed point $\lim_{t \rightarrow \infty} u^*(t) = u^*$. Also, the aggregate utility $u^*(t)$ obtained in iteration t is a non-decreasing sequence, i.e., $u^*(t) \leq u^*(t+1)$. The improvement in $u^*(t)$ in phase III is more significant than that in phase I. In phase I, the transmitters may transmit in all channels, i.e., a significant amount of interference is generated. However, in phase III, since the users have selected to transmit and listen to only one channel, the number of potential interferers in each channel is reduced. As a result, the improvement in $u^*(t)$ is more significant.

In Fig. 4, we consider the case $N = 10$, $C = 4$, and $\alpha = 0$ when the set of data channels \mathcal{C} changes due to dynamic spectrum leasing. Specifically, we assume that two

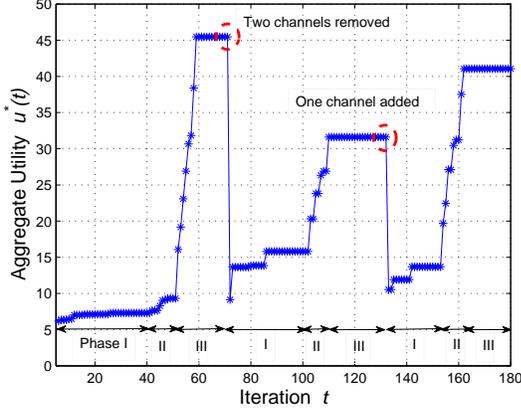


Fig. 4. The change in aggregate utility $u^*(t)$ when the set of data channels \mathcal{C} changes due to dynamic spectrum leasing. Initially, we assume that there are four data channels available. We assume that two data channels are removed from \mathcal{C} and then one data channel is added back to \mathcal{C} . We run the three-phase distributed algorithm (i.e., Algorithm 1) based on the previous solution after set \mathcal{C} has changed. We can see that $u^*(t)$ converges again quickly to a fixed point when the set \mathcal{C} changes.

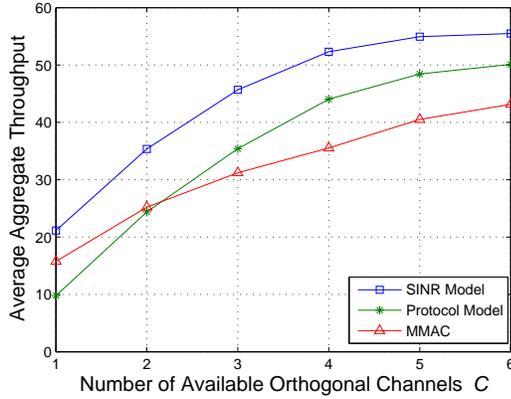


Fig. 5. Average aggregate throughput versus the number of orthogonal channels available for Algorithm 1 using the SINR model, the protocol model, and the MMAC [29]. Notice that the design based on the SINR model achieves the highest aggregate throughput.

channels are removed from \mathcal{C} when the lease expires, and one new channel is leased and added back to \mathcal{C} later. As we can see, by running Algorithm 1 based on \mathbf{p}^* from the previous solution after each change in set \mathcal{C} , the solution converges quickly to a fixed point again and adapts to these dynamic network changes.

Next, we compare the aggregate throughput achieved with Algorithm 1 for the SINR model (using $M_i = M_{i,SINR}, \forall i \in \mathcal{N}$) and protocol model (using $M_i = M_{i,PTC}, \forall i \in \mathcal{N}$), and the multi-channel MAC (MMAC) protocol [29] for $N = 10$ and $\alpha = 0$ averaged over 100 different random topologies when the number of orthogonal channels C varies. The MMAC protocol is a multi-channel extension of the IEEE 802.11 distributed coordination function and is suitable for the spectrum leasing model in CR networks. In MMAC, the users first select the channel with the least scheduled traffic, and then contend for it by using the carrier sense multiple access

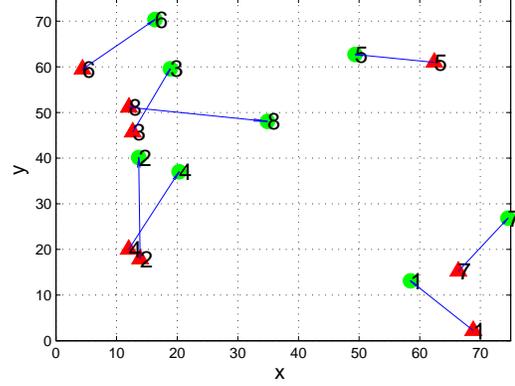


Fig. 6. A CR network with eight users. User 5 generates and receives the least amount of interference due to its isolated position.

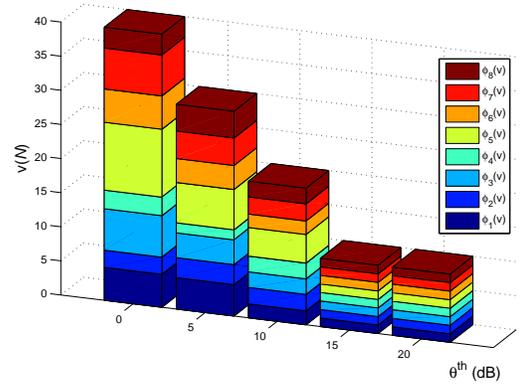


Fig. 7. Aggregate utility of the grand coalition $v(\mathcal{N}) = \sum_{i \in \mathcal{N}} \phi_i(v)$ and the Shapley value $\phi_i(v)$ for the secondary users in Fig. 6. When θ^{th} is increased, $v(\mathcal{N})$ is decreased because the interference range is increased. When θ^{th} is increased to 15 dB, $v(\mathcal{N})$ is equally shared among these one-hop neighbours as stated in Theorem 8. Notice that user 5 has the largest share of payoff for $\theta^{th} < 15$ dB due to its large marginal contribution to different coalitions.

with collision avoidance (CSMA/CA) protocol. We assume that the channel is sensed busy if any one-hop neighbour transmits. In other words, the sensing is based on the protocol model. Since Algorithm 1 obtains a locally optimal solution from a given starting point, we execute Algorithm 1 from thirteen randomly generated feasible starting points ($\mathbf{p}^*, \mathbf{q}^*$), and record the solution that yields the maximum aggregate utility to obtain a solution that is close to the globally optimal one. As shown in Fig. 5, when C increases, less interference is experienced by each user, so the overall system throughput is increased. Also, we notice that the design based on the SINR model always achieves a higher throughput than that using the protocol model and the MMAC protocol.

Finally, we investigate the payoff distribution for the Shapley value and the existence of the core for the network scenario shown in Fig. 6. Eight secondary users are randomly placed in a $75 \text{ m} \times 75 \text{ m}$ open area, $\alpha = 0$, and the peak data rate of each user is fixed to 10 Mbps. The minimum SNR is guaranteed to be at least 20 dB and we consider

different SINR thresholds θ^{th} , e.g., for different bit error rate requirements. The aggregate utility of the grand coalition $v(\mathcal{N}) = \sum_{i \in \mathcal{N}} \phi_i(v)$ and the Shapley value $\phi(v)$ are shown in Fig. 7. By increasing θ^{th} , the receivers become less tolerant to interference from other users, so the interference range is increased and the spatial reuse factor is reduced. As a result, $v(\mathcal{N})$ is reduced as shown in Fig. 7. From (5), when θ^{th} is increased up to a certain value, all users are one-hop neighbours to each other. This holds true in this setting when $\theta^{th} \geq 15$ dB. As expected from Theorem 8, all the users equally share the aggregate utility in this case. Also, notice that user 5 generates and receives the least amount of interference due to its isolated position. Thus, it has a large marginal contribution to different coalitions, and receives the largest proportion of the payoff for $\theta^{th} < 15$ dB. Moreover, it can be shown that the constraints in (31) can be satisfied and the core exists in this example for all the values of θ^{th} that we have studied. However, the Shapley value lies only in the core for $\theta^{th} \geq 10$ dB, which includes the cases $\theta^{th} \geq 15$ dB where all the users are one-hop neighbours to each other as stated in Theorem 8.

VII. CONCLUSIONS

In this paper, we have studied random access in CR networks using the SINR model. For cooperative users in a multi-channel model, a three-phase distributed algorithm has been proposed to obtain a near-optimal solution for the formulated non-convex NUM problem. It converges readily to a close-to-optimal value even when the set of data channels changes due to dynamic spectrum leasing. For rational users in a single-channel model, we have used the core and the Shapley value to characterize the stability and fair allocation of the payoff among the users, respectively. To the best of our knowledge, this is the first work that applies coalitional game theory to study the incentive issues of rational user cooperation in random access under the SINR model. In our system model, we have assumed that (a) the set of users \mathcal{N} is fixed and (b) the transmission between the transmitter and receiver of each user is only single-hop. For (a), it can be shown that by running Algorithm 1 starting from the previous solution after \mathcal{N} has changed, the solution converges readily again to a fixed point. For (b), we may consider the multi-hop setting by introducing binary routing variables and flow conservation constraints as in [30]. Interesting topics for future work are the extension of the proposed framework to include the transmit powers as optimization variables, a CSMA/CA-based multi-channel MAC, and rational user cooperation in multiple channels.

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