Risk-Averse Forward Contract for Electric Vehicle Frequency Regulation Service

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Abstract—Electric vehicles (EVs) can be coordinated by an aggregator to participate in the electricity day-ahead market (DAM) and provide frequency regulation service. In the DAM, a short-term forward contract is made between the aggregator and an independent system operator (ISO). The contract specifies the contract size, which is the amount of regulation capacity provided by the aggregator for each hour of the next day. However, since the capacity of the aggregator is provided by many EVs instead of a single source, the challenge is how to efficiently aggregate the small and uncertain individual capacity and determine the optimal size of the forward contract. We consider two cases for the contract between the aggregator and the ISO. In the first case, the aggregator needs to ensure that the capacity provided by the EVs on the next day will meet the amount specified in the contract. In contrast, in the second case, the ISO allows an update of the contract size. A stochastic program is formulated to determine the contract size for both cases. Our problem formulation incorporates risk management using the conditional value at risk (CVaR). Chance constraints are embedded when the contract size is fixed. We tackle the chance constraints using the Markov inequality and propose an efficient algorithm. The EV charging data collected in the province of British Columbia, Canada, is used to evaluate the performance of the proposed algorithm. Simulation results show that the proposed algorithm improves the revenue of the aggregator compared to an existing algorithm from the literature.

I. INTRODUCTION

In recent years, there has been a growing interest in utilizing the idle battery systems of parked electric vehicles (EVs) to provide frequency regulation service. The power grid needs this service to compensate the mismatch between generation and load, and to maintain the utility frequency around a nominal value (e.g., 60 Hertz). EVs can provide efficient frequency regulation service by changing their real-time charging and discharging power rapidly. The economic analyses in [1], [2] show that EVs have the potential to provide revenue to their owners by selling regulation capacity to an independent system operator (ISO), such as the New York ISO (NYISO).

EVs are intrinsically dispersed and each EV has limited regulation capacity. An *aggregator* is typically used to coordinate a large fleet of EVs in order to satisfy the minimum capacity required to enter the wholesale market of an ISO [3]–[10]. As the hourly regulation capacity is the commodity traded in the market, it is necessary to schedule the hourly capacity for each EV in order to improve the revenue. To this end, a dynamic algorithm for scheduling of the hourly regulation capacity of EVs is proposed in [3]. In [4], a framework for the EV frequency regulation service with unidirectional chargers is provided. An algorithm which takes into account the uncertainty of the regulation signal is reported in [5]. In [6], an algorithm for allocating regulation tasks among EVs is proposed. A multi-level architecture for the aggregator is considered in [7]. In [8], the potential of EVs to provide frequency regulation and voltage support services in microgrids is investigated. In [9], [10], algorithms are proposed to schedule the hourly EV frequency regulation capacities. The works in [3]–[10] focus on an aggregator optimizing the operation of the EVs. On the other hand, an aggregator also needs to participate in the market to sell the regulation capacity.

The forward contract and the *day-ahead market* (DAM) are widely used for electricity-related trades. In the United States, the ISOs purchase the regulation capacity using forward contracts in the DAM [11]. The contracts help the ISOs to reduce their financial risk and their exposure to the volatility of the prices [12]. In a forward contract, a service provider (typically a power generator in the current power systems) and an ISO reach an agreement that the generator will provide a certain amount of regulation capacity (i.e., the contract size) at a specified future time. In the current DAM, a power generator first reports its available regulation capacity to the ISO. The capacity is then used to determine the size of the forward contract. In the emerging smart grid, EV aggregators are expected to be new participants in the DAM. In this paper, we are in particular interested in the following question: *How does an EV aggregator decide its contract size in the DAM?*

The optimal contract size of an aggregator depends on two factors. The first factor is the available regulation capacity of the EVs. In [13], a stochastic model for EVs' regulation capacity based on the hourly probability that an EV is connected with the power grid is proposed. On the other hand, the second factor is the revenue of the aggregator. The monetary revenue can be characterized by its expected value and the financial risk [14]–[16]. The risk is the uncertainty that the revenue is lower than the expected value or even becomes negative. For an aggregator, the risk may arise from the possible mismatch between the contract size and the instantaneous capacity available from the EVs. The financial risk can be measured using the *conditional value at risk* (CVaR) [14], which is the expected revenue in the low rewarding cases (i.e., when the revenue is not more than a certain quantile). The CVaR has been widely used to study the financial risk of market participation of different entities, such as power generators [15] and microgrids [16].
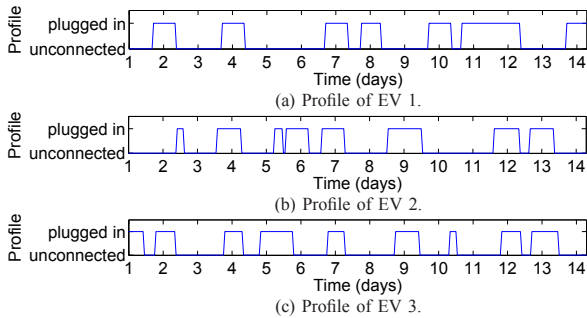
Fig. 1. Sample profiles of EVs' connection with the power grid. The data was collected in the province of British Columbia, Canada in Jan. 2015 [17].

The capacity of an aggregator is made up of the scattered, uncertain, and small-scale regulation capacity of many EVs, which makes the aggregator different from other market participants (e.g., power generators). Hence, the EV aggregator requires novel algorithms for optimization of its forward contract size in order to improve its revenue. To be specific, Fig. 1 shows sample profiles of the EV connection periods with the power grid. As shown in Fig. 1, the EV charging sessions are stochastic and different EVs tend to have different charging periods. Thus, it is necessary to account for the uncertainty of the available capacity of EVs in the forward contract. Moreover, the effect of the financial risk on the forward contract size of an EV aggregator has yet to be explored. The main contributions of this paper are summarized as follows:

- We analytically model an aggregator which participates in the DAM to obtain a forward contract for providing frequency regulation service. We study two cases. In the first case, the contract size is fixed and the aggregator has to ensure that the capacity of the EVs will be sufficient to satisfy the contract. In the second case, an update of the contract size is allowed by the ISO.
- For both cases, we formulate a stochastic optimization problem to determine the size of the forward contract. Risk management is incorporated into the problem formulation using the CVaR. Chance constraints are embedded when the contract size is fixed. The Markov inequality is used to tackle the chance constraints and an efficient algorithm is proposed.
- We evaluate the performance of the proposed algorithm based on real EV charging data, collected in the province of British Columbia, Canada. Simulation results show that the proposed algorithm improves the revenue compared to an existing algorithm from the literature.

The rest of the paper is organized as follows. The system model is introduced in Section II. In Section III, we present the problem formulation and develop a forward contract algorithm. Simulation results are presented in Section IV. Conclusion is given in Section V.

II. System Model

We consider an EV aggregator which provides frequency regulation service. Fig. 2 shows a framework for an aggregator which aggregates the regulation capacity of many EVs and
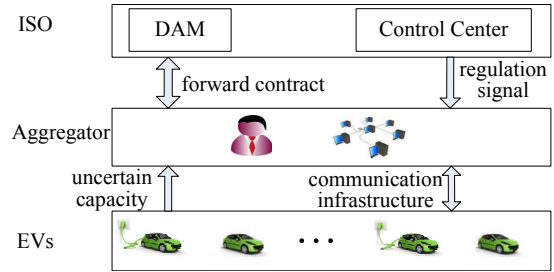

Fig. 2. The aggregator coordinates the EVs to participate in the DAM and provides frequency regulation service to the ISO. In the DAM, there is uncertainty regarding the available regulation capacity on the next day. This is because the EV owners use EVs for driving and may plug in and unplug EV chargers at different time of the day, c.f., Fig. 1.

sells the capacity to an ISO. The aggregator participates in the DAM and obtains a forward contract from the ISO. The contract allows the aggregator to provide frequency regulation service on the next day and ensures that the aggregator will be reimbursed for the service. On the next day, the aggregator coordinates the EVs to provide frequency regulation service by changing the EVs' real-time charging rate according to a regulation signal issued by the ISO.

The concept of a forward contract is widely used in the trading of electricity-related commodities (e.g. energy, frequency regulation service). This is because these commodities are physically difficult to store as inventory. The ISO uses the forward contract to ensure that sufficient supply of regulation capacity is available to satisfy its demand on the next day. According to the contract, the aggregator is responsible for providing a certain amount of hourly regulation capacity in each hour of the next day. In particular, the regulation up capacity and regulation down capacity are the amount by which the EVs' real-time charging power can be decreased and increased, respectively. We use $\mathcal{H} = \{1, \ldots, H\}$ to denote the set of operation hours on the next day, where $H = 24$ as we consider the DAM. Let $v_{fc}^u(h)$ and $v_{fc}^d(h)$ denote the amount of regulation up capacity and regulation down capacity specified in the forward contract for hour $h \in \mathcal{H}$, respectively.

The available capacity of the EVs is *uncertain* while the aggregator is obliged to provide a *certain* amount of capacity according to the forward contract. Let $\mathcal{M} = \{1, \ldots, M\}$ denote the set of EVs. We denote the regulation up capacity and regulation down capacity of EV $i \in \mathcal{M}$ at hour $h \in \mathcal{H}$ by $v_i^u(h)$ and $v_i^d(h)$, respectively. The values of $v_i^u(h)$ and $v_i^d(h)$ are uncertain in the DAM because the periods when the chargers of individual EVs will be plugged-in and unplugged on the next day are unknown. Hence, the available capacity of the EVs is stochastic and can be either higher or lower than the amount specified in the forward contract.

The ISOs have different rules regarding the mismatch between the size of the forward contract and the available capacity. We study two cases for the forward contract and use a binary parameter $\theta \in \{0, 1\}$ to distinguish between them. In the first case, the contract size is fixed and the aggregator needs to ensure that sufficient capacity is provided by the EVs. In this case, we set $\theta = 0$ and have the following constraints

$$\mathbb{P}\Big(v_{fc}^u(h) \le \sum_{i \in \mathcal{M}} v_i^u(h)\Big) \ge \gamma^u(1-\theta), \quad h \in \mathcal{H}, \qquad (1)$$

$$\mathbb{P}\Big(v_{fc}^d(h) \le \sum_{i \in \mathcal{M}} v_i^d(h)\Big) \ge \gamma^d(1-\theta), \quad h \in \mathcal{H}, \qquad (2)$$

where $\mathbb{P}(A)$ denotes the probability of the event A. Parameters $\gamma^u, \gamma^d \in (0,1)$ are the required confidence level with which the capacity from EVs is sufficient compared to the contract size. They take values close to 1 (e.g., $\gamma^u = \gamma^d = 0.95$). We use constraints (1) and (2) to ensure the aggregator makes a forward contract which it is able to fulfill with high probability. Otherwise, if an aggregator fails to provide sufficient capacity and violates its contract repeatedly, the ISO may forbid the aggregator to participate in the market. Note that we have $(1-\theta)$ on the right hand side of (1) and (2) since we need the chance constraints to take effect only when $\theta = 0$.

In the second case, we consider an ISO which allows an update on the size of the forward contract. In particular, *spot markets* are used by the ISOs to ensure that the supply satisfies the demand for the regulation service in real-time. The spot market is a place where the participants trade and deliver frequency regulation capacity in real-time. It is different from the DAM where the capacity is delivered on the next day. Some ISOs may allow the aggregator to update its contract size according to its available capacity in the spot markets. We set $\theta = 1$ in this case. When $\theta = 1$, constraints (1) and (2) are always satisfied because their right hand side become 0. The NYISO is an example for an ISO which allows an update of the contract size [11]. Note that the update of the contract size may incur a charge for the aggregator, which needs to be considered to calculate the revenue of the aggregator.

The revenue of the aggregator has three components. First, the ISO pays for the forward contract according to the prices specified in the contract. The prices in the forward contract are denoted by $p_{fc}^u(h)$ and $p_{fc}^d(h)$ for the regulation up capacity and regulation down capacity at hour $h \in \mathcal{H}$, respectively. Second, the ISO may charge the aggregator if the aggregator updates its contract size in the spot markets. This charge is referred to as the capacity balancing charge by NYISO [11]. Note that this charge can be either positive or negative. If the aggregator reduces the size of the forward contract, it needs to pay a positive charge to the ISO. On the other hand, if the aggregator increases the size of the forward contract, the ISO provides an additional payment to the aggregator according to the prices in the spot markets. Let $p_{st}^u(h)$ and $p_{st}^d(h)$ denote the prices in the spot markets at hour h for the regulation up capacity and the regulation down capacity, respectively. Third, the aggregator may make payments to the EVs to reimburse their regulation capacity. We use $p_{ev}^u(h)$ and $p_{ev}^d(h)$ to denote the prices with which the aggregator pays for the regulation up capacity and regulation down capacity of an EV at hour h, respectively. The values of $p_{ev}^u(h)$ and $p_{ev}^d(h)$ depend on the agreement between the aggregator and the EVs. In this paper, we consider a case when the aggregator and EVs share the revenue from the frequency regulation service. Let r denote the revenue of the aggregator, which is given by

$$\begin{aligned}
r = \sum_{h \in \mathcal{H}} \Big(& p_{fc}^u(h)v_{fc}^u(h) + p_{fc}^d(h)v_{fc}^d(h) \\
& - \theta\Big(p_{st}^u(h)(v_{fc}^u(h) - \sum_{i \in \mathcal{M}} v_i^u(h)) \\
& + p_{st}^d(h)(v_{fc}^d(h) - \sum_{i \in \mathcal{M}} v_i^d(h))\Big) \\
& - \Big(p_{ev}^u(h)\sum_{i \in \mathcal{M}} v_i^u(h) + p_{ev}^d(h)\sum_{i \in \mathcal{M}} v_i^d(h)\Big)\Big). \quad (3)
\end{aligned}$$

The second and third lines in (3) denote the charge for updating the contract size. The charge occurs when an update of the contract size is allowed, i.e., $\theta = 1$ in the second case. The capacities $v_{fc}^u(h)$ and $v_{fc}^d(h)$ are control variables for the aggregator. The prices $(p_{fc}^u(h), p_{fc}^d(h))$ and $(p_{st}^u(h), p_{st}^d(h))$ are announced by the ISO in the DAM and spot markets, respectively. Hence, $p_{fc}^u(h)$, $p_{fc}^d(h)$, $p_{st}^u(h)$, and $p_{st}^d(h)$ are uncertain input parameters for the aggregator. We model the aggregator as a price-taker, i.e., the capacity of the aggregator is relatively small compared to the total capacity traded in the markets and the aggregator cannot affect the prices.

In the DAM, the aggregator determines the size of its forward contract for a certain optimality criterion regarding its revenue. In this paper, we consider two metrics for the revenue, namely the expected revenue and the financial risk, where the risk is measured by the CVaR. In economics, risk aversion is used to model the attitude of market participants who are reluctant to an uncertain revenue with financial risk rather than a steady payoff. Risk aversion is accounted for in our problem formulation.

III. PROBLEM FORMULATION AND ALGORITHM

In this section, we formulate an optimization problem that allows the aggregator to determine the size of the day-ahead forward contract. Our formulation takes into account the uncertainty regarding the available capacity of the EVs and the prices. The CVaR is incorporated into the problem formulation to account for risk management. We consider an aggregator which may have risk aversion. Both the expected revenue and the CVaR are included in the objective function. Let $\alpha \in (0,1)$ denote an arbitrary confidence level. The value at risk (VaR_α) is the quantile for which the probability that the revenue in (3) is larger than or equal to the quantile (i.e., $r \ge \text{VaR}_\alpha$) is α (i.e., $\mathbb{P}(r \ge \text{VaR}_\alpha) = \alpha$). The VaR_α is a metric for the financial risk but the CVaR_α is usually preferred because it accounts for the revenue beyond the confidence level α [14]. In particular, the CVaR_α is defined based on the VaR_α as the expected revenue for the cases when the revenue is not more than VaR_α. That is,

$$\text{CVaR}_\alpha = \mathop{\mathbb{E}}_{r \le \text{VaR}_\alpha}(r), \qquad (4)$$

where $\mathbb{E}$ denotes the expectation with respect to the random variables $p_{fc}^u(h)$, $p_{fc}^d(h)$, $p_{st}^u(h)$, $p_{st}^d(h)$, $\sum_{i \in \mathcal{M}} v_i^u(h)$, and $\sum_{i \in \mathcal{M}} v_i^d(h)$, $h \in \mathcal{H}$. In the DAM, the forward contract is made daily and the contract size is determined for the 24 hours

of the next day. The problem to determine the contract size of an aggregator is formulated as follows

$$\underset{v_{fc}^u(h),\, v_{fc}^d(h),\, h \in \mathcal{H}}{\text{maximize}} \quad \Big((1-\beta)\,\mathbb{E}(r) + \beta\,\text{CVaR}_\alpha \Big) \tag{5a}$$

$$\text{subject to} \quad v_{fc}^u(h),\, v_{fc}^d(h) \geq 0, \quad h \in \mathcal{H}, \tag{5b}$$

$$\text{constraints (1) and (2)}, \tag{5c}$$

where $\beta \in [0,1]$ is a tunable parameter which adjusts the weight of the expected revenue $\mathbb{E}(r)$ and CVaR_α. Increasing β puts more weight on CVaR_α, makes the aggregator more conservative, and improves the revenue in the low rewarding cases. Decreasing β puts more weight on the expected revenue and less weight on the CVaR_α. For $\beta = 0$, the aggregator aims to maximize the expected revenue and neglects the financial risk. This is a special case which can be used to model a risk-neutral aggregator. Without loss of generality, we use β to model risk-averse and risk-neutral aggregators.

Problem (5) has CVaR_α in its objective function. The CVaR_α is typically calculated in a scenario-based manner [14]–[16]. A scenario is a possible realization of random variables $p_{fc}^u(h)$, $p_{fc}^d(h)$, $p_{st}^u(h)$, $p_{st}^d(h)$, $\sum_{i \in \mathcal{M}} v_i^u(h)$, and $\sum_{i \in \mathcal{M}} v_i^d(h)$. As we consider the DAM, the historical prices in the past days from the ISO [18] can be used to generate the values of the prices $p_{fc}^u(h)$, $p_{fc}^d(h)$, $p_{st}^u(h)$, and $p_{st}^d(h)$ in different scenarios. On the other hand, we assume that the EVs report possible values of their available capacity to the aggregator based on historical records of EVs' charging sessions. Algorithms for an EV to determine its available capacity under a given charging session can be found in [3]–[5], [9], [10]. Let $\mathcal{K}$ denote the set of scenarios and $|\mathcal{K}|$ as its cardinality. We use $\boldsymbol{\omega}_k$, $k \in \mathcal{K}$, to denote an arbitrary scenario. $r(\boldsymbol{\omega}_k)$ is the revenue under scenario $\boldsymbol{\omega}_k$. We introduce an auxiliary variable η and rewrite problem (5) as follows [14]

$$\underset{\substack{\eta,\, v_{fc}^u(h),\\ v_{fc}^d(h),\, h \in \mathcal{H}}}{\text{maximize}} \quad \Big((1-\beta)\frac{1}{|\mathcal{K}|}\sum_{k \in \mathcal{K}} r(\boldsymbol{\omega}_k) \\ + \beta\big(\eta - \frac{1}{|\mathcal{K}|(1-\alpha)}\sum_{k \in \mathcal{K}}(\eta - r(\boldsymbol{\omega}_k))^+\big)\Big) \tag{6a}$$

$$\text{subject to} \quad \text{constraints (1), (2), and (5b)}, \tag{6b}$$

where $(x)^+ = \max(x, 0)$. Auxiliary variable η represents the value of VaR_α. Hence, the term $(\eta - r(\boldsymbol{\omega}_k))^+$ is the gap between VaR_α and the revenue under scenario $\boldsymbol{\omega}_k$, when the revenue is lower than VaR_α. Note that the probability for the revenue to be lower than VaR_α is $(1-\alpha)$, according to the definition of VaR_α. The expected number of scenarios with a revenue lower than VaR_α is $|\mathcal{K}|(1-\alpha)$. Thus, the expression $\frac{1}{|\mathcal{K}|(1-\alpha)}\sum_{k \in \mathcal{K}}(\eta - r(\boldsymbol{\omega}_k))^+$ is the expected value of the gap between VaR_α and the revenue, when the revenue is lower than VaR_α. Now recall the definition of CVaR_α in (4), i.e., the expected revenue when revenue is lower than VaR_α. Therefore, the term $\frac{1}{|\mathcal{K}|(1-\alpha)}\sum_{k \in \mathcal{K}}(\eta - r(\boldsymbol{\omega}_k))^+$ is indeed the gap between VaR_α and CVaR_α, i.e., $(\text{VaR}_\alpha - \text{CVaR}_\alpha)$. Hence, $\eta - \frac{1}{|\mathcal{K}|(1-\alpha)}\sum_{k \in \mathcal{K}}(\eta - r(\boldsymbol{\omega}_k))^+$ can be used to replace CVaR_α in (5a).

We introduce auxiliary variables $\phi(\boldsymbol{\omega}_k)$ to represent the

value of $(\eta - r(\boldsymbol{\omega}_k))^+$. Problem (6) can be rewritten as

$$\underset{\substack{\eta,\, \phi(\boldsymbol{\omega}_k),\, k \in \mathcal{K},\\ v_{fc}^u(h),\, v_{fc}^d(h),\\ h \in \mathcal{H}}}{\text{maximize}} \quad \Big((1-\beta)\frac{1}{|\mathcal{K}|}\sum_{k \in \mathcal{K}} r(\boldsymbol{\omega}_k) \\ + \beta\big(\eta - \frac{1}{|\mathcal{K}|(1-\alpha)}\sum_{k \in \mathcal{K}}\phi(\boldsymbol{\omega}_k)\big)\Big) \tag{7a}$$

$$\text{subject to} \quad \phi(\boldsymbol{\omega}_k) \geq \eta - r(\boldsymbol{\omega}_k), \quad k \in \mathcal{K}, \tag{7b}$$

$$\phi(\boldsymbol{\omega}_k) \geq 0, \qquad k \in \mathcal{K}, \tag{7c}$$

$$\text{constraints (1), (2), and (5b)}. \tag{7d}$$

Constraints (7b) and (7c) ensure that $\phi(\boldsymbol{\omega}_k) \geq (\eta - r(\boldsymbol{\omega}_k))^+$. As the objective function (7a) is decreasing with respect to $\phi(\boldsymbol{\omega}_k)$, the optimal solution of problem (7) is obtained only if either $\phi(\boldsymbol{\omega}_k) = \eta - r(\boldsymbol{\omega}_k)$ or $\phi(\boldsymbol{\omega}_k) = 0$. Hence, constraints (7b) and (7c) can be used to ensure $\phi(\boldsymbol{\omega}_k) = (\eta - r(\boldsymbol{\omega}_k))^+$ in problem (7). We study two cases for problem (7) and its chance constraints (1) and (2). First, when $\theta = 1$, then the right hand side of constraints (1) and (2) become 0 and the constraints are always satisfied. However, when $\theta = 0$, i.e., the contract size is fixed, chance constraints (1) and (2) make problem (7) difficult to solve. The probabilities in constraints (1) and (2) are not amenable to an efficient solution and would require an exhaustive search, and as a result, incur a high computational complexity. In order to reduce the computational complexity, we use the Markov inequality and convex approximation [19] to replace the chance constraints. In particular, a chance constraint is replaced by a convex constraint such that if the convex constraint is satisfied, then the chance constraint is also satisfied. We introduce auxiliary positive variables $\zeta^u(h)$ and $\zeta^d(h), h \in \mathcal{H}$. Then, we obtain the following problem:

$$\underset{\substack{\eta,\, \phi(\boldsymbol{\omega}_k),\, k \in \mathcal{K},\\ v_{fc}^u(h),\, v_{fc}^d(h),\\ h \in \mathcal{H}}}{\text{maximize}} \quad \Big((1-\beta)\frac{1}{|\mathcal{K}|}\sum_{k \in \mathcal{K}} r(\boldsymbol{\omega}_k) \\ + \beta\big(\eta - \frac{1}{|\mathcal{K}|(1-\alpha)}\sum_{k \in \mathcal{K}}\phi(\boldsymbol{\omega}_k)\big)\Big) \tag{8a}$$

$$\text{subject to} \quad \inf_{\zeta^u(h)>0}\Big(\mathbb{E}\big[\big(\zeta^u(h)+v_{fc}^u(h)-\sum_{i \in \mathcal{M}}v_i^u(h)\big)^+\big] \\ - \big(1-\gamma^u(1-\theta)\big)\zeta^u(h)\Big) \leq 0, \quad h \in \mathcal{H}, \tag{8b}$$

$$\inf_{\zeta^d(h)>0}\Big(\mathbb{E}\big[\big(\zeta^d(h)+v_{fc}^d(h)-\sum_{i \in \mathcal{M}}v_i^d(h)\big)^+\big] \\ - \big(1-\gamma^d(1-\theta)\big)\zeta^d(h)\Big) \leq 0, \quad h \in \mathcal{H}, \tag{8c}$$

$$\text{constraints (5b), (7b), and (7c)}. \tag{8d}$$

In problem (8), constraints (8b) and (8c) are introduced to replace the chance constraints (1) and (2), respectively.

Proposition 1: If an aggregator selects $v_{fc}^u(h)$ and $v_{fc}^d(h)$ for hour $h \in \mathcal{H}$ such that constraints (8b) and (8c) are satisfied, then chance constraints (1) and (2) are also satisfied.

Proof: Given constraint (8b) is satisfied, we have the following inequality

$$\mathbb{P}\Big(v_{fc}^u(h) \leq \sum_{i \in \mathcal{M}}v_i^u(h)\Big)$$

$$= 1 - \mathbb{P}\Big(\zeta^u(h) + v_{fc}^u(h) - \sum_{i \in \mathcal{M}} v_i^u(h) > \zeta^u(h)\Big)$$

$$\geq 1 - \mathbb{P}\Big(\big(\zeta^u(h) + v_{fc}^u(h) - \sum_{i \in \mathcal{M}} v_i^u(h)\big)^+ > \zeta^u(h)\Big)$$

$$\overset{(a)}{\geq} 1 - \frac{\mathbb{E}\Big[\big(\zeta^u(h) + v_{fc}^u(h) - \sum_{i \in \mathcal{M}} v_i^u(h)\big)^+\Big]}{\zeta^u(h)}$$

$$\overset{(b)}{\geq} 1 - (1 - \gamma^u(1 - \theta)) = \gamma^u(1 - \theta), \tag{9}$$

where inequality (a) is due to the Markov inequality. Inequality (b) is obtained from constraint (8b). From (9), if constraint (8b) is satisfied, then constraint (1) is satisfied. Similarly, by replacing $v_{fc}^u(h)$, $v_i^u(h)$, and γ^u with $v_{fc}^d(h)$, $v_i^d(h)$, and γ^d in (9) and repeating the above steps, we can show that constraint (2) is satisfied when (8c) is satisfied. ∎

As shown in Proposition 1, the solution obtained by solving problem (8) is in the feasible set of problem (7). In other words, the objective value of problem (8) is a lower bound for the optimal value of problem (7). The gap between the lower bound and the optimal value is studied via simulations. We found that the gap is less than 4% of the revenue in our simulation results. Regarding problem (8), tackling constraints (8b) and (8c) directly may be challenging. Hence, we introduce auxiliary variables $\psi^u(h, \boldsymbol{\omega}_k)$ and $\psi^d(h, \boldsymbol{\omega}_k), h \in \mathcal{H}, k \in \mathcal{K}$, and rewrite problem (8) as

$$\underset{\substack{\zeta^u(h), \zeta^d(h) \\ \eta, \psi^u(h, \boldsymbol{\omega}_k), \\ \psi^d(h, \boldsymbol{\omega}_k), \phi(\boldsymbol{\omega}_k), \\ v_{fc}^u(h), v_{fc}^d(h), \\ k \in \mathcal{K}, h \in \mathcal{H}}}{\text{maximize}} \quad \Big((1 - \beta) \frac{1}{|\mathcal{K}|} \sum_{k \in \mathcal{K}} r(\boldsymbol{\omega}_k) \\ + \beta \big(\eta - \frac{1}{|\mathcal{K}|(1 - \alpha)} \sum_{k \in \mathcal{K}} \phi(\boldsymbol{\omega}_k)\big)\Big) \tag{10a}$$

$$\text{subject to} \quad \frac{1}{|\mathcal{K}|} \sum_{k \in \mathcal{K}} \psi^u(h, \boldsymbol{\omega}_k) - (1 - \gamma^u(1 - \theta))\zeta^u(h) \leq 0,$$
$$h \in \mathcal{H}, \tag{10b}$$

$$\frac{1}{|\mathcal{K}|} \sum_{k \in \mathcal{K}} \psi^d(h, \boldsymbol{\omega}_k) - (1 - \gamma^d(1 - \theta))\zeta^d(h) \leq 0,$$
$$h \in \mathcal{H}, \tag{10c}$$

$$\psi^u(h, \boldsymbol{\omega}_k) \geq \zeta^u(h) + v_{fc}^u(h) - \sum_{i \in \mathcal{M}} v_i^u(h, \boldsymbol{\omega}_k),$$
$$h \in \mathcal{H}, \quad k \in \mathcal{K}, \tag{10d}$$

$$\psi^d(h, \boldsymbol{\omega}_k) \geq \zeta^d(h) + v_{fc}^d(h) - \sum_{i \in \mathcal{M}} v_i^d(h, \boldsymbol{\omega}_k),$$
$$h \in \mathcal{H}, \quad k \in \mathcal{K}, \tag{10e}$$

$$\psi^u(h, \boldsymbol{\omega}_k), \psi^d(h, \boldsymbol{\omega}_k) \geq 0, \quad h \in \mathcal{H}, \quad k \in \mathcal{K}, \tag{10f}$$

$$\zeta^u(h), \zeta^d(h) > 0, \quad h \in \mathcal{H}, \tag{10g}$$

$$\text{constraints (5b), (7b), and (7c),} \tag{10h}$$

where $\sum_{i \in \mathcal{M}} v_i^u(h, \boldsymbol{\omega}_k)$ and $\sum_{i \in \mathcal{M}} v_i^d(h, \boldsymbol{\omega}_k)$ are the aggregate regulation up capacity and regulation down capacity in hour h under scenario $\boldsymbol{\omega}_k$, respectively. Problem (10) is a linear program which can be solved efficiently. The algorithm executed by the aggregator to determine the contract size in the DAM is presented in Algorithm 1.

Algorithm 1 Forward contract algorithm executed by the aggregator to participate in the DAM

1: Initialize θ, γ^u, γ^d, α, β, $\mathcal{H}$, $\mathcal{M}$, $\mathcal{K}$
2: Construct scenarios $\boldsymbol{\omega}_k, k \in \mathcal{K}$, based on historical prices and EV charging data
3: **if** $\theta = 1$ **then**
4:     Solve problem (7) to obtain $v_{fc}^u(h)$ and $v_{fc}^d(h)$, $h \in \mathcal{H}$
5: **else**
6:     Solve problem (10) to obtain $v_{fc}^u(h)$ and $v_{fc}^d(h)$, $h \in \mathcal{H}$
7: **end if**
8: Submit $v_{fc}^u(h)$ and $v_{fc}^d(h)$ to the ISO in the DAM

IV. PERFORMANCE EVALUATION

We evaluate the performance of the proposed algorithm with data collected in the province of British Columbia, Canada [17]. The data was collected by smart chargers installed in the province which are capable to send the records of EV charging sessions to a database via the cellular network. There were 2026 records available for our study. Each record contains several tags including the EV charger time of plug in, time of unplug, and the amount of charged energy. These records are used in our simulation study to mimic the charging sessions of the EVs. We consider a fleet of 1000 EVs. Each EV has a battery capacity of 24 kWh [20]. The maximum charging rate is assumed to be 6.2 kW, which is the typical charging rate of the smart chargers in the province of British Columbia. We use the historical prices in Jan. 2015 from NYISO [18]. 100 scenarios are generated for our simulation. In this case, problem (10) has 5097 variables and 7496 constraints and is solved in 10.35 seconds with a desktop computer which has a quad-core CPU and 16 GB memory.

We compare with a benchmark algorithm from [13], which is referred to as the CPC (contract power capacity) algorithm. We compare with the CPC algorithm because both our proposed algorithm and the CPC algorithm aim to determine the contract size. The CPC algorithm calculates the contract size based on the hourly probabilities that EVs are connected with the power grid. Unless stated otherwise, we set $\alpha = 0.8$, $\beta = 0.2$, and $\gamma^u = \gamma^d = 0.95$ for the proposed algorithm. We study both cases when $\theta = 0$ and $\theta = 1$. The CPC algorithm obtains a contract size using a cumulative density function when $\theta = 0$. We extend the CPC algorithm to the case when $\theta = 1$ and assume it obtains a contract size with the expected capacity. Fig. 3 shows the revenue as a function of the maximum charging rate for the proposed algorithm and the CPC algorithm. As shown in Fig. 3, our proposed algorithm achieves a higher revenue compared to the benchmark CPC algorithm. In particular, when EV chargers have 6 kW maximum charging rate and $\theta = 1$, the expected daily revenue r is increased from \$141 to \$179. Note that the revenue is calculated according to (3) and we assume $p_{ev}^u(h) = 0.8\mathbb{E}(p_{fc}^u(h))$ and $p_{ev}^d(h) = 0.8\mathbb{E}(p_{fc}^d(h))$. The proposed algorithm outperforms the CPC algorithm for two reasons. First, the proposed algorithm takes into account the
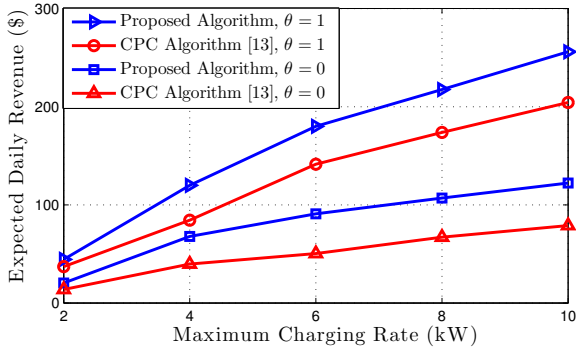
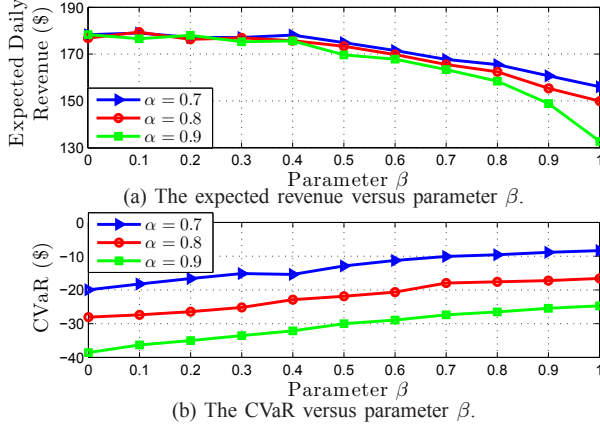Fig. 3. Expected revenue versus maximum charging rate of EV chargers.

(a) The expected revenue versus parameter β.

(b) The CVaR versus parameter β.

Fig. 4. The tradeoff between the expected revenue and the CVaR tuned by parameter β.

market rules of the forward contract, especially the update of the contract size in the spot market. Second, the charging demand and historical charging data are used to model the capacity of the EVs in the proposed algorithm. In contrast, the CPC algorithm only considers the hourly probability that EVs are connected with the power grid.

We also analyze the impact of parameter β on the performance of the proposed algorithm. β is an important design parameter tuned by the aggregator. If the aggregator is risk neutral, it can set $\beta = 0$ to improve its expected revenue. On the other hand, if the aggregator has risk aversion and intends to reduce its financial risk, β can be increased. Fig. 4 shows the expected revenue and the CVaR versus parameter β for $\theta = 1$. As β increases, the expected revenue decreases while the CVaR increases, i.e., the financial risk is reduced. In Fig. 4(b), the CVaR decreases as α increases because the CVaR is the expected revenue in the $(1 - \alpha)$ lowest rewarding cases.

V. CONCLUSION

In this paper, we studied the day-ahead forward contract for the frequency regulation service between an aggregator and an ISO. The market rules of the ISOs were considered and two cases were analyzed, namely the case when the contract size was fixed in the DAM, and the case when an update on the contract size in the spot market was allowed by the ISO. We formulated a stochastic optimization problem to determine the optimal contract size. Risk management was taken into account via the CVaR. Chance constraints were embedded in

the problem formulation when the contract size was fixed. We tackled the chance constraints using the Markov inequality and an efficient algorithm was proposed. Simulation results showed that the proposed algorithm enables the aggregator to achieve a higher revenue compared to an existing algorithm from the literature. For future work, an interesting extension is to jointly consider the financial forward contract and the physical constraints of the EVs aggregate charging power.

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