A Robust Design of Electric Vehicle Frequency Regulation Service

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Abstract—Electric vehicles (EVs) have the potential to provide frequency regulation service to an independent system operator (ISO) by changing their real-time charging or discharging power according to an automatic generation control (AGC) signal. Recently, the Federal Energy Regulatory Commission has issued an order to ISOs to introduce a *performance-based* compensation paradigm. In this new compensation scheme, the ISOs make payments to the EVs for providing the frequency regulation capacity and the real-time dispatch when following the AGC signal. In this paper, we propose a robust EV frequency regulation (REF) algorithm to determine the hourly regulation capacity for each EV considering the randomness of the AGC signal. The proposed REF algorithm is based on a robust optimization problem formulation. It enables EVs to follow the random AGC signal reliably and to improve the EV frequency regulation revenue. Simulation results show that the proposed REF algorithm obtains a higher revenue compared with a benchmark algorithm from the literature under a performance-based compensation paradigm.

I. INTRODUCTION

Electric vehicles (EVs) are among the potential candidates to replace combustion engine vehicles for reducing the emission of CO_2 and other greenhouse gases. When EVs are connected with the grid, they can be regarded as energy storage systems and can be coordinated to provide *frequency regulation* service to independent system operators (ISOs), such as the California ISO (CAISO). Frequency regulation service helps ISOs keep the utility frequency within an acceptable range by maintaining the instantaneous balance between generation and load. The pilot projects in [1], [2] show that EVs are able to provide frequency regulation service by following an *automatic generation control* (AGC) signal issued by the ISO. Fast ramping resources such as EVs can reduce the overall frequency regulation capacity requirement for ISOs and lead to lower costs for ISOs and consumers [3].

The market-based EV frequency regulation service has received significant attention [4]–[8]. The ISO purchases the hourly frequency regulation capacity from EVs on the market and the participating EVs are obliged to follow the AGC signal and provide hourly regulation capacity. An *aggregator* is typically used to serve as an agent between the ISO and the fleet of EVs. The works in [4]–[6] propose algorithms for the aggregator to distribute regulation tasks among EVs in realtime. An EV frequency regulation algorithm is proposed in [7] for the aggregator to determine the charging schedule of each EV and the hourly regulation capacity. A unidirectional frequency regulation algorithm is proposed in [8] for EVs to provide frequency regulation service by varying the charging power around a set point. The works in [7], [8] consider capacity-based frequency regulation compensation, where the revenue only depends on the frequency regulation capacity.

The Federal Energy regulatory Commission (FERC) issued Order 755 [9] in Oct. 2011, which requires ISOs to introduce performance-based compensation for the frequency regulation service. The ISOs are required to make payments for frequency regulation capacity and frequency regulation performance. Performance is evaluated based on the real-time dispatch (i.e., changing charging or discharging power around a baseline) when EVs follow the AGC signal. Performance-based frequency regulation compensation has been implemented by some ISOs, e.g., the Pennsylvania Jersey Maryland Interconnection (PJM). Considering the trend towards performancebased compensation, it is desirable that the EVs follow the random AGC signal reliably to improve their revenue.

Each EV has limited battery capacity. An EV cannot follow the AGC signal to charge if it is fully charged and it cannot discharge if its battery is depleted. The challenge of using EVs to provide reliable frequency regulation service has caught the attention in both the academia [10], [11] and ISOs [2]. The works in [2], [10], [11] focus on designing a special AGC signal for which the regulation up and regulation down components are almost identical for each hour. As stated in [11], the AGC signal approach is only applicable for ISOs which request EVs to have the same amount for regulation up and regulation down capacity, such as PJM. However, this approach is difficult to apply for ISOs which have separate regulation up and regulation down markets, such as CAISO and Electric Reliability Council of Texas (ERCOT). In those markets, the EVs may have different regulation up and regulation down capacity and the AGC signal approach cannot be applied. On the other hand, it is desirable to provide reliable EV frequency regulation service in order to improve the performance-based frequency regulation revenue.

In this paper, we tackle the above issue and propose a robust design of the EV frequency regulation service by taking into account the randomness of the AGC signal. The contributions of this paper are as follows:

- We propose a *robust EV frequency regulation* (REF) algorithm to improve the revenue of EVs for performancebased compensation.
- We use robust optimization approach to formulate an EV

frequency regulation problem. We transform the formulated combinatorial optimization problem into a linear program based on duality.

• We evaluate the performance of the proposed REF algorithm with a historical AGC signal from PJM. A comparison with the benchmark OptMaxReg algorithm in [8] shows that the proposed REF algorithm improves the performance-based frequency regulation revenue.

This paper is organized as follows. We introduce the system model in Section II. We present the problem formulation and the REF algorithm in Section III. Numerical results are presented in Section IV. Conclusions are given in Section V.

II. SYSTEM MODEL

The considered EV frequency regulation scheme is illustrated in Fig. 1. A two-way communication system is implemented to enable information exchange between the ISO, aggregator, and EVs. The aggregator coordinates the EVs to provide frequency regulation service to the ISO. First, the aggregator aggregates the hourly frequency regulation capacity of the EVs. The ISO purchases the hourly capacity and the aggregator enters into a contract with the ISO to provide frequency regulation service. During operation, the aggregator retrieves the AGC signal from the ISO every few seconds (e.g., 2-6 seconds, depending on the ISO's requirement) and broadcasts the AGC signal to EVs. The EVs are obliged to provide frequency regulation service by changing their realtime charging or discharging power based on the AGC signal.

We denote the operation hours by $\mathcal{H} = \{1, \ldots, H\}$. The set of EVs is denoted by $\mathcal{N} = \{1, \ldots, N\}$. Each EV $i \in \mathcal{N}$ has a baseline charging power $x_i(h)$, regulation up capacity $v_i^u(h)$, and regulation down capacity $v_i^d(h)$ for each hour $h \in \mathcal{H}$. The aggregator determines the values of $x_i(h)$, $v_i^u(h)$, and $v_i^d(h)$ to maximize the frequency regulation revenue.

The AGC signal is generated by the ISO according to the real-time unbalance between generation and load in the grid. The AGC signal is updated in short intervals. We divide one hour into multiple time slots. Each time slot corresponds to the duration of one interval of the AGC signal, e.g., one time slot lasts 4 seconds for PJM. Let $\mathcal{T} = \{1, \ldots, T\}$ denote the set of time slots in one hour, i.e., the duration of one time slot of the AGC signal is $\frac{1}{T}$. The AGC signal in time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$ is denoted by $q(h, t) \in [-1, 1]$.

The AGC signal indicates the amount by which EV $i \in \mathcal{N}$ should increase or decrease its charging power, compared to the baseline charging power $x_i(h)$. A negative AGC signal (i.e., q(h,t) < 0) indicates the power generation in the grid is lower than the load. EV *i* provides regulation up by multiplying the AGC signal with its regulation up capacity $v_i^u(h)$ and decreasing the charging power accordingly. A positive AGC signal (i.e., q(h,t) > 0) indicates the generation is higher than the load in the grid. EV *i* provides regulation down by multiplying the AGC signal with its regulation down capacity $v_i^d(h)$ and increasing the charging power accordingly. Note that for chargers that comply with the Society of Automotive Engineers (SAE) J1772 standard [12], EVs are able to change



Fig. 1. The block diagram for EVs to provide frequency regulation service by following an AGC signal from the ISO. The AGC signal is generated by the ISO in real-time and is random.

their charging power by adjusting the pilot signal duty cycle [4]. We analyze the AGC signal on an hourly basis as ISOs typically purchase the hourly capacity. We denote the hourly regulation up component and the hourly regulation down component of the AGC signal within hour h by $f^u(h)$ and $f^d(h)$, respectively. We have

$$f^{u}(h) = \frac{1}{T} \sum_{t \in \mathcal{T}} \left[-q(h,t) \right]^{+},$$
 (1)

$$f^{d}(h) = \frac{1}{T} \sum_{t \in \mathcal{T}} [q(h,t)]^{+},$$
 (2)

where $[x]^+ = \max\{x, 0\}$. Let $e_i(h)$ denote the charged energy for EV *i* within hour *h*. Then, for EV $i \in \mathcal{N}$, hour $h \in \mathcal{H}$, $e_i(h)$ can be represented as

$$e_{i}(h) = \frac{1}{T} \sum_{t \in \mathcal{T}} \left(x_{i}(h) - v_{i}^{u}(h) \left[-q(h,t) \right]^{+} + v_{i}^{d}(h) \left[q(h,t) \right]^{+} \right)$$

= $x_{i}(h) - v_{i}^{u}(h) f^{u}(h) + v_{i}^{d}(h) f^{d}(h).$ (3)

The terms $v_i^d(h)f^d(h)$ and $v_i^u(h)f^u(h)$ represent the charged and discharged energy due to following AGC signal during hour *h*, respectively. Note that we assume EV *i* needs to follow the AGC signal in every time slot because EV is a fast ramping resource and it has a zero lost opportunity cost [10], [13].

The performance-based compensation paradigm comprises payments of two parts. The first part of payment is the reward for providing the regulation up and regulation down capacity. The ISO pays $p^u(h)$ to each EV for providing regulation up capacity and price $p^d(h)$ for providing regulation down capacity at hour h. The second part of payment is performance related. The performance is typically evaluated based on the real-time dispatch when EVs follow the AGC signal. We denote the performance price (e.g., the regulation market performance clearing price (RMPCP) in PJM) at hour h as $p^p(h)$. Let $m^u(h)$ and $m^d(h)$ denote the summation of absolute changes of the regulation up and regulation down components of the AGC signal, respectively. We have

$$m^{u}(h) = \sum_{t \in \mathcal{T}} \left| \left[-q(h,t) \right]^{+} - \left[-q(h,t-1) \right]^{+} \right|, \ h \in \mathcal{H}, \ (4)$$

$$m^{d}(h) = \sum_{t \in \mathcal{T}} \left| \left[q(h,t) \right]^{+} - \left[q(h,t-1) \right]^{+} \right|, \ h \in \mathcal{H},$$
 (5)

where |x| denotes the absolute value of x. The payment for the performance at hour h is based on the performance price and

the summation of absolute changes of the real time charging or discharging power [13]. We assume the communication links between the ISO, aggregator, and EVs are reliable. If EV i follows the AGC signal at hour h, it receives a payment based on the performance as follows

$$p^{p}(h) \sum_{t \in \mathcal{T}} \left| \left(x_{i}(h) - v_{i}^{u}(h) \left[-q(h,t) \right]^{+} + v_{i}^{d}(h) \left[q(h,t) \right]^{+} \right) - \left(x_{i}(h) - v_{i}^{u}(h) \left[-q(h,t-1) \right]^{+} + v_{i}^{d}(h) \left[q(h,t-1) \right]^{+} \right) \right|$$

$$= p^{p}(h) \sum_{t \in \mathcal{T}} \left(v_{i}^{u}(h) \Big| \left[-q(h,t) \right]^{+} - \left[-q(h,t-1) \right]^{+} \Big| \right)$$

$$+ v_{i}^{d}(h) \Big| \left[q(h,t) \right]^{+} - \left[q(h,t-1) \right]^{+} \Big| \right)$$

$$= p^{p}(h) \left(v_{i}^{u}(h) m^{u}(h) + v_{i}^{d}(h) m^{d}(h) \right).$$
(6)

EVs have to pay for charging energy from the grid. We denote the price for energy charging at hour h by $p^e(h)$. Note that ISOs typically calculate revenue on an hourly basis. Let $r_i(v_i^u(h), v_i^d(h), x_i(h))$ represent the revenue for EV i at hour h. It can be written as

$$r_{i}(v_{i}^{u}(h), v_{i}^{d}(h), x_{i}(h)) = p^{u}(h)v_{i}^{u}(h) + p^{d}(h)v_{i}^{d}(h) + p^{p}(h)\left(v_{i}^{u}(h)m^{u}(h) + v_{i}^{d}(h)m^{d}(h)\right)\mathbf{1}_{i,h} - p^{e}(h)\left(x_{i}(h) - v_{i}^{u}(h)f^{u}(h) + v_{i}^{d}(h)f^{d}(h)\right).$$
(7)

The first term on the right hand side of (7) is the payment for capacity, whereas the second term is the payment for performance, and the third term is the cost for charging energy. $\mathbf{1}_{i,h}$ is an indicator function which is equal to 1 when EV *i* follows the AGC signal in hour *h*, and is equal to 0 otherwise.

The limited battery capacity is a challenge for EVs to follow the AGC signal reliably. EVs cannot follow the AGC signal to charge when it is fully charged and it cannot discharge when its battery is depleted. EVs may fail to follow the AGC signal occasionally because the randomness of the AGC signal regulation up and regulation down components ($f^u(h)$ and $f^d(h)$) can lead to an uncertain charged or discharged energy, see (3). We studied the statistical joint distribution of $f^u(h)$ and $f^d(h)$, i.e., $\mathbb{P}(f^u(h), f^d(h))$, by analyzing the AGC signal data from PJM [14], for the period from January 1, 2012 to March 31, 2012. The results in Fig. 2 show that $f^u(h)$ and $f^d(h)$ may deviate significantly from their expected values. In the next section, we use robust optimization to determine the hourly regulation capacity which enables EVs to follow the AGC signal reliably (i.e., $\mathbf{1}_{i,h} = 1$).

III. ROBUST EV FREQUENCY REGULATION

In this section, a robust EV frequency regulation algorithm is proposed. The algorithm aims to obtain a *reliable solution* of $v_i^u(h)$, $v_i^d(h)$, and $x_i(h)$ to enable EVs to follow the AGC signal reliably. In particular, we use the robust optimization approach [15], [16] to analyze the cases when the unknown parameters $f^u(h)$ and $f^d(h)$ change adversely (i.e., take values such that EVs tend to miss the AGC signal) within $0 \le f^u(h) \le \zeta^u, 0 \le f^d(h) \le \zeta^d$. Constants $\zeta^u, \zeta^d \in [0, 1]$ represent the maximum value of the AGC signal regulation up



Fig. 2. Joint distribution of the hourly regulation up component and the hourly regulation down component of an AGC signal. The distribution is obtained by analyzing the AGC signal data for 2,160 hours from [14]. There are two types of AGC signals in [14] and this figure is obtained by analyzing the RegA signal. The figure shows the randomness of the regulation up component and regulation down component. Note the distribution can be different for other AGC signals. Our approach is applicable to any distribution.

component and regulation down component, respectively. ζ^{u} and ζ^{d} can be selected based on the historical AGC data.

An integer parameter $\eta \in \{0, 1, \dots, H\}$ is introduced to adjust the level of robustness. We aim to find a solution which enables EVs to follow AGC signal reliably when the unknown parameters $f^{u}(h)$ and $f^{d}(h)$ change adversely in at most η hours and take expected values in other hours. Note that the worst case of the unknown parameters (e.g., $f^{u}(h) = \zeta^{u}, f^{d}(h) = 0$ does happen for certain hours and our approach has no compromise on the range in which the unknown parameters change. However, it is unlikely that the AGC signal will have the worst case consecutively in all operation hours. If we set $\eta = H$, we are considering the case where the AGC signal changes adversely in all operation hours, which may lead to a conservative solution. On the other hand, if we set $\eta = 0$, the randomness of the AGC signal is ignored and the solution is unreliable. Therefore, parameter η is introduced to enable EVs follow the AGC signal reliably without making the solution conservative.

Let τ represent an arbitrary hour in set $\{1, \ldots, h\}$. We use S to denote the set for the hours when the unknown parameters $f^u(\tau)$ and $f^d(\tau)$ change adversely. The cardinality of set S is denoted by |S|. The expected value of $f^u(\tau)$ and $f^d(\tau)$ are denoted by μ^u and μ^d , respectively. We consider two cases for the uncertain parameters $f^u(\tau)$ and $f^d(\tau)$. In the first case, $f^u(\tau)$ and $f^d(\tau)$ change adversely ($f^u(\tau) = 0, f^d(\tau) = \zeta^d$) to *increase* the *state of charge* (SOC) of EVs at the end of hour h. Let $s_i(0)$ denote the initial SOC of EV i at the beginning of operation hours. We denote the battery capacity of EV i as B_i . We assume the charging efficiency for both charger and battery is close to one. We have the following constraint

$$s_{i}(0) + \max_{\{\mathcal{S} \subseteq \{1,...,h\} \mid |\mathcal{S}| \le \eta\}} \frac{1}{B_{i}} \sum_{\tau \in \mathcal{S}} \left(x_{i}(\tau) - v_{i}^{u}(\tau)0 + v_{i}^{d}(\tau)\zeta^{d} \right) \\ + \frac{1}{B_{i}} \sum_{\tau \in \{1,...,h\} \setminus \mathcal{S}} \left(x_{i}(\tau) - v_{i}^{u}(\tau)\mu^{u} + v_{i}^{d}(\tau)\mu^{d} \right) \le 1,$$
(8)

where $\{1, \ldots, h\} \setminus S$ represents the relative complement of set S with respect to set $\{1, \ldots, h\}$, i.e., the hours when the unknown parameters $f^u(\tau)$ and $f^d(\tau)$ take expected values. The operator $\max_{\{S \subseteq \{1, \ldots, h\} \mid |S| \le \eta\}}$ will select at most η

hours from set $\{1, \ldots, h\}$. In the selected hours $\tau \in S$, the unknown parameters $f^u(\tau)$ and $f^d(\tau)$ change adversely (i.e., $f^u(\tau) = 0, f^d(\tau) = \zeta^d$) to increase the SOC of EV *i*, see the first term following the max in (8). In other hours (i.e., $\tau \in \{1, \ldots, h\} \setminus S$), the unknown parameters $f^u(\tau)$ and $f^d(\tau)$ take the expected values, see the last term on the left hand side of (8). Constraint (8) can be equivalently written as

$$s_{i}(0) + \frac{1}{B_{i}} \sum_{\tau=1}^{u} \left(x_{i}(\tau) - v_{i}^{u}(\tau)\mu^{u} + v_{i}^{d}(\tau)\mu^{d} \right)$$

$$- \frac{1}{B_{i}} \min_{\left\{ S \subseteq \{1,...,h\} \mid |S| \le \eta \right\}} \sum_{\tau \in S} \left(-v_{i}^{u}(\tau)\mu^{u} \right)$$

$$+ v_{i}^{d}(\tau)(\mu^{d} - \zeta^{d}) \le 1, \quad i \in \mathcal{N}, \ h \in \mathcal{H}.$$

$$(9)$$

Eq. (9) is obtained by adding component $\frac{1}{B_i} \sum_{\tau \in S} (x_i(\tau) - v_i^u(\tau)\mu^u + v_i^d(\tau)\mu^d)$ on the last term of left hand side in (8) and removing the same component on the first term after max.

In the second case, the unknown parameters change adversely (i.e., $f^u(\tau) = \zeta^u$, $f^d(\tau) = 0$) to *reduce* the SOC of EV. The following constraint to keeps the SOC to be above zero

$$s_{i}(0) + \frac{1}{B_{i}} \sum_{\tau=1}^{n} \left(x_{i}(\tau) - v_{i}^{u}(\tau)\mu^{u} + v_{i}^{d}(\tau)\mu^{d} \right)$$

+
$$\frac{1}{B_{i}} \min_{\left\{ S \subseteq \{1,...,h\} \mid |S| \le \eta \right\}} \sum_{\tau \in S} \left(v_{i}^{u}(\tau)(\mu^{u} - \zeta^{u}) - v_{i}^{d}(\tau)\mu^{d} \right) \ge 0, \quad i \in \mathcal{N}, \ h \in \mathcal{H}.$$
(10)

Constraints (9) and (10) confine the SOC of EV *i* to be within [0, 1] at the end of hour *h*. Note the SOC within hour *h* is bounded by the SOC at the end of hour in the considered cases when the unknown parameters change adversely. Hence, in this paper, we use constraints (9) and (10) to enable EVs follow AGC signal reliably (i.e., $\mathbf{1}_{i,h} = 1$). We denote the expected value of $m^u(\tau)$ and $m^d(\tau)$ as λ^u and λ^d , respectively. We formulate a robust EV frequency regulation problem as follows

$$\underset{i \in \mathcal{N}, h \in \mathcal{H}}{\text{maximize}} \sum_{\substack{h \in \mathcal{H} \\ i \in \mathcal{N}, h \in \mathcal{H}}} \sum_{\substack{h \in \mathcal{H} \\ i \in \mathcal{N}}} \sum_{\substack{i \in \mathcal{N} \\ i \in \mathcal{N}, h \in \mathcal{H}}} \sum_{\substack{h \in \mathcal{H} \\ i \in \mathcal{N}}} \sum_{\substack{i \in \mathcal{N} \\ i \in \mathcal{N}, h \in \mathcal{H}}} \sum_{\substack{h \in \mathcal{H} \\ i \in \mathcal{N}}} \sum_{\substack{i \in \mathcal{N} \\ i \in \mathcal{N}, h \in \mathcal{H}}} \sum_{\substack{h \in \mathcal{H} \\ i \in \mathcal{N}, h \in \mathcal{H}, h \in$$

subject to
$$x_i(h) + v_i^d(h) \le E_i^{\max}, \ i \in \mathcal{N}, \ h \in \mathcal{H},$$
 (11b)

$$v_i(h) - v_i^u(h) \ge E_i^{\min}, \ i \in \mathcal{N}, \ h \in \mathcal{H},$$
 (11c)

$$v_i^u(h), v_i^d(h) \ge 0, \quad i \in \mathcal{N}, \ h \in \mathcal{H},$$
(11d)

$$s_{i}(0) + \frac{1}{B_{i}} \sum_{\tau=1}^{d_{i}-1} \left(x_{i}(\tau) - v_{i}^{u}(\tau) \mu^{u} + v_{i}^{d}(\tau) \mu^{d} \right)$$

$$\geq s_{i}^{d}, \quad i \in \mathcal{N}, \tag{11e}$$

The objective function (11a) represents an upper bound of the expected aggregate revenue of EVs. An upper bound is considered because using (7) as the objective function leads to an untractable problem. Note the upper bound is close to the expected revenue as we use constraints (9) and (10) to ensure $\mathbf{1}_{i,h} = 1$ in most cases. The gap between the upper bound and the expected revenue can be rewritten as $(1 - \mathbb{P}(\mathbf{1}_{i,h}))\mathbb{E}(p^p(h))(v_i^u(h)\lambda^u + v_i^d(h)\lambda^d)$, where $\mathbb{P}(\mathbf{1}_{i,h})$ represents the probability for $\mathbf{1}_{i,h} = 1$. Our simulation results show the gap is a small value when we choose $\eta > 0$ to improve robustness (e.g., the gap is less than 3% of the optimal value when $\eta = 1$). The gap decreases when we choose η to be a larger value. We consider the case when the aggregator aims to maximize the aggregate revenue of EVs in the objective function (11a) and keeps a percentage (e.g., 10%) of the aggregate revenue. $\mathbb{E}(p^u(h)), \mathbb{E}(p^d(h)), \mathbb{E}(p^p(h)),$ and $\mathbb{E}(p^e(h))$ in (11a) represent the expected price for regulation up capacity, regulation down capacity, regulation performance, and charged energy at hour h. We consider the expected values in the objective function because the uncertainty in the revenue does not affect whether EVs follow the AGC signal reliably or not. Constraints (11b) and (11c) guarantee that the real-time charging power of EV i in hour h is within the maximum and minimum charging power of EV *i*. E_i^{\max} and E_i^{\min} represent the maximum and minimum charging power of EV i, respectively. Constraint (11d) guarantees the frequency regulation capacities are non-negative values. Constraint (11e) depicts the charging demand of EV *i*. d_i and s_i^d represent the expected departure hour and the expected SOC for EV i upon departure, respectively. We assume EV i is connected to the grid from the beginning of operation hours until d_i . The values of s_i^d and d_i are set by EV owners.

Problem (11) is a non-convex combinatorial optimization problem because constraints (9), (10) have nonconvex components. We first analyze constraint (9). The optimal value of the non-convex component of (9) $\min_{\{S \subseteq \{1,...,h\}} |S| \le \eta\} \sum_{\tau \in S} \left(-v_i^u(\tau)\mu^u + v_i^d(\tau)(\mu^d - \zeta^d)\right) \text{ is } \{S \subseteq \{1,...,h\} |S| \le \eta\}$ equivalent to the optimal value of the following problem [16]

$$\underset{w_{\tau}, \tau \in \{1, \dots, h\}}{\text{minimize}} \quad \sum_{\tau \in \mathcal{H}} w_{\tau} \Big(-v_i^u(\tau)\mu^u + v_i^d(\tau) \left(\mu^d - \zeta^d\right) \Big)$$
(12a)

subject to $0 \le w_{\tau} \le 1, \ \tau \in \{1, ..., h\},$ (12b)

$$\sum_{\tau \in \mathcal{H}} w_{\tau} \le \eta, \tag{12c}$$

where $w_{\tau} \in [0, 1]$ for $\tau \in \{1, \ldots, h\}$ are variables in problem (12). $v_i^u(\tau)$ and $v_i^d(\tau)$ take arbitrary finite values. Problem (12) is a linear program which is both feasible (e.g., $w_{\tau} = 0, \tau \in \mathcal{H}$) and bounded (e.g., $\sum_{\tau \in \{1,\ldots,h\}} \min\{-v_i^u(\tau)\mu^u + v_i^d(\tau)(\mu^d - \zeta^d), 0\}$ is a lower bound of the objective function). From strong duality, the optimal values of problem (12) and its dual problem are the same. The dual problem of (12) can be written as

$$\underset{y_{\tau}, \tau \in \{1, \dots, h\}, z}{\text{maximize}} \quad -z\eta - \sum_{\tau \in \{1, \dots, h\}} y_{\tau} \tag{13a}$$

subject to

$$y_{\tau} + z \ge v_i^u(\tau)\mu^u - v_i^d(\tau) \left(\mu^d - \zeta^d\right), \tau \in \{1, \dots, h\}, \quad (13b) y_{\tau}, z \ge 0, \ \tau \in \{1, \dots, h\}, \quad (13c)$$

where y_{τ} and z are dual variables for constraints (12b) and

(12c), respectively. Now we replace the non-convex components in constraint (9) with the objective function in problem (13) and add all constraints in (13) to problem (11). We use a similar approach to convert the non-convex component in constraint (10) in three steps. First, we convert the non-convex components in constraint (10) into a linear program by substituting $(-v_i^u(\tau)\mu^u + v_i^d(\tau)(\mu^d - \zeta^d))$ in problem (12) with $v_i^u(\tau)(\mu^u - \zeta^u) - v_i^d(\tau)\mu^d$. Then, we convert the linear program obtained in the first step into its dual problem by substituting $(-v_i^u(\tau)\mu^u + v_i^d(\tau)(\mu^d - \zeta^d))$ in problem (13) (the substitution is the same as in the first step). Finally, we substitute the non-convex component in constraint (10) with the objective function of the dual problem obtained in the second step and add the constraints of the dual problem to problem (11). We have the following problem

$$\begin{array}{l} \underset{v_{i}^{u}(h), v_{i}^{d}(h), x_{i}(h)}{\underset{z_{i,h}^{1,h}, z_{i,h}^{2}}{\underset{z_{i,h}^{1,h}, z_{i,h}^{2}}{\underset{y_{i,h}^{1,h}(\tau), y_{i,h}^{2}(\tau)}} + \underbrace{\sum_{i \in \mathcal{N}} v_{i}^{u}(h) \left(p^{u}(h) + p^{p}(h) \lambda^{u} + p^{e}(h) \mu^{u} \right)}_{i \in \mathcal{N}, (h, \tau) \in \{(h, \tau) \\ \mid h \in \mathcal{H}, \tau \in \{1, \dots, h\}\}} + \underbrace{v_{i}^{d}(h) \left(p^{d}(h) + p^{p}(h) \lambda^{d} - p^{e}(h) \mu^{d} \right)}_{(14a)}$$

subject to
$$s_i(0) + \frac{1}{B_i} \sum_{\tau=1}^h \left(x_i(\tau) - v_i^u(\tau) \mu^u + v_i^d(\tau) \mu^d \right)$$

 $+ \frac{1}{B_i} \left(z_{i,h}^1 \eta + \sum_{\tau=1}^h y_{i,h}^1(\tau) \right) \le 1, \ i \in \mathcal{N}, \ h \in \mathcal{H},$ (14b)

$$z_{i,h}^{1} + y_{i,h}^{1}(\tau) \ge \left(v_{i}^{u}(\tau)\mu^{u} - v_{i}^{d}(\tau)(\mu^{d} - \zeta^{d})\right), \\ i \in \mathcal{N}, \ (h,\tau) \in \{(h,\tau) \mid h \in \mathcal{H}, \tau \in \{1,\dots,h\}\},$$
(14c)

$$s_{i}(0) + \frac{1}{B_{i}} \sum_{\tau=1}^{h} \left(x_{i}(\tau) - v_{i}^{u}(\tau) \mu^{u} + v_{i}^{d}(\tau) \mu^{d} \right) - \frac{1}{B_{i}} \left(z_{i,h}^{2} \eta + \sum_{\tau=1}^{h} y_{i,h}^{2}(\tau) \right) \geq 0, \ i \in \mathcal{N}, \ h \in \mathcal{H},$$
(14d)

$$z_{i,h}^{2} + y_{i,h}^{2}(\tau) \ge \left(v_{i}^{u}(\tau)(\zeta^{u} - \mu^{u}) + v_{i}^{d}(\tau)\mu^{d}\right), \\ i \in \mathcal{N}, \ (h,\tau) \in \{(h,\tau) \mid h \in \mathcal{H}, \tau \in \{1,\dots,h\}\}$$
(14e)

$$z_{i,h}^{1}, z_{i,h}^{2}, y_{i,h}^{1}(\tau), y_{i,h}^{2}(\tau) \ge 0, i \in \mathcal{N}, (h, \tau) \in \{(h, \tau) \mid h \in \mathcal{H}, \tau \in \{1, \dots, h\}\},$$
(14f)
Constraints (11b)-(11e). (14g)

where $(z_{i,h}^1, y_{i,h}^1(\tau))$ and $(z_{i,h}^2, y_{i,h}^2(\tau))$ are dual variables corresponding to the non-convex components in constraints (9) and (10), respectively. Linear constraints (14b) and (14d) replace the non-convex constraints (9) and (10) by substituting the combinatorial optimization components with the corresponding linear dual problems. Constraints (14c) and (14e) are obtained from the constraints in the dual problems. Problem (14) is a linear program and can be solved efficiently.

The robust EV frequency regulation (REF) algorithm is presented in Algorithm 1. In the algorithm, the aggregator first

initializes the parameters and then solves problem (14). The results are sent to each EV and the aggregate results are sent to the ISO.

Algorithm 1 REF algorithm executed by the aggregator at the beginning of operation hours

- 1: Initialize $\eta, \mathcal{N}, \mathcal{H}, p^{e}(h), p^{u}(h)$, and $p^{d}(h), h \in \mathcal{H}$
- 2: Collect s_i^d , d_i , $s_i(0)$, E_i^{\max} , E_i^{\min} , and B_i from each EV $i \in \mathcal{N}$ 3: Solve problem (14) to obtain $v_i^u(h), v_i^d(h)$, and $x_i(h), h \in \mathcal{H}$ for each EV $i \in \mathcal{N}$
- 4: Send $v_i^u(h), v_i^d(h)$, and $x_i(h)$ to each EV $i \in \mathcal{N}$ and report $\sum_{i \in \mathcal{N}} v_i^u(h), \sum_{i \in \mathcal{N}} v_i^d(h)$, and $\sum_{i \in \mathcal{N}} x_i(h), h \in \mathcal{H}$ to the ISO

IV. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed REF algorithm under the historical record of the AGC signal from PJM. We conducted a statistical analysis of the historical AGC signal data record [14] and obtained the parameters $\zeta^u = 0.473$, $\zeta^d = 0.483$, $\mu^u = 0.116$, $\mu^d = 0.113$, $\lambda^u = 13.852$, and $\lambda^d = 13.805$. We compare our proposed REF algorithm with the OptMaxReg algorithm from [8]. The OptMaxReg algorithm assumes parameters $f^u(h)$ and $f^d(h)$ take expected values and neglects their uncertainty. The performance-based frequency regulation prices are obtained from [14], [17]. The average price over the operation period for the hourly energy consumption $p^e(h)$, regulation up capacity $p^u(h)$, regulation down capacity $p^d(h)$, and the performance $p^p(h)$ are \$42.4/megawatt per hour (MWh), \$4.9/MWh, \$7.2/MWh, and \$3.2/MWh, respectively.

We consider a fleet of 100 EVs. Each EV is connected with a bidirectional level-2 charger. The maximum charging power is 3.3 kW. EVs are able to discharge power to the grid. Note that the battery capacity of EV may vary from several kWh (e.g., 4.4 kWh for a Toyota Prius) to tens of kWh (e.g., 20 kWh for a Honda Fit). We assume the battery capacity is 12 kWh for simulation purposes. We consider an overnight charging case where EVs charges in the night and is used for driving on the next day. EV's charging deadline d_i is generated based on the 2009 national household travel survey [18].

For performance metrics, we determine the *revenue* and *reliability* of EV frequency regulation service by testing the algorithm outputs (i.e., $x_i(h)$, $v_i^u(h)$, $v_i^d(h)$) with the historical AGC signal. The EVs receive the payment for their performance if they follow the AGC signal in hour h. If an EV i fails to follow the AGC signal in hour h, its frequency regulation capacity in hour h is regarded as *forced derated* [19] (i.e., the capacity from EV i is discarded by the aggregator in hour h). We denote \mathcal{D}_i as the set of hours when the frequency regulation capacity from EV i is forced derated. The reliability of EV frequency regulation service is evaluated by $1 - \frac{EFDH}{H}$ [11], [20], where the equivalent forced derated hours (*EFDH*) is [19]

$$EFDH = \frac{\sum_{i \in \mathcal{N}} \sum_{h \in \mathcal{D}_i} (v_i^u(h) + v_i^d(h))}{\frac{1}{H} \sum_{i \in \mathcal{N}} \sum_{h \in \mathcal{H}} (v_i^u(h) + v_i^d(h))}.$$



Fig. 3. The reliability and revenue with respect to parameter η . We choose $\eta = 1$ for our proposed algorithm and $\eta = 2$ for the benchmark algorithm.



Fig. 4. Comparison of the proposed REF algorithm with OptMaxReg algorithm [8] under performance-based compensation.

Fig. 3 shows the revenue and reliability for the proposed REF algorithm. We compare with a benchmark robust algorithm, which considers the case when $f^{u}(h)$ and $f^{d}(h)$ change adversely in $\left[\mu^{u} - \frac{\eta}{H}\mu^{u}, \mu^{u} + \frac{\eta}{H}(\zeta^{u} - \mu^{u})\right],$ $\left[\mu^{d} - \frac{\eta}{H}\mu^{d}, \mu^{d} + \frac{\eta}{H}(\zeta^{d} - \mu^{d})\right]$ for all hours. As shown in the figure, both the revenue and reliability improve as η increases at the beginning because EVs earn more revenue for the performance (i.e., $\mathbf{1}_{i,h} = 1$ for more hours). When η further increases, the revenue decreases as the obtained solution is more conservative which leads to lower revenue for the capacity. Our proposed REF algorithm outperforms the benchmark robust algorithm. Our proposed REF algorithm selects $\eta = 1$ to have 97% reliability and \$224 frequency regulation revenue for each day. The benchmark robust algorithm selects $\eta = 2$ to achieve \$192 revenue with 94% reliability. Note the revenue is obtained from (7). For the proposed REF algorithm, the gap between the revenue (7) and the considered objective function (11a) is \$2.13 when $\eta = 1$.

We compare the proposed REF algorithm with the benchmark robust algorithm, and the OptMaxReg algorithm [8] in Fig. 4. The figure reveals that the proposed REF algorithm outperforms the OptMaxReg algorithm [8] with a higher revenue under performance-based compensation. This is because the proposed algorithm enables EVs to follow the AGC signal reliably and to earn more revenue for performance.

V. CONCLUSION

In this paper, we proposed a robust EV frequency regulation algorithm to improve revenue under performance-based compensation. A robust optimization problem is formulated to enable EVs to follow the AGC signal reliably. We transformed the formulated combinatorial optimization problem into a linear program based on the duality. We performed numerical experiments on the historical record of the AGC signal from PJM. Simulation results show that our proposed algorithm achieves a higher revenue compared with a benchmark algorithm from the literature. For future work, an interesting topic is the extension of the proposed algorithm to the case when the mobility behavior of EV is unknown.

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