

Energy Detection Based Spectrum Sensing with Random Arrival and Departure of Primary User's Signal

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Abstract—Energy detection (ED) is a popular spectrum sensing technique for cognitive radios. The study of ED which takes into account the dynamic of traffic patterns of primary users, in the form of random signal arrival and departure, is of both theoretical and practical importance. Some of the existing works, however, resort to certain approximation techniques to characterize the detection performance. In this paper, given a pair of arrival and departure time instants, we first derive an exact expression for the conditional detection probability. The exact mean detection probability is then obtained via an average operation over the random arrival and departure times. To improve the robustness of the detection performance against random signal arrival and departure, we further propose a Bayesian-based ED scheme. We present simulation results to validate our analytic study, and show the performance gain of our proposed Bayesian approach.

I. INTRODUCTION

Spectrum sensing is a key technique for enabling opportunistic spectrum access in cognitive radio (CR) systems [1], [2]. Among different spectrum sensing schemes, energy detection (ED) has received considerable attention due to its implementation simplicity [1], [3]. While there are various studies of ED in the context of cognitive radios, most of them consider the following standard binary hypothesis test:

$H_0 : x[n] = v[n], \quad 0 \leq n \leq N - 1$ (primary user is absent),
and

$H_1 : x[n] = s[n] + v[n], \quad 0 \leq n \leq N - 1$
(primary user is present), (1)

where N is the length of the data record, $s[n]$, $x[n]$, and $v[n]$ are, respectively, the signal of the primary user, the received signal at the CR terminal, and the measurement noise.

Performance analysis of ED under the model (1) is well documented in the literature [3]. The alternative hypothesis of (1) assumes that the traffic of the primary user is static during the sensing period. Such an assumption, however, is not realistic in certain scenarios. For example, when primary users can dynamically enter and leave the network, random arrival and/or departure observed in the sensing period is unavoidable, especially when a long sensing duration is adopted for obtaining good sensing performance [4], [5].

Recently, there have been several works which address the effect of primary user traffic patterns on the detection

performance of ED. In [6], Beaulieu *et al.* considered the random arrival or departure of the primary user's signal, and proposed a Bayesian-based ED which exploits the distributions of the arrival and departure times. The impact of signal arrival/departure on the sensing-throughput tradeoff is addressed in [7]. In [8], the impact of the primary user traffic on the detection performance is investigated. It is worthy of nothing that, in [6-8], the characterization of the detection/false-alarm probabilities resort to certain approximation techniques, e.g., the central limit theorem or the Chi-square approximation. In [9], the authors focused on the case with random arrival, and derived the exact detection probability of ED; a Bayesian-based ED is also proposed to improve the robustness of the detection performance against random signal arrival.

In this paper, we generalize the result in [9] by further taking into account the effect of random departure of the primary user's signal. Specifically, we consider the following model for the binary hypothesis test:

$H_0 : x[n] = v[n], \quad 0 \leq n \leq N - 1$ (primary user is absent),
and

$H_1 : \begin{cases} x[n] = v[n], & 0 \leq n < n_0, \\ x[n] = s[n] + v[n], & n_0 \leq n \leq n_1, \\ x[n] = v[n], & n_1 + 1 \leq n \leq N - 1, \end{cases}$
(primary user is present), (2)

where n_0 and n_1 denote, respectively, the arrival and departure time of the primary signal.

Given a pair of (n_0, n_1) , we derive the *exact expression* of the conditional detection probability. The average detection probability can then be obtained by taking average of the conditional probability over the distributions of n_0 and n_1 . To the best of our knowledge, the results presented in this paper are the first report in the literature that provides exact performance analysis for ED under both arrival and departure of the primary user's signal. By leveraging the statistical characterizations of n_0 and n_1 , a Bayesian-based ED is further proposed for improving the detection performance in the considered scenario. Simulation results validate the derived analytic formula, and justify the advantages of the proposed Bayesian test as compared to the conventional ED.

II. PERFORMANCE ANALYSIS

The test statistic of the conventional ED is given by

$$T = \sum_{n=0}^{N-1} |x[n]|^2. \quad (3)$$

Under the alternative hypothesis \mathcal{H}_1 in (3) and conditioned on a fixed pair of n_0 and n_1 , the test statistic T can be decomposed into

$$T = \underbrace{\sum_{n=0}^{n_0-1} |x[n]|^2}_{\triangleq T_1} + \underbrace{\sum_{n=n_0}^{n_1} |x[n]|^2}_{\triangleq T_2} + \underbrace{\sum_{n=n_1+1}^{N-1} |x[n]|^2}_{\triangleq T_3}. \quad (4)$$

Based on (4), we first derive the exact expression of the conditional detection probability; the average detection probability can then be obtained by taking the expectation with respect to certain distributions of n_0 and n_1 .

A. Conditional Detection Probability

We assume that (i) the signal $s[n]$ and noise $v[n]$ are zero-mean white sequences with variances given by σ_s^2 and σ_v^2 , respectively; (ii) $s[n]$ and $v[n]$ are independent. With T_1 , T_2 , and T_3 defined in (4), it can be verified that

$$z_1 \triangleq T_1/\sigma_v^2 \sim \chi_{n_0}^2,$$

$$z_2 \triangleq T_2/(\sigma_v^2 + \sigma_s^2) \sim \chi_{n_1-n_0+1}^2,$$

and

$$z_3 \triangleq T_3/\sigma_v^2 \sim \chi_{N-n_1-1}^2,$$

where the symbol χ_m^2 denotes the central Chi-square distribution with degrees-of-freedom equal to m .

The associated probability density functions (pdf) are

$$f_{z_1}(x) = \frac{x^{(n_0/2)-1} e^{-x/2}}{\sqrt{2^{n_0}} \Gamma(n_0/2)} u(x), \quad (5)$$

$$f_{z_2}(x) = \frac{x^{((n_1-n_0+1)/2)-1} e^{-x/2}}{\sqrt{2^{(n_1-n_0+1)}} \Gamma((n_1-n_0+1)/2)} u(x), \quad (6)$$

$$f_{z_3}(x) = \frac{x^{((N-n_1-1)/2)-1} e^{-x/2}}{\sqrt{2^{(N-n_1-1)}} \Gamma((N-n_1-1)/2)} u(x), \quad (7)$$

where $u(x)$ is the unit step function.

To simplify notation, let us consider the equivalent test statistic

$$\begin{aligned} \bar{T} &\triangleq \frac{T}{\sigma_v^2} \\ &= \frac{T_1}{\sigma_v^2} + \frac{T_2}{\sigma_v^2} + \frac{T_3}{\sigma_v^2} \\ &= z_1 + \left(\frac{\sigma_s^2 + \sigma_v^2}{\sigma_v^2} \right) z_2 + z_3 \\ &= z_1 + (1 + SNR) z_2 + z_3, \end{aligned} \quad (8)$$

where $SNR \triangleq \sigma_s^2/\sigma_v^2$. Since random variables z_1 , z_2 , and z_3 are independent, the pdf of \bar{T} is given by

$$f_{\bar{T}}(x) = f_{z_1}(x) * \left(\frac{1}{1 + SNR} \right) f_{z_2} \left(\frac{x}{1 + SNR} \right) * f_{z_3}(x) \quad (9)$$

where $*$ denotes the convolution operator. By taking Laplace transform, we have

$$\begin{aligned} F_{\bar{T}}(s) &= F_{z_1}(s) \mathcal{L} \left\{ \left(\frac{1}{1 + SNR} \right) f_{z_2} \left(\frac{x}{1 + SNR} \right) \right\} F_{z_3}(s) \\ &= F_{z_1}(s) F_{z_2}((1 + SNR)s) F_{z_3}(s), \end{aligned} \quad (10)$$

where $\mathcal{L}(\cdot)$ denotes the Laplace transform operator, and the second equality follows since $\mathcal{L}\{f(ax)\} = (a)^{-1} F(s/a)$ [13].

By taking the inverse Laplace transform of (10) and through further analysis, an explicit expression for $f_{\bar{T}}(x)$ is given in the following proposition (see Appendix A for a proof).

Proposition 1: Let $f_{\bar{T}}(x)$ be defined in (9). Then, we have

$$f_{\bar{T}}(x) = \frac{(1 + SNR)^{-\frac{(n_1-n_0+1)}{2}}}{\sqrt{2^N} \Gamma(N/2)} e^{-\frac{x}{2}} x^{\frac{N}{2}-1} \sum_{i=0}^{\infty} a_i x^i, \quad (11)$$

where

$$\begin{aligned} a_0 &= 1, \\ a_1 &= \frac{(n_1 - n_0 + 1)/2}{N/2} \cdot \frac{SNR}{1! [2(1 + SNR)]}, \\ a_2 &= \frac{((n_1 - n_0 + 1)/2)((n_1 - n_0 + 1)/2 + 1)}{(N/2)(N/2 + 1)} \\ &\quad \cdot \frac{(SNR)^2}{2! [2(1 + SNR)]^2}, \dots \end{aligned} \quad (12)$$

Based on (11), the detection probability conditioned on a pair of fixed n_0 and n_1 , denoted by $P_D(n_0, n_1)$, is derived in the following theorem (see Appendix B for a proof).

Theorem 1: The conditional probability $P_D(n_0, n_1)$ is given by

$$\begin{aligned} P_D(n_0, n_1) &= \frac{(1 + SNR)^{-(n_1-n_0+1)/2}}{\Gamma(N/2)} \sum_{i=0}^{\infty} a_i 2^i \Gamma \left(\frac{N}{2} + i, \frac{\gamma}{2} \right), \end{aligned} \quad (13)$$

where $\Gamma(\cdot, \cdot)$ is the incomplete Gamma function [11] and a_i 's are defined in (12).

Remark : When n_1 is equal to $N - 1$, the signal model (2) is reduced to the random arrival case considered in [9]. The conditional probability $P_D(n_0, n_1)$ in (13) is simplified to

$$\begin{aligned} P_D(n_0, N - 1) &= \frac{(1 + SNR)^{-(N-n_0)/2}}{\Gamma(N/2)} \sum_{i=0}^{\infty} a_i 2^i \Gamma \left(\frac{N}{2} + i, \frac{\gamma}{2} \right), \end{aligned} \quad (14)$$

which is the result reported in [9, eq. (2.16)].

B. Average Detection Probability

The average detection probability, denoted by P_D , can be obtained by averaging $P_D(n_0, n_1)$ with respect to the distributions of n_0 and n_1 . We adopt the common assumption that the arrival and departure times are exponentially distributed [6]. Thus, the pdf of n_0 is

$$f(n_0) = \lambda_0 e^{-\lambda_0 n_0} u(n_0), \quad \text{for } \lambda_0 > 0. \quad (15)$$

The conditioned pdf of n_1 given n_0 is

$$f(n_1 | n_0) = \lambda_1 e^{-\lambda_1(n_1 - n_0)} u(n_1 - n_0), \quad \text{for } \lambda_1 > 0. \quad (16)$$

In (15) and (16), λ_0 and λ_1 are the mean arrival and departure rates. The probability mass function (pmf) of n_0 , and the pmf of n_1 for a given n_0 are, respectively,

$$p(n_0) = e^{-n_0 \lambda_0 T_s} - e^{-(n_0+1) \lambda_0 T_s}, \quad (17)$$

and

$$p(n_1 | n_0) = e^{n_0 \lambda_1 T_s} \left(e^{-n_1 \lambda_1 T_s} - e^{-(n_1+1) \lambda_1 T_s} \right), \quad (18)$$

where T_s is the sampling interval. Hence, the average detection probability can be obtained by averaging $P_D(n_0, n_1)$ over the joint distribution of n_0 and n_1 as

$$\begin{aligned} P_D &= \sum_{n_0, n_1} P_D(n_0, n_1) p(n_0, n_1) \\ &= \sum_{n_0, n_1} P_D(n_0, n_1) \left(\sum_{n_0} p(n_1 | n_0) p(n_0) \right) \\ &= (1 - e^{-\lambda_0 T_s}) (1 - e^{-\lambda_1 T_s}) \sum_{n_0=0}^{N-1} e^{-n_0(\lambda_0 - \lambda_1) T_s} \\ &\quad \times \sum_{n_1=n_0}^{N-1} e^{-n_1 \lambda_1 T_s} \frac{(1 + SNR)^{-\frac{(n_1 - n_0 + 1)}{2}}}{\Gamma(N/2)} \\ &\quad \times \sum_{i=0}^{\infty} a_i 2^i \Gamma\left(\frac{N}{2} + i, \frac{\gamma}{2}\right). \end{aligned} \quad (19)$$

C. Simulation Results

Simulations are conducted to validate our theoretical study in Section II. The duration of the sensing window N is 200, the sampling interval T_s is 1.25 ms, and $\lambda_0 = \lambda_1 = 100$ seconds. For false-alarm probability $P_f = 0.05$, Fig. 1 shows the average detection probability computed according to (20) and the simulated results. Fig. 2 plots the receiver operating characteristics (ROC) curves for $SNR = 5$ dB. Both figures show that the analytical and simulation results are well matched.

III. PROPOSED BAYESIAN DETECTOR

A. Bayesian Decision Rule

The performance of ED will degrade significantly in the presence of signal arrival and departure [5]. Following [9], in this section we propose a Bayesian-based ED by exploiting the

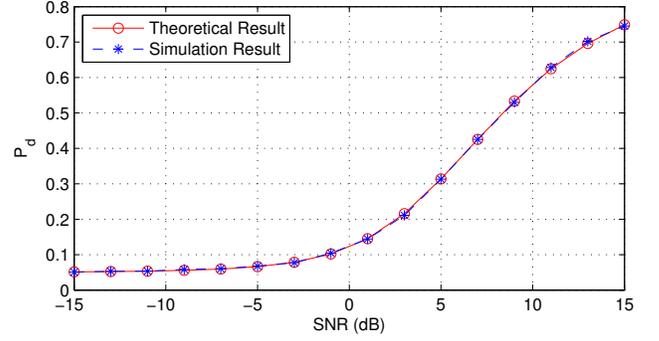


Fig. 1. Detection probability P_d versus SNR ($P_f = 0.05$).

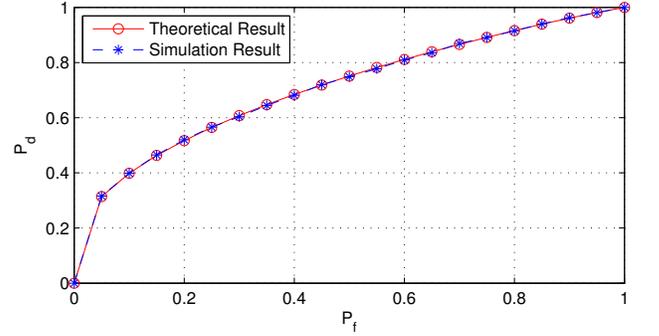


Fig. 2. Detection probability P_d versus false-alarm probability P_f .

statistical knowledge of n_0 and n_1 . The conditional pmf of the data samples under hypotheses \mathcal{H}_0 and \mathcal{H}_1 are, respectively,

$$p(\mathbf{x}; \mathcal{H}_0) = \frac{1}{(2\pi\sigma_v^2)^{N/2}} \exp\left(\frac{-1}{2\sigma_v^2} \sum_{n=0}^{N-1} |x[n]|^2\right), \quad (20)$$

and

$$\begin{aligned} p(\mathbf{x}; n_0, n_1, \mathcal{H}_1) &= \frac{1}{(2\pi\sigma_v^2)^{n_0/2}} e^{\frac{-1}{2\sigma_v^2} \sum_{n=0}^{n_0-1} |x[n]|^2} \\ &\quad \times \frac{1}{(2\pi(\sigma_v^2 + \sigma_s^2))^{\frac{n_1 - n_0 + 1}{2}}} e^{\frac{-1}{2(\sigma_v^2 + \sigma_s^2)} \sum_{n=n_0}^{n_1-1} |x[n]|^2} \\ &\quad \times \frac{1}{(2\pi\sigma_v^2)^{\frac{N - n_1 - 1}{2}}} e^{\frac{-1}{2\sigma_v^2} \sum_{n=n_1+1}^{N-1} |x[n]|^2}. \end{aligned} \quad (21)$$

We define

$$p(\mathbf{x}; \mathcal{H}_1) = \sum_{n_0=0}^{N-1} p(n_0) \sum_{n_1=n_0}^{N-1} p(n_1 | n_0) p(\mathbf{x}; n_0, n_1, \mathcal{H}_1). \quad (22)$$

Based on [3], the Bayesian decision rule chooses the hypothesis \mathcal{H}_1 if

$$\frac{p(\mathbf{x}; \mathcal{H}_1)}{p(\mathbf{x}; \mathcal{H}_0)} \geq \gamma, \quad (23)$$

where γ is the decision threshold, $p(\mathbf{x}; \mathcal{H}_0)$ and $p(\mathbf{x}; \mathcal{H}_1)$ are defined in (20) and (22), respectively.

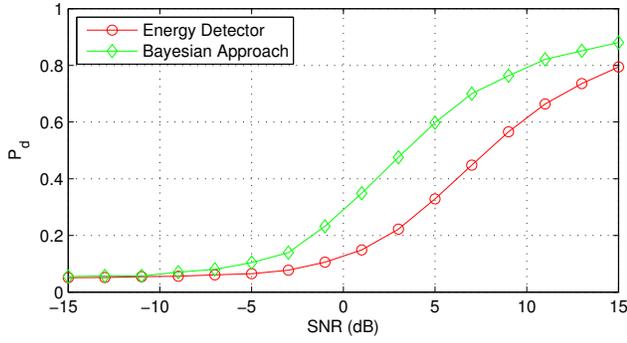


Fig. 3. Detection probability P_d v.s. SNR curves obtained by two methods ($P_f = 0.05$).

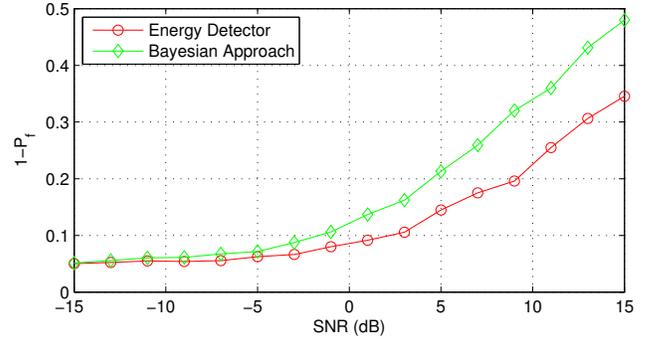


Fig. 4. $(1 - P_f)$ v.s. SNR curves obtained by two methods ($P_d = 0.95$).

B. Simulation Results and Discussions

To illustrate the performance of the proposed Bayesian ED in (23), Fig. 3 compares the proposed Bayesian ED (23) with the conventional ED in terms of the average detection probability. The setup is the same as that in Section II-C, and the false-alarm probability is P_f is chosen to be 0.05. It can be seen from the figure that the proposed Bayesian-based solution provides an improved detection performance. With detection probability P_d chosen to be 0.95, Fig. 4 shows the achievable $1 - P_f$, which is a measure of the channel utilization efficiency of secondary users [10], [12], attained by the two methods. The results show that the Bayesian test (23) yields a larger $1 - P_f$, thereby yielding better channel utilization.

IV. CONCLUSION

In this paper, we presented an exact detection performance analyses for energy detection (ED) in the presence of random arrival and departure of the primary user's signal for cognitive radio systems. A Bayesian-based ED is also proposed to improve the detection performance against random signal arrival and departure. These results can be directly generalized to the scenario of multi-status change considered in [5]. Exact characterization of the detection probability of the proposed Bayesian ED (23) is currently under investigation.

APPENDIX

A. Proof of Proposition 1

To prove the proposition, we need the following lemmas.

Lemma A.1: For $\lambda > 0$, we have [11]

$$\mathcal{L}\{x^{\lambda-1}e^{-ax}u(x)\} = \Gamma(\lambda)(s+a)^{-\lambda}.$$

Lemma A.2: For $\nu > 0$ and $\mu > 0$, it follows that [11]

$$\int_0^x t^{\nu-1}(x-t)^{\mu-1}e^{\delta t}dt = B(\mu, \nu)x^{\mu+\nu-1}\Phi(\nu, \mu + \nu; \delta x), \quad (\text{A.1})$$

where $B(\cdot, \cdot)$ is the beta function, and $\Phi(\cdot, \cdot; \cdot)$ is the confluent hyper-geometric function defined as

$$\begin{aligned} \Phi(\alpha, \gamma, z) = & 1 + \frac{\alpha}{\gamma} \cdot \frac{z}{1!} + \frac{\alpha(\alpha+1)}{\gamma(\gamma+1)} \cdot \frac{z^2}{2!} \\ & + \frac{\alpha(\alpha+1)(\alpha+2)}{\gamma(\gamma+1)(\gamma+2)} \cdot \frac{z^3}{3!} + \dots \end{aligned} \quad (\text{A.2})$$

From Lemma A.1, it follows readily that

$$F_{z_1}(s) = \frac{(s+1/2)^{-\frac{n_0}{2}}}{\sqrt{2^{n_0}}}, \quad (\text{A.3})$$

$$F_{z_2}(s) = \frac{(s+1/2)^{-(n_1-n_0+1)/2}}{\sqrt{2^{(n_1-n_0+1)}}}, \quad (\text{A.4})$$

and

$$F_{z_3}(s) = \frac{(s+1/2)^{-(N-n_1-1)/2}}{\sqrt{2^{(N-n_1-1)}}}. \quad (\text{A.5})$$

As a result, we have

$$\begin{aligned} F_{\bar{T}}(s) = & \frac{1}{\sqrt{2^N}} \left(s + \frac{1}{2}\right)^{-\frac{n_0}{2}} \times \\ & \left((1 + SNR)s + \frac{1}{2} \right)^{-\frac{(n_1-n_0+1)}{2}} \left(s + \frac{1}{2}\right)^{-\frac{(N-n_1-1)}{2}} \\ = & \frac{(1 + SNR)^{-\frac{(n_1-n_0+1)}{2}}}{\sqrt{2^N}} \left(s + \frac{1}{2}\right)^{-\frac{N-n_1+n_0-1}{2}} \\ & \times \left(s + \frac{1}{2(1 + SNR)}\right)^{-\frac{n_1-n_0+1}{2}}. \end{aligned} \quad (\text{A.6})$$

The inverse Laplace transform of $F_{\bar{T}}(s)$ is

$$\begin{aligned}
f_{\bar{T}}(x) &= \frac{(1 + SNR)^{-(n_1 - n_0 + 1)/2}}{\sqrt{2^N}} \\
&\times \left\{ \mathcal{L}^{-1} \left\{ \left(s + \frac{1}{2} \right)^{\frac{N - n_1 + n_0 - 1}{2}} \right\} \right. \\
&\quad \left. * \mathcal{L}^{-1} \left\{ \left(s + \frac{1}{2(1 + SNR)} \right)^{\frac{-(n_1 - n_0 + 1)}{2}} \right\} \right\} \\
&\stackrel{(a)}{=} \frac{(1 + SNR)^{-(n_1 - n_0 + 1)/2}}{\sqrt{2^N}} \times \\
&\quad \left\{ \left[\frac{x^{[(N - n_1 + n_0 - 1)/2] - 1} e^{-\frac{x}{2}} u(x)}{\Gamma\left(\frac{N - n_1 + n_0 - 1}{2}\right)} \right] \right. \\
&\quad \left. * \left[\frac{x^{[(n_1 - n_0 + 1)/2] - 1} e^{-x/[2(1 + SNR)]} u(x)}{\Gamma\left(\frac{n_1 - n_0 + 1}{2}\right)} \right] \right\}, \tag{A.7}
\end{aligned}$$

where (a) follows from Lemma A.1. To determine the convolution part in (A.7), we have

$$\begin{aligned}
&x^{(N - n_1 + n_0 - 1)/2 - 1} e^{-x/2} u(x) \\
&\quad * x^{(n_1 - n_0 + 1)/2 - 1} e^{-x/[2(1 + SNR)]} u(x) \\
&= e^{-x/2} \int_0^x (x - \tau)^{(N - n_1 + n_0 - 1)/2 - 1} \times \\
&\quad \tau^{(n_1 - n_0 + 1)/2 - 1} e^{SNR\tau/[2(1 + SNR)]} d\tau \\
&\stackrel{(b)}{=} e^{-x/2} B\left(\frac{n_1 - n_0 + 1}{2}, \frac{N - n_1 + n_0 - 1}{2}\right) x^{\frac{N}{2} - 1} \\
&\quad \times \Phi\left(\frac{n_1 - n_0 + 1}{2}, \frac{N}{2}; \frac{SNR}{2(1 + SNR)} x\right) \\
&= e^{-x/2} B\left(\frac{n_1 - n_0 + 1}{2}, \frac{N - n_1 + n_0 - 1}{2}\right) \\
&\quad \times x^{\frac{N}{2} - 1} \sum_{i=0}^{\infty} a_i x^i \\
&= e^{-x/2} B\left(\frac{n_1 - n_0 + 1}{2}, \frac{N - n_1 + n_0 - 1}{2}\right) \\
&\quad \times \sum_{i=0}^{\infty} a_i x^{\frac{N}{2} + i - 1}, \tag{A.8}
\end{aligned}$$

where (b) follows from Lemma A.2, and the sequence $\{a_i\}$ in (A.8) is given by (12).

Based on (A.7) and (A.8), we have

$$\begin{aligned}
f_{\bar{T}}(x) &= \frac{(1 + SNR)^{-(n_1 - n_0 + 1)/2}}{\sqrt{2^N} \Gamma\left(\frac{N - n_1 + n_0 - 1}{2}\right) \Gamma\left(\frac{n_1 - n_0 + 1}{2}\right)} e^{-x/2} \\
&\times B\left(\frac{n_1 - n_0 + 1}{2}, \frac{N - n_1 + n_0 - 1}{2}\right) \\
&\times \sum_{i=0}^{\infty} a_i x^{\frac{N}{2} + i - 1} \\
&\stackrel{(c)}{=} \frac{(1 + SNR)^{-(n_1 - n_0 + 1)/2}}{\sqrt{2^N} \Gamma\left(\frac{N}{2}\right)} e^{-\frac{x}{2}} x^{\frac{N}{2} - 1} \sum_{i=0}^{\infty} a_i x^i, \tag{A.9}
\end{aligned}$$

where (c) holds by the relationship between gamma function and beta function, and is given by

$$\begin{aligned}
&B\left(\frac{n_1 - n_0 + 1}{2}, \frac{N - n_1 + n_0 - 1}{2}\right) \\
&= \frac{\Gamma\left(\frac{n_1 - n_0 + 1}{2}\right) \Gamma\left(\frac{N - n_1 + n_0 - 1}{2}\right)}{\Gamma\left(\frac{N}{2}\right)}. \tag{A.10}
\end{aligned}$$

The assertion follows directly from (A.9) and (A.10). ■

B. Proof of Theorem 1

Based on (11), it follows that $P_D(n_0, n_1)$

$$\begin{aligned}
&= \int_{\gamma}^{\infty} f_{\bar{T}}(x) dx \\
&= \frac{(1 + SNR)^{-(n_1 - n_0 + 1)/2}}{\sqrt{2^N} \Gamma(N/2)} \sum_{i=0}^{\infty} a_i \left(\int_{\gamma}^{\infty} e^{-x/2} x^{N/2 + i - 1} dx \right) \\
&\stackrel{(d)}{=} \frac{(1 + SNR)^{-(n_1 - n_0 + 1)/2}}{\Gamma(N/2)} \sum_{i=0}^{\infty} a_i 2^i \Gamma\left(\frac{N}{2} + i, \frac{\gamma}{2}\right),
\end{aligned}$$

where (d) follows from the definition of the incomplete gamma function $\Gamma(\cdot, \cdot)$ defined in [11, p. 899], with together some manipulations. ■

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REFERENCES

- [1] E. Hossain and V. K. Bhargava, *Cognitive Wireless Communication Networks*, Springer, 2007.
- [2] F. H. P. Fitzek and M. D. Katz, *Cognitive Wireless Networks: Concepts, Methodologies, and Visions Inspiring the Age of Enlightenment of Wireless Communications*, Springer 2007.
- [3] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*, Prentice-Hall PTR, 1998.
- [4] T. S. Shehata and M. El-Tanany, "A novel adaptive structure of the energy detector applied to cognitive radio networks," in *Proc. of Canadian Workshop on Information Theory*, Ottawa, Ontario, pp. 95-98, May 2009.
- [5] L. Tang, Y. Chen, E. L. Hines, and M. S. Alouini, "Performance analysis of spectrum sensing with multiple status changes in primary user traffic," *IEEE Communications Letters*, vol. 16, no. 6, pp. 874-877, June 2011.
- [6] N. C. Beaulieu and Y. Chen, "Improved energy detectors for cognitive radios with randomly arriving and departing primary users," *IEEE Signal Processing Letters*, vol. 17, no. 10, pp. 867-870, Oct. 2010.
- [7] L. Tang, Y. Chen, E. L. Hines, and M. S. Alouini, "Effect of primary user traffic on sensing-throughput tradeoff for cognitive radios," *IEEE Trans. on Wireless Commun.*, vol. 10, no. 4, pp. 1063-1068, April 2011.
- [8] M. E. Sahin, I. Guvenc, and H. Arslan, "Opportunity detection for OFDMA-based cognitive radio systems with timing misalignment," *IEEE Trans. on Wireless Commun.*, vol. 8, no. 10, pp. 5530-5532, Oct. 2009.
- [9] J. Y. Wu, C. H. Wang, and T. Y. Wang, "Performance analysis of energy detection based spectrum sensing with unknown primary signal arrival time," *IEEE Trans. on Commun.*, vol. 59, no. 7, pp. 1779-1784, July 2011.
- [10] Y. C. Liang, Y. Zeng, E. Y. C. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Trans. on Wireless Commun.*, vol. 7, no. 4, pp. 1326-1337, April 2008.
- [11] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*, 7th ed., Academic Press, 2007.
- [12] E. Biglieri, A. J. Goldsmith, L. J. Greenstein, N. B. Mandayam, and H. V. Poor, *Principles of Cognitive Radio*, Cambridge, 2013.