Market Participation of Energy Storage Systems for Frequency Regulation Service: A Bi-level Model

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Abstract—This paper examines the prospect of using the energy storage systems (ESSs) in the distribution network for frequency regulation service under the two-settlement market mechanism. A bi-level problem is formulated to determine the bidding strategy for the ESS which provides regulation service for the system operator in the day-ahead and real-time markets, where the upper-level problem maximizes the ESS' revenue from frequency regulation and the lower-level problem models the system operator's market clearing. The problem is rendered applicable for the ESSs in the distribution network by addressing the power flow constraints. The uncertainty associated with other competitive ESSs and the system frequency deviations are incorporated by using scenarios for possible realizations. The formulated problem is transformed to a mixed-integer linear program by replacing the lower-level problem with the Karush-Kuhn-Tucker (KKT) optimality conditions and tackling the nonconvexity in the objective function based on strong duality. Case studies are carried out on an IEEE 37-bus test feeder by using market data from California Independent System Operator (CAISO). The results demonstrate that the ESS can increase its revenue from frequency regulation by using our proposed method to determine the bidding strategy.

I. INTRODUCTION

Frequency regulation is an important application of energy storage systems (ESSs) for power grid to enhance system reliability. To encourage the use of ESSs for frequency regulation service, the Federal Energy Regulatory Commission (FERC) has been redesigning the regulations to accommodate the adoption of ESSs under the market-based framework [1]. The recent Order 841 has removed the barriers for the ESSs to participate in the ancillary services market, which recognizes the ESSs' viability to provide frequency regulation [2].

There is a rich body of literature on exploiting ESSs for frequency regulation. Prior research works mainly investigate the application of the front-of-the-meter ESSs owned by the power utilities. The integration of ESSs with conventional generators and renewable resources to improve system operability has been studied in [3], [4]. Various ESSs control schemes that address the issue with limited storage capacity have been proposed in [5], [6]. Some recent efforts shift the focus to the behind-the-meter ESSs implemented by industrial customers. The operation strategy and economic benefit of using these ESSs for the regulation service have been investigated in [7]–[9]. With the increasing deployment of such ESSs at the commercial sectors, new opportunity opens up for the system

operator to obtain frequency regulation service from the distribution network. It is necessary to examine the prospect of the ESSs at the distribution level to provide frequency regulation for the system operator under a market mechanism.

In our prior work [10], we have demonstrated from the system operator's perspective that the distribution network constraints need to be considered when using the ESSs for frequency regulation. In this paper, we focus on the decisionmaking from the ESSs' perspective which participate in the regulation service, where the distribution network constraints are incorporated by the system operator. We optimize the bidding decision for a *strategic* ESS by using a bi-level model to maximize its revenue from providing the regulation service under the two-settlement market mechanism, i.e., the dayahead market (DAM) and the real-time market (RTM) [11]. Bi-level programming has been widely used in various aspects of the ESSs' application in power systems [12], [13]. Our paper is different from these aforementioned works in that we consider the specific use case of ESSs at the distribution level and render the solution applicable by incorporating the distribution network constraints. The main contributions are summarized below:

- To examine the ESSs' participation at the distribution level for frequency regulation service, we formulate a bilevel problem to determine the ESS's bidding strategy in the DAM, which also considers the ESS's opportunity to participate in the RTM to provide the regulation service.
- Our bi-level problem addresses the distribution network constraints that affect the ESS's bidding strategy for the regulation service. The effect of the uncertainty associated with other competitive ESSs' bids and the system frequency deviations has also been incorporated by using scenarios to represent the possible realizations.
- We demonstrate the effectiveness of our proposed method by carrying out case studies on an IEEE 37-bus test feeder. Results demonstrate the ESS of interest can increase its revenue from the regulation service using our method to strategically determine the bids.

The rest of the paper is organized as follows. The system model, including the ESS operation model and the market clearing model, is presented in Section II. The bi-level program to optimize the strategic ESS's bidding decision is discussed in Section III. Section IV provides the case studies on an IEEE 37-bus test feeder. Section V draws the conclusion.

II. SYSTEM MODEL

Consider the ESSs' participation in the DAM and RTM for frequency regulation. Let \mathcal{N} and \mathcal{L} denote the set of buses and branches in the distribution network, respectively, where $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$. The ESSs are located at buses in set $\mathcal{N}^{s} \subseteq$ \mathcal{N} . The DAM is operated one day prior on an hourly basis, where the ESSs submit bids for regulation service and are awarded a contract if selected by the system operator. The RTM is operated on 15-minute intervals to balance the DAM schedules to the actual power usage on the day of operation, where the ESSs can submit bids to sell extra capacity (if any) to the system operator. We denote the set of hours in one day and time slots in one hour by $h \in \mathcal{H} = \{1, \ldots, H\}$ and $t \in \mathcal{T} = \{1, \ldots, T\}$, respectively.

In the following subsections, we present the ESS operation model and the market clearing process of the system operator.

A. ESS Operation Model

Let $p_n^{s, \max}$ and $p_n^{s, \min}$ denote the maximum and minimum power demand of the ESS at bus $n \in \mathcal{N}^s$, respectively. The ESS power demand $p_n^s(h, t)$ in time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$ at bus $n \in \mathcal{N}^s$ is bounded by

$$p_n^{s,\min} \le p_n^s(h,t) \le p_n^{s,\max}.$$
 (1)

Note that we define $p_n^s(h,t) > 0$ when the ESS at bus n is discharging, and $p_n^s(h,t) < 0$ when the ESS at bus n is charging. Let $E_n^{s,\text{init}}$ and $E_n^{s,\max}$ denote the initial energy level and the maximum storage capacity of the ESS at bus $n \in \mathcal{N}^s$, respectively. Let Δt denote the time interval between two consecutive time slots of hour $h \in \mathcal{H}$. The energy level $E_n^s(h,t)$ of ESS at bus $n \in \mathcal{N}^s$ at the end of time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$ is given by

$$E_{n}^{s}(h,t) = E_{n}^{s}(h,t-1) - p_{n}^{s}(h,t)\Delta t,$$
(2a)

$$E_n^{\rm s}(h,0) = E_n^{\rm s}(h-1,0) - \sum_{t=1}^{5} p_n^{\rm s}(h-1,t)\Delta t, \ h \in \mathcal{H} \setminus \{1\},$$
(2b)

$$0 \le E_n^{\mathsf{s}}(h,t) \le E_n^{\mathsf{s},\max}, \ t \in \mathcal{T} \cup \{0\},$$

$$(2c)$$

where $E_n^{\rm s}(1,0) = E_n^{\rm s,init}$. Constraints (1) and (2a)–(2c) for the ESS at bus $n \in \mathcal{N}^{\rm s}$ need to be satisfied for every time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$.

B. Market Clearing Model

We assume that the ESSs at the distribution level only participate in frequency regulation service. Moreover, we assume that the regulation service can be fully provided by those ESSs, i.e., the required capacity for frequency regulation can be cleared using the ESSs in the distribution network. DAM is cleared one day before the actual energy delivery takes place. The system operator selects the ESSs based on their bids, such that at the minimum cost the resulting capacity matches the required capacity on an hourly basis [14]. Let $p^{\text{req}}(h) \in \mathbb{R}$ denote the required regulation capacity of the system operator in hour $h \in \mathcal{H}$. A positive regulation capacity $p^{\rm req}(h) > 0$ indicates that the system operator wants to procure generation capacity $p_n^{\rm s,DAM}(h) > 0$ in hour $h \in \mathcal{H}$ from the ESS at bus $n \in \mathcal{N}^{\rm s}$, whereas a negative regulation capacity $p^{\rm req}(h) < 0$ indicates that the system operator wants to procure load capacity $p_n^{\rm s,DAM}(h) < 0$ in hour $h \in \mathcal{H}$ from the ESS at $n \in \mathcal{N}^{\rm s}$. Let $\alpha_n^{\rm s,DAM}(h), p_n^{\rm min,DAM}(h)$, and $p_n^{\rm max,DAM}(h)$ denote the bidding price, minimum, and maximum power of hour $h \in \mathcal{H}$ in the bid for DAM submitted by the ESS at bus $n \in \mathcal{N}^{\rm s}$, respectively. Note that the ESS specifies $\alpha_n^{\rm s,DAM}(h) > 0$, $p_n^{\rm min,DAM}(h) = 0$, and $p_n^{\rm max,DAM}(h) > 0$ when the system needs a positive regulation capacity $p^{\rm req}(h) > 0$, and $\alpha_n^{\rm s,DAM}(h) < 0$, $p_n^{\rm min,DAM}(h) < 0$, and $p_n^{\rm max,DAM}(h) = 0$ when the system needs a negative regulation capacity $p^{\rm req}(h) < 0$ in the submitted bid. The DAM market can be cleared by the system operator solving the following problem

$$\min_{\substack{p_n^{\text{s,DAM}}(h),\\ n \in \mathcal{N}^{\text{s}}, h \in \mathcal{H}}} \sum_{h=1}^{H} \sum_{n \in \mathcal{N}^{\text{s}}} \alpha_n^{\text{s,DAM}}(h) p_n^{\text{s,DAM}}(h)$$
(3a)

subject to
$$\sum_{n \in \mathcal{N}^{s}} p_{n}^{s, \text{DAM}}(h) = p^{\text{req}}(h), h \in \mathcal{H},$$
(3b)
$$p_{n}^{\min, \text{DAM}}(h) \leq p_{n}^{s, \text{DAM}}(h) \leq p_{n}^{\max, \text{DAM}}(h),$$
$$n \in \mathcal{N}^{s}, h \in \mathcal{H}.$$
(3c)

The hourly market clearing prices are the dual variables associated with constraint (3b), denoted by $\lambda^{\text{DAM}}(h)$, $h \in \mathcal{H}$, which result in the lowest cost for the system operator to meet the regulation requirement on an hourly basis. If the ESS at bus $n \in \mathcal{N}^{\text{s}}$ is selected by the system operator, it will be paid at the market clearing price $\lambda^{\text{DAM}}(h)$ to provide $p_n^{\text{s},\text{DAM}}(h)$ for frequency regulation in hour $h \in \mathcal{H}$ of the next day.

In the RTM, the system operator mitigates the discrepancies between the scheduled and actual demand by purchasing additional frequency regulation capacity if needed. ESSs can participate by submitting bids of additional demand change (if any) that they can provide for the regulation service. RTM requires actual energy delivery when the market clears. The system operator needs to ensure that the distribution network constraints are satisfied when using the ESSs. As such, we present the constraints below for every bus $n \in \mathcal{N}$, every branch $(n, j) \in \mathcal{L}$, and each time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$ [10].

$$\begin{bmatrix} \boldsymbol{p}^{\text{inj}}(h,t) \\ \boldsymbol{q}^{\text{inj}}(h,t) \end{bmatrix} = \begin{bmatrix} -\boldsymbol{B}' & \boldsymbol{G}' \\ -\boldsymbol{G} & -\boldsymbol{B} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}(h,t) \\ \boldsymbol{v}(h,t) \end{bmatrix}, \qquad (4a)$$
$$p_{nj}(h,t) =$$

$$\frac{R_{nj}(|v_n(h,t)| - |v_j(h,t)|) + X_{nj}(\theta_n(h,t) - \theta_j(h,t))}{R_{nj}^2 + X_{nj}^2}, \quad (4b)$$

$$\frac{q_{nj}(n,t) =}{\frac{X_{nj}\left(|v_n(h,t)| - |v_j(h,t)|\right) - R_{nj}(\theta_n(h,t) - \theta_j(h,t))}{R_{nj}^2 + X_{nj}^2}}, \quad (4c)$$

$$p_{nj}(h,t)\cos(\eta\gamma) + q_{nj}(h,t)\sin(\eta\gamma) \le s_{nj}^{\max},$$
(4d)

$$v_n^{\min} \le |v_n(h,t)| \le v_n^{\max},\tag{4e}$$

where $\eta = \{0, 1, ..., 2\pi/\gamma\}$. $p^{inj}(h, t), q^{inj}(h, t), v(h, t)$, and

 $\theta(h, t)$ denote the vectors of the injected active power, reactive power, voltage magnitude, and phase angle in time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$, respectively. Matrices G, G' as well as B, B'denote the real and imaginary parts corresponding to the voltage magnitude and phase angles in the bus admittance matrix, respectively. $p_{nj}(h,t)$, $q_{nj}(h,t)$, s_{nj}^{\max} denote the linearized active power flow, reactive power flow, and power flow limit of the distribution line $(n, j) \in \mathcal{L}$, respectively, where the line resistance and reactance are denoted by R_{nj} and X_{nj} , respectively. v_n^{\min} and v_n^{\max} denote the lower and upper limit of the voltage magnitude at bus $n \in \mathcal{N}$, respectively.

In time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$, let $\Delta \omega(h, t) \in \mathcal{R}$ denote the frequency deviation that the system operator aims to regulate. If $\Delta \omega(h, t) < 0$, the system operator needs regulation up service in time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$ from the ESSs, which indicates that the ESS at bus $n \in \mathcal{N}^{s}$ if selected will be dispatched by the system operator to provide a positive demand change $p_n^{s,RTM}(h,t) > 0$. If $\Delta \omega(h,t) > 0$, the system operator needs regulation down service in time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$ from the ESSs, which indicates that the ESS at bus $n \in \mathcal{N}^{s}$ if selected will be dispatched by the system operator to provide a negative demand change $p_n^{s,RTM}(h,t) < 0$. Let $\alpha_n^{s,RTM}(h,t)$, $p_n^{\min,RTM}(h,t)$, and $p_n^{\max,RTM}(h,t)$ denote the bidding price, minimum, and maximum demand change submitted by the ESS at bus $n \in \mathcal{N}^{s}$ for RTM, respectively. Similarly, the ESS specifies $\alpha_n^{s, \text{RTM}}(h, t) > 0, p_n^{\min, \text{RTM}}(h, t) =$ 0, and $p_n^{\max,\,\mathrm{RTM}}(h,t)\,>\,0$ when the system operator needs regulation up service, and $\alpha_n^{\rm s,RTM}(h,t) < 0, \, p_n^{\min,\,{\rm RTM}}(h,t) <$ 0, and $p_n^{\max, \text{RTM}}(h, t) = 0$ when the system operator needs regulation down service in the submitted bid. Note that the change in the ESSs demand $p_n^{\text{s,RTM}}(h,t), n \in \mathcal{N}^{\text{s}}$ for frequency regulation service in time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$ affects the change in the distribution network power flow, i.e., the change in the injected active power $\Delta p_n^{\text{inj}}(h,t)$ of bus $n \in \mathcal{N}$. To determine the ESSs dispatch for the regulation service that also results in feasible power flow changes, the system operator solves the following problem in the RTM for any given $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$

$$\underset{\substack{p_n^{\text{s,RTM}}(h,t),\\n\in\mathcal{N}^{\text{s}}}}{\min}\sum_{n\in\mathcal{N}^{\text{s}}}\alpha_n^{\text{s,RTM}}(h,t)p_n^{\text{s,RTM}}(h,t)$$
(5a)

subject to
$$\beta_n \Delta \omega(h,t) = \Delta p_n^{\text{inj}}(h,t) - p_n^{\text{s,RTM}}(h,t), n \in \mathcal{N}^{\text{s}},$$
(5b)

$$\beta_n \Delta \omega(h,t) = \Delta p_n^{\text{inj}}(h,t), \, n \in \mathcal{N} \setminus \mathcal{N}^{\text{s}}, \qquad (5c)$$

$$p_n^{\min, \operatorname{RTM}}(h, t) \le p_n^{\operatorname{s, RTM}}(h, t) \le p_n^{\max, \operatorname{RTM}}(h, t),$$
$$n \in \mathcal{N}^{\operatorname{s}} \quad (5d)$$

constraints
$$(4a)-(4e)$$
, (5e)

where β_n is the frequency bias factor of bus $n \in \mathcal{N}$. Similarly, the market clearing prices are the dual variables associated with constraint (5b), denoted by $\lambda_n^{\text{RTM}}(h,t), n \in \mathcal{N}^{\text{s}}, t \in \mathcal{T}, h \in \mathcal{H}$. If the ESS at bus $n \in \mathcal{N}^{\text{s}}$ is selected by the system operator to provide a demand change $p_n^{\text{s,RTM}}(h,t)$ in time slot t of hour h, it will be paid at the market clearing price of $\lambda_n^{\text{RTM}}(h,t)$ for the regulation service.



Fig. 1. A bi-level model for the strategic ESS to determine its bids for frequency regulation service.

III. PROBLEM FORMULATION AND SOLUTION METHOD

In this section, we present the bi-level problem to determine the bids in the DAM for a *strategic* ESS which aims to maximize its revenue, considering the ESS also has the opportunity to participate in the RTM. The ESS at the upper level makes its bidding decision by anticipating the market clearing outcome of the system operator at the lower level. As illustrated in Fig. 1, the upper-level problem aims to maximize the revenue of the strategic ESS from providing the regulation service. Such decision-making problem of the ESS is subject to the lower-level problem that models the market clearing process of the system operator. The resulting market price and regulation capacity at the lower level are influenced by the ESS's decision made at the upper level, which in turn are used to determine the bids of the ESS for frequency regulation.

The bi-level model of the strategic ESS requires the bidding information from other competitive ESSs and the system frequency deviations, which may not be available to the ESS when making the bidding decision. However, by retrieving public market information from such as California Independent System Operator (CAISO) Open Access Same-Time Information System (OASIS) [15], scenarios can be generated to represent possible realizations. We assume that different ESSs submit different bids in terms of the price and quantity for frequency regulation, as their operation objectives and constraints are generally different. Moreover, we assume that all the ESSs will place their bids truthfully. This assumption is valid, as for example, CAISO will analyze the bids by using a market power mitigation (MPM) process before clearing the market. The MPM process will detect and counteract any actions of the market participants that are inconsistent with competitive markets [16].

A. Bi-Level Formulation

Assuming the strategic ESS is located at bus m and has perfect information about the other ESSs' bids and system frequency deviations. The strategic ESS aims to determine the bidding strategy in the DAM, such that its revenue from providing the regulation service in both the DAM and RTM can be maximized. Therefore, it solves the following problem to optimize its decision for the bids in the DAM

$$\begin{array}{l} \underset{\substack{p_{m}^{\text{s,DAM}}(h), p_{m}^{\text{s,RTM}}(h,t), \\ \alpha_{m}^{\text{DAM}}(h), \alpha_{m}^{\text{s,RTM}}(h,t), \\ t \in \mathcal{T}, h \in \mathcal{H}}}{\underset{m}{\text{maximize}}} \sum_{h=1}^{H} p_{m}^{\text{s,DAM}}(h) \lambda^{\text{DAM}}(h) \\ + \sum_{h=1}^{H} \sum_{t=1}^{T} p_{m}^{\text{s,RTM}}(h,t) \lambda_{m}^{\text{RTM}}(h,t) \end{array} \tag{6a}$$

subject to
$$p_m^{s}(h,t) = p_m^{s,DAM}(h) + p_m^{s,RTM}(h,t),$$

 $t \in \mathcal{T}, h \in \mathcal{H},$ (6b)
 $n^{s,\min} < n^s (h, t) < n^{s,\max}, t \in \mathcal{T}, h \in \mathcal{H}$ (6c)

$$E_m^{\rm s}(h,t) = E_m^{\rm s}(h,t-1) - p_m^{\rm s}(h,t)\Delta t,$$

$$t \in \mathcal{T}, h \in \mathcal{H}.$$
(6d)

$$E_{m}^{s}(h,0) = E_{m}^{s}(h-1,0) - \sum_{t=1}^{T} p_{m}^{s}(h-1,t)\Delta t, \ h \in \mathcal{H} \setminus \{1\},$$
(6c)
(6c)

$$0 \le E_m^{\mathrm{s}}(h,t) \le E_m^{\mathrm{s},\max}, \ t \in \mathcal{T} \cup \{0\}, \ h \in \mathcal{H},$$

(6f)

$$\underset{p_{n}^{s,\text{DAM}}(h),}{\operatorname{argmin}} \quad \sum_{h=1}^{H} \sum_{n \in \mathcal{N}^{s}} \alpha_{n}^{s,\text{DAM}}(h) p_{n}^{s,\text{DAM}}(h), \quad (\text{6g})$$

$$n \in \mathcal{N}^{\circ}, h \in \mathcal{H}$$

subject to constraints (3b)–(3c) (6)

subject to constraints
$$(3b)-(3c)$$
, (6h)

$$\underset{\substack{p_n^{s,\text{RTM}}(h,t), n \in \mathcal{N}^s \\ t \in \mathcal{T}, h \in \mathcal{H}}}{\operatorname{argmin}} \sum_{h=1}^{n} \sum_{t=1}^{r} \sum_{n \in \mathcal{N}^s} \alpha_n^{s,\text{RTM}}(h,t) p_n^{s,\text{RTM}}(h,t),$$
(6i)

subject to constraints (5b)–(5e) for time slots

$$t \in \mathcal{T}, h \in \mathcal{H}.$$
 (6j)

The upper-level problem (6a)-(6f) represents the revenue maximization of the strategic ESS at bus m. The lower-level problems (6g)-(6h) and (6i)-(6j) represent the market clearing of the system operator in the DAM and RTM, respectively. Note that problem (6) is nonconvex, which is difficult to solve.

B. Mixed-Integer Linear Programming Problem

Given that the lower-level problems are convex, problem (6) can be transformed into a mixed-integer linear programming problem (MILP) by replacing the lower-level problems with their corresponding Karush-Kuhn-Tucker (KKT) conditions and applying linearization techniques [17]. Let $\overline{\delta}_n^{\text{DAM}}(h)$ and $\underline{\delta}_n^{\text{DAM}}(h), n \in \mathcal{N}^{\text{s}}, h \in \mathcal{H}$ denote the Lagrange multipliers associated with constraint (3c). We also introduce the binary variables $\overline{\phi}_n^{\text{DAM}}(h), \underline{\phi}_n^{\text{DAM}}(h), n \in \mathcal{N}^{\text{s}}, h \in \mathcal{H}$, and a large constant M. Thus, the DAM problem (6g)–(6h) can be replaced by

$$\alpha_n^{\mathrm{s,DAM}}(h) - \lambda^{\mathrm{DAM}}(h) - \underline{\delta}_n^{\mathrm{DAM}}(h) + \overline{\delta}_n^{\mathrm{DAM}}(h) = 0, \ n \in \mathcal{N}^{\mathrm{s}},$$
(7a)

$$\underline{\delta}_{n}^{\text{DAM}}(h) \leq \left(1 - \underline{\phi}_{n}^{\text{DAM}}(h)\right) M, \ n \in \mathcal{N}^{\text{s}},\tag{7b}$$

$$p_n^{\text{s,DAM}}(h) - p_n^{\min,\text{DAM}}(h) \le \underline{\phi}_n^{\text{DAM}}(h)M, \ n \in \mathcal{N}^{\text{s}}, \tag{7c}$$

$$\overline{\delta}_{n}^{\text{DAM}}(h) \leq \left(1 - \overline{\phi}_{n}^{\text{DAM}}(h)\right) M, \ n \in \mathcal{N}^{\text{s}},\tag{7d}$$

$$p_n^{\max, \text{DAM}}(h) - p_n^{\text{s, DAM}}(h) \le \overline{\phi}_n^{\text{DAM}}(h)M, \ n \in \mathcal{N}^{\text{s}}, \tag{7e}$$

$$\frac{\delta_n^{\text{DAM}}(h)}{\delta_n^{\text{DAM}}(h)}, \overline{\delta}_n^{\text{DAM}}(h) \ge 0, \ n \in \mathcal{N}^{\text{s}}, \tag{7f}$$

$$\underline{\phi}_{n}^{\text{DAM}}(h), \overline{\phi}_{n}^{\text{DAM}}(h) \in \{0, 1\}, n \in \mathcal{N}^{\text{s}},$$
(7g)

Let $\underline{\delta}_n^{\text{RTM}}(h,t)$ and $\overline{\delta}_n^{\text{RTM}}(h,t)$, $n \in \mathcal{N}^{\text{s}}$ denote the Lagrange multipliers associated with constraint (5d). Let $\lambda_n^{q,\text{RTM}}(h,t)$, $\delta_{nj}^{\eta}(h,t)$, $(n,j) \in \mathcal{L}$, $\eta = \{0,1,\ldots,2\pi/\gamma\}$, $\underline{\delta}_n^{v}(h,t)$ and $\overline{\delta}_n^{v}(h,t)$, $n \in \mathcal{N}$ denote the Lagrange multipliers associated with constraints (4a), (4d), and (4e), respectively. Let $\underline{\phi}_n^{\text{RTM}}(h,t), \overline{\phi}_n^{\text{RTM}}(h,t), \underline{\phi}_n^{v}(h,t)$ and $\overline{\phi}_n^{v}(h,t)$ denote the binary variables for every bus $n \in \mathcal{N}$. Let $\phi_{nj}^{\eta}(h,t)$ denote the binary variables for every line $(n, j) \in \mathcal{L}$. Note that all these Lagrange multipliers and binary variables are defined for each time slot $t \in \mathcal{T}$ of hour $h \in \mathcal{H}$. Similarly, we can replace the RTM problem (6i)–(6j) by

$$\begin{aligned} &\alpha_n^{\mathrm{s},\mathrm{RTM}}(h,t) - \lambda_n^{\mathrm{RTM}}(h,t) - \underline{\delta}_n^{\mathrm{RTM}}(h,t) + \overline{\delta}_n^{\mathrm{RTM}}(h,t) = 0, \quad (8a) \\ &- \sum_{j \in \mathcal{N}} \lambda_j^{\mathrm{RTM}}(h,t) B'_{jn} + \sum_{j \in \mathcal{N}} \lambda_j^{q,\mathrm{RTM}}(h,t) G'_{jn} \\ &+ \sum_{\substack{j > n, \\ (n,j) \in \mathcal{L}}} \frac{X_{nj} \mathrm{cos}(\eta \gamma) - R_{nj} \mathrm{sin}(\eta \gamma)}{R_{nj}^2 + X_{nj}^2} \delta_{nj}^{\eta}(h,t) \\ &- \sum_{\substack{j < n, \\ (n,j) \in \mathcal{L}}} \frac{X_{jn} \mathrm{cos}(\eta \gamma) - R_{jn} \mathrm{sin}(\eta \gamma)}{R_{jn}^2 + X_{jn}^2} \delta_{jn}^{\eta}(h,t) = 0, \quad (8b) \\ &- \sum_{\substack{\lambda_j^{\mathrm{RTM}}(h,t) \in \mathcal{L}}} \lambda_j^{\mathrm{RTM}}(h,t) G_{jn} - \sum_{\substack{\lambda_j^{q,\mathrm{RTM}}(h,t) \in \mathcal{L}}} \lambda_j^{q,\mathrm{RTM}}(h,t) B_{jn} \end{aligned}$$

$$\sum_{j \in \mathcal{N}} \lambda_{j} - (n,t) \mathcal{L}_{jn} - \sum_{j \in \mathcal{N}} \lambda_{j} - (n,t) \mathcal{L}_{jn} + \sum_{j \geq n, \atop (n,j) \in \mathcal{L}} \frac{R_{nj} \cos(\eta\gamma) + X_{nj} \sin(\eta\gamma)}{R_{nj}^{2} + X_{nj}^{2}} \delta_{nj}^{\eta}(h,t) - \sum_{j \geq n, \atop (n,j) \in \mathcal{L}} \frac{R_{jn} \cos(\eta\gamma) + X_{jn} \cos(\eta\gamma)}{R_{nj}^{2} + X_{nj}^{2}} \delta_{jn}^{\eta}(h,t)$$

$$-\sum_{\substack{j < n, \\ (n,j) \in \mathcal{L}}} \frac{-jn^{-v}(\mu) + jn^{-v}(\mu)}{R_{jn}^2 + X_{jn}^2} \delta_{jn}^{\prime\prime}(h,t)$$
$$-\delta^{\prime\prime}(h,t) + \overline{\delta}^{\prime\prime}(h,t) = 0$$
(8c)

$$-\underline{\delta}_{n}^{*}(h,t) + \delta_{n}(h,t) = 0, \tag{8c}$$

$$\underline{\delta}_{n}^{\mathsf{KIM}}(h,t) \leq \left(1 - \underline{\phi}_{n}^{\mathsf{KIM}}(h,t)\right) M,\tag{8d}$$

$$p_n^{\text{s,RTM}}(h,t) - p_n^{\min,\text{RTM}}(h,t) \le \underline{\phi}_n^{\text{RTM}}(h,t)M, \tag{8e}$$

$$\overline{\delta}_{n}^{\text{RTM}}(h,t) \le \left(1 - \overline{\phi}_{n}^{\text{RTM}}(h,t)\right) M,\tag{8f}$$

$$p_n^{\max, \operatorname{RTM}}(h, t) - p_n^{\operatorname{s, RTM}}(h, t) \le \overline{\phi}_n^{\operatorname{RTM}}(h, t) M, \tag{8g}$$

$$\underline{\delta}_{n}^{v}(h,t) \leq \left(1 - \underline{\phi}_{n}^{v}(h,t)\right) M,\tag{8h}$$

$$|v_n(h,t)| - v_n^{\min} \le \underline{\phi}_n^v(h,t)M, \tag{8i}$$

$$\delta_n(h,t) \le \left(1 - \phi_n(h,t)\right) M, \tag{8j}$$

$$\begin{aligned} v_n^{m-1} &= |v_n(h,t)| \le \phi_n(h,t)M, \\ \delta_{ni}^{\eta}(h,t) \le \left(1 - \phi_{ni}^{\eta}(h,t)\right)M, \end{aligned} \tag{8k} \\
\delta_{ni}^{\eta}(h,t) \le \left(1 - \phi_{ni}^{\eta}(h,t)\right)M, \end{aligned} \tag{8l}$$

$$\frac{R_{nj}\left(|v_n(h,t)| - |v_j(h,t)|\right) + X_{nj}(\theta_n(h,t) - \theta_j(h,t))}{R_{nj}^2 + X_{nj}^2} \cos(\eta\gamma)$$

$$+\frac{X_{nj}\left(|v_{n}(h,t)|-|v_{j}(h,t)|\right)-R_{nj}(\theta_{n}(h,t)-\theta_{j}(h,t))}{R_{nj}^{2}+X_{nj}^{2}}\sin(\eta\gamma)$$

$$-s_{nj}^{\max} \le \phi_{nj}^{\eta}(h,t)M,\tag{8m}$$

The nonlinear objective of the revenue in the DAM can be replaced by

$$p_{m}^{\text{s,DAM}}(h)\lambda^{\text{DAM}}(h) = \lambda^{\text{DAM}}(h)p_{n}^{\text{req}}(h) + \sum_{n \in \mathcal{N}^{\text{s}} \setminus \{m\}} \underline{\delta}_{n}^{\text{DAM}}(h)p_{n}^{\text{min,DAM}}(h) - \sum_{n \in \mathcal{N}^{\text{s}} \setminus \{m\}} \overline{\delta}_{n}^{\text{DAM}}(h)p_{n}^{\text{s,DAM}}(h) - \sum_{n \in \mathcal{N}^{\text{s}} \setminus \{m\}} \alpha_{n}^{\text{s,DAM}}(h)p_{n}^{\text{s,DAM}}(h).$$
(9)

Similarly, the nonlinear objective of the revenue in the RTM can be replaced by

$$\begin{split} p_{m}^{\text{s,RTM}}(h,t)\lambda_{m}^{\text{RTM}}(h,t) \\ &= \sum_{n \in \mathcal{N}} \left(\lambda_{n}^{\text{RTM}}(h,t) p_{n}^{\text{load}}(h,t) + \lambda_{n}^{q,\text{RTM}}(h,t) q_{n}^{\text{load}}(h,t) \right) \\ &+ \sum_{n \in \mathcal{N}} \left(\underline{\delta}_{n}^{v}(h,t) v_{n}^{\min} - \overline{\delta}_{n}^{v}(h,t) v_{n}^{\max} - \beta_{n} \Delta \omega(h,t) \right) \\ &+ \sum_{n \in \mathcal{N}^{s} \setminus \{m\}} \underline{\delta}_{n}^{\text{RTM}}(h,t) p_{n}^{\min,\text{RTM}}(h,t) - \overline{\delta}_{n}^{\text{RTM}}(h,t) p_{n}^{\max,\text{RTM}}(h,t) \\ &+ \sum_{(n,j) \in \mathcal{L}} \underline{\delta}_{nj}^{\eta}(h,t) s_{nj}^{\max} - \sum_{n \in \mathcal{N}^{s} \setminus \{m\}} \alpha_{n}^{s,\text{RTM}}(h,t) p_{n}^{s,\text{RTM}}(h,t). \end{split}$$
 (10)

By replacing (6a) with (9) and (10), replacing (6g)–(6h) with (7a)-(7h), as well as replacing (6i)–(6j) with (8a)–(8n), problem (6) is transformed into a MILP, which can be straightforwardly solved.

C. Consider the Uncertainty

The bidding decisions of the strategic ESS are made by anticipating the market clearing outcome in the DAM and RTM, which depends on the other competitive ESSs' bids and system frequency deviations. In general, such information is not available to the ESS when submitting the bids in the DAM. Thus, we extend the bi-level problem by considering a stochastic formulation that applies a scenario-based approach to model the possible realizations. Let $k \in \mathcal{K} = \{1, \ldots, K\}$ and Π_k denote the scenario index and the realization probability, respectively. The lower-level problem of the market clearing in the DAM of scenario $k \in \mathcal{K}$ can be given by

$$\underset{\substack{p_{nk}^{\text{s,DAM}}(h),\\n \in \mathcal{N}^{\text{s}}, h \in \mathcal{H},}}{\operatorname{argmin}} \quad \sum_{h=1}^{H} \sum_{n \in \mathcal{N}^{\text{s}}} \alpha_{nk}^{\text{s,DAM}}(h) p_{nk}^{\text{s,DAM}}(h),$$
(11a)

subject to

to
$$\sum_{n \in \mathcal{N}^{s}} p_{nk}^{s, \text{DAM}}(h) = p_{k}^{\text{req}}(h), \ h \in \mathcal{H},$$
(11b)
$$p_{nk}^{\min, \text{DAM}}(h) \le p_{nk}^{s, \text{DAM}}(h) \le p_{nk}^{\max, \text{DAM}}(h),$$
$$n \in \mathcal{N}^{s}, \ h \in \mathcal{H}.$$
(11c)

The lower-level problem of the market clearing in the RTM of scenario $k \in \mathcal{K}$ can be given by

$$\underset{\substack{p_{nk}^{\text{s,RTM}}(h,t), n \in \mathcal{N}^{\text{s}}, \\ t \in \mathcal{T}, h \in \mathcal{H}, }}{\operatorname{argmin}} \sum_{h=1}^{H} \sum_{t=1}^{T} \sum_{n \in \mathcal{N}^{\text{s}}} \alpha_{nk}^{\text{s,RTM}}(h,t) p_{nk}^{\text{s,RTM}}(h,t), \quad (12a)$$

subject to
$$\beta_n \Delta \omega_k(h,t) = \Delta p_{nk}^{\text{inj}}(h,t) - p_{nk}^{\text{s,RIM}}(h,t)$$

$$\begin{split} n \in \mathcal{N}^{\mathrm{s}}, t \in \mathcal{T}, h \in \mathcal{H}, \quad (12\mathrm{b}) \\ \beta_n \Delta \omega_k(h, t) &= \Delta p_{nk}^{\mathrm{inj}}(h, t), n \in \mathcal{N} \setminus \mathcal{N}^{\mathrm{s}}, \\ t \in \mathcal{T}, h \in \mathcal{H}, \quad (12\mathrm{c}) \\ p_{nk}^{\mathrm{min}, \mathrm{RTM}}(h, t) &\leq p_{nk}^{\mathrm{s}, \mathrm{RTM}}(h, t) \leq p_{nk}^{\mathrm{max}, \mathrm{RTM}}(h, t), \\ n \in \mathcal{N}^{\mathrm{s}}, t \in \mathcal{T}, h \in \mathcal{H}, \quad (12\mathrm{d}) \\ \mathrm{distribution network constraints for time slots} \end{split}$$

$$t \in \mathcal{T}, \ h \in \mathcal{H},\tag{12e}$$

where the distribution network constraints of scenario $k \in \mathcal{K}$ can be given by

$$\begin{bmatrix} \boldsymbol{p}_{k}^{\text{inj}}(h,t) \\ \boldsymbol{q}_{k}^{\text{inj}}(h,t) \end{bmatrix} = \begin{bmatrix} -\boldsymbol{B}' & \boldsymbol{G}' \\ -\boldsymbol{G} & -\boldsymbol{B} \end{bmatrix} \begin{bmatrix} \boldsymbol{\theta}_{k}(h,t) \\ \boldsymbol{v}_{k}(h,t) \end{bmatrix}, \quad (13a)$$
$$p_{nik}(h,t) =$$

$$\frac{R_{nj}\left(|v_{nk}(h,t)| - |v_{jk}(h,t)|\right) + X_{nj}(\theta_{nk}(h,t) - \theta_{jk}(h,t))}{R_{nj}^2 + X_{nj}^2},$$
(13b)

$$\frac{q_{njk}(h,t) =}{\frac{X_{nj}\left(|v_{nk}(h,t)| - |v_{jk}(h,t)|\right) - R_{nj}(\theta_{nk}(h,t) - \theta_{jk}(h,t))}{R_{nj}^2 + X_{nj}^2}},$$
(13c)

$$p_{njk}(h,t)\cos(\eta\gamma) + q_{njk}(h,t)\sin(\eta\gamma) \le s_{nj}^{\text{min}}, \quad (13d)$$

× .

$$v_n^{\min} \le |v_{nk}(h,t)| \le v_n^{\max}.$$
(13e)

Let $\delta_{kk'}^{\lambda}(h), \delta_{kk'}^{p}(h), \delta_{kk'}(h), \delta_{kk'}^{1}(h), \delta_{kk'}^{2}(h), h \in \mathcal{H}$ denote the binary variables. We have

$$\underset{\substack{p_{mk}^{\text{s,DAM}}(h), p_{mk}^{\text{s,RIM}}(h,t), \\ \alpha_{mk}^{\text{DAM}}(h), \alpha_{mk}^{\text{RIM}}(h,t), \\ t \in \mathcal{T}, h \in \mathcal{H}, k \in \mathcal{K}}}{\underset{k \in \mathcal{K}}{\overset{K}{\prod_{k=1}^{K} \prod_{k=1}^{K} \prod_{k=1}^{K} \prod_{k=1}^{K} p_{mk}^{\text{s,DAM}}(h) \lambda_{k}^{\text{DAM}}(h)}}$$

$$+\sum_{h=1}^{H}\sum_{t=1}^{I}p_{mk}^{s,\text{RTM}}(h,t)\lambda_{mk}^{\text{RTM}}(h,t)\right) \quad (14a)$$

subject to
$$p_{mk}^{s}(h,t) = p_{mk}^{s,DAM}(h) + p_{mk}^{s,RIM}(h,t),$$

 $t \in \mathcal{T}, h \in \mathcal{H}, k \in \mathcal{K},$ (14b)

$$p_m^{\text{s,min}} \le p_{mk}^{\text{s}}(h,t) \le p_m^{\text{s,max}},$$

$$\in \mathcal{T}, h \in \mathcal{H}, k \in \mathcal{K},$$
 (14c)

$$E_{mk}^{s}(h,t) = E_{mk}^{s}(h,t-1) - p_{mk}^{s}(h,t)\Delta t,$$

$$t \in \mathcal{T}, h \in \mathcal{H}, k \in \mathcal{K},$$
(14d)

$$E_{mk}^{s}(h,0) = E_{mk}^{s}(h-1,0) - \sum_{k=1}^{T} p_{mk}^{s}(h-1,t)\Delta t,$$

$$h \in \mathcal{H} \setminus \{1\}, \, k \in \mathcal{K}, \tag{14e}$$

$$0 \le E_{mk}^{s}(h,t) \le E_{m}^{s,\max}, t \in \mathcal{T} \cup \{0\}, h \in \mathcal{H},$$

$$k \in \mathcal{K}.$$
(14f)

DAM market clearing problem (11), $k \in \mathcal{K}$, (14g) RTM market clearing problem (12), $k \in \mathcal{K}$, (14h) $\lambda_k^{\text{DAM}}(h) - \lambda_{k'}^{\text{DAM}}(h) \le \delta_{kk'}^{\lambda}(h)M, \forall k > k',$ $k, k' \in \mathcal{K}$, (14i)



where constraints (14i)-(14m) ensure the nondecreasing characteristics of the DAM bidding curve, while constraints (14n)-(14s) force the non-anticipativity of the DAM decisions.

IV. CASE STUDIES

In this section, case studies of our proposed method are presented by considering ESSs in an IEEE 37-bus distribution test feeder [18]. The voltage magnitudes of the 37-bus test feeder are in per-unit (pu) with a 4.8 kV base. The base power of the system is in 100 kVA. The slack bus is the substation bus 37, i.e., its voltage magnitude is 1 pu and its phase angle is zero [10]. The frequency bias factor is set to be $\beta_n =$ 0, $n = 1, \ldots, 36$, and $\beta_{37} = 3.483$ [10]. We use frequency measurements from [19] to generate the required capacity and frequency deviations that the system operator aims to clear in the DAM and RTM. We consider 5 ESSs located at buses 2, 13, 17, 23, and 33 to participate in frequency regulation, among which the ESS at bus 2 is the strategic ESS of our interest to design the bids. We use the quarterly and weighted average prices in CAISO DAM and RTM markets to generate the bidding prices for other competitive ESSs [20], which we consider 20 scenarios with equal realization probability.

We first present the result for how the bidding strategy of the strategic ESS by using our proposed method can affect the market clearing prices, as shown in Fig. 2. We consider the deterministic case where the ESS at bus 2 has the complete knowledge about the system operator's requirement as well as other ESSs' bidding information in the DAM and RTM. We generate two cases where the system operator has the same requirement for frequency regulation, but receives different bidding prices from other competitive ESSs. Based



Fig. 2. Impact on DAM market clearing prices with 5 ESSs $E_2^{\text{max}} = 150 \text{ kWh}$, $E_{13}^{\text{max}} = 50 \text{ kWh}$, $E_{17}^{\text{max}} = 100 \text{ kWh}$, $E_{23}^{\text{max}} = 50 \text{ kWh}$, and $E_{33}^{\text{max}} = 100 \text{ kWh}$ when ESS at bus 2 participates strategically and competitively. For cases a) and b), the system operator has the same requirement for frequency regulation but receives different bids from other competitive ESSs.

on economies of scale, we associate higher prices with ESSs of smaller capacity when generating the bids. For both cases, we compare the market clearing prices resulted from when the ESS at bus 2 strategically determines the bids by using our proposed method and when it participates as other competitive ESSs. It can be observed that the ESS at bus 2 manages to increase the market clearing prices of certain hours in the DAM in both cases. This is due to the fact that the system operator needs to fully procure the required frequency regulation capacity in the DAM. The ESS at bus 2 thus is able to strategically bid into the regulation markets to increase the market prices, considering the bidding price and capability of other competitive ESSs which also participate in the regulation service. It can also be observed that the hours when the market clearing price increases vary drastically. For example, the market price of hour 5 in the DAM increases from 4 to 12 \$/kWh in Fig. 2(a), while it remains the same in Fig. 2(b). We note that the strategic ESS' bids can be largely affected by other competitive ESSs' bidding information, which will affect the resulted market clearing prices.

Next we present the results of the revenue for the ESS at bus 2 to provide the regulation service by using our proposed method, as shown in Fig. 3. We consider the stochastic case where the ESS at bus 2 is uncertain about the system operator's requirement as well as other ESSs' bidding information in the



DAM and RTM. We compare the revenues of the ESS at bus 2 for the case that it strategically submits the bids and that it participates as other competitive ESSs, when different number of ESSs participate in the DAM and RTM for providing the regulation service. It can be observed that the ESS at bus 2 can increase its revenue by strategically bidding into the frequency regulation markets in all three cases. However, such strategic bidding gain for the ESS at bus 2 shrinks drastically as the number of ESSs that participate in the regulation service increases. The strategic bidding gain for the ESS at bus 2 is significantly higher if fewer ESSs compete in providing the regulation service. This indicates the large market power permits the ESS at bus 2 to better maximize its revenue from providing frequency regulation service. Overall, we conclude that our proposed method is effective in increasing the revenue that the strategic ESS can receive from participating in the regulation service.

V. CONCLUSION

In this paper, we have examined using the ESSs at the distribution level to provide frequency regulation service for the system operator under the two-settlement market mechanism. We have formulated a bi-level problem for a strategic ESS to determine the bids. The formulated problem includes the operation constraints imposed by the distribution network and the uncertainty associated with other ESSs' bids. The upper level problem maximizes the revenue of the strategic ESS for providing the regulation service, which depends on the market prices cleared from the lower-level problem by the system operator. The case studies for an IEEE 37-bus test feeder with five ESSs participating in the regulation service validate the effectiveness of our proposed method. Results demonstrate that the strategic ESS manages to increase its revenue by exercising the market power to increase the market prices. Further research is needed to examine the strategic collaboration among the ESSs in the distribution network and the impact on the market price for frequency regulation service.

REFERENCES

- M. Kintner-Meyer, "Regulatory policy and markets for energy storage in North America," *Proceedings of the IEEE*, vol. 102, no. 7, pp. 1065– 1072, Jul. 2014.
- [2] U.S. Federal Energy Regulatory Commission (FERC), "Electric storage participation in markets organized by regional transmission organizations and independent system operators," *Washington, DC, FERC 841, Dockets RM16-23-000 AD16-20-000*, Feb. 2018.
- [3] X. Xie, Y. Guo, B. Wang, Y. Dong, L. Mou, and F. Xue, "Improving AGC performance of coal-fueled thermal generators using multi-MW scale BESS: A practical application," *IEEE Trans. on Smart Grid*, vol. 9, no. 3, pp. 1769–1777, May 2018.
- [4] H. Zhao, M. Hong, W. Lin, and K. A. Loparo, "Voltage and frequency regulation of microgrid with battery energy storage systems," *IEEE Trans. on Smart Grid*, vol. 10, no. 1, pp. 414–424, Jan. 2019.
- [5] D. Fooladivanda, C. Rosenberg, and S. Garg, "Energy storage and regulation: An analysis," *IEEE Trans. on Smart Grid*, vol. 7, no. 4, pp. 1813–1823, Jul. 2016.
- [6] V. Knap, S. K. Chaudhary, D. Stroe, M. Swierczynski, B. Craciun, and R. Teodorescu, "Sizing of an energy storage system for grid inertial response and primary frequency reserve," *IEEE Trans. on Power Systems*, vol. 31, no. 5, pp. 3447–3456, Sept. 2016.
- [7] Y. Kim, G. Del-Rosario-Calaf, and L. K. Norford, "Analysis and experimental implementation of grid frequency regulation using behindthe-meter batteries compensating for fast load demand variations," *IEEE Trans. on Power Systems*, vol. 32, no. 1, pp. 484–498, Jan. 2017.
- [8] H. Hao, D. Wu, J. Lian, and T. Yang, "Optimal coordination of building loads and energy storage for power grid and end user services," *IEEE Trans. on Smart Grid*, vol. 9, no. 5, pp. 4335–4345, Sep. 2018.
- [9] D. Zhu and Y. A. Zhang, "Optimal coordinated control of multiple battery energy storage systems for primary frequency regulation," *IEEE Trans. on Power Systems*, vol. 34, no. 1, pp. 555–565, Jan. 2019.
- [10] Y. Sun, S. Bahrami, V.W.S. Wong, and L. Lampe, "Application of energy storage systems for frequency regulation service," in *Proc. of IEEE SmartGridComm*, Dresden, Germany, Oct. 2017.
- [11] B. Xu, Y. Dvorkin, D. S. Kirschen, C. A. Silva-Monroy, and J.-P. Watson, "A comparison of policies on the participation of storage in US frequency regulation markets," in *Proc. of IEEE Power and Energy Society General Meeting*, Boston, MA, Jul. 2016.
- [12] H. Mohsenian-Rad, "Coordinated price-maker operation of large energy storage units in nodal energy markets," *IEEE Trans. on Power Systems*, vol. 31, no. 1, pp. 786–797, Jan. 2016.
- [13] M. Yazdani-Damavandi and N. Neyestani and M. Shafie-khah and J. Contreras and J. P. S. Catalão, "Strategic behavior of multi-energy players in electricity markets as aggregators of demand side resources using a bi-level approach," *IEEE Trans. on Power Systems*, vol. 33, no. 1, pp. 397–411, Jan. 2018.
- [14] S. Stoft, Power System Economics: Designing Markets for Electricity. Wiley-IEEE Press, 2002.
- [15] California ISO, "Open Access Same-Time Information System (OASIS)." [Online]. Available: http://oasis.caiso.com/mrioasis/logon.do
- [16] —, "Operating procedure: Day-Ahead market," Jan. 2019. [Online]. Available: http://www.caiso.com/documents/1210.pdf
- [17] C. Ruiz and A. J. Conejo, "Pool strategy of a producer with endogenous formation of locational marginal prices," *IEEE Trans. on Power Systems*, vol. 24, no. 4, pp. 1855–1866, Nov. 2009.
- [18] "IEEE Power and Energy Society distribution test feeders," 2016. [Online]. Available: http://ewh.ieee.org/soc/pes/dsacom/testfeeders/ index.html
- [19] FINGRID, "Frequency measurement data." [Online]. Available: https://data.fingrid.fi/en/dataset/frequency-historical-data
- [20] California ISO, "Annual and quarterly reports." [Online]. Available: http://www.caiso.com/market/Pages/MarketMonitoring/ AnnualQuarterlyReports/Default.aspx