Adaptive Energy Consumption Scheduling with Load Uncertainty for the Smart Grid

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***Abstract*—In this paper, we propose a novel real-time energy consumption scheduling algorithm that takes into account *load uncertainty* to minimize the energy payment for each user. We formulate the problem of load scheduling as an optimization problem. To reduce the computational complexity, we devise an *approximate dynamic programming* approach to schedule the operation of appliances. In our problem formulation, we consider different sets of appliances including *must-run* and *controllable*. Unlike most of the existing demand side management algorithms that assume perfect knowledge of users' energy needs, our design only requires knowledge of some estimates of the future demand. Simulation results confirm that the proposed energy scheduling algorithm can benefit both the users by reducing their energy expenses and the utility companies by improving the peak-to-average ratio in load demand.**

I. Introduction

To enable more efficient grid operation, different programs have been proposed to shape the daily energy consumption pattern of the users. These programs are referred to as demand side management (DSM). Among different techniques considered for DSM (e.g., voluntary load management programs [1]–[3] and direct load control [4]), *smart pricing* is known as an effective means to encourage users to consume wisely and more efficiently. By reflecting the hourly changes in the wholesale electricity price to the demand side, users pay what the electricity is worth at different times of day. They are encouraged to reduce their load at peak hours. In general, it is difficult for consumers to follow the real time prices and respond to their frequent variations. This aspect and other issues of manual control of appliances are discussed in [5], [6]. Another approach is to equip users with automated control units that respond to real-time price signals. Methods to avoid efficiency loss in the system due to enhanced rationality levels of users has been discussed in [7]. The effect of *load synchronization*, i.e., the concentration of a large portion of energy consumption in low-price hours, has been studied in [8]. Load synchronization can be avoided by adopting pricing tariffs with *inclining block rates* (IBRs), where the marginal price increases when the load increases [9].

Most of the existing work in the DSM literature assumes that the list of all appliances to be scheduled and the price information are known a priori [7], [10]–[13]. Different methods have been proposed that consider the effect of price uncertainties [14], [15]. In this paper, we focus on developing an automated residential load scheduling algorithm in a retail electricity market that takes into account *load uncertainties*.

In general, making decisions about the operating state of different appliances for the current time slot depends on the information about the user's energy needs available at the current time slot and the *expected schedule* determined in the upcoming time slots. This usually includes solving a mixed-integer program [16]. However, in situations where computational complexity or convergence time of the algorithm are critical such as in real-time application, a suboptimal but faster scheme that establishes a balance between simple implementation and adequate performance is more desired.

In our study, we devise an approximate dynamic programing approach with elaborate mathematical analysis which takes into account estimates for the future load and has less computational complexity compared to the design in [16]. The contributions of this paper are as follows.

- We propose a real-time energy consumption scheduling algorithm which takes into account *load uncertainty* for DSM. Our algorithm is based on solving an approximate dynamic program to minimize users' electricity bill payments. Each appliance sends an *admission request* to the *energy consumption control* (ECC) unit. The start operation of each appliance is subject to the decision made by the ECC unit. By running a centralized algorithm, the ECC unit determines the optimal operation schedule of each appliance in each time slot.
- We study operation constraints to model a variety of appliances including *must-run* and *controllable* appliances.
- Simulation results show that our proposed adaptive scheduling algorithm reduces the energy payment of the users in presence of load uncertainty, compared to the case where no scheduling algorithm is adopted. It also improves the overall power system performance by reducing the peak-to-average ratio (PAR) in aggregate load demand. Compared to [16], our method has much less complexity while [16] has a slightly better performance.

The rest of this paper is organized as follows. The system model is introduced in Section II. The problem formulation and the description of the proposed load scheduling algorithm are presented in Section III. Simulation results are provided in Section IV. The paper is concluded in Section V.

II. System Model

Consider a residential unit that participates in a DSM program. This unit is equipped with an ECC device to schedule and adjust the household energy consumption. Let $\mathcal{A}$ denote the set of appliances. Each appliance $a \in \mathcal{A}$ is either *must-run* or *controllable*. Must-run appliances such as TV and PC need to start working immediately. In contrast, the operation of controllable appliances can be *delayed* or *interrupted* if necessary. Plug-in electric vehicle (PEV) and washing machine are two examples of interruptible and non-interruptible controllable appliances, respectively.

We divide the intended operation cycle into T time slots. Each time slot begins with an *admission control* phase. In this phase, to start the operation of an appliance, an *admission request* is sent to the ECC unit. Once an admission request is submitted, the state of the appliance changes from *sleep* to *awake*. The appliance remains awake until its operation is *finished*. However, the *operation* of an awake appliance is subject to the acceptance of its admission request and specification of its operation schedule by the ECC unit. The decisions regarding the admission of the requests and the adjustment of the operation of different awake appliances are updated periodically in each admission control phase.

An awake appliance a can be either *inactive* (with zero power consumption) or *active* (operating at nominal power γ_a). Different operating states of must-run and controllable appliances are shown in Fig. 1. The admission request of each appliance a specifies the total energy E_a needed to finish the operation of the appliance, the operating power γ_a, and whether the appliance is must-run or controllable. For controllable appliances, the deadline before which the operation of the appliance has to be finished, denoted by β_a, and whether it is interruptible or not, are the additional information to be included in the admission request submitted by the appliance. For a controllable appliance a, if it is non-interruptible, the ECC may only delay its operation. However, for interruptible appliances, the operation can not only be postponed but also be interrupted and later restored, if needed.

We define binary variable $x_t^a \in \{0,1\}$ as the state of power consumption of appliance $a \in \mathcal{A}$ at time slot $t \in \{1,\ldots,T\}$. We set $x_t^a = 1$ if appliance a is admitted to operate at time slot t (i.e., *active*), otherwise, we set $x_t^a = 0$ (i.e., *inactive*). Let E_t^a denote the remaining amount of energy required to finish the operation of appliance a when the current time slot is t. Given E_t^a, for each future time slot $k > t > 0$, we have

$$E_k^a = \left[E_t^a - \gamma_a \sum_{i=t}^{k-1} x_i^a \right]^+ . \tag{1}$$

For each *non-interruptible* controllable appliance a that is active at current time slot t, we have

$$x_k^a = 1, \quad \forall\, k \in \{t,\ldots,\beta_a\},\ 0 < E_k^a < E_a. \tag{2}$$

Let $l_t \triangleq \sum_{a\in\mathcal{A}} \gamma_a x_t^a$ denote the total household power consumption at time slot t. We consider a pricing function $\lambda_t(l_t)$ which represents the price of electricity at each time slot t as a function of the user's total power consumption at that time slot l_t. For combined real-time pricing (RTP) and IBR pricing tariffs, the price function $\lambda_t(l_t)$ is defined as [8]:

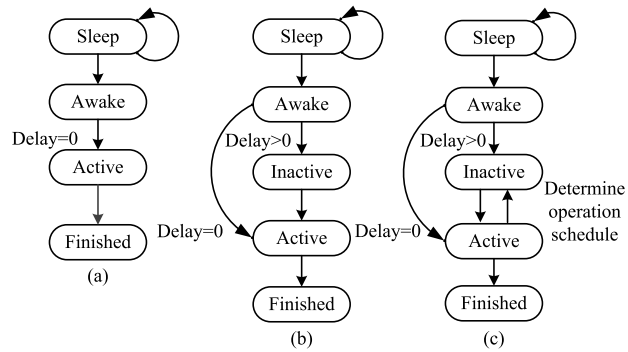

Fig. 1. Different operating states of (a) must-run, (b) non-interruptible controllable, and (c) interruptible controllable appliances.

$$\lambda_t(l_t) = \begin{cases} m_t, & \text{if } \ 0 \le l_t \le b_t, \\ n_t, & \text{if } \ l_t > b_t, \end{cases} \tag{3}$$

where m_t, n_t, and b_t are price parameters, and $m_t \le n_t$.

III. Problem Formulation and Algorithm Description

In this section, we consider the problem of *efficient power scheduling* such that the electricity payment of each user is minimized. We assume that only some statistical demand information are known ahead of time. The *demand information*, i.e., the information about the list of appliances that are awake at each time slot, whether they are must-run or controllable, and the deadline by which the operation of each appliance should be finished is revealed only gradually over time. An update is given at the beginning of each time slot, and the operation schedule of each appliance is adapted accordingly.

We define the *state* of the system at each time slot t as $\mathbb{S}_t \triangleq (\mathbf{E}_t, \mathbf{I}_t)$, where $\mathbf{E}_t \triangleq (E_t^1,\ldots,E_t^{|\mathcal{A}|})$, and $\mathbf{I}_t \triangleq (\tilde{\mathcal{M}}_t, \tilde{\mathcal{C}}_t, \mathcal{S}_t, \lambda_t(\cdot))$. The definitions of the sets of appliances $\tilde{\mathcal{M}}_t$, $\tilde{\mathcal{C}}_t$, and $\mathcal{S}_t$ are presented in Table I. At each time slot t, we seek to minimize the *expected* energy payment of the user with respect to demand uncertainties:

$$\underset{\substack{\mathbf{x}_t^a,\ \forall\, a\in\tilde{\mathcal{C}}_k,\\ \forall\, k\in\{t,\ldots,T\}}}{\text{minimize}} \quad g_t\big(\mathbb{S}_t, L_t\big) + \mathbb{E}\left\{ \sum_{k=t+1}^{T} g_k\big(\mathbb{S}_k, L_k \mid \mathbf{I}_t\big) \right\} \tag{4}$$

$$\begin{aligned} \text{subject to} \ \ & x_k^a \in \{0,1\}, \quad \forall\, a\in\tilde{\mathcal{C}}_k,\ \forall\, k\in\{t,\ldots,T\}, \\ & \gamma_a \sum_{i=k}^{\beta_a} x_i^a = E_k^a,\ \forall\, a\in\tilde{\mathcal{C}}_k,\ \forall\, k\in\{t,\ldots,T\}, \\ & x_k^a = 1, \quad \forall\, a\in\tilde{\mathcal{N}}_k, \forall\, k\in\{t,\ldots,\beta_a\}, \\ & \qquad\qquad 0 < E_k^a < E_a, \end{aligned}$$

where $\mathbb{E}\{\cdot\}$ denotes mathematical expectation and we have

$$g_t\big(\mathbb{S}_t, L_t\big) \triangleq L_t \lambda_t\big(L_t\big), \tag{5}$$

$$g_k\big(\mathbb{S}_k, L_k \mid \mathbf{I}_t\big) \triangleq L_{k,t} \lambda_k\big(L_{k,t}\big), \tag{6}$$

TABLE I
NOTATIONS USED FOR DIFFERENT SETS OF APPLIANCES.

$\mathcal{M}_{k,t}$	Must-run appliances that are awake at time slot t and remain awake at time slot $k \geq t$.
$\hat{\mathcal{M}}_{k,t}$	Must-run appliances that are asleep at time slot t and will be awake at time slot $k \geq t$.
$\tilde{\mathcal{M}}_k$	All must-run appliances that are awake at time slot k ($\mathcal{M}_{k,t} \cup \hat{\mathcal{M}}_{k,t}$).
$\mathcal{C}_{k,t}$	Controllable appliances that are awake at time slot t and remain awake at time slot $k \geq t$.
$\hat{\mathcal{C}}_{k,t}$	Controllable appliances that are asleep at time slot t and will be awake at time slot $k \geq t$.
$\tilde{\mathcal{C}}_k$	All controllable appliances that are awake at time slot k ($\mathcal{C}_{k,t} \cup \hat{\mathcal{C}}_{k,t}$).
$\mathcal{N}_{k,t}$	Non-interruptible appliances in $\mathcal{C}_{k,t}$.
$\hat{\mathcal{N}}_{k,t}$	Non-interruptible appliances in $\hat{\mathcal{C}}_{k,t}$.
$\tilde{\mathcal{N}}_k$	All non-interruptible controllable appliances that are awake at time slot k ($\mathcal{N}_{k,t} \cup \hat{\mathcal{N}}_{k,t}$).
$\mathcal{S}_t$	All appliances that are sleeping at time slot t.

$$L_t = \sum_{a \in \tilde{\mathcal{M}}_t} \gamma_a + \sum_{a \in \tilde{\mathcal{C}}_t} \gamma_a x_t^a, \tag{7}$$

$$L_{k,t} = \sum_{a \in \mathcal{M}_{k,t}} \gamma_a + \sum_{a \in \hat{\mathcal{M}}_{k,t}} \gamma_a + \sum_{a \in \mathcal{C}_{k,t}} \gamma_a x_k^a + \sum_{a \in \hat{\mathcal{C}}_{k,t}} \gamma_a x_k^a, \tag{8}$$

$\mathbf{x}_t^a \triangleq (x_t^a, \ldots, x_T^a)$, E_k^a is as in (1), and the definitions of the different sets of appliances $\mathcal{M}_{k,t}$, $\hat{\mathcal{M}}_{k,t}$, $\tilde{\mathcal{M}}_t$, $\mathcal{C}_{k,t}$, $\hat{\mathcal{C}}_{k,t}$, $\tilde{\mathcal{C}}_k$, and $\tilde{\mathcal{N}}_k$ are presented in Table I. We note that E_t^a is known at time slot t, must-run appliances are *active* as long as they are *awake*, $\mathcal{C}_{k,t} = \mathcal{C}_{t,t}$ for all $k \geq t$, and as the demand information is known up to time slot t, $\hat{\mathcal{M}}_{t,t} = \hat{\mathcal{C}}_{t,t} = \emptyset$. The first term in the objective function in (4) is the payment of the user in the current time slot t for the known load L_t, while the second term is the expected cost of energy in the upcoming time slots. We will refer to the latter as *cost-to-go* function. Each appliance can be either *on* or *off*. This is indicated by the first constraint. The second constraint implies that the operation of each appliance should be finished by its deadline. The last constraint guarantees that the operation of non-interruptible appliances will continue once they become active until they finish their job.

A. Approximate Dynamic Programming Approach

Problem (4) in its current form is difficult to solve as it requires the computation of the expected schedule for currently sleeping appliances. To tackle this problem, we minimize an *upper bound* of the objective function. That is, we assume that all appliances that become awake in future time slots are must-run appliances and we cannot control their operation to reduce the cost. Thus, we schedule the operation only for currently awake controllable appliances $\tilde{\mathcal{C}}_t$:

$$\underset{\mathbf{x}_t^a,\, \forall\, a \in \tilde{\mathcal{C}}_t}{\text{minimize}} \;\; g_t\big(\mathbb{S}_t, L_t\big) + \mathbb{E}\left\{ \sum_{k=t+1}^{T} g_k\big(\mathbb{S}_k, \bar{L}_k \mid \mathbf{I}_t\big) \right\} \tag{9}$$

$$\begin{aligned} \text{subject to} \;\; & x_k^a \in \{0,1\}, \quad \forall\, a \in \tilde{\mathcal{C}}_t, \forall\, k \in \{t, \ldots, T\}, \\ & \gamma_a \sum_{k=t}^{\beta_a} x_k^a = E_t^a, \quad \forall\, a \in \tilde{\mathcal{C}}_t, \\ & x_k^a = 1, \quad \forall\, a \in \tilde{\mathcal{N}}_t, \forall\, k \in \{t, \ldots, \beta_a\}, \\ & \qquad\qquad 0 < E_k^a < E_a, \end{aligned}$$

where $g_k\big(\mathbb{S}_k, \bar{L}_k \mid \mathbf{I}_t\big) \triangleq \bar{L}_{k,t} \lambda_k\big(\bar{L}_{k,t}\big)$, and

$$\bar{L}_{k,t} = \sum_{a \in \mathcal{M}_{k,t}} \gamma_a + \sum_{a \in \hat{\mathcal{M}}_{k,t} \cup \hat{\mathcal{C}}_{k,t}} \gamma_a + \sum_{a \in \mathcal{C}_{k,t}} \gamma_a x_k^a \tag{10}$$

denotes the expected load (on worst case) at time slot $k > t$.

Problem (9) determines the operating schedule for the current time slot as well as the operating schedule for future time slots to evaluate the cost-to-go. In its current form, problem (9) is difficult to solve as it requires the calculation of the expected value of the cost-to-go for a nonlinear function. The nonlinearity of the cost-to-go function couples the task of calculating the expectation with the task of calculating the operating schedule of controllable appliances.

To tackle these computational difficulties, we use the *certainty equivalent* approximation, i.e., all uncertainties are fixed at their expected value [17]. This technique allows to separate the task of scheduling the operation of controllable appliances from the task of calculating the expectation. Moreover, by fixing the uncertainties in their expected values, it is possible to transfer the whole problem into a linear mixed integer program. Linear mixed integer programs in general are known to be NP-complete, and the level of their computational complexity depends on the number of integer variables and the number of constraints. One promising technique to simplify the complexity of problem (9) is to approximate the cost-to-go by formulating a similar problem which has a less complex structure [17].

To this end, first, we relax the binary constraint on variables x_k^a and let $0 \leq x_k^a \leq 1$ for each $a \in \tilde{\mathcal{C}}_t$ and any $k \in \{t+1, \ldots, T\}$. We relax the constraint that in the upcoming time slots the operation of non-interruptible appliances should be continued if they start operation, i.e., we remove the last constraint. We note that as we mark the non-interruptible appliances as must-run if they start operation, the elimination of the last constraint does not affect the continuation of their operation. These changes significantly simplify the computational complexity of the problem while preserving its structure. Thus, we formulate the problem of power scheduling in the form of *approximate dynamic programming* and determine the power schedule for the current time slot as the solution of the following mixed-integer programming problem:

$$\underset{\mathbf{x}_t^a,\, \forall\, a \in \tilde{\mathcal{C}}_t}{\text{minimize}} \;\; g_t\big(\mathbb{S}_t, L_t\big) + \sum_{k=t+1}^{T} g_k\big(\mathbb{S}_k, \mathbb{E}\{\bar{L}_k \mid \mathbf{I}_t\}\big) \tag{11}$$

$$\text{subject to} \quad x_t^a \in \{0,1\}, \quad \forall\, a \in \tilde{\mathcal{C}}_t, \tag{12}$$

$$0 \le x_k^a \le 1, \; \forall\, a \in \tilde{\mathcal{C}}_t, \forall\, k \in \{t+1, \ldots, T\}, \tag{13}$$

$$\gamma_a \sum_{k=t}^{\beta_a} x_k^a = E_t^a, \quad \forall\, a \in \tilde{\mathcal{C}}_t. \tag{14}$$

For the price function in (3), since $m_t \le n_t$, for a total load l_t at time slot t, the user's payment $l_t \lambda_t(l_t)$ is determined as the maximum of two intersecting lines [8]:

$$l_t \lambda_t(l_t) = \max\big\{m_t l_t, \; n_t l_t + (m_t - n_t) b_t\big\}. \tag{15}$$

Therefore, problem (11) can be reformulated as

$$\begin{aligned}
\underset{\mathbf{x}_t^a, \, \forall\, a \in \tilde{\mathcal{C}}_t}{\text{minimize}} \quad & \max\Bigg\{ m_t \Bigg(\sum_{a \in \tilde{\mathcal{C}}_t} \gamma_a x_t^a + \sum_{a \in \tilde{\mathcal{M}}_t} \gamma_a \Bigg), \\
& \qquad n_t \Bigg(\sum_{a \in \tilde{\mathcal{C}}_t} \gamma_a x_t^a + \sum_{a \in \tilde{\mathcal{M}}_t} \gamma_a \Bigg) + (m_t - n_t) b_t \Bigg\} \\
& + \sum_{k=t+1}^{T} \max\Bigg\{ \mathbb{E}\Bigg\{ m_k \Bigg(\sum_{a \in \tilde{\mathcal{C}}_t} \gamma_a x_k^a + l_{k,t} \Bigg), \\
& \qquad n_k \Bigg(\sum_{a \in \tilde{\mathcal{C}}_t} \gamma_a x_k^a + l_{k,t} \Bigg) + (m_k - n_k) b_k \Bigg\} \Bigg\}
\end{aligned} \tag{16}$$

$$\text{subject to} \quad (12) - (14),$$

where

$$l_{k,t} \triangleq \sum_{a \in \mathcal{M}_{k,t}} \gamma_a + \sum_{a \in \hat{\mathcal{M}}_{k,t} \cup \hat{\mathcal{C}}_{k,t}} \gamma_a. \tag{17}$$

Finally, by introducing another auxiliary variable, ν_k, for each time slot k, we can re-write problem (16) as

$$\underset{\boldsymbol{\nu}_t, \mathbf{x}_t^a, \, \forall\, a \in \tilde{\mathcal{C}}_t}{\text{minimize}} \sum_{k=t}^{T} \nu_k \tag{18}$$

$$\text{subject to} \quad (12) - (14),$$

$$\begin{aligned}
& m_t \Bigg(\sum_{a \in \tilde{\mathcal{C}}_t} \gamma_a x_t^a + \sum_{a \in \tilde{\mathcal{M}}_t} \gamma_a \Bigg) \le \nu_t, \\
& n_t \Bigg(\sum_{a \in \tilde{\mathcal{C}}_t} \gamma_a x_t^a + \sum_{a \in \tilde{\mathcal{M}}_t} \gamma_a \Bigg) + (m_t - n_t) b_t \le \nu_t, \\
& m_k \Bigg(\sum_{a \in \tilde{\mathcal{C}}_t} \gamma_a x_k^a + \hat{l}_{k,t} \Bigg) \le \nu_k, \\
& \qquad\qquad \forall\, k \in \{t+1, \ldots, T\}, \\
& n_k \Bigg(\sum_{a \in \tilde{\mathcal{C}}_t} \gamma_a x_k^a + \hat{l}_{k,t} \Bigg) + (m_k - n_k) b_k \le \nu_k, \\
& \qquad\qquad \forall\, k \in \{t+1, \ldots, T\},
\end{aligned}$$

where $\boldsymbol{\nu}_t \triangleq (\nu_t, \ldots, \nu_T)$, and $\hat{l}_{k,t} \triangleq \mathbb{E}\big\{l_{k,t}\big\}$, the estimate of the power consumption of must-run appliances in an upcoming time slot $k \ge t$, will be calculated in the next sub-section.

Algorithm 1: Energy consumption scheduling algorithm in the presence of load uncertainty to be executed at the beginning of each time slot t.

1: Receive admission requests.
2: Label received requests either as must-run or controllable.
3: Activate must-run appliances (activate / remain active).
4: Update $\hat{l}_{\tau,t}$ according to (19).
5: Update E_k^a according to (1).
6: Solve (18) to activate / deactivate controllable appliances.
7: **if** activated device is non-interruptible
8: Mark it as must-run.
9: **end if**

Problem (18) is a mixed-binary linear program and can be solved efficiently by using MOSEK optimization software [18]. The solution of problem (18) determines the appropriate schedule for the operation of controllable appliances. However, for interruptible appliances, only the operation schedule of the current time slot t will be executed, and the schedule of the future time slots $t+1, \ldots, T$ may change when the optimization problem is solved again in the next time slot as new information about the future load becomes available.

B. Load Estimation

In our system model, we assume that the demand information of the appliances is *not* known ahead of time, i.e., in (17), the set of awake appliances in the upcoming time slots $k > t$ that are currently sleeping, i.e., set $\hat{\mathcal{M}}_{k,t} \cup \hat{\mathcal{C}}_{k,t}$, is not known. Instead, only the probability that a currently *sleeping* appliance will be *active* in time slot $k > t$, $\delta_{k,t}^a$, is known at the beginning of the current time slot t. Interested readers may refer to [16] for more information about how to calculate $\delta_{k,t}^a$. By conditioning on the event of observing a currently sleeping appliance *active* in an upcoming time slot k, while the system is at time slot t, the estimate of the power consumption required in (18) becomes:

$$\hat{l}_{k,t} = \mathbb{E}\left\{l_{k,t}\right\} = \sum_{a \in \mathcal{M}_{k,t}} \gamma_a + \sum_{a \in \mathcal{S}_t} \gamma_a \delta_{k,t}^a, \tag{19}$$

where $\mathcal{S}_t$ is defined in Table I.

C. Algorithm Description

In this section, we explain different steps of the proposed energy consumption scheduling algorithm (Algorithm 1) in presence of load uncertainty to be executed at each time slot t. At the beginning of the admission control phase at each time slot, all received admission requests are labeled as either *must-run* or *controllable* (Lines 1 and 2). Must-run appliances $a \in \tilde{\mathcal{M}}_t$ are activated right away (Line 3). That is, their operation starts or continues at the requested power γ_a. The current information is used to calculate the expected load in the upcoming time slots using (19) as indicated in Line 4. The remaining required energy of each appliance, E_k^a, is updated at the beginning of the current time slot using (1) (Line 5). Next, the "on" / "off" state of each awake controllable appliance

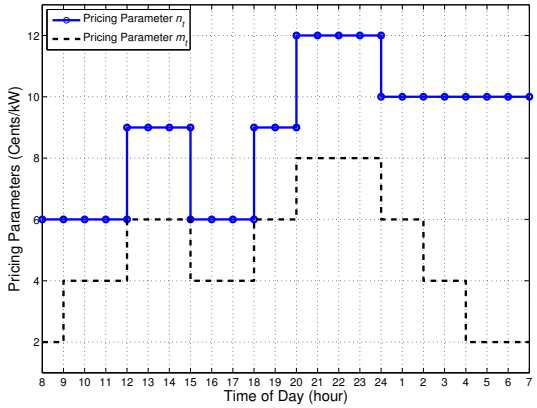

Fig. 2. The pricing parameters used based on the combined RTP and IBR pricing model in (3). Parameter $b_t = 3.5$ kW is fixed for all time slots.

is set for the rest of the time slots by solving problem (18) (Line 6). In Lines 7 to 9, if any non-interruptible controllable appliance becomes active (i.e., it switches from *off* to *on*), it is removed from the list of controllable appliances and it is added to the list of must-run devices as it should remain on until it finishes its operation.

IV. Performance Evaluation

In this section, we present simulation results and assess the performance of our proposed energy consumption scheduling algorithm. We run the simulation multiple times (i.e., 100 times) with different patterns for the times at which the appliances become awake. We then present the average results. Unless stated otherwise, the simulation setting is as follows. We assume that the general RTP method combined with IBR is adopted as in (3). Fig. 2 illustrates the variation of parameters m_t and n_t of the price function over one day. We consider a single household with various must-run and controllable appliances. Controllable appliances can be either interruptible or non-interruptible. Non-interruptible appliances include: Electric stove ($E_a = 4.5$ kWh, $\gamma_a = 1.5$ kW), clothes dryer ($E_a = 1$ kWh, $\gamma_a = 0.5$ kW), and vacuum cleaner ($E_a = 3$ kWh, $\gamma_a = 1.5$ kW). Interruptible appliances include: Refrigerator ($E_a = 2.5$ kWh, $\gamma_a = 0.125$ kW), air conditioner ($E_a = 6$ kWh, $\gamma_a = 1.5$ kW), dishwasher ($E_a = 2$ kWh, $\gamma_a = 1$ kW), heater ($E_a = 4$ kWh, $\gamma_a = 1$ kW), water heater ($E_a = 2$ kWh, $\gamma_a = 1$ kW), pool pump ($E_a = 4$ kWh, $\gamma_a = 2$ kW), and PEV ($E_a = 10$ kWh, $\gamma_a = 2.5$ kW). Must-run appliances include: Lighting ($E_a = 3$ kWh, $\gamma_a = 0.5$ kW), TV ($E_a = 1$ kWh, $\gamma_a = 0.25$ kW), PC ($E_a = 1.5$ kWh, $\gamma_a = 0.25$ kW), ironing appliance ($E_a = 2$ kWh, $\gamma_a = 1$ kW), hairdryer ($E_a = 1$ kWh, $\gamma_a = 1$ kW), and others ($E_a = 6$ kWh, $\gamma_a = 1.5$ kW). The time slot at which each appliance becomes awake is selected randomly from a pre-determined time interval, e.g. [6:00, 14:00] for electric stove and [16:00, 24:00] for PEV.

A. Performance Gains of Users and Utility Company

To have a baseline to compare with, we consider a system without ECC deployment, where each appliance a is assumed to start operation right after it becomes awake. Similar to [14], we consider a system in which the effect of IBR is ignored and only the basic price in each time slot is taken into account to schedule the operation of different appliances. As an upper bound, we also consider the scheme in [16] in which problem (9) is solved to schedule the operation of controllable appliances. In our simulation model, we set $b_t = 3.5$ kW in (3) for all time slots. Simulation results show that, to reduce electricity payment, the ECC unit shifts the load to time slots with lower prices such as the few first hours after midnight. However, the high price penalty for exceeding the b_t threshold prevents *load synchronization* as discussed in Section I. The simulation results show that exploiting the use of ECC unit reduces the average daily payment of the user from \$4.76 to \$4.10. If the effect of IBR is ignored, the average daily payment of the user is \$4.15. The average daily payment of the users for the load control algorithm in [16] is \$4.01. Our proposed algorithm also helps reduce the average PAR of the system from 2.64 to 2.47 (6.4% reduction) compared to the system without ECC deployment. The average PAR of the system in which the effect of IBR is ignored is 2.93. The average PAR of the system with ECE deployment as in [16] is 1.98. We can see that the efficiency loss in our proposed scheme compared to the one in [16] is insignificant. Yet, our design has less computational complexity as we explain next.

TABLE II
Performance measures of different algorithms.

Average run time of the algorithm (in seconds).

	$\|\mathcal{A}\|$=20	$\|\mathcal{A}\|$=25	$\|\mathcal{A}\|$=35
Proposed algorithm	0.7346	0.7769	0.7928
Algorithm in [16]	1.2066	6.5016	14.2728

Average number of integer variables.

	$\|\mathcal{A}\|$=20	$\|\mathcal{A}\|$=25	$\|\mathcal{A}\|$=35
Proposed algorithm	4	6	8
Algorithm in [16]	57	89	106

Average number of constraints.

	$\|\mathcal{A}\|$=20	$\|\mathcal{A}\|$=25	$\|\mathcal{A}\|$=35
Proposed algorithm	6	8	10
Algorithm in [16]	28	30	33

In general, integer programs with n integer variables and m constraints are known to be NP-complete. However, there exist pseudo-polynomial algorithms for solving $m \times n$ integer programs with fixed m which have the order of complexity of $O(n^{2m+2}(m\alpha)^{(m+1)(2m+1)} \log(n^2(m\alpha^2)^{2m+3}))$, where α is the maximum coefficient in the set of constraints [19]. A complete discussion of the complexity of the algorithms is out of the scope of this paper. However, to have a rough estimate on the order of complexity of our proposed algorithm compared to the one in [16], simulation results on the average run time of the algorithm, the number of integer variables, and the number of constraints for the proposed algorithm compared to the one in [16] are summarized in Table II.

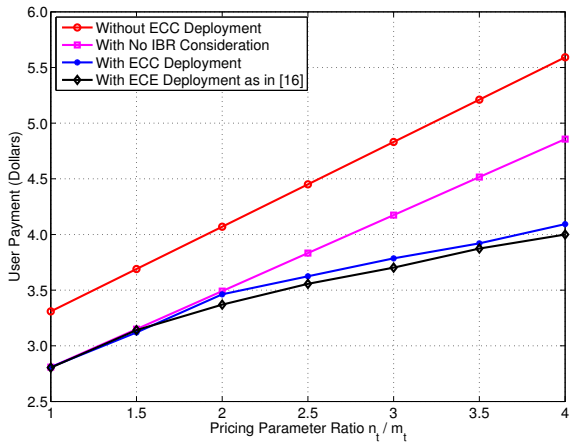

Fig. 3. The daily payment of the user for different values of n_t/m_t.

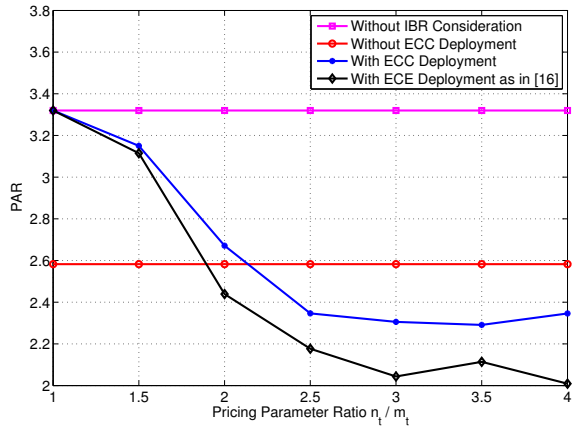

Fig. 4. PAR of the system for different values of n_t/m_t.

B. The Impact of Adopting Inclining Block Rates

In this section, we examine the impact of pricing parameters m_t and n_t on the performance of the system. In our simulation model, parameter m_t changes as shown in Fig. 2. We set $b_t =$ 3 kW for all time slots. Simulation results for the average daily payment of the user as well as the average PAR of the system for different choices of parameter ratio n_t/m_t are depicted in Figs. 3 and 4, respectively. By increasing the ratio n_t/m_t, the payment of the user will increase, as the user has to pay more every time its load exceeds threshold b_t as shown in Fig. 3. From Fig. 4, increasing n_t/m_t improves the PAR of the system, as load synchronization is prevented. However, for the system without IBR consideration, changes of the pricing parameter ratio n_t/m_t do not affect the PAR.

V. Conclusions

In this paper, we proposed a residential load control algorithm for DSM in presence of load uncertainty. We formulated the problem as an approximate dynamic program to minimize the electricity payment of users in situations where only an estimate of the future load is available. We employed RTP combined with IBRs to balance residential load to achieve a low PAR. Simulation results showed that our proposed algorithm reduces the energy cost of users, encouraging them to participate in DSM. Exploiting IBR with RTP tariffs can help avoid load synchronization, and the combination of the general RTP method with our algorithm can also reduce the PAR of the total load. The latter provides incentives for utilities to support implementing the proposed DSM algorithm.

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