AoI-Driven Client Scheduling for Federated Learning: A Lagrangian Index Approach

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***Abstract*—Federated learning (FL) is a distributed learning framework where clients jointly train a global model without sharing their local datasets. In randomized client sampling, a subset of clients are uniformly chosen to participate in training in each communication round of FL. Recent research has shown that by jointly considering the age of information (AoI) and channel state information (CSI) of each client, the convergence of FL can be improved. In this paper, we formulate a joint AoI and CSI-based client scheduling problem as a constrained Markov decision process. We propose a low-complexity and scalable algorithm based on the Lagrangian index approach. Simulation results show that the proposed Lagrangian index-based approach achieves near-optimal performance. For FL tasks with the CIFAR-10 dataset, our results show that the proposed algorithm can speed up the convergence of FL by 40%, by reducing the duration of uplink transmission, when compared with two state-of-the-art FL algorithms.**

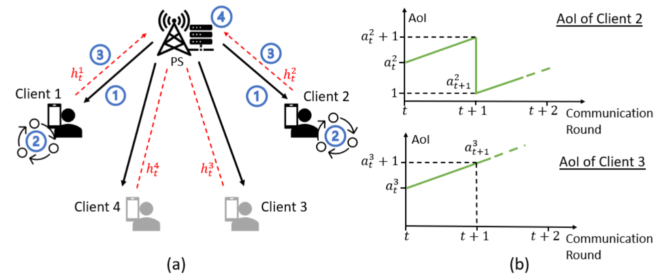

Fig. 1: (a) Illustration of a federated learning (FL) system with one parameter server (PS) and four clients. In communication round t, two clients are scheduled to participate in FL training. The solid arrows represent the downlink broadcasting of the model parameters. The red dashed lines represent the wireless channels of the clients. (b) Plots of the evolution of two clients' age of information (AoI).

I. Introduction

Federated learning (FL) [1] is a distributed learning framework, where multiple mobile clients are orchestrated by a parameter server (PS) to train a deep learning (DL) model. For the federated averaging (FedAvg) algorithm proposed in [1], multiple clients are connected to the PS through wireless links. The training phase of FL involves multiple communication rounds. At the beginning of each communication round, the PS broadcasts the updated DL model and schedules a subset of clients to participate in training (step ① in Fig. 1). The scheduled clients perform gradient-based learning using their local datasets (step ② in Fig. 1) and transmit the updated model back to the PS (step ③ in Fig. 1). Finally, the PS aggregates the received model updates by averaging them (step ④ in Fig. 1).

Client scheduling is crucial for the convergence of FL. Some of the previous works on FL (*e.g.*, [1], [2]) use randomized client scheduling. In [1], a subset of clients are selected uniformly at random from the set of clients in each communication round. In [2], the authors proposed an unbiased multinomial distribution (MD) sampling scheme, where the probability of a client being scheduled is proportional to the size of its local dataset. Recent works have shown that incorporating channel-aware scheduling in FL can reduce the duration of uplink transmission in each communication round, which in turn improves convergence. In [3], the authors proposed a Lyapunov optimization-based scheduling algorithm which selects the clients based on their instantaneous channel state information (CSI). In [4], the authors proposed a client association and resource allocation scheme based on CSI and the local data distribution. Due to the temporal and spatial correlation of wireless channels, greedy scheduling based on CSI only may lead to a subset of users being repeatedly scheduled exclusively, which can degrade the convergence performance of FL [5]. In [6], the authors proposed to use the age of update as a metric to accelerate FL training.

In this paper, we investigate the joint age of information (AoI) [7] and CSI-based client scheduling problem. Due to the temporal dependencies of the decisions made in different time instants, optimization problems employing AoI as part of the objective function or resource constraints are sequential decision problems, which are often formulated as constrained Markov decision processes (CMDPs). When the number of clients is large, the computational complexity for determining the optimal CMDP policy becomes high. Therefore, some recent works have proposed scalable suboptimal scheduling algorithms (*e.g.*, [8]) based on the Whittle index approach [9], which has been proven to be asymptotically optimal when the number of clients approach infinity and the percentage of scheduled clients remains constant. However, only a special class of problems (known as Whittle indexable) can be solved by the Whittle index approach. Even for a problem that can be proven to be Whittle indexable, deriving the Whittle index is still hard, and is often impossible for practical problems with large state space. To address this issue, in this paper, we propose a Lagrangian index-based approach [10] for client scheduling in FL systems. The proposed Lagrangian index-based approach was proven to be asymptotically optimal, enjoys similar scalability as the Whittle index-based approach [10], and does not require the underlying CMDP problem to satisfy any special property. The contributions of this paper are as follows:

- We formulate the joint CSI and AoI optimization problem as a CMDP and propose an asymptotically optimal solution based on the Lagrangian index.
- For problems with a long time horizon, we propose a low-complexity stationary Lagrangian index algorithm based on an infinite-horizon approximation.
- Simulation results show that both the Lagrangian index scheduling algorithm and its infinite-horizon approximation achieve near-optimal performance.
- We evaluate the proposed Lagrangian index-based algorithm in large-scale FL experiments. Simulation results show that the proposed algorithm outperforms the Fed-Avg [1] and MD sampling [2] algorithms by 40% in terms of the average uplink transmission time, achieving a better convergence performance.

Notations: We use $\mathbb{C}$ to denote the set of complex numbers and $\mathbb{Z}_+$ to denote the set of non-negative integers. We use $\mathbb{E}[\cdot]$ to denote the expectation of a random variable.

II. System Model and Problem Formulation

A. System Model

We consider a system with a PS and N clients, *cf.* Fig. 1. The set of clients is denoted by $\mathcal{N} = \{1, 2, \dots, N\}$. Each client has its own local dataset. We consider a time-slotted system with a finite time horizon, where the set of communication rounds is denoted by $\mathcal{T} = \{1, 2, \dots, T\}$.

1) Scheduling Decision Vector: In communication round $t \in \mathcal{T}$, let $\mathbf{u}_t = (u_t^1, \dots, u_t^N)$ denote the scheduling decision vector determined by the PS, where

$$\text{C1: } u_t^n \in \mathcal{U}_n \triangleq \{0, 1\}, \quad n \in \mathcal{N}, t \in \mathcal{T}. \tag{1}$$

Client n participates in training and sends its updated network parameters to the PS in communication round t when u_t^n is equal to one. The PS has limited capacity and can aggregate parameters of the neural network from at most M clients in each communication round. Thus, the scheduling decision vector has to be chosen from the following feasible set

$$\text{C2: } \mathbf{u}_t \in \mathcal{U} \triangleq \Big\{ \mathbf{u}_t \in \{0,1\}^N \mid \sum_{n=1}^{N} u_t^n \le M \Big\}, \quad t \in \mathcal{T}.$$

Constraint C2 is also referred to as the linking constraint.

2) Duration of Uplink Transmission in Each Communication Round: In FL, after local training in communication round $t \in \mathcal{T}$, each scheduled client $n \in \{j \mid u_t^j = 1, j \in \mathcal{N}\}$ needs to send its updated network parameters to the PS. Similar to [3], in this paper, we consider time-division multiple access (TDMA), where the scheduled clients perform uplink transmission sequentially using a fixed transmit power. The time it takes for client n to send its updated parameters successfully to the PS depends on the CSI between the client and the PS. In communication round t, let $h_t^n \in \mathbb{C}$ denote the instantaneous CSI between client n and the PS. Given the system bandwidth W and transmit power P_n of client n, its instantaneous transmission rate can be expressed as

$$r(h_t^n) = W \log_2 \left(1 + |h_t^n|^2 P_n / \sigma_n^2\right), \quad n \in \mathcal{N}, t \in \mathcal{T}, \tag{2}$$

where σ_n^2 denotes the received noise power of client n. We discretize the possible values of h_t^n into a finite set $\mathcal{H}_n = \{h^{n,1}, \dots, h^{n,\max}\}$, $n \in \mathcal{N}$, and define the CSI vector $\mathbf{h}_t = (h_t^1, \dots, h_t^N)$, $t \in \mathcal{T}$, where $h_t^n \in \mathcal{H}_n$. Let ζ denote the packet size (in bits) needed to transmit the updated model parameters to the PS. In an FL system, the updates from all clients have the same size ζ. The time it takes for scheduled client n to send its updated model parameters to the PS is given by

$$y(h_t^n) = \zeta / r(h_t^n), \quad n \in \mathcal{N}, t \in \mathcal{T}. \tag{3}$$

The duration of uplink transmission in communication round $t \in \mathcal{T}$ is equal to $\tau(\mathbf{h}_t, \mathbf{u}_t) = \sum_{n=1}^{N} y(h_t^n) u_t^n$.

B. Problem Formulation

In this subsection, we formulate the joint CSI and AoI optimization problem as a finite-horizon CMDP.

1) Decision Epochs and Actions: We consider a finite-horizon CMDP where each decision epoch corresponds to a communication round. We use the set of communication rounds $\mathcal{T} = \{1, \dots, T\}$ as the set of decision epochs of the CMDP. In decision epoch $t \in \mathcal{T}$, the action vector corresponds to a feasible scheduling decision for all the N clients. That is, the action vector is $\mathbf{u}_t \in \mathcal{U}$, where the feasible action set $\mathcal{U}$ is defined by constraint C2.

2) States: In decision epoch t, let a_t^n denote the AoI of client $n \in \mathcal{N}$, which represents the number of communication rounds that have elapsed since client n sent its updated model to the PS. To obtain a finite state space, we set an upper limit $A_{\max}$ for the AoI. That is, we set $a_t^n = A_{\max}$ for any client n that has not updated its model for more than $A_{\max}$ communication rounds. We have $a_t^n \in \mathcal{A} = \{1, 2, \dots, A_{\max}\}$. Let $\mathbf{a}_t = (a_t^1, \dots, a_t^N)$ denote the AoI vector in decision epoch t. The state vector in decision epoch t can be represented as $\mathbf{s}_t = (\mathbf{a}_t, \mathbf{h}_t) \in \mathcal{S} \triangleq \prod_{n \in \mathcal{N}} (\mathcal{A} \times \mathcal{H}_n)$, $t \in \mathcal{T}$.

3) State Transition Probability: Given a_t^n and u_t^n, the AoI of client n in the next decision epoch, a_{t+1}^n, is a deterministic value, which is given by

$$\begin{aligned} \mathbb{P}(a_{t+1}^n \mid a_t^n, u_t^n) = \; & \mathbf{1}(a_{t+1}^n = 1)\mathbf{1}(u_t^n = 1) \\ & + \mathbf{1}(a_{t+1}^n = \min(A_{\max}, a_t^n + 1))\, \mathbf{1}(u_t^n = 0), \end{aligned} \tag{4}$$

for all $n \in \mathcal{N}$, $t \in \mathcal{T}$, where $\mathbf{1}$ denotes the indicator function. The first term on the right-hand side corresponds to the case when client n is selected to participate in training in decision epoch t, whereas the second term accounts for the alternative case. We consider a wireless channel that evolves according to a stochastic random process. At the beginning of each decision epoch t, the CSI of client n, h_t^n, is revealed to the PS. We consider the case where $h_t^n \in \mathcal{H}_n$ is distributed according to probability $\mathbb{P}(h_t^n)$, $n \in \mathcal{N}$, $t \in \mathcal{T}$.

4) Client Scheduling Policy: A client scheduling policy π is defined as a mapping from state space $\mathcal{S}$ and the set of decision epochs $\mathcal{T}$ to action space $\mathcal{U}$. Let $(\mathbf{s}_1^\pi, \dots, \mathbf{s}_T^\pi)$ denote the state evolution under policy π, where $\mathbf{s}_t^\pi = (\mathbf{a}_t^\pi, \mathbf{h}_t)$, $t \in \mathcal{T}$. Let $(\mathbf{u}_1^\pi, \dots, \mathbf{u}_T^\pi)$ denote the action taken under policy π, where $\mathbf{u}_t^\pi = (u_t^{1,\pi}, \dots, u_t^{N,\pi})$, $t \in \mathcal{T}$. In decision epoch $t \in \mathcal{T}$, given state vector $\mathbf{s}_t \in \mathcal{S}$, the PS chooses an action $\pi_t(\mathbf{s}_t) = \mathbf{u}_t^\pi$. Let Π denote the set of all deterministic policies that satisfy constraint C2. Thus, $\pi \in \Pi$ if and only if $\mathbf{u}_t^\pi \in \mathcal{U}$, $\forall t \in \mathcal{T}$.

5) Cost Function: Let $k^n \in \mathbb{Z}_+$ denote the number of data samples in the local dataset of client $n \in \mathcal{N}$. Let K denote the total number of data samples from all the clients, *i.e.*, $K = \sum_{n=1}^N k^n$. In decision epoch t, we use the following cost function that jointly considers the weighted aggregate AoI and the uplink transmission time $\tau(\mathbf{h}_t, \mathbf{u}_t)$

$$c(\mathbf{s}_t, \mathbf{u}_t) = \sum_{n \in \mathcal{N}} \left[\frac{N k^n}{K} a_t^n + \xi y(h_t^n) u_t^n \right], \tag{5}$$

where ξ is a non-negative weight parameter. In (5), the weight coefficient $\frac{Nk^n}{K}$ places a higher weight on clients with more data samples, which encourages these clients to be scheduled more frequently, in order to reduce their AoI.

6) CMDP Problem Formulation: The optimal policy π^* is defined as the policy that minimizes the expected total cost. The CMDP can be formulated as follows

$$\begin{aligned} \underset{\pi}{\text{minimize}} \quad & \sum_{t=1}^{T} \mathbb{E}\left[c(\mathbf{s}_t^\pi, \mathbf{u}_t^\pi) \right], \\ \text{subject to} \quad & \text{C2a: } \mathbf{u}_t^\pi \in \mathcal{U}, \quad t \in \mathcal{T}. \end{aligned} \tag{6}$$

Problem (6) is a finite-horizon CMDP, whose optimal solution can be found in principle by solving the Bellman equation iteratively using value iteration. However, this approach cannot be applied to problems with a large number of clients, due to its high computational complexity $\mathcal{O}(T|\mathcal{S}||\mathcal{U}|^2) = \mathcal{O}(T(H_{\max}|\mathcal{A}|)^N)$, where $H_{\max} = \max_{n \in \mathcal{N}} |\mathcal{H}_n|$.

III. Low-complexity Solution to Problem (6)

In this section, we propose a low-complexity asymptotically optimal algorithm for problem (6) based on the Lagrangian index [10]. For problems with long time-horizon, we develop an infinite-horizon approximation of the proposed algorithm in order to further reduce the computational complexity.

A. Lagrangian Index Algorithm

It has been shown in [10] that by relaxing the linking constraint C2a so that it holds on average, problem (6) can be decomposed into N client-specific CMDP problems. In this subsection, we will adopt this approach, and obtain an upper bound for the solution to problem (6).

1) Relaxation of the Linking Constraint: Let Φ denote the class of (possibly randomized) policies that satisfy the linking constraint C2a in expectation. Note that Φ is different from the class of policies $\Pi \subset \Phi$ that satisfy the linking constraint C2a in each decision epoch. Given a policy ϕ, we have $\phi \in \Phi$ if and only if

$$\text{C1a: } \mathbf{u}_t^\phi \in \{0,1\}^N \ \text{ and C2b: } \mathbb{E}\left[\sum_{n \in \mathcal{N}} u_t^{n,\phi} \right] \leq M, \quad t \in \mathcal{T}.$$

The expectation in constraint C2b is taken with respect to the stochasticity of state $\mathbf{s}_t^\phi$ and policy ϕ. After relaxing constraint C2a so that it holds in expectation, problem (6) becomes

$$\underset{\phi \in \Phi}{\text{minimize}} \quad \sum_{t=1}^{T} \mathbb{E}\left[c(\mathbf{s}_t^\phi, \mathbf{u}_t^\phi) \right]. \tag{7}$$

Since any policy that satisfies constraint C2a also satisfies constraints C1a and C2b, the optimal value of problem (7) is a lower bound of the optimal value of problem (6).

2) Optimal Solution to Problem (7) via linear programming (LP): Let us define the local state of client $n \in \mathcal{N}$ in decision epoch $t \in \mathcal{T}$ as $\mathbf{s}_t^n = (a_t^n, h_t^n) \in \mathcal{S}_n \triangleq \mathcal{A} \times \mathcal{H}_n$. Let Φ_n denote the class of policies with scheduling decision $u_t^n = \phi_t^n(\mathbf{s}_t^n)$ for client n in state $\mathbf{s}_t^n \in \mathcal{S}_n$, $n \in \mathcal{N}$, $t \in \mathcal{T}$, and satisfy $u_t^n \in \{0,1\}$, $t \in \mathcal{T}$. In this way, a policy $\phi = \prod_{n \in \mathcal{N}} \phi^n \in \prod_{n \in \mathcal{N}} \Phi_n \subset \Phi$ can be constructed by combining the policies of all N clients, where $\mathbf{u}_t = \phi_t(\mathbf{s}_t) = (\phi_t^1(\mathbf{s}_t^1), \cdots, \phi_t^N(\mathbf{s}_t^N))$, $t \in \mathcal{T}$. In the following, we adopt the LP approach for obtaining the optimal solution to problem (7), by first finding the optimal policies for the N client-specific CMDPs. For the client-specific CMDP $n \in \mathcal{N}$, we use the expected fraction of time that client n sojourns in state $\mathbf{s}_t^n$ and selects action u_t^n, for all $\mathbf{s}_t^n \in \mathcal{S}_n$, $u_t^n \in \mathcal{U}_n$, $n \in \mathcal{N}$, $t \in \mathcal{T}$, as the optimization variables. Given policies $\phi^n \in \Phi_n$, $n \in \mathcal{N}$, and $\phi = \prod_{n \in \mathcal{N}} \phi^n$, we define $\nu_t^{n,\phi}(\mathbf{s}_t^n, u_t^n)$ as the probability that client n is in state $\mathbf{s}_t^n$ and selects action u_t^n in decision epoch t. We have

$$\nu_t^{n,\phi}(\mathbf{s}_t^n, u_t^n) = \mathbb{E}\left[\mathbf{1}(\mathbf{s}_t^{n,\phi} = \mathbf{s}_t^n, u_t^{n,\phi} = u_t^n) \right], \tag{8}$$

where $\mathbf{s}_t^{n,\phi}$ and $u_t^{n,\phi}$ denote client n's state and action taken in decision epoch t under policy ϕ. We define the cost function related to client n in decision epoch t as

$$c^n(\mathbf{s}_t^n, u_t^n) = \frac{Nk^n}{K} a_t^n + \xi y(h_t^n) u_t^n, \quad n \in \mathcal{N}, t \in \mathcal{T}. \tag{9}$$

Now, problem (7) can be reformulated to the following LP [10]

$$\begin{aligned} \underset{\substack{\nu_t^{n,\phi}(\mathbf{s}_t^n, u_t^n), \\ \mathbf{s}_t^n \in \mathcal{S}_n, u_t^n \in \mathcal{U}_n, \\ n \in \mathcal{N}, t \in \mathcal{T}}}{\text{minimize}} \quad & \sum_{t \in \mathcal{T}} \sum_{n \in \mathcal{N}} \sum_{\mathbf{s}_t^n \in \mathcal{S}_n} \sum_{u_t^n \in \mathcal{U}_n} c^n(\mathbf{s}_t^n, u_t^n) \nu_t^{n,\phi}(\mathbf{s}_t^n, u_t^n) \\ \text{subject to} \quad & \text{C2c: } \sum_{n \in \mathcal{N}} \sum_{\mathbf{s}_t^n \in \mathcal{S}_n} \nu_t^{n,\phi}(\mathbf{s}_t^n, 1) \leq M, t \in \mathcal{T}, \\ & \text{C3: } \sum_{u_t^n \in \mathcal{U}_n} \nu_t^{n,\phi}(\mathbf{s}_t^n, u_t^n) = \sum_{\mathbf{s}_{t-1}^n \in \mathcal{S}_n} \sum_{u_{t-1}^n \in \mathcal{U}_n} \mathbb{P}(\mathbf{s}_t^n \mid \mathbf{s}_{t-1}^n, u_{t-1}^n) \\ & \qquad \times \nu_{t-1}^{n,\phi}(\mathbf{s}_{t-1}^n, u_{t-1}^n), \quad \mathbf{s}_t^n \in \mathcal{S}_n, n \in \mathcal{N}, t \in \mathcal{T} \setminus \{1\}, \\ & \text{C4: } \sum_{u_1^n \in \mathcal{U}_n} \nu_1^{n,\phi}(\mathbf{s}_1^n, u_1^n) = 1, \quad \mathbf{s}_1^n \in \mathcal{S}_n, n \in \mathcal{N}, \\ & \text{C5: } \nu_t^{n,\phi}(\mathbf{s}_t^n, u_t^n) \geq 0, \mathbf{s}_t^n \in \mathcal{S}_n, u_t^n \in \mathcal{U}_n, n \in \mathcal{N}, t \in \mathcal{T}. \end{aligned} \tag{10}$$

The objective function of problem (10) corresponds to the expected total cost. The left-hand side of constraint C2c corresponds to the expected number of clients that are scheduled to participate in FL in decision epoch $t \in \mathcal{T}$. Thus, constraints C2b and C2c are equivalent. Constraints C3 and C4 are the flow conservation conditions that ensure the solution satisfies the state transition probability (4). Constraint C5 ensures that the optimization variables, which are probabilities of events, are non-negative. Given the optimal solution to problem (10) $\nu_t^{n,\phi^*}(\mathbf{s}_t^n, u_t^n)$, we can design a randomized policy $\phi^* = \prod_{n\in\mathcal{N}} \phi^{n,*}$, where $\phi_t^{n,*}(\mathbf{s}_t^n)$ is a random variable with probability distribution

$$\mathbb{P}(\phi_t^{n,*}(\mathbf{s}_t^n) = u_t^n) = \begin{cases} \frac{\nu_t^{n,\phi^*}(\mathbf{s}_t^n,u_t^n)}{\nu_t^{n,\phi^*}(\mathbf{s}_t^n)}, & \text{if } u_t^n \in \mathcal{U}_n \text{ and } \mathbf{s}_t^n \in \mathcal{V}_t^n, \\ \frac{1}{2}, & \text{if } u_t^n \in \mathcal{U}_n \text{ and } \mathbf{s}_t^n \in \mathcal{S}_n \setminus \mathcal{V}_t^n, \\ 0, & \text{otherwise,} \end{cases}$$

where $\nu_t^{n,\phi^*}(\mathbf{s}_t^n) \triangleq \sum_{u_t^n\in\mathcal{U}_n} \nu_t^{n,\phi^*}(\mathbf{s}_t^n, u_t^n)$. The set $\mathcal{V}_t^n = \left\{\mathbf{s}_t^n \in \mathcal{S}_n \mid \nu_t^{n,\phi^*}(\mathbf{s}_t^n) > 0\right\}$ represents the states that are likely being visited under the randomized policy $\phi_t^{n,*}(\mathbf{s}_t^n)$. In [10], it was shown that client scheduling policy ϕ^* is the optimal solution to problem (7).

3) Lagrangian Index-based Feasible Solution: Although policy ϕ^* achieves the optimal solution to problem (7), it may not be a feasible solution to problem (6) since constraint C2a may not always be satisfied. In [10], the authors proposed a low-complexity asymptotically-optimal feasible heuristic solution to problem (6) called the *Lagrangian index* scheduling policy. We now introduce the steps for finding the Lagrangian index by first solving the dual problem of problem (7). Let $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_T) \succeq \mathbf{0}$ denote the vector of Lagrange multipliers corresponding to constraint C2b. Then, the dual problem of problem (7) can be expressed as

$$\begin{aligned} \underset{\boldsymbol{\lambda}\succeq\mathbf{0}}{\text{maximize}} \;\; \underset{\psi\in\Phi'}{\text{minimize}} \sum_{t=1}^{T} \Bigg\{ & \mathbb{E}\left[c(\mathbf{s}_t^\psi, \mathbf{u}_t^\psi)\right] \\ & + \lambda_t \left(\sum_{n=1}^{N} \mathbb{E}[u_t^{n,\psi}] - M \right) \Bigg\}, \end{aligned} \tag{11}$$

where Φ' denotes the class of policies that satisfy constraint C1a. Given the optimal Lagrange multiplier vector $\boldsymbol{\lambda}^* = (\lambda_1^*, \dots, \lambda_T^*)$, the optimal value of problem (11) can be found recursively by backward induction [10][1]. To this end, we first initialize $L_{T+1}^{\boldsymbol{\lambda}^*}(\mathbf{s}_{T+1}) = 0$, for $\mathbf{s}_{T+1} \in \mathcal{S}$, and then recursively compute

$$\begin{aligned} L_t^{\boldsymbol{\lambda}^*}(\mathbf{s}_t) = \min_{\mathbf{u}_t\in\mathcal{U}} \Big\{ & c(\mathbf{s}_t, \mathbf{u}_t) + \mathbb{E}\left[L_{t+1}^{\boldsymbol{\lambda}^*}(\mathbf{s}_{t+1}) \mid \mathbf{s}_t, \mathbf{u}_t\right] \\ & + \lambda_t^* \big(\sum_{n=1}^{N} u_t^n - M \big) \Big\}, \quad t \in \mathcal{T}, \mathbf{s}_t \in \mathcal{S}. \end{aligned}$$

Here, we use the following short-hand notation

$$\mathbb{E}\left[f(\mathbf{s}_{t+1}) \mid \mathbf{s}, \mathbf{u}\right] \triangleq \sum_{\mathbf{s}'\in\mathcal{S}} f(\mathbf{s}')\mathbb{P}(\mathbf{s}_{t+1} = \mathbf{s}' \mid \mathbf{s}_t = \mathbf{s}, \mathbf{u}_t = \mathbf{u}),$$

where $f(\cdot)$ is any function of $\mathbf{s}_{t+1}$. Given the optimal Lagrange multiplier vector $\boldsymbol{\lambda}^*$ and the initial state $\mathbf{s}_1$, $L_1^{\boldsymbol{\lambda}^*}(\mathbf{s}_1)$ is the optimal value of problem (11). Furthermore, given $\boldsymbol{\lambda}^*$, $L_t^{\boldsymbol{\lambda}^*}(\mathbf{s})$ can be expressed as follows

$$L_t^{\boldsymbol{\lambda}^*}(\mathbf{s}_t) = -\sum_{i=t}^{T} \lambda_i^* M + \sum_{n=1}^{N} V_t^{n,\boldsymbol{\lambda}^*}(\mathbf{s}_t^n),$$

for all $\mathbf{s}_t \in \mathcal{S}$, $t \in \mathcal{T}$, where $V_t^{n,\boldsymbol{\lambda}^*}(\mathbf{s}_t^n)$ denotes the value function of the client-specific MDP. It can similarly be derived using backward induction, by first initializing $V_{T+1}^{n,\boldsymbol{\lambda}^*}(\mathbf{s}_{T+1}^n) = 0$, for $\mathbf{s}_{T+1}^n \in \mathcal{S}_n$, $n \in \mathcal{N}$. Then, $V_t^{n,\boldsymbol{\lambda}^*}(\mathbf{s}_t^n)$, $\mathbf{s}_t^n \in \mathcal{S}_n$, are found iteratively for $t = T, T-1, \dots, 1$, by calculating

$$\begin{aligned} V_t^{n,\boldsymbol{\lambda}^*}(\mathbf{s}_t^n) = \min_{u_t^n\in\mathcal{U}_n} \Big\{ & c^n(\mathbf{s}_t^n, u_t^n) \\ & + \mathbb{E}\left[V_{t+1}^{n,\boldsymbol{\lambda}^*}(\mathbf{s}_{t+1}^n) \mid \mathbf{s}_t^n, u_t^n\right] + \lambda_t^* u_t^n \Big\}. \end{aligned} \tag{12}$$

The value functions of the client-specific MDPs can be utilized to derive the Lagrangian index for each client.

Definition 1 (Lagrangian Index)**.** In decision epoch $t \in \mathcal{T}$, the Lagrangian index for client n in state $\mathbf{s}_t^n$ is defined as

$$\begin{aligned} i_t^n(\mathbf{s}_t^n) = & \left(c^n(\mathbf{s}_t^n, 1) + \mathbb{E}\left[V_{t+1}^{n,\boldsymbol{\lambda}^*}(\mathbf{s}_{t+1}^n) \mid \mathbf{s}_t^n, 1\right] \right) \\ & - \left(c^n(\mathbf{s}_t^n, 0) + \mathbb{E}\left[V_{t+1}^{n,\boldsymbol{\lambda}^*}(\mathbf{s}_{t+1}^n) \mid \mathbf{s}_t^n, 0\right] \right), \quad \mathbf{s}_t^n \in \mathcal{S}_n, n \in \mathcal{N}. \end{aligned}$$

We now introduce the Lagrangian index scheduling policy.

Definition 2 (Lagrangian Index Scheduling)**.** Under the Lagrangian index scheduling policy, in decision epoch $t \in \mathcal{T}$, given the Lagrangian indices $i_t^n(\mathbf{s}_t^n)$, $n \in \mathcal{N}$, the M clients with the smallest non-positive Lagrangian indices are scheduled to participate in FL. In the case where multiple clients have the same Lagrangian indices, policy ϕ^* is utilized to break the tie. Let $\tilde{\pi} \in \Pi$ denote the Lagrangian index policy. In each decision epoch, we have

$$\tilde{\pi}_t(\mathbf{s}_t) = \arg\min_{\mathbf{u}_t\in\mathcal{U}} \sum_{n\in\mathcal{N}} u_t^n i_t^n(\mathbf{s}_t^n). \tag{13}$$

Since constraint C2a is satisfied, the optimal solution of problem (13) is a feasible solution to CMDP problem (6). It can be proven that by increasing the number of clients in the FL system, while a fixed percentage of clients are scheduled in each decision epoch, the Lagrangian index-based scheduling policy achieves asymptotically optimal performance [10].

B. Infinite-horizon Approximation of the Lagrangian Index

Some of the FL tasks may have a long time-horizon T, which will increase the dimensionality of problem (10), making it more computationally complex to solve. In this subsection, we propose a *stationary Lagrangian index scheduling* algorithm, which has a lower complexity for problems with long time-horizon. It can be interpreted as an infinite-horizon approximation of the original Lagrangian index scheduling approach. Given a stationary policy $\bar{\phi}$, let us define the probability that client $n \in \mathcal{N}$ is in state $\mathbf{s}^n$ and selects action u^n in any decision epoch $t \in \mathcal{T}$ as

$$\mu^{n,\bar{\phi}}(\mathbf{s}^n, u^n) = \mathbb{E}\left[\mathbf{1}(\mathbf{s}_t^{n,\bar{\phi}} = \mathbf{s}^n, u_t^{n,\bar{\phi}} = u^n)\right]. \tag{14}$$

The equivalent LP for problem (10) becomes

$$\underset{\substack{\mu^{n,\bar{\phi}}(\mathbf{s}^n,u^n),\\ \mathbf{s}^n\in\mathcal{S}_n, u^n\in\mathcal{U}_n,\\ n\in\mathcal{N}}}{\text{minimize}} \quad \sum_{n\in\mathcal{N}} \sum_{\mathbf{s}^n\in\mathcal{S}_n} \sum_{u^n\in\mathcal{U}_n} c^n(\mathbf{s}^n, u^n)\mu^{n,\bar{\phi}}(\mathbf{s}^n, u^n) \tag{15}$$

$$\text{subject to} \quad \text{C2d: } \sum_{n\in\mathcal{N}} \sum_{\mathbf{s}^n\in\mathcal{S}_n} \mu^{n,\bar{\phi}}(\mathbf{s}^n, 1) \leq M,$$

[1]Since constraints C2b and C2c are equivalent, $\boldsymbol{\lambda}^*$ can be found by solving the dual problem of problem (10), for the dual variables associated with constraint C2c.

$$\text{C3a: } \sum_{u^n \in \mathcal{U}_n} \mu^{n,\bar{\phi}}(\mathbf{s}^n, u^n) = \sum_{\mathbf{s}_{t-1}^n \in \mathcal{S}_n} \sum_{u_{t-1}^n \in \mathcal{U}_n} \mathbb{P}(\mathbf{s}^n \mid \mathbf{s}_{t-1}^n, u_{t-1}^n) \times \mu^{n,\bar{\phi}}(\mathbf{s}_{t-1}^n, u_{t-1}^n), \quad \mathbf{s}^n \in \mathcal{S}_n, n \in \mathcal{N},$$

$$\text{C4a: } \sum_{\mathbf{s}^n \in \mathcal{S}_n} \sum_{u^n \in \mathcal{U}_n} \mu^{n,\bar{\phi}}(\mathbf{s}^n, u^n) = 1, \quad n \in \mathcal{N},$$

$$\text{C5a: } \mu^{n,\bar{\phi}}(\mathbf{s}^n, u^n) \geq 0, \quad \mathbf{s}^n \in \mathcal{S}_n, u^n \in \mathcal{U}_n, n \in \mathcal{N}.$$

Problem (15) has a computational complexity of $\mathcal{O}((NH_{\max}|\mathcal{A}|)^{2.5}\log(NH_{\max}|\mathcal{A}|/\delta))$, where δ denotes the relative accuracy of the solver [11]. Let λ_{inf}^* denote the optimal Lagrange multiplier related to constraint C2d, which can be found by solving the dual problem of problem (15). Let $V^{n,\lambda_{\text{inf}}^*}(\mathbf{s}^n)$ denote the infinite-horizon average-cost value function of state $\mathbf{s}^n$ of the client-specific MDP, given λ_{inf}^*. $V^{n,\lambda_{\text{inf}}^*}(\mathbf{s}^n)$ can be obtained using the relative value iteration algorithm (RVIA) [12, Proposition 5.3.2], by first initializing $V_0^{n,\lambda_{\text{inf}}^*}(\mathbf{s}^n) = 0$, $\mathbf{s}^n \in \mathcal{S}_n, n \in \mathcal{N}$ and then iteratively calculating, for all $j = 1, 2, \ldots$

$$V_j^{n,\lambda_{\text{inf}}^*}(\mathbf{s}^n) = \min_{u^n \in \mathcal{U}_n} \Big\{ c^n(\mathbf{s}^n, u^n) + \lambda_{\text{inf}}^* u^n + \mathbb{E}\big[V_{j-1}^{n,\lambda_{\text{inf}}^*}(\mathbf{s}_{\text{next}}^n) \mid \mathbf{s}^n, u^n\big] - V_j^{n,\lambda_{\text{inf}}^*}(\mathbf{s}_{\text{ref}}^n) \Big\}, \tag{16}$$

for all $\mathbf{s}^n \in \mathcal{S}_n$, $n \in \mathcal{N}$, where $\mathbf{s}_{\text{ref}}^n$ is a fixed reference state. Since RVIA is guaranteed to converge [12], we let $V^{n,\lambda_{\text{inf}}^*}(\mathbf{s}^n) = \lim_{j\to\infty} V_j^{n,\lambda_{\text{inf}}^*}(\mathbf{s}^n)$. We subsequently define the stationary Lagrangian index in Definition 3 and its corresponding scheduling policy in Definition 4.

Definition 3 (Stationary Lagrangian Index)**.** The stationary Lagrangian index for client n in state $\mathbf{s}^n \in \mathcal{S}_n$ is defined as

$$i_{\text{inf}}^n(\mathbf{s}^n) = \Big(c^n(\mathbf{s}^n, 1) + \mathbb{E}\Big[V^{n,\lambda_{\text{inf}}^*}(\mathbf{s}_{\text{next}}^n) \mid \mathbf{s}^n, 1\Big]\Big) - \Big(c^n(\mathbf{s}^n, 0) + \mathbb{E}\Big[V^{n,\lambda_{\text{inf}}^*}(\mathbf{s}_{\text{next}}^n) \mid \mathbf{s}^n, 0\Big]\Big), n \in \mathcal{N}. \tag{17}$$

Definition 4 (Stationary Lagrangian Index Scheduling)**.** Under the stationary Lagrangian index scheduling policy, in each decision epoch, given the stationary Lagrangian indices $i_{\text{inf}}^n(\mathbf{s}^n)$, $n \in \mathcal{N}$, the M clients with the smallest non-positive stationary Lagrangian indices are scheduled to participate in FL. Let $\tilde{\pi}^{\text{inf}} \in \Pi$ denote the stationary Lagrangian index scheduling policy. Given current state $\mathbf{s} \in \mathcal{S}$, in each decision epoch,

$$\tilde{\pi}^{\text{inf}}(\mathbf{s}) = \operatorname*{arg\,min}_{\mathbf{u}\in\mathcal{U}} \sum_{n\in\mathcal{N}} u^n i_{\text{inf}}^n(\mathbf{s}^n). \tag{18}$$

Compared with problem (13), we can obtain a stationary policy for each state $\mathbf{s} \in \mathcal{S}$ from (18). This allows us to solve problems with long time-horizon since the dimension of problem (15) does not increase with T. We will refer to this algorithm as the stationary Lagrangian index-based client scheduling algorithm. The key steps in the planning and deployment stages are presented in Algorithm 1.

IV. Performance Evaluation and Comparison

We consider a communications scenario where the set of N users are uniformly distributed within a ring with inner radius $L_i = 10$ m and outer radius $L_o = 1.5$ km. The PS

Algorithm 1 Stationary Lagrangian Index-based Client Scheduling Algorithm

```
Planning Stage:
Solve the dual problem of problem (15) to obtain λ*_inf.
Initialization: T_iter, V_0^{n,λ*_inf}(s^n) := 0, for all s^n ∈ S_n, n ∈ N.
for n = 1 to N do
    Set j := 0.
    while j ≤ T_iter do
        Calculate V_j^{n,λ*_inf}(s^n), for all s^n ∈ S_n, from (16).
        Set j := j + 1.
    end while
    Compute and store the stationary Lagrangian index i^n_inf(s^n) from (17),
    for all s^n ∈ S_n.
end for
Deployment Stage:
Set t := 1
while t ≤ T do
    for n = 1 to N do
        Observe s^n_t := (a^n_t, h^n_t)
        Retrieve the stored stationary Lagrangian index i^n_inf(s^n_t).
    end for
    Obtain u_t by solving problem (18).
    Set t := t + 1.
end while
```

is located at the centre of the ring. The system bandwidth W is equal to 50 MHz. The transmit power of each client P_n is set to 28 dBm, and the noise variance σ_n^2 is set to -97 dBm. We consider a channel model where the pathloss for client $n \in \mathcal{N}$ can be expressed as $128.1 + 37.6\log_{10}(l^n)$, and l^n denotes the distance between client n and the PS in kilometers [4]. We assume Rayleigh fading when computing the instantaneous channel gain of each client. We discretize the CSI into $|\mathcal{H}_n| = \bar{H}$ levels, for all $n \in \mathcal{N}$, based on its empirical cumulative distribution function. For the stationary Lagrangian indices, we set T_{iter} to be 1,000. We consider the CIFAR-10 dataset consisting of photos of 10 classes of objects. The number of samples owned by the set of clients $\mathcal{N}$ follows a Zipf distribution with parameter κ. That is, the number of samples owned by client n is $k^n = \left\lceil \frac{n^{-\kappa}K}{\sum_{i\in\mathcal{N}} i^{-\kappa}} \right\rceil$, $n \in \mathcal{N}$, where κ represents the degree of difference between the amount of data owned by different clients. Among the k^n samples owned by client n, the composition of classes follows a Dirichlet distribution with parameter α. Let $x_1^n, \ldots, x_{10}^n$ denote the number of samples from the 10 classes of the dataset owned by client n. We have $\mathbb{P}(\mathbf{x}^n) \propto \prod_{i=1}^{10}(x_i^n)^{\alpha-1}$, and $\sum_{i=1}^{10} x_i^n = k^n$, $n \in \mathcal{N}$. Each client performs 50 stochastic gradient descent steps before sending the updated model back to the PS. We consider a convolutional neural network (CNN) with three convolutional layers, two fully-connected layers, and a dropout layer, The size of the neural network ζ is equal to 159.8 kB. The simulation code was programmed in Python 3.8 using PyTorch.

In Fig. 2, we compare the performance of the proposed finite-horizon Lagrangian index-based algorithm and the proposed infinite-horizon approximation stationary Lagrangian index-based algorithm. In Fig. 2(a), we compare the Lagrange multiplier vector $\boldsymbol{\lambda}^* = (\lambda_1^*, \ldots, \lambda_T^*)$ obtained from the dual problem of problem (10) and the Lagrange multiplier λ_{inf}^* from the dual problem of problem (15). The results show that

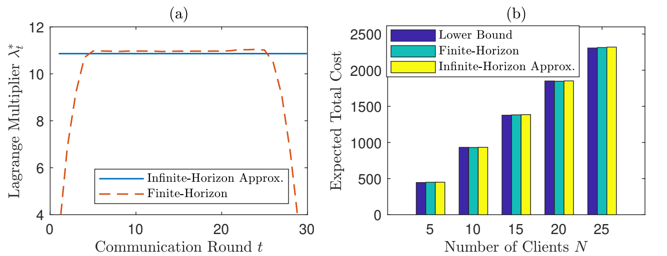

Fig. 2: Performance comparison between the proposed Lagrangian index-based algorithm (finite-horizon) and the proposed stationary Lagrangian index-based algorithm (infinite-horizon approximation) in an FL system with $\bar{H} = 3$, $T = 30$, $\kappa = 0$, $\xi = 200$, and $A_{\max} = 30$. (a) The Lagrange multiplier vector $\boldsymbol{\lambda}^*$ obtained from the dual problem of problem (10), and λ^*_{inf} obtained from the dual problem of problem (15), where $N = 25$ and $M = 5$. (b) The expected total cost obtained by the two algorithms and the lower bound.

λ_t^* differs from λ^*_{inf} only significantly at the beginning and close to the end of the entire time-horizon. This justifies the choice of using the infinite-horizon approximation. The lower bound for the optimal value of problem (6) is obtained by the optimal solution to problem (7), *i.e.*, policy ϕ^*. We vary the number of clients N in Fig. 2(b), and consider the case where one fifth of the clients are scheduled to participate in FL in each communication round. Each experiment was repeated 1,000 times with different random seeds. The expected total cost obtained using the finite-horizon and infinite-horizon approximation algorithms are compared. They show similar performance. These results show that both approaches achieve near optimal performance.

Next, we deploy the proposed algorithm in a large-scale FL system with 100 clients. The infinite-horizon approximation of the Lagrangian index is adopted. In Fig. 3, we show the performance of the proposed algorithm in terms of two metrics. The *average duration of one communication round* corresponds to $\tau(\mathbf{h}_t, \mathbf{u}_t)$. Similar to [3], we assume that the duration of a communication round can be approximated by the total uplink transmission time of all clients that are scheduled to participate in FL. The *testing accuracy* refers to the mean accuracy of all clients on the testing dataset. We compare the results obtained by the proposed algorithm with the FedAvg [1] and the MD sampling algorithms [2][2]. The results in Fig. 3(a) show that the proposed algorithm can reduce the average duration of each communication round by up to 40%, when compared with the baseline algorithms. This is due to the inclusion of the duration of the uplink transmission time in the objective function. Consequently, the testing accuracy of the proposed algorithm converges faster compared with the baseline algorithms, as shown in Fig. 3(b). Therefore, the proposed algorithm can accelerate the convergence in FL training.

V. Conclusion

In this paper, we designed a joint AoI and CSI optimization framework to address the client scheduling problem in FL. We formulated the client scheduling problem as a CMDP and proposed a low-complexity Lagrangian index solution. The

[2]Since [6] considered frequency division multiple access whereas we consider TDMA in this paper, a direct comparison with [6] may not be fair.

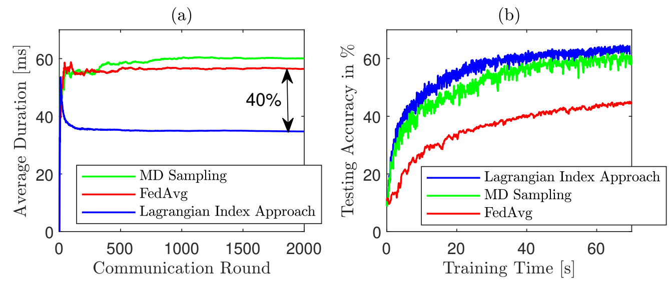

Fig. 3: Comparison of (a) the average duration of each communication round, and (b) the testing accuracy between the proposed Lagrangian index approach and the FedAvg [1] and MD sampling [2] algorithms. We set $N = 100$, $M = 10$, $\kappa = 0.2$, $\alpha = 0.1$, $\bar{H} = 5$, $A_{\max} = 50$, and $\xi = 200$.

proposed Lagrangian index-based approach has the potential to be applied to other optimization problems with AoI as part of the objective function or constraints. Simulation results showed that the proposed algorithm achieves near-optimal performance. It can reduce the duration of the uplink transmission time in FL training by up to 40%, when compared with two baseline algorithms. In this way, the convergence speed of the testing accuracy during FL training is improved. In the journal extension [13] of this work, we consider the case where the clients' local datasets are not independent and identically distributed (non-i.i.d.) and the diversity information can be inferred from the representative gradients of all clients.

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