An Optimal Energy Allocation Algorithm for Energy Harvesting Wireless Sensor Networks

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Abstract—**With the use of energy harvesting technologies, the lifetime of a wireless sensor network (WSN) can be prolonged significantly. Unlike a traditional WSN powered by non-rechargeable batteries, the energy management policy of an energy harvesting WSN needs to take into account the energy replenishment process. In this paper, we study the energy allocation for sensing and transmission in an energy harvesting sensor node with a rechargeable battery and a finite data buffer. The sensor node aims to maximize the total throughput in a finite horizon subject to time-varying energy harvesting rate, energy availability in the battery, and channel fading. We formulate the energy allocation problem as a sequential decision problem and propose an optimal energy allocation (OEA) algorithm using dynamic programming. We conduct simulations to compare the performance between our proposed OEA algorithm and the channel-aware energy allocation (CAEA) algorithm from [1]. Simulation results show that the OEA algorithm achieves a higher throughput than the CAEA algorithm under different settings.**

I. Introduction

A wireless sensor network (WSN) is composed of a large number of sensor nodes that are deployed for environmental sensing, monitoring, and maintenance. Traditionally, a sensor node is mainly powered by a non-rechargeable battery, which has a limited energy storage capacity. As a result, a WSN can only function for a limited amount of time. A lot of research efforts have been dedicated to prolong the lifetime of a WSN by improving its energy efficiency [2]–[5].

Alternatively, the idea of *energy harvesting* was proposed to address the problem of finite lifetime in a WSN by enabling the wireless sensor nodes to replenish energy from ambient sources, such as solar, wind, and vibrations [6], [7]. The design considerations of an energy harvesting WSN are different from a non-rechargeable battery powered WSN in many ways. First, with a potentially infinite amount of energy available to the sensor nodes, an energy harvesting WSN can remain functional for a long period of time. Hence, energy conservation is not the prime design issue. Second, the energy management strategy for an energy harvesting WSN needs to take into account the energy replenishment process. For example, an overly conservative energy expenditure may limit the throughput by failing to take the full advantage of the energy harvesting process. On the other hand, an overly aggressive use of energy may result in an energy outage, which prevents some sensor nodes from functioning properly. Third, the energy availability constraint, which requires the energy consumption to be less than the energy stored in the battery, must be met at all time. This constraint complicates the design of an energy management policy, since the current energy consumption decision would affect the outcome in the future.

Some of the recent works on energy harvesting WSNs have formulated the energy management problem as a dynamic programming (DP) [8], [9] problem. Ho *et al.* in [1] proposed a throughput-optimal energy allocation algorithm for a time-slotted system under time-varying fading channel and energy source by using DP. In [10], a throughput-optimal energy allocation policy was derived in a continuous time model and some suboptimal online waterfilling schemes were proposed to address the dimensionality problem inherent in the DP solution. Chen *et al.* in [11] studied the energy allocation problem of a single node using the shortest path approach. A simple distributed heuristic scheme was proposed that solved the joint energy allocation and routing problem in a rechargeable WSN.

Sharma *et al.* in [12] proposed some energy management schemes for a single energy harvesting sensor node that achieved the maximum throughput and minimum mean delay. Gatzianas *et al.* in [13] presented an online adaptive transmission scheme for wireless networks with rechargeable batteries that maximizes total system utility and stabilizes the data queue using Lyapunov techniques. In [14], utility-optimal energy allocation algorithms were proposed for systems with predictable or stochastic energy availability.

Most of these results from [1], [10]–[14] assumed either an infinitely long data backlog or data buffer. Yet, it is more reasonable if a *finite data buffer* is considered. Besides, the energy consumed in *data sensing* has always been overlooked in the literature. This motivates us to design an optimal energy allocation (OEA) algorithm for energy harvesting WSNs which takes into account both the data sensing energy consumption and the finite capacity of the data buffer. However, these considerations introduce new challenges. For instance, if the sensor node consumes an insufficient amount of energy for sensing but an excessive amount of energy for transmission, then the data buffer may be empty, which leads to a reduction in throughput. Thus, the sensor node needs to maintain a good *balance* between the energy consumed for sensing and the energy for transmission.

In this paper, we consider a point-to-point wireless link between an energy harvesting sensor node and the sink. The channel and energy harvesting rate may vary over time. The sensor node has a rechargeable battery and a data buffer with finite capacity. Our objective is to maximize throughput over a finite horizon. The sensor node needs to decide the amount of energy it should allocate for sensing and transmission in each time slot by taking into account the battery energy level, data buffer level, energy harvesting rate, and channel condition.

The main contributions of our work are as follows:

- We study the energy allocation problem for sensing and transmission in a energy harvesting WSN. We formulate it as a finite-horizon sequential decision problem under channel fluctuations and energy variations in a time-slotted system.
- We obtain the optimal energy allocation policy and propose the OEA algorithm by using DP.
- We provide extensive simulation results to compare the performance of the OEA algorithm and the channel-aware energy allocation (CAEA) algorithm from [1]. The results show that the OEA algorithm achieves a higher throughput than the CAEA algorithm under different settings. We also study the impact of the data-sensing efficiency (i.e., the amount of data that the sensor can sense per unit energy) on the throughput performance.

Unlike the existing works in the literature [1], [10]–[14], we take into account a finite data buffer and the energy consumed for sensing. The rest of the paper is organized as follows: We describe the system model in Section II and formulate our problem in Section III. In Section IV, we propose the OEA algorithm that maximizes the expected throughput over a finite horizon using DP. In Section V, we evaluate the performance of the OEA algorithm and compare it with the CAEA algorithm. Conclusion is given in Section VI.

II. System Model

As shown in Fig. 1, we consider a single energy harvesting sensor node, which contains a rechargeable battery with capacity b_{max} Joule and a data buffer with size q_{max} Mbits. We assume that the system is time-slotted with K time slots and the duration of a time slot is D sec. We let $k \in \mathcal{K} \triangleq \{0, 1, \dots, K-1\}$ be the time slot index. The sensor node performs sensing in the field, stores the sensed data in the buffer, and transmits the data to the receiver $\mathbf{Rx}$ of the sink over a wireless channel. We consider an additive white Gaussian noise (AWGN) channel with block flat fading. That is, the channel remains constant for the duration of each time slot, but may change at the slot boundaries. Let α_k be the channel gain in time slot k.

We assume that the sink sends delayed channel state information (CSI) of the previous time slot back to the sensor node. In other words, at the beginning of time slot k, the sensor node only knows the value of α_{k-1}, but not α_k. At the beginning of time slot k, the stored battery level is b_k and the amount of stored data in the data buffer is q_k. During the whole time slot k, the sensor node is able to replenish energy by h_k, which can be used for sensing or transmission in time slot $k+1$ onward. As a result, the sensor node does not know the value of h_k until the next time slot $k+1$. In other words, at the beginning of time slot k, the sensor node knows the value of h_{k-1}, but not h_k.

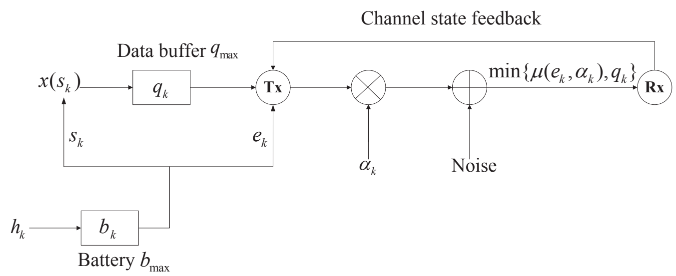

Fig. 1. The system model of an energy harvesting wireless sensor node transmitting data to the receiver $\mathbf{Rx}$ of the sink. At time slot k, the random variables are the energy to be harvested h_k and the channel gain α_k. Due to channel feedback delay and the time required to track the energy harvesting rate, we assume that the values of α_{k-1} and h_{k-1} are only known at the beginning of time slot k. The optimization variables (or actions) are the energy consumed for transmission e_k and sensing s_k. The stored battery level is b_k and the amount of data available in the buffer is q_k. $x(s_k)$ is the amount of data obtained by using s_k amount of energy. The throughput in time slot k is $\min\{\mu(e_k, \alpha_k), q_k\}$. Our problem is to optimally allocate e_k and s_k such that the total expected throughput in a total of K time slots is maximized.

If the channel gain is α_k and the allocated transmission energy is e_k in time slot k, then the instantaneous transmission power is $\frac{e_k}{D}$, and the sensor node is able to transmit $\mu(e_k, \alpha_k)$ bits of data in time slot k. In general, $\mu(e_k, \alpha_k)$ is a monotonically non-decreasing and concave function in e_k given α_k. One such function is given by [15]:

$$\mu(e_k, \alpha_k) = DW \log_2 \left(1 + \frac{\alpha_k e_k}{N_0 W D}\right) \text{ bits,} \tag{1}$$

where N_0 is the power spectral density of the Gaussian noise, and W is the bandwidth of the channel.

For sensing in time slot k, we let $x(s_k)$ be the amount of data generated when s_k units of energy are used for sensing. In general, $x(s_k)$ is a monotonically non-decreasing and concave function in s_k. The data obtained by sensing in time slot k will be stored in the data buffer until it is transmitted in the following time slots. Except for sensing and transmission, we assume that other circuits in the sensor node consume negligible energy.

The sensor node needs to decide on e_k and s_k, for all $k \in \mathcal{K}$ such that the overall expected throughput in the K time slots is maximized. To achieve this goal, it has to maintain a good *tradeoff* between the energy allocation for e_k and s_k. Given a fixed energy budget in a time slot, if e_k is too small, then the throughput in time slot k will be small. However, if e_k is too large, then s_k will be small that insufficient amount of sensing data is stored in the buffer for transmission in the next time slot, which leads to a reduction in throughput. In addition, the total energy budget $e_k + s_k$ in time slot k should also be carefully controlled. If the energy management policy is overly aggressive such that the rate of energy consumption is greater than the rate of energy harvesting, it may result in an energy outage, which prevents the sensor node from functioning properly. On the other hand, an overly conservative energy management policy would limit the throughput in each time slot. Thus it is a challenging problem to decide the values of e_k and s_k optimally in each time slot $k \in \mathcal{K}$.

III. Problem Formulation

In this section, we formulate the problem of finding the optimal energy allocation for sensing and transmission as a *finite-horizon sequential decision* problem [8], [9], which consists of five elements: decision epochs, states, actions, state transition probabilities, and rewards. The *decision epochs* are

$$k \in \mathcal{K} = \{0, 1, \dots, K-1\}. \tag{2}$$

At the beginning of time slot k, the *state* of the system is denoted as

$$\boldsymbol{y}_k = (b_k, q_k, h_{k-1}, \alpha_{k-1}), \tag{3}$$

which includes the battery energy state b_k and data buffer state q_k for the current time slot k, as well as the energy harvesting state h_{k-1} and channel state α_{k-1} in the previous time slot $k-1$. First, for the battery energy state in time slot k, the sensor node harvests h_k units of energy from the environment. On the other hand, it consumes e_k units of energy for data transmission and s_k units of energy for sensing. Since the battery has a finite capacity b_{max}, the energy stored in the battery is updated as

$$b_{k+1} = \min\{b_k - (e_k + s_k) + h_k, b_{max}\}, \forall\, k \in \mathcal{K}. \tag{4}$$

It ensures that the maximum stored energy b_{max} is not exceeded. We assume that the initial energy b_0 is known and satisfies the constraint that $0 \le b_0 \le b_{max}$. Moreover, the amount of energy consumed for sensing and transmission must be no more than the battery level:

$$e_k + s_k \le b_k, \; \forall\, k \in \mathcal{K}. \tag{5}$$

Second, for the data buffer state in time slot k, $x(s_k)$ amount of sensing data are generated and queued up in the data buffer if s_k units of energy are allocated for sensing. On the other hand, $\mu(e_k, \alpha_k)$ amount of data are transmitted and removed from the data buffer if e_k units of energy are used for transmission. However, since the data available in the data buffer for transmission at time slot k is q_k, the throughput at time slot k is given by $\min\{\mu(e_k, \alpha_k), q_k\}$. Since the data buffer is finite with capacity q_{max}, the amount of data in the buffer is then updated as:

$$q_{k+1} = \min\{[q_k - \mu(e_k, \alpha_k)]^+ + x(s_k), q_{max}\}, \forall\, k \in \mathcal{K}, \tag{6}$$

with $[z]^+ = \max[z, 0]$. We assume that the initial amount of data in the data buffer q_0 is known and satisfies $0 \le q_0 \le q_{max}$. Equation (6) implies that if the sensor allocates too much energy for transmission so that $\mu(e_k, \alpha_k) > q_k$, then energy is wasted. On the other hand, if the sensor allocates too much energy for sensing so that $x(s_k) > q_{max}$, then the data buffer overflows and energy is wasted. Thus the sensor should make a proper energy allocation decision at each time slot. Third, since the energy harvesting rate and the current channel state information at time slot k is not known to the sensor, we use two independent first-order stationary Markovian models to model h_k and α_k. The random variable h_k takes values in some finite set $\mathcal{H} = \{H_1, H_2, \dots, H_N\}$. The random variable α_k takes values in some finite set $\mathcal{A} = \{A_1, A_2, \dots, A_M\}$. The transition probability of these two independent random variables are denoted as $\mathbb{P}(h_k \,|\, h_{k-1})$ and $\mathbb{P}(\alpha_k \,|\, \alpha_{k-1})$.

Based on the current state $\boldsymbol{y}_k$ at time slot k, the sensor will choose to consume e_k units of energy for data transmission and s_k units of energy for sensing. That is, an *action* (e_k, s_k) is taken for transmission and sensing energy allocation from its feasible set $U_k(\boldsymbol{y}_k)$. We have

$$(e_k, s_k) \in U_k(\boldsymbol{y}_k) = \{(e, s) \,|\, e + s \le b_k, e \ge 0, s \ge 0\}, \tag{7}$$

where $U_k(\boldsymbol{y}_k)$ represents the feasible set of (e_k, s_k) at time slot k. The constraint $e_k + s_k \le b_k, \forall\, k \in \mathcal{K}$ ensures that the amount of energy consumed for sensing and transmission must be no more than the battery level. In addition, it is possible to impose additional constraints on (e_k, s_k). For example, a constraint on the minimum amount of energy for sensing or transmission to ensure a minimum amount of sensed data or transmitted data for each time slot, respectively. Also, the maximum transmission power constraint can be imposed.

The *state transition probability* $\mathbb{P}(\boldsymbol{y}_{k+1} \,|\, \boldsymbol{y}_k, e_k, s_k)$ is the probability that the system will go into state $\boldsymbol{y}_{k+1}$ if action (e_k, s_k) is taken at state $\boldsymbol{y}_k$ at time slot k. Due to the independence between (b_{k+1}, h_k) and (q_{k+1}, α_k) for all $k \in \mathcal{K}$, we can simplify the state transition probability as

$$\begin{aligned}
&\mathbb{P}(\boldsymbol{y}_{k+1} \,|\, \boldsymbol{y}_k, e_k, s_k) \\
&= \mathbb{P}(b_{k+1}, q_{k+1}, h_k, \alpha_k \,|\, b_k, q_k, h_{k-1}, \alpha_{k-1}, e_k, s_k) \\
&= \mathbb{P}(b_{k+1}, h_k \,|\, b_k, h_{k-1}, e_k, s_k)\mathbb{P}(q_{k+1}, \alpha_k \,|\, q_k, \alpha_{k-1}, e_k, s_k) \\
&= \mathbb{P}(b_{k+1} \,|\, b_k, h_k, e_k, s_k)\mathbb{P}(h_k \,|\, h_{k-1}) \\
&\quad \times \mathbb{P}(q_{k+1} \,|\, q_k, \alpha_k, e_k, s_k)\mathbb{P}(\alpha_k \,|\, \alpha_{k-1}),
\end{aligned} \tag{8}$$

where

$$\mathbb{P}(b_{k+1} \,|\, b_k, h_k, e_k, s_k) = \begin{cases} 1, & \text{if (4) is satisfied,} \\ 0, & \text{otherwise,} \end{cases} \tag{9}$$

$$\mathbb{P}(q_{k+1} \,|\, q_k, \alpha_k, e_k, s_k) = \begin{cases} 1, & \text{if (6) is satisfied,} \\ 0, & \text{otherwise.} \end{cases} \tag{10}$$

$\mathbb{P}(h_k \,|\, h_{k-1})$ and $\mathbb{P}(\alpha_k \,|\, \alpha_{k-1})$ are defined according to the corresponding Markov model.

Given the current state $\boldsymbol{y}_k$ and the action (e_k, s_k), $\mathcal{E}_{\alpha_k}[\mu(e_k, \alpha_k)]$ is the expected amount of data that can be transmitted when e_k units of energy are used for transmission. However, since the data available in the data buffer for transmission at time slot k is q_k, the expected throughput (i.e., the amount of data transmitted in the time slot) at time slot k is given by $\mathcal{E}_{\alpha_k}[\min\{\mu(e_k, \alpha_k), q_k\}]$. We define the *expected reward* at time slot k to be the expected throughput. That is,

$$\begin{aligned}
&\mathcal{E}_{\alpha_k}[\min\{\mu(e_k, \alpha_k),\; q_k\}] \\
&= \min\left\{ \sum_{\alpha \in \mathcal{A}} \mu(e_k, \alpha)\mathbb{P}(\alpha \,|\, \alpha_{k-1}),\; q_k \right\}.
\end{aligned} \tag{11}$$

Let $\pi = \{(e_k(\boldsymbol{y}_k), s_k(\boldsymbol{y}_k)), \forall\, \boldsymbol{y}_k, k \in \mathcal{K}\}$ be the power allocation *policy*, where $(e_k(\boldsymbol{y}_k), s_k(\boldsymbol{y}_k))$ is the transmission and sensing power allocation at state $\boldsymbol{y}_k$ under policy π. A *feasible* policy should satisfy (7) in all the time slots. Let Π be the feasible set of π. Since α_k and h_k are random variables, the sensor node aims to find an optimal and feasible sensing and transmit power allocation policy π^* that maximizes the total *expected* reward, i.e., the total expected throughput summed over a finite horizon of K time slots. That is, given the initial state $\boldsymbol{y}_0 = (b_0, q_0, h_{-1}, \alpha_{-1})$ in the first time slot, it aims to solve the following optimization problem

$$\mathcal{T}^* = \max_{\pi \in \Pi} \sum_{k=0}^{K-1} \mathcal{E}\Big[\min\{q_k, \mu(e_k, \alpha_k)\} \Big| \boldsymbol{y}_0, \pi\Big], \qquad (12)$$

where $\mathcal{E}[\cdot]$ denotes the statistical expectation taken over all relevant random variables given initial state $\boldsymbol{y}_0$ and policy π.

In general, the optimization problem in (12) cannot be solved independently for each time slot due to the causality constraints on different variables. For example, the current energy consumption affects the energy availability in the next time slot, and thus affects the future energy allocation. Also, the energy allocated for sensing at the current time slot affects the amount of data in the queue for transmission in the next time slot. For such sequential optimization problem (12) under channel condition and energy harvesting rate uncertainties, we can solve it *optimally* using finite-horizon DP.

IV. Finite-Horizon Dynamic Programming

In this section, we solve problem (12) by using finite-horizon DP. An OEA algorithm is proposed that achieves the maximal expected throughput in problem (12).

Let $J_k(b_k, q_k, h_{k-1}, \alpha_{k-1})$ be the maximum expected throughput from time slot k to $K-1$, given that the system is in state $(b_k, q_k, h_{k-1}, \alpha_{k-1})$ immediately before the decision at time slot k. The *Bellman's equations* are given by the following recursive equations starting from $k = K-1$ to $k = 0$.

For $k = K-1$, we have

$$J_{K-1}(b_{K-1}, q_{K-1}, h_{K-2}, \alpha_{K-2}) = \max_{(e_{K-1}, s_{K-1}) \in U_{K-1}(\boldsymbol{y}_{K-1})} \mathcal{E}_{\alpha_{K-1}}[\min\{\mu(e_{K-1}, \alpha_{K-1}), q_{K-1}\} \mid \alpha_{K-2}]. \qquad (13a)$$

For $k = K-2, \ldots, 0$, we have

$$J_k(b_k, q_k, h_{k-1}, \alpha_{k-1}) = \max_{(e_k, s_k) \in U_k(\boldsymbol{y}_k)} \Big\{ \mathcal{E}_{\alpha_k}[\min\{\mu(e_k, \alpha_k), q_k\} \mid \alpha_{k-1}] + \mathcal{E}_{h_k, \alpha_k}[J_{k+1}(b_{k+1}, q_{k+1}, h_k, \alpha_k) \mid h_{k-1}, \alpha_{k-1}] \Big\}, \qquad (13b)$$

where b_{k+1} and q_{k+1} are updated as in (4) and (6), respectively. Notice that if the feasible set of (e_k, s_k) is $U_k(\boldsymbol{y}_k)$ as defined in (7), then (13a) can be simplified as

$$J_{K-1}(b_{K-1}, q_{K-1}, h_{K-2}, \alpha_{K-2}) = \mathcal{E}_{\alpha_{K-1}}[\min\{\mu(b_{K-1}, \alpha_{K-1}),\ q_{K-1}\} \mid \alpha_{K-2}]. \qquad (14)$$

Algorithm 1 Optimal Energy Allocation (OEA) Algorithm for Energy Harvesting Sensor Node.

Planning Phase:
Set $J_{K-1}(b_{K-1}, q_{K-1}, h_{K-2}, \alpha_{K-2})$, $\forall b_{K-1}$, $\forall q_{K-1}$, $\forall h_{K-2}$, $\forall \alpha_{K-2}$, using (13a).
Set $k := K-2$.
while $k \geq 0$ **do**
  Calculate $J_k(b_k, q_k, h_{k-1}, \alpha_{k-1}), \forall b_k, \forall q_k, \forall h_{k-1}, \forall \alpha_{k-1}$, using (13b).
  Find the optimal action $(e_k^*(\boldsymbol{y}_k), s_k^*(\boldsymbol{y}_k))$, using (15).
  Set $k := k-1$.
end while
Sensing and Transmission Phase:
Set $k := 0$.
while $k \leq K-1$ **do**
  Track the energy harvesting rate of the previous time slot h_{k-1}.
  Track the energy available for use in the battery b_k.
  Track the amount of data in the buffer q_k.
  Obtain the channel feedback α_{k-1} from the sink.
  Set $\boldsymbol{y}_k := (b_k, q_k, h_{k-1}, \alpha_{k-1})$.
  Obtain $(e_k^*(\boldsymbol{y}_k), s_k^*(\boldsymbol{y}_k))$ based on optimal policy π^*.
  Consume $e_k^*(\boldsymbol{y}_k)$ amount of energy for transmission and $s_k^*(\boldsymbol{y}_k)$ amount of energy for sensing.
  Update battery energy b_{k+1} by using (4) and the amount of data in the buffer q_{k+1} by using (6).
  Set $k := k+1$.
end while

That is, we use all the available energy for transmission in the final time slot. Thus the optimal energy allocation for the final time slot is $(e_{K-1}^*, s_{K-1}^*) = (b_{K-1}, 0)$. For (13b), the first and second terms on the right hand side represent, respectively, the expected *immediate* throughput for time slot k and the expected total *future* throughput for time slot $k+1$ to $K-1$ if action (e_k, s_k) is chosen. Hence, the equation in (13b) describes the *tradeoff* between the current rewards and the future rewards.

Theorem 1: The optimal policy of problem (12) is $\pi^* = \{(e_k^*(\boldsymbol{y}_k), s_k^*(\boldsymbol{y}_k)), \forall \boldsymbol{y}_k, k \in \mathcal{K}\}$, where

$$(e_k^*(\boldsymbol{y}_k), s_k^*(\boldsymbol{y}_k)) = \arg\max_{(e_k, s_k) \in U_k(\boldsymbol{y}_k)} \Big\{ \mathcal{E}_{\alpha_k}[\min\{\mu(e_k, \alpha_k), q_k\} \mid \alpha_{k-1}] + \mathcal{E}_{h_k, \alpha_k}[J_{k+1}(b_{k+1}, q_{k+1}, h_k, \alpha_k) \mid h_{k-1}, \alpha_{k-1}] \Big\}. \qquad (15)$$

Moreover, for every initial state $\boldsymbol{y}_0 = (b_0, q_0, h_{-1}, \alpha_{-1})$, the maximum throughput $\mathcal{T}^*$ is given by $J_0(b_0, q_0, h_{-1}, \alpha_{-1})$.

Proof: The proof follows by applying the Bellman's equations and backward induction [8] and using (4) and (6). ■

We then propose our OEA algorithm in Algorithm 1. In the planning phase, the sensor solves for the optimal policy π^* and records it as a lookup table. In the sensing and transmission phase, the sensor first tracks the energy harvesting rate of the previous time slot h_{k-1}, the battery energy level b_k, the amount of data in the buffer q_k, and obtains the channel feedback α_{k-1} from the sink. Then, the sensor chooses the action (e_k^*, s_k^*) based on current system state $\boldsymbol{y}_k$ and the optimal policy π^*. That is, it consumes e_k^* and s_k^* amount of energy for transmission and sensing, respectively.

V. Performance Evaluation

In this section, we evaluate the performance of our OEA algorithm by comparing its achieved throughput with that of the CAEA algorithm from [1]. We consider a band-limited AWGN channel, where the channel bandwidth is $W = 100$ KHz and the noise power spectral density is $N_0 = 10^{-18}$ W/Hz. The channel state can be "G = Good", "N = Normal", or "B = Bad". It evolves according to the three-state Markov chain as shown in Fig. 2 [16] with the transition matrix of the Markov chain given by

$$P_\alpha = \begin{bmatrix} P_{BB} & P_{BN} & P_{BG} \\ P_{NB} & P_{NN} & P_{NG} \\ P_{GB} & P_{GN} & P_{GG} \end{bmatrix} = \begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.7 & 0.3 \end{bmatrix}, \quad (16)$$

where P_{XY} represents the probability of the channel state going from state X to state Y, X and $Y \in \{B, N, G\}$. The channel gain α is 0.5×10^{-13}, 1×10^{-13}, and 1.5×10^{-13} when the channel state is "Bad", "Normal", and "Good", respectively. The battery buffer size b_{max} is set to be 100 Joules, and the data buffer size q_{max} is set to be 1 Mbits. For tractability, we assume that the energy harvesting state h_k takes values from the finite set $\mathcal{H} = \{H_1, H_2, H_3, H_4\}$ and evolves according to the four-state *Markov chain* with the state transition probability given by

$$\begin{aligned} P_h &= \begin{bmatrix} P_{H_1H_1} & P_{H_1H_2} & P_{H_1H_3} & P_{H_1H_4} \\ P_{H_2H_1} & P_{H_2H_2} & P_{H_2H_3} & P_{H_2H_4} \\ P_{H_3H_1} & P_{H_3H_2} & P_{H_3H_3} & P_{H_3H_4} \\ P_{H_4H_1} & P_{H_4H_2} & P_{H_4H_3} & P_{H_4H_4} \end{bmatrix} \\ &= \begin{bmatrix} 0.3 & 0.7 & 0 & 0 \\ 0.25 & 0.5 & 0.25 & 0 \\ 0 & 0.25 & 0.5 & 0.25 \\ 0 & 0 & 0.7 & 0.3 \end{bmatrix}, \end{aligned} \quad (17)$$

where $P_{H_iH_j}$ represents the probability of the energy harvesting state going from state H_i to state H_j, $\forall\, i, j \in \{1, 2, 3, 4\}$. The steady state probability is then given by $[P_{H_1}\, P_{H_2}\, P_{H_3}\, P_{H_4}] = [0.13\ 0.37\ 0.37\ 0.13]$. $x(s_k)$ is assumed to be a linear function of s_k [11], given by

$$x(s_k) = \gamma s_k, \quad (18)$$

where γ is the data-sensing efficiency parameter. We denote the average throughput as $\bar{\mathcal{T}}$, which is given by

$$\bar{\mathcal{T}} = \frac{\mathcal{T}^*}{K}, \quad (19)$$

where $\mathcal{T}^*$ is the maximal total expected throughput over K time slots defined in (12).

The CAEA algorithm in [1] assumed infinite backlogged data and neglected the sensing energy. For a fair comparison, we modify the CAEA algorithm by allowing the data buffer to be finite with size q_{max}. We assume that the sensor allocates a fixed percentage of energy available in the battery for sensing

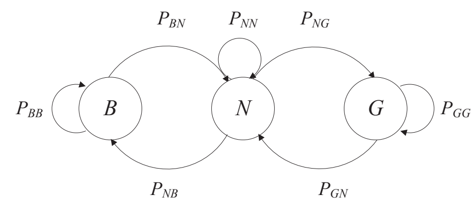

Fig. 2. A three-state Markov chain for the channel gain, where "B", "N", and "G" represent the channel in the bad, normal, and good state, respectively.

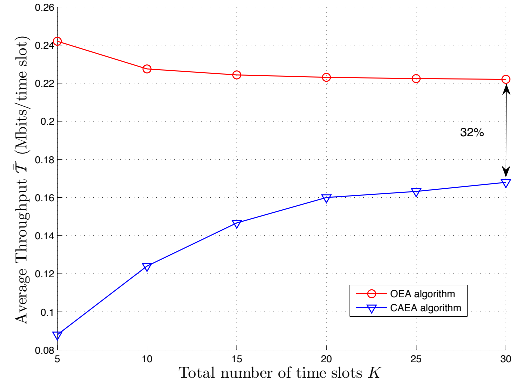

Fig. 3. Average throughput of the two algorithms for different number of total time slots K.

in each time slot, and optimizes the energy allocated for transmission to achieve the maximal expected throughput.

We start by examining the average throughput $\bar{\mathcal{T}}$ of the OEA algorithm and the CAEA algorithm with different number of total time slots K. The data-sensing efficiency parameter γ is set to be 0.02 Mbits/J. We set the fixed percentage of energy for sensing to be 10% in the CAEA algorithm, which is reasonable in WSNs. The value of energy harvesting rate is taken from the set $\mathcal{H} = \{H_1, H_2, H_3, H_4\} = \{6, 12, 18, 24\}$ J/time slot. As shown in Fig. 3, our proposed OEA algorithm outperforms the CAEA algorithm in terms of the achieved average throughput. For example, our OEA algorithm achieves 32% higher average throughput than the CAEA algorithm when $K = 30$. The reason is that in the CAEA algorithm, the sensor just optimally controls the energy for transmission, while the sensing energy is fixed. However, in our OEA algorithm, both the sensing and transmitting energy is optimally allocated, which results in a better performance than the CAEA algorithm.

Next, we consider the performance of the two algorithms under different average energy harvesting rates $\bar{H}$, where $\bar{H} = \sum_{i=1}^{4} P_{H_i} H_i$. In Fig. 4, we plot the average throughput against the average energy harvesting rate when the total number of time slots $K = 30$. We observe that our OEA algorithm performs much better than the CAEA algorithm, especially when the average energy harvesting rate $\bar{H}$ is high. As shown in Fig. 4, our OEA algorithm achieves 105% higher average throughput than the CAEA algorithm when the average energy harvesting rate $\bar{H} = 35$ J/time slot. Moreover, the throughput of the CAEA algorithm saturates very quickly as the average harvesting rate is increased. It is because the harvested energy cannot be accommodated, and more and

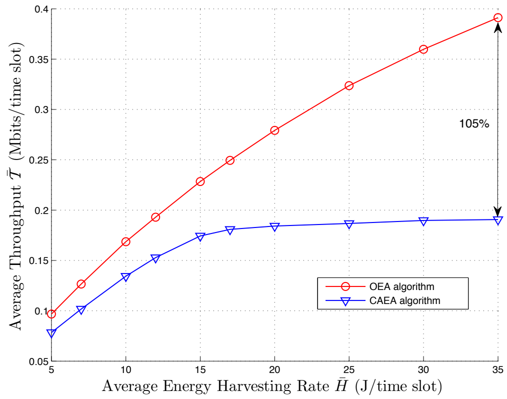

Fig. 4. Average throughput of the two algorithms for different energy harvesting rates when $K = 30$ and $B_{max} = 100$ Joules.

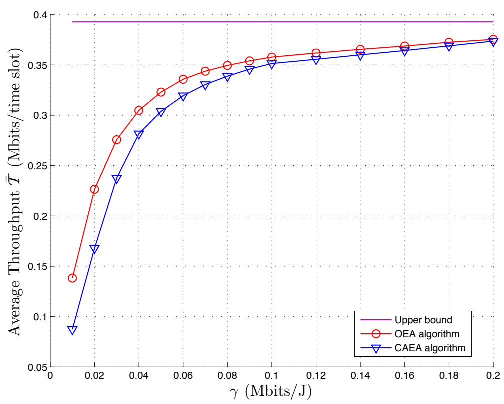

Fig. 5. Average throughput of the two algorithms for different values of data-sensing efficiency parameter γ.

more energy is lost due to the overflow of the battery energy. However, in our algorithm, energy wastage will not occur as long as the harvesting rate is less than b_{max} and the data buffer is large enough. The reason is that under the OEA algorithm, the sensor node maintains a good balance between the energy allocated for sensing and transmission, and thus achieves a better performance.

Finally, we study the impact of the data-sensing efficiency on the average throughput. We fix K to be 30 and examine the throughput under different values of γ. A larger value of γ corresponds to a higher data-sensing efficiency, since the sensor node spends less energy for sensing the same amount of data. As shown in Fig. 5, when γ is increased, the throughput increases as well, because more energy is available for data transmission. However, the performance saturates as γ is increased beyond a certain value. When γ approaches infinity, it corresponds to the case where the sensing is extremely efficient. The throughput of this case provides an upper bound for the performance of the OEA algorithm for sensor nodes with different sensing efficiency.

VI. Conclusion

In this paper, we studied the problem of maximizing the finite horizon expected throughput for an energy harvesting sensor node under energy harvesting rate variations and channel fluctuations in a time-slotted system. A finite data buffer and the energy consumed for sensing data were considered for the first time. In this case, the sensor should achieve a good tradeoff between the energy consumed for sensing and transmission so as to achieve a high throughput. We obtained the optimal energy allocation policy using DP and proposed an OEA algorithm. Finally, we provided extensive simulation results to compare the performances of the OEA algorithm and the CAEA algorithm, and studied the impact of data-sensing efficiency on the throughput. The results showed that the OEA algorithm achieved a much higher throughput than the CAEA algorithm under different settings. An interesting topic for future work is the extension of our model to a multi-hop setting for data transmission.

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