A Joint Angle and Distance based User Pairing Strategy for Millimeter Wave NOMA Networks

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***Abstract*—In this paper, we consider downlink non-orthogonal multiple access (NOMA) transmission in millimeter wave (mmWave) networks with spatially random users. To facilitate NOMA transmission in mmWave networks, we propose a novel joint angle and distance based user pairing strategy. In particular, the user located nearest to the base station (BS) is paired with another user that is located within a distance threshold from the BS and has the minimum relative spatial angle difference. In consideration of the directional beamforming and the randomness of user locations, the BS opportunistically chooses to enable NOMA or orthogonal multiple access (OMA) based on the instantaneous spatial angle difference between the paired users. The proposed scheme fully exploits the antenna array gain for the paired NOMA users. By using tools from stochastic geometry, we derive the coverage probability of the proposed scheme. Simulations validate the theoretical analysis. Results reveal that the proposed scheme outperforms the angle-based NOMA, distance-based NOMA, and OMA schemes, confirming the importance of exploiting both the angle and distance information for user pairing in mmWave networks. Results also show that there exists an optimal value of the distance threshold that maximizes the coverage probability.**

I. Introduction

Non-orthogonal multiple access (NOMA) has the potential to enhance the spectrum efficiency and improve user fairness of the fifth generation (5G) wireless networks [1]. Various NOMA schemes have recently been proposed, including power-domain NOMA and sparse code multiple access [2]. For power-domain NOMA in the downlink, the base station (BS) simultaneously serves multiple users in the same physical resource block by applying superposition coding at the BS and performing successive interference cancellation (SIC) at all those paired users except the one being allocated with the highest transmit power [3].

The authors in [1] evaluated the system-level performance of NOMA in cellular networks and demonstrated the importance of user pairing. The coverage performance of NOMA with distance-based user pairing was analyzed in [4], which showed that the performance gain of NOMA over orthogonal multiple access (OMA) is larger when the users with more diverse channel conditions are being paired. The performance of NOMA was also analyzed for networks with cooperative transmission [5], unsaturated traffic [6], multiple-input multiple-output (MIMO) transmission [7], and uplink transmission [8].

Millimeter wave (mmWave) communication is an enabling technology for 5G networks [9]. By taking the advantage of short wavelength at mmWave frequencies, antenna arrays can be deployed to perform directional beamforming to exploit the array gain. The authors in [10] and [11] analyzed the coverage probability of OMA transmission in mmWave networks, where the actual beamforming pattern was approximated by the sector and sinc antenna pattern models. Applying NOMA in mmWave networks can further enhance the network performance [12]–[15]. Specifically, the authors in [12] analyzed the performance of NOMA in mmWave networks with random beamforming. The performance of NOMA in clustered mmWave networks was analyzed in [13] and [14], where random and distance-based user pairing strategies were adopted, respectively. However, these studies did not take into account directional transmission in the design of user pairing strategies, and hence the array gain may not be fully exploited by all the paired users. In [15], we proposed an angle-based user pairing strategy to increase the probability that the paired users are covered by the BS's main beam. However, a simplified sector antenna pattern model was adopted and the distance information, which is also important for user pairing, was not considered in [15].

In this paper, we consider downlink NOMA transmission in mmWave networks, taking into account the beamforming pattern generated by a uniform linear array (ULA), link blockage characterized by the line-of-sight (LoS) ball model, and spatially random users. To fully exploit the antenna array gain and facilitate NOMA transmission in mmWave networks, we propose a joint angle and distance based user pairing strategy. Specifically, the user located nearest to the BS is paired with another user that is located within a certain distance from the BS and has the minimum relative spatial angle difference. To account for the directional beamforming and the randomness of the spatial angle difference, the BS opportunistically chooses to enable NOMA or OMA. The main contributions of this paper are summarized as follows.

- We propose a novel joint angle and distance based user pairing strategy, which fully exploits the antenna array gain and the benefits of NOMA by taking into account the key features of mmWave networks (e.g., directional beamforming).
- We analyze the distributions of the distance and the minimum spatial angle difference of the paired users chosen by our proposed user pairing strategy, and derive the corresponding coverage probabilities by using tools from stochastic geometry.
- Simulations validate the theoretical analysis. Results show that the proposed scheme achieves a better performance than

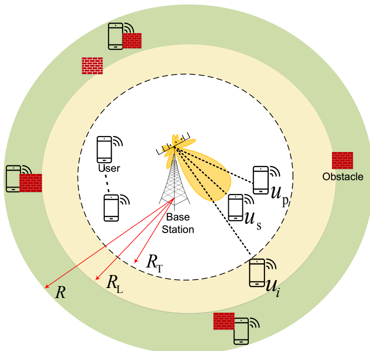

Fig. 1: Topology of an mmWave network under consideration.

the angle-based NOMA, distance-based NOMA, and OMA schemes. Results also confirm the importance of taking into account both angle and distance information for user pairing.

The rest of this paper is organized as follows. We present the system model and the proposed scheme in Section II. Section III analyzes the coverage probability of the proposed scheme. Performance evaluation is presented in Section IV. Section V concludes this paper. The proofs are given in the Appendices.

II. System Model

A. System Description

Consider the downlink power-domain NOMA transmission of an mmWave cellular network, where one BS serves multiple spatially random users, as shown in Fig. 1. The BS, equipped with N antennas, is located at the center of a circular network coverage area with radius R. The single-antenna users form a homogeneous Poisson point process (PPP), denoted as $\Phi = \{x_1, x_2, \ldots\}$, where x_i is the spatial location of user u_i. The spatial density of users is denoted as λ. Time is slotted into constant durations.

For outdoor transmission at mmWave frequencies, each link is sensitive to blockage and can be modeled as a LoS or non-LoS (NLoS) link [9]. We adopt the LoS ball model, which has been widely used in the literature and validated in real-world scenarios [10]. The probability of a link with length r being LoS is $p_{\mathrm{L}}(r) = \mathbb{1}(r \le R_{\mathrm{L}})$, where $\mathbb{1}(\cdot)$ denotes the indicator function and R_{L} denotes the maximum length of LoS links. As demonstrated in [10], the correlation of the blockage effects among different links can be neglected as it only causes a minor loss in the accuracy of the performance analysis.

At mmWave frequencies, the NLoS links suffer from severe path loss and the channel gain of NLoS links can be 20 dB lower than that of LoS links [16]. Hence, we focus on the LoS links, as in [12]. We denote $l(r) = C_{\mathrm{L}} r^{-\beta_{\mathrm{L}}}$ as the path loss of a LoS link with length r, where C_{L} and β_{L} denote the constant parameter and the path loss exponent, respectively. Due to limited scattering, each link is assumed to suffer from independent Nakagami-m fading [10]–[15]. The channel gain between the BS and user u_i, denoted as $|h_i|^2$, is a normalized Gamma random variable, i.e., $\mathrm{Gamma}(M_{\mathrm{L}}, 1/M_{\mathrm{L}})$, where M_{L} is assumed to be a positive integer for simplicity.

To accurately characterize the directional transmission, we adopt the actual array pattern of the ULA. The array response vector corresponding to the spatial angle of departure (AoD) ϑ_i is $\mathbf{a}(\vartheta_i) = \frac{1}{\sqrt{N}} \left[1, \ldots, \mathrm{e}^{j2\pi k \rho \vartheta_i}, \ldots, \mathrm{e}^{j2\pi (N-1) \rho \vartheta_i}\right]^{\mathrm{T}}$, where $(\cdot)^{\mathrm{T}}$ denotes the transpose, $\vartheta_i = \cos\phi_i$ is the spatial AoD of the channel between the BS and user u_i, ϕ_i is the physical AoD in radian, ρ is the ratio of antenna spacing to wavelength, and $k \in \{0, 1, \ldots, N-1\}$ is the antenna index. To avoid grating lobes, the antennas are usually densely placed. As shown in [11], the spatial AoD ϑ_i can be approximated as a uniformly distributed random variable over $[-1, 1]$. To reduce the hardware cost and power consumption due to digital signal processing, we consider analog beamforming. The analog beamforming vector is given by $\mathbf{w} = \mathbf{a}(\theta)$, where θ denotes the spatial AoD of the channel between the BS and its intended user and is also uniformly distributed over $[-1, 1]$. Hence, the array gain obtained by user u_i, i.e., $|\mathbf{a}^{\mathrm{H}}(\vartheta_i)\mathbf{w}|^2$, is

$$G(\vartheta_i - \theta) = \frac{\sin^2(\pi \rho N (\vartheta_i - \theta))}{N^2 \sin^2(\pi \rho (\vartheta_i - \theta))} \triangleq \frac{\sin^2(\pi \rho N \varphi_i)}{N^2 \sin^2(\pi \rho \varphi_i)}, \quad (1)$$

where $\vartheta_i - \theta$ is replaced with φ_i for notational ease. As proved in [16], φ_i is also uniformly distributed over $[-1, 1]$ due to the uniform distributions of ϑ_i and θ. With beam tracking techniques, the BS can adjust its beam orientation to be aligned with the channel vector between itself and its intended user.

B. Proposed NOMA Transmission

We consider two-user NOMA transmission. To enhance the network performance, the user that is located nearest to the BS is selected and denoted as u_{s}. To account for directional transmission in mmWave networks, we propose a joint angle and distance based user pairing strategy. The *paired user*, denoted as u_{p}, is selected based on the following criterion

$$x_{\mathrm{p}} = \arg \min_{x_i \in \Phi(R_{\mathrm{T}}) \setminus \{x_{\mathrm{s}}\}} \varphi_{i\mathrm{s}}, \quad (2)$$

where x_{p} denotes the location of user u_{p}, R_{T} is the distance threshold, $\Phi(R_{\mathrm{T}}) = \{x_i \mid r_i < R_{\mathrm{T}}\}$ with r_i being the distance between the BS and user u_i, and $\varphi_{i\mathrm{s}} = |\varphi_i - \varphi_{\mathrm{s}}|$ is the absolute value of the spatial angle difference between users u_i and u_{s}. Specifically, the proposed user pairing strategy pairs the nearest user with another user from the set $\Phi(R_{\mathrm{T}})$ and has the minimum relative spatial angle difference. We assume that the users' location information is known at the BS as in [12]. This assumption is practical because the location of each user changes much slower than the instantaneous channel gain. The conventional distance-based user pairing strategy (e.g., [14]) only exploits the distance information, which is suitable for sub-6 GHz networks, but ignores the key feature of mmWave networks (i.e., directional transmission). Without taking into account the angle information, the probability that both NOMA users are simultaneously covered by the directional beam of the BS can be low, and hence, the antenna array gain cannot be fully exploited. On the other hand, the conventional angle-based user pairing strategy (e.g., [15]) fully exploits the antenna array gain, but does not take into account the distance information, which may cause a large propagation loss in mmWave networks. The proposed user pairing strategy captures the advantages of both the distance-based and angle-based user pairing strategies to fully exploit the benefits of NOMA in mmWave networks.

Due to the randomness of the spatial angle difference (i.e., $\varphi_{\rm ps}$), we consider an opportunistic transmission scheme, which can be categorized into the following three cases. Case 1: The BS serves both users $u_{\rm s}$ and $u_{\rm p}$ using NOMA if the spatial angle difference $\varphi_{\rm ps} \leq \epsilon$, where ϵ is a predetermined angle threshold. Case 2: If the spatial angle difference $\varphi_{\rm ps} > \epsilon$, then the BS is not able to simultaneously provide high array gains for both users $u_{\rm s}$ and $u_{\rm p}$ using NOMA, and hence, the BS serves both users using OMA. Case 3: If user $u_{\rm p}$ does not exist, then the BS only serves user $u_{\rm s}$ using OMA. The power allocation coefficients for users $u_{\rm s}$ and $u_{\rm p}$ are denoted as $\alpha_{\rm s}$ and $\alpha_{\rm p}$, respectively, with $\alpha_{\rm s} + \alpha_{\rm p} = 1$ and $\alpha_{\rm p} > \alpha_{\rm s}$.

1) NOMA Transmission: In Case 1, the signal intended for user $u_{\rm p}$ is decoded first. The BS adjusts its beam orientation towards user $u_{\rm s}$. Hence, the array gain between the BS and user $u_{\rm s}$ is $NG(0)$, which is equal to N since $G(0) = 1$. The signal-to-interference-plus-noise ratio (SINR) of the signal intended for user $u_{\rm p}$ and decoded by user $u_{\rm s}$ is given by

$$\Gamma_{\rm p\to s} = \frac{\alpha_{\rm p} N P_{\rm B} |h_{\rm s}|^2 l(r_{\rm s})}{\alpha_{\rm s} N P_{\rm B} |h_{\rm s}|^2 l(r_{\rm s}) + \sigma^2}, \tag{3}$$

where $P_{\rm B}$ denotes the transmit power of the BS and σ^2 denotes the variance of the additive white Gaussian noise (AWGN). If user $u_{\rm s}$ fails to decode the signal intended for user $u_{\rm p}$, then it cannot decode its own signal. Otherwise, user $u_{\rm s}$ decodes the signal intended for itself with signal-to-noise ratio (SNR) $\Gamma_{\rm s} = \alpha_{\rm s} N P_{\rm B} |h_{\rm s}|^2 l(r_{\rm s})/\sigma^2$. On the other hand, user $u_{\rm p}$ treats the signal intended for user $u_{\rm s}$ as noise and decodes its own signal with the following SINR

$$\Gamma_{\rm p|s} = \frac{\alpha_{\rm p} N P_{\rm B} |h_{\rm p}|^2 l(r_{\rm p}) G(\varphi_{\rm ps})}{\alpha_{\rm s} N P_{\rm B} |h_{\rm p}|^2 l(r_{\rm p}) G(\varphi_{\rm ps}) + \sigma^2}, \tag{4}$$

where $\varphi_{\rm ps} = |\varphi_{\rm p} - \varphi_{\rm s}|$ and $G(\varphi_{\rm ps})$ is the array gain between the BS and user $u_{\rm p}$ when the spatial angle difference between users $u_{\rm s}$ and $u_{\rm p}$ is $\varphi_{\rm ps}$.

2) OMA Transmission: When either Case 2 or Case 3 occurs, OMA transmission is enabled. The BS adjusts its beam orientation to be aligned with the channel vector between itself and its intended receiver. Hence, the SNR observed at user $u_i, i \in \{{\rm s}, {\rm p}\}$ is $\Gamma_i^{\rm OMA} = NP_{\rm B}|h_i|^2 l(r_i)/\sigma^2$. Note that only the selected user (i.e., $u_{\rm s}$) is served in Case 3.

III. Analysis of Coverage Probability

In this section, we derive the probability density functions (PDFs) of the random variables involved in SINRs and analyze the coverage probabilities of users $u_{\rm s}$ and $u_{\rm p}$ of the proposed scheme. We present the PDF of distance $r_{\rm s}$ in Lemma 1.

Lemma 1. *Given that there are $K \geq 1$ users in set $\Phi(R_{\rm T})$, the PDF of the distance between the BS and user $u_{\rm s}$ is*

$$f_{r_{\rm s}}(r) = 2KrR_{\rm T}^{-2}\left(1 - r^2/R_{\rm T}^2\right)^{K-1}, \ 0 < r < R_{\rm T}. \tag{5}$$

Based on the user pairing strategy given in (2), user $u_{\rm p}$ must be different from user $u_{\rm s}$ and its distance to the BS should be greater than that of user $u_{\rm s}$, i.e., $r_{\rm p} > r_{\rm s}$. Note that the distance distribution of user $u_{\rm p}$ depends on $r_{\rm s}$. The conditional PDF of distance $r_{\rm p}$ is given in Lemma 2.

Lemma 2. *If the paired user $u_{\rm p}$ exists, then the conditional PDF of the distance between the BS and user $u_{\rm p}$ is given by*

$$f_{r_{\rm p}|r_{\rm s}}(r) = \begin{cases} 2r(R_{\rm T}^2 - r_{\rm s}^2)^{-1}, & r_{\rm s} < r < R_{\rm T}, \\ 0, & r \leq r_{\rm s}. \end{cases} \tag{6}$$

Due to the randomness of user locations, the spatial angle difference $\varphi_{\rm ps}$ is also a random variable. With the user pairing strategy given in (2), the PDF of the spatial angle difference $\varphi_{\rm ps}$ is presented in the following lemma.

Lemma 3. *Given that there are $K \geq 1$ users in set $\Phi(R_{\rm T})$, with the user pairing strategy given in (2), the PDF of the spatial angle difference between users $u_{\rm s}$ and $u_{\rm p}$ is given by*

$$f_{\varphi_{\rm ps}}(\varphi) = (K-1)\left(1 - \varphi/2\right)^{2K-3}, \ 0 \leq \varphi \leq 2. \tag{7}$$

According to Lemmas 1 and 3, the PDFs of distance $r_{\rm s}$ and the spatial angle difference $\varphi_{\rm ps}$ depend on the number of users in set $\Phi(R_{\rm T})$ (i.e., K), which in turn depends on user density λ and radius $R_{\rm T}$. Specifically, a larger K leads to a higher probability of distance $r_{\rm s}$ and angle $\varphi_{\rm ps}$ being small.

For NOMA transmission, the reception thresholds for users $u_{\rm s}$ and $u_{\rm p}$ are $\tau_{\rm s} = 2^{\nu_{\rm s}} - 1$ and $\tau_{\rm p} = 2^{\nu_{\rm p}} - 1$, respectively, where $\nu_{\rm s}$ and $\nu_{\rm p}$ denote the corresponding target data rates. As user $u_{\rm s}$ is located closer to the BS, it can decode its own signal if it successfully performs SIC and the SNR of its own signal is greater than the reception threshold. Thus, the coverage probability of user $u_{\rm s}$ can be expressed as $P_{\rm NOMA}^{\rm s} = \mathbb{P}(\Gamma_{\rm s} > \tau_{\rm s}, \Gamma_{\rm p\to s} > \tau_{\rm p}, \varphi_{\rm ps} \leq \epsilon)$, where $\varphi_{\rm ps} \leq \epsilon$ is the condition that NOMA is enabled. Proposition 1 provides the coverage probability of user $u_{\rm s}$ when NOMA is enabled.

Proposition 1. *Given that there are $K \geq 2$ users in set $\Phi(R_{\rm T})$ and the spatial angle difference $\varphi_{\rm ps}$ is not greater than ϵ, i.e., $\varphi_{\rm ps} \leq \epsilon$, the coverage probability of user $u_{\rm s}$ is given by*

$$\begin{aligned} P_{\rm NOMA}^{\rm s}(K) = & \sum_{n=0}^{M_{\rm L}-1} \sum_{m=0}^{K-1} \frac{\binom{K-1}{m} 2K(-1)^m}{n! \beta_{\rm L} R_{\rm T}^{2m+2}} \left(\frac{M_{\rm L}\xi_{\rm s}}{C_{\rm L}}\right)^{-\frac{2m+2}{\beta_{\rm L}}} \\ & \times \gamma\left(n + \frac{2m+2}{\beta_{\rm L}}, \frac{M_{\rm L}\xi_{\rm s} R_{\rm T}^{\beta_{\rm L}}}{C_{\rm L}}\right)\left(1 - \left(1 - \frac{1}{2}\epsilon\right)^{2K-2}\right), \end{aligned} \tag{8}$$

if $\alpha_{\rm p} > \tau_{\rm p}\alpha_{\rm s}$, otherwise $P_{\rm NOMA}^{\rm s}(K) = 0$, where $\xi_{\rm s} = \max\left\{\frac{\tau_{\rm p}\sigma^2}{(\alpha_{\rm p} - \tau_{\rm p}\alpha_{\rm s})NP_{\rm B}}, \frac{\tau_{\rm s}\sigma^2}{\alpha_{\rm s} N P_{\rm B}}\right\}$.

Based on Proposition 1, $P_{\rm NOMA}^{\rm s}(K)$ is an increasing function of K and ϵ, and depends on the target data rates and power allocation coefficients of both users. The power allocation coefficients should be appropriately set to satisfy inequality $\alpha_{\rm p} > \tau_{\rm p}\alpha_{\rm s}$, so as to ensure successful SIC at user $u_{\rm s}$.

The paired user $u_{\rm p}$ treats the signal intended for user $u_{\rm s}$ as noise and can successfully decode its own signal if $\Gamma_{\rm p|s} > \tau_{\rm p}$. Proposition 2 presents the coverage probability of user $u_{\rm p}$.

Proposition 2. *Given that there are $K \geq 2$ users in set $\Phi(R_{\rm T})$ and the spatial angle difference $\varphi_{\rm ps}$ is not greater than ϵ, i.e., $\varphi_{\rm ps} \leq \epsilon$, the coverage probability of the paired user $u_{\rm p}$ can be approximated as (9), shown at the top of this page, if $\alpha_{\rm p} > \tau_{\rm p}\alpha_{\rm s}$, otherwise $P_{\rm NOMA}^{\rm p}(K) = 0$, where $\xi_{\rm p} =$*

$$P_{\mathrm{NOMA}}^{\mathrm{p}}(K) \approx \sum_{n=0}^{M_{\mathrm{L}}-1} \sum_{q_1=1}^{Q_1} \sum_{q_2=1}^{Q_2} \left(\frac{M_{\mathrm{L}}\xi_{\mathrm{p}}}{C_{\mathrm{L}}}\right)^{-\frac{2}{\beta_{\mathrm{L}}}} \frac{K(K-1)\epsilon\pi^2 r_{\mathrm{s},q_2}\sqrt{1-\zeta_{q_1}^2}}{n! Q_1 Q_2 R_{\mathrm{T}}\beta_{\mathrm{L}}(R_{\mathrm{T}}^2 - r_{\mathrm{s},q_2}^2)} \sqrt{1-\eta_{q_2}^2}\, G^{\frac{2}{\beta_{\mathrm{L}}}}(\varphi_{\mathrm{ps},q_1}) \left(1 - \frac{\varphi_{\mathrm{ps},q_1}}{2}\right)^{2K-3} \times \left(1 - \frac{r_{\mathrm{s},q_2}^2}{R_{\mathrm{T}}^2}\right)^{K-1} \left(\gamma\left(n+\frac{2}{\beta_{\mathrm{L}}}, \frac{M_{\mathrm{L}}\xi_{\mathrm{p}} R_{\mathrm{T}}^{\beta_{\mathrm{L}}}}{G(\varphi_{\mathrm{ps},q_1})C_{\mathrm{L}}}\right) - \gamma\left(n+\frac{2}{\beta_{\mathrm{L}}}, \frac{M_{\mathrm{L}}\xi_{\mathrm{p}} r_{\mathrm{s},q_2}^{\beta_{\mathrm{L}}}}{G(\varphi_{\mathrm{ps},q_1})C_{\mathrm{L}}}\right)\right) \quad (9)$$

$$P_{\mathrm{OMA}}^{\mathrm{p}}(K) \approx \sum_{n=0}^{M_{\mathrm{L}}-1} \sum_{q_2=1}^{Q_2} \left(\frac{M_{\mathrm{L}}\xi_{\mathrm{p}}^{O}}{C_{\mathrm{L}}}\right)^{-\frac{2}{\beta_{\mathrm{L}}}} \frac{2K\pi r_{\mathrm{s},q_2}\sqrt{1-\eta_{q_2}^2}}{n! Q_2 R_{\mathrm{T}}\beta_{\mathrm{L}}(R_{\mathrm{T}}^2 - r_{\mathrm{s},q_2}^2)} \left(1-\frac{1}{2}\epsilon\right)^{2K-2} \left(1 - \frac{r_{\mathrm{s},q_2}^2}{R_{\mathrm{T}}^2}\right)^{K-1} \times \left(\gamma\left(n+\frac{2}{\beta_{\mathrm{L}}}, \frac{M_{\mathrm{L}}\xi_{\mathrm{p}}^{O} R_{\mathrm{T}}^{\beta_{\mathrm{L}}}}{C_{\mathrm{L}}}\right) - \gamma\left(n+\frac{2}{\beta_{\mathrm{L}}}, \frac{M_{\mathrm{L}}\xi_{\mathrm{p}}^{O} r_{\mathrm{s},q_2}^{\beta_{\mathrm{L}}}}{C_{\mathrm{L}}}\right)\right). \quad (11)$$

$\frac{\tau_{\mathrm{p}}\sigma^2}{(\alpha_{\mathrm{p}}-\tau_{\mathrm{p}}\alpha_{\mathrm{s}})NP_{\mathrm{B}}}$, $\zeta_{q_1} = \cos\left(\frac{2q_1-1}{2Q_1}\pi\right)$, $\varphi_{\mathrm{ps},q_1} = \frac{\epsilon}{2}(\zeta_{q_1}+1)$, $\eta_{q_2} = \cos\left(\frac{2q_2-1}{2Q_2}\pi\right)$, $r_{\mathrm{s},q_2} = \frac{R_{\mathrm{T}}}{2}(\eta_{q_2}+1)$, and Q_1 and Q_2 are parameters introduced by Gauss-Chebyshev quadrature.

Similarly, $P_{\mathrm{NOMA}}^{\mathrm{p}}(K)$ is also increasing with K, but does not depend on the target data rate of user u_{s} as SIC is not required at user u_{p}. The approximation in (9) is due to the use of Gauss-Chebyshev quadrature for the reduction of computational complexity. Results in Section IV show that small values of Q_1 and Q_2 (e.g., 30) guarantee the accuracy of the approximation, and hence (9) can be efficiently calculated.

When $\varphi_{\mathrm{ps}} > \epsilon$, OMA is enabled to serve users u_{s} and u_{p} in the first and second half of a time slot, respectively. To ensure that NOMA and OMA achieve the same target data rate, the SINR thresholds required for successful signal reception at users u_{s} and u_{p} when OMA is enabled are denoted as $\tau_{\mathrm{s}}' = 2^{2\nu_{\mathrm{s}}} - 1$ and $\tau_{\mathrm{p}}' = 2^{2\nu_{\mathrm{p}}} - 1$, respectively. On the other hand, when the paired user does not exist, OMA is enabled to serve user u_{s} only and the corresponding SNR reception threshold is τ_{s}. Proposition 3 presents the coverage probabilities of users u_{s} and u_{p} for OMA transmission.

Proposition 3. *When the paired user exists (i.e., $K \geq 2$) but $\varphi_{\mathrm{ps}} > \epsilon$, the coverage probabilities of users u_{s} and u_{p} can, respectively, be expressed as*

$$P_{\mathrm{OMA}}^{\mathrm{s}}(K) = \sum_{n=0}^{M_{\mathrm{L}}-1} \sum_{m=0}^{K-1} \frac{\binom{K-1}{m} 2K(-1)^m}{n!\beta_{\mathrm{L}} R_{\mathrm{T}}^{2m+2}} \left(\frac{M_{\mathrm{L}}\xi_{\mathrm{s}}^{O}}{C_{\mathrm{L}}}\right)^{-\frac{2+2m}{\beta_{\mathrm{L}}}} \times \gamma\left(n + \frac{2m+2}{\beta_{\mathrm{L}}}, \frac{M_{\mathrm{L}}\xi_{\mathrm{s}}^{O} R_{\mathrm{T}}^{\beta_{\mathrm{L}}}}{C_{\mathrm{L}}}\right)\left(1-\frac{1}{2}\epsilon\right)^{2K-2}, \quad (10)$$

and (11), shown at the top of this page, where $\xi_{\mathrm{s}}^{O} = \frac{\tau_{\mathrm{s}}'\sigma^2}{NP_{\mathrm{B}}}$, $\xi_{\mathrm{p}}^{O} = \frac{\tau_{\mathrm{p}}'\sigma^2}{NP_{\mathrm{B}}}$, and η_{q_2} and r_{s,q_2} are given in Proposition 2.

When the paired user does not exist (i.e., $K = 1$), the coverage probability of user u_{s} is given by

$$P_{\mathrm{OMA}}^{\mathrm{s},1} = \sum_{n=0}^{M_{\mathrm{L}}-1} \left(\frac{2}{n!\beta_{\mathrm{L}} R_{\mathrm{T}}^2}\right)\left(\frac{M_{\mathrm{L}}\tau_{\mathrm{s}}\sigma^2}{NP_{\mathrm{B}}C_{\mathrm{L}}}\right)^{-\frac{2}{\beta_{\mathrm{L}}}} \times \gamma\left(n+\frac{2}{\beta_{\mathrm{L}}}, \frac{M_{\mathrm{L}}\tau_{\mathrm{s}}\sigma^2 R_{\mathrm{T}}^{\beta_{\mathrm{L}}}}{NP_{\mathrm{B}}C_{\mathrm{L}}}\right). \quad (12)$$

Proof. Following the steps as in Appendices D and E, we obtain (10)–(12). Due to space limitation, the details of the proof are omitted. □

The coverage probabilities achieved by the proposed NOMA scheme are presented in Theorem 1.

Theorem 1. *For the proposed NOMA scheme in mmWave networks, the coverage probabilities of users u_{s} and u_{p}, denoted as $P_{\mathrm{s}}^{\mathrm{cov}}$ and $P_{\mathrm{p}}^{\mathrm{cov}}$, are given by*

$$P_{\mathrm{s}}^{\mathrm{cov}} = \Omega(1)P_{\mathrm{OMA}}^{\mathrm{s},1} + \sum_{K=2}^{\infty}\Omega(K)\left(P_{\mathrm{NOMA}}^{\mathrm{s}}(K) + P_{\mathrm{OMA}}^{\mathrm{s}}(K)\right),$$

$$P_{\mathrm{p}}^{\mathrm{cov}} = \sum_{K=2}^{\infty}\Omega(K)\left(P_{\mathrm{NOMA}}^{\mathrm{p}}(K) + P_{\mathrm{OMA}}^{\mathrm{p}}(K)\right), \quad (13)$$

where $\Omega(K) = \frac{(\lambda\pi R_{\mathrm{T}}^2)^K}{K!}\exp(-\lambda\pi R_{\mathrm{T}}^2)$ is the probability that K users are within set $\Phi(R_{\mathrm{T}})$, and $P_{\mathrm{NOMA}}^{\mathrm{s}}(K)$, $P_{\mathrm{OMA}}^{\mathrm{s}}(K)$, $P_{\mathrm{NOMA}}^{\mathrm{p}}(K)$, $P_{\mathrm{OMA}}^{\mathrm{p}}(K)$, and $P_{\mathrm{OMA}}^{\mathrm{s},1}$ are given in (8)–(12).

Proof. By combining the coverage probabilities of all the NOMA and OMA cases, we directly obtain the main result on the coverage probabilities given in (13). □

IV. Performance Evaluation

In this section, we present the simulation and numerical results of the proposed scheme, which are compared with the results of the distance-based NOMA [14], angle-based NOMA [15], and OMA schemes. In [14], the BS selects the user that is located nearest to the BS and pairs it with a randomly selected user. The BS pairs a randomly selected user with another user that has the minimum relative angle difference in [15]. In addition, the BS serves two users in the first and second half of a time slot in OMA for fair comparison. The transmit power of the BS is $P_{\mathrm{B}} = -15$ dBm and the noise power is $\sigma^2 = -80$ dBm. According to recent measurements [11], the value of radius of the LOS ball should be around hundred meters. The radii R and R_{L} are set to 200 m and 130 m, respectively. Unless specified otherwise, we set $\lambda = 0.0004$ nodes/m^2, $\beta_{\mathrm{L}} = 3$, $N = 8$, $C_{\mathrm{L}} = 2$, $M_{\mathrm{L}} = 3$, $\epsilon = 0.4$, $R_{\mathrm{T}} = 100$ m, $\alpha_{\mathrm{p}} = 0.9$, $\alpha_{\mathrm{s}} = 0.1$, $Q_1 = Q_2 = 30$, $\nu_{\mathrm{s}} = 5$ bits per channel use (BPCU), and $\nu_{\mathrm{p}} = 3$ BPCU.

Fig. 2 shows the impact of the transmit power (i.e., P_{B}) on the coverage probability of the paired user. The simulation

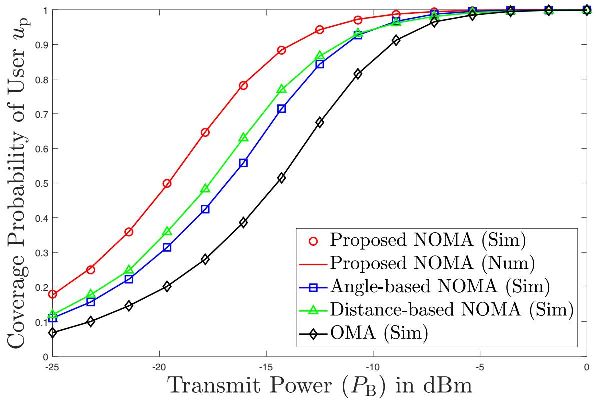

Fig. 2: Coverage probability of the paired user u_{p} versus transmit power.

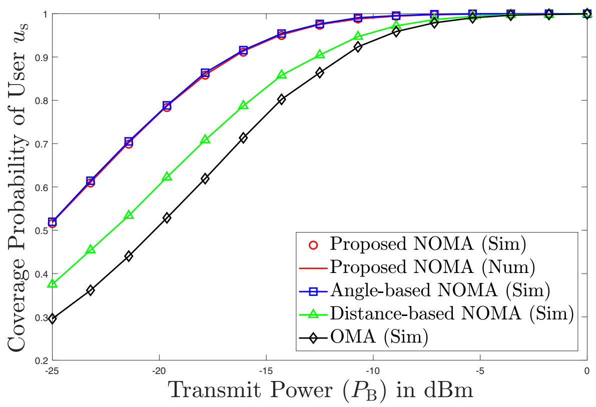

Fig. 3: Coverage probability of user u_{s} versus transmit power.

(Sim) results match well with the numerical (Num) results, which validate the theoretical analysis. As P_{B} increases, the received SINR at user u_{p} increases, which in turn increases the coverage probabilities of the paired user for all considered schemes. The coverage probability of the proposed scheme is higher than that of all other schemes. This is because the proposed joint angle and distance based user pairing strategy captures the advantages of both distance-based and angle-based user pairing strategies.

We plot the coverage probability of user u_{s} versus the transmit power in Fig. 3. The coverage probabilities of all schemes increase with the transmit power. However, the coverage probability of user u_{s} of the proposed scheme is slightly smaller than that of the angle-based NOMA scheme. This is due to the fact that the proposed scheme, with an objective to pair the user located within a certain distance from the BS, limits the spatial region from which the paired user can be selected, while the user pairing strategy of the angle-based NOMA scheme does not depend on the distance and has a higher probability of enabling NOMA. Based on Figs. 2 and 3, the proposed scheme outperforms the angle-based NOMA scheme in terms of the overall network performance.

Fig. 4 illustrates the impact of distance threshold R_{T} and user density λ on the coverage probability of the paired user. When λ increases, the coverage probabilities of both schemes increase, as the probability that the spatial angle difference between users u_{s} and u_{p} being small also increases and in turn a higher antenna array gain can be achieved. With the increase

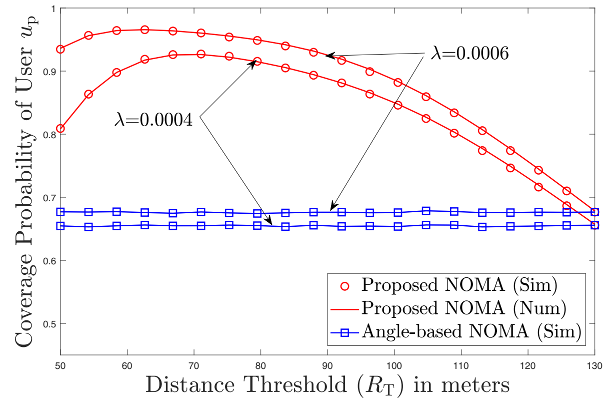

Fig. 4: Coverage probability of the paired user u_{p} versus distance threshold.

of R_{T}, the coverage probability of the proposed scheme first increases as the probability of enabling NOMA is increased. When R_{T} exceeds a certain threshold, the coverage probability decreases because of a higher probability of selecting a user located far away from the BS. Thus, there exists an optimal value of R_{T} that maximizes the coverage probability of the paired user. The coverage probability of the angle-based NOMA scheme does not change with R_{T}. Moreover, the proposed scheme achieves a better performance than the angle-based NOMA scheme, except when $R_{\mathrm{T}} = R_{\mathrm{L}} = 130$ m.

V. Conclusions

In this paper, we studied the downlink NOMA transmission in mmWave networks with spatially random users. We proposed a joint angle and distance based user pairing strategy to facilitate NOMA transmission in mmWave networks. The proposed user pairing strategy fully exploits the antenna array gain and the spectrum efficiency gain of NOMA. We derived the coverage probabilities of the proposed scheme by using tools from stochastic geometry, while taking into account the actual beamforming pattern. Simulations validated the performance analysis. Results showed that the proposed scheme achieves a better performance than three baseline schemes in terms of the coverage probabilities and confirmed the importance of considering both the angle and distance information for user pairing in mmWave networks. Moreover, results also showed that there exists an optimal value of the distance threshold that maximizes the coverage probability.

Appendix

A. *Proof of Lemma 1*

With the LoS ball model, the LoS users follow a homogeneous PPP with density λ. Hence, given K users within set $\Phi(R_{\mathrm{T}})$, the cumulative distribution function (CDF) of the distance (e.g., r_i) between a randomly chosen user and the BS is $F_{r_i}(r) = \frac{r^2}{R_{\mathrm{T}}^2}\mathbb{1}(r < R_{\mathrm{T}})$. As user u_{s} is the one that is nearest to the BS among K users, the PDF of distance r_{s} can be obtained based on order statistics [17] as $f_{r_{\mathrm{s}}}(r) = K(1-F_{r_i}(r))^{K-1} f_{r_i}(r), 0 < r < R_{\mathrm{T}}$, where $f_{r_i}(r) = \frac{2r}{R_{\mathrm{T}}^2}\mathbb{1}(r < R_{\mathrm{T}})$ is the first derivative of $F_{r_i}(r)$. By substituting $f_{r_i}(r)$ and $F_{r_i}(r)$ into $f_{r_{\mathrm{s}}}(r)$, we obtain (5).

B. Proof of Lemma 2

As user $u_{\rm s}$ is located nearest to the BS, the probability that $r_{\rm p} \leq r_{\rm s}$ is zero. Hence, the CDF of distance $r_{\rm p}$ conditioning on $r_{\rm p} > r_{\rm s}$ can be expressed as

$$F_{r_{\rm p}|r_{\rm s}}(r) = \frac{\mathbb{P}(r_{\rm p} \leq r, r_{\rm p} > r_{\rm s})}{\mathbb{P}(r_{\rm p} > r_{\rm s})} = \frac{r^2 - r_{\rm s}^2}{R_{\rm T}^2 - r_{\rm s}^2}, \; r_{\rm s} < r < R_{\rm T}.$$

By taking the first derivative of $F_{r_{\rm p}|r_{\rm s}}(r)$, we obtain the conditional PDF of distance $r_{\rm p}$ in (6).

C. Proof of Lemma 3

As mentioned in Section II-B, $\varphi_i = \vartheta_i - \theta$ is uniformly distributed over $[-1, 1]$ due to the uniform distributions of spatial AoD ϑ_i and θ. Hence, the PDF of the spatial angle difference between user $u_{\rm s}$ and a randomly selected user is $f_{\varphi_{is}}(\varphi) = 1 - \frac{1}{2}\varphi, 0 \leq \varphi \leq 2$. For the proposed user pairing strategy, user $u_{\rm p}$ is the one that has the minimum relative spatial angle difference, the CDF of the minimum spatial angle difference $\varphi_{\rm ps}$ is given by $F_{\varphi_{\rm ps}}(\varphi) = 1 - \left(1 - \frac{1}{2}\varphi\right)^{2K-2}$. By taking the first derivative of $F_{\varphi_{\rm ps}}(\varphi)$, we obtain the PDF of the spatial angle difference $\varphi_{\rm ps}$ in (7).

D. Proof of Proposition 1

Given that there are $K \geq 2$ users within set $\Phi(R_{\rm T})$, the coverage probability of user $u_{\rm s}$ can be expressed as

$$\begin{aligned}
&P_{\rm NOMA}^{\rm s}(K) \overset{(a)}{=} \mathbb{P}\left(|h_{\rm s}|^2 > \frac{\xi_{\rm s}}{l(r_{\rm s})}\right) \mathbb{P}(\varphi_{\rm ps} \leq \epsilon) \\
&\overset{(b)}{=} \sum_{n=0}^{M_{\rm L}-1} \frac{(M_{\rm L}\xi_{\rm s})^n}{n!} \mathbb{E}_{\{r_{\rm s}\}}\left(\frac{\exp\left(-\frac{M_{\rm L}\xi_{\rm s}}{l(r_{\rm s})}\right)}{(l(r_{\rm s}))^n}\right) F_{\varphi_{is}}(\epsilon) \\
&\overset{(c)}{=} \sum_{n=0}^{M_{\rm L}-1} \frac{(M_{\rm L}\xi_{\rm s}/C_{\rm L})^n}{n!} \int_0^{R_{\rm T}} \exp\left(-\frac{M_{\rm L}\xi_{\rm s} r_{\rm s}^{\beta_{\rm L}}}{C_{\rm L}}\right) r_{\rm s}^{n\beta_{\rm L}} \\
&\quad \times f_{r_{\rm s}}(r) {\rm d}r_{\rm s} F_{\varphi_{is}}(\epsilon) \\
&\overset{(d)}{=} \sum_{n=0}^{M_{\rm L}-1} \frac{(M_{\rm L}\xi_{\rm s}/C_{\rm L})^n}{n!} \sum_{m=0}^{K-1} \binom{K-1}{m} \left(-\frac{1}{R_{\rm T}^2}\right)^m \\
&\quad \times \int_0^{R_{\rm T}} \exp\left(-\frac{M_{\rm L}\xi_{\rm s} r_{\rm s}^{\beta_{\rm L}}}{C_{\rm L}}\right) r_{\rm s}^{n\beta_{\rm L}+1+2m} \frac{2K}{R_{\rm T}^2} {\rm d}r_{\rm s} F_{\varphi_{is}}(\epsilon),
\end{aligned}$$

where (a) follows from the independence between $\varphi_{\rm ps}$ and other random variables and by denoting $\xi_{\rm s} = \max\left\{\frac{\tau_{\rm p}\sigma^2}{(\alpha_{\rm p} - \tau_{\rm p}\alpha_{\rm s})NP_{\rm B}}, \frac{\tau_{\rm s}\sigma^2}{\alpha_{\rm s}NP_{\rm B}}\right\}$, (b) holds as $|h_{\rm s}|^2$ follows the normalized Gamma distribution, (c) follows by taking expectation over $r_{\rm s}$, and (d) follows by taking a binomial expansion. After solving the integral, we obtain (8).

E. Proof of Proposition 2

With the joint angle and distance based user pairing strategy, the coverage probability of the paired user $u_{\rm p}$ is given by

$$\begin{aligned}
&P_{\rm NOMA}^{\rm p}(K) = \mathbb{P}(\Gamma_{\rm p|s} > \tau_{\rm p}, \varphi_{\rm ps} \leq \epsilon) \\
&\overset{(a)}{=} \sum_{n=0}^{M_{\rm L}-1} \frac{(M_{\rm L}\xi_{\rm p})^n}{n!} \mathbb{E}_{\{\varphi_{\rm ps}, r_{\rm s}, r_{\rm p}\}}\left(\frac{\exp\left(-\frac{M_{\rm L}\xi_{\rm p}}{G(\varphi_{\rm ps})l(r_{\rm p})}\right)}{(G(\varphi_{\rm ps})l(r_{\rm p}))^n}\right), \quad (14)
\end{aligned}$$

where (a) follows by denoting $\xi_{\rm p} = \frac{\tau_{\rm p}\sigma^2}{(\alpha_{\rm p} - \tau_{\rm p}\alpha_{\rm s})NP_{\rm B}}$ and by taking expectation over $|h_{\rm p}|^2$. We denote the expectation in (14) as Θ. Based on the PDFs of distance $r_{\rm s}$, distance $r_{\rm p}$, and the spatial angle difference $\varphi_{\rm ps}$, we have

$$\begin{aligned}
\Theta &= \int_0^{\epsilon} \int_0^{R_{\rm T}} \int_{r_2}^{R_{\rm T}} \exp\left(-\frac{M_{\rm L}\xi_{\rm p} r_1^{\beta_{\rm L}}}{G(\varphi)C_{\rm L}}\right) (G(\varphi)C_{\rm L} r_1^{-\beta_{\rm L}})^{-n} \\
&\quad \times f_{r_{\rm p}|r_{\rm s}}(r_1) f_{r_{\rm s}}(r_2) f_{\varphi_{\rm ps}}(\varphi) {\rm d}r_1 {\rm d}r_2 {\rm d}\varphi \\
&\overset{(a)}{=} \frac{4K}{(M_{\rm L}\xi_{\rm p})^n R_{\rm T}^2 \beta_{\rm L}} \left(\frac{M_{\rm L}\xi_{\rm p}}{C_{\rm L}}\right)^{-\frac{2}{\beta_{\rm L}}} \int_0^{\epsilon} \int_0^{R_{\rm T}} \\
&\quad \left(\gamma\left(n + \frac{2}{\beta_{\rm L}}, \frac{M_{\rm L}\xi_{\rm p} R_{\rm T}^{\beta_{\rm L}}}{G(\varphi)C_{\rm L}}\right) - \gamma\left(n + \frac{2}{\beta_{\rm L}}, \frac{M_{\rm L}\xi_{\rm p} r_2^{\beta_{\rm L}}}{G(\varphi)C_{\rm L}}\right)\right) \\
&\quad \times \frac{r_2}{R_{\rm T}^2 - r_2^2} \left(1 - \frac{r_2^2}{R_{\rm T}^2}\right)^{K-1} G(\varphi)^{\frac{2}{\beta_{\rm L}}} f_{\varphi_{\rm ps}}(\varphi) {\rm d}r_2 {\rm d}\varphi, \quad (15)
\end{aligned}$$

where (a) follows from the Fubini's theorem. By applying Gauss-Chebyshev quadrature in (15), we obtain (9).

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