Direct Energy Trading of Microgrids in Distribution Energy Market

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***Abstract*—Recent advancement of distributed renewable generation has motivated microgrids to trade energy directly with one another, as well as with the utility, in order to minimize their operational costs. Energy trading among microgrids, however, confronts challenges such as reaching a fair trading price, maximizing participants' profit, and satisfying power network constraints. In this paper, we formulate the direct energy trading among multiple microgrids as a generalized Nash bargaining (GNB) problem that involves the distribution network's operational constraints (e.g., power balance equations, voltage limits). We prove that solving the GNB problem maximizes the social welfare and also distributes the revenue among the microgrids based on their market power. To address the nonconvexity of the GNB problem, we propose a two-phase approach. The first phase involves solving the optimal power flow problem in a distributed fashion using the alternative direction method of multipliers to determine the amount of energy trading. The second phase determines the market clearing price and mutual payments of the microgrids. Simulation results on an IEEE 33-bus system with four microgrids show that the proposed framework substantially reduces total network cost by 37.2%. Our results suggest direct trading need to be enforced by the regulators in order to maximize the social welfare.**

I. Introduction

Recent development of renewable generations (e.g., wind turbine, photovoltaic (PV) panel) has made sustainable energy economically viable. Unlike the conventional large-scale generators, renewable generators are often small-scale, and thus appropriate for serving microgrids. However, the stochastic nature of renewable energy sources and the fluctuations in load demand can cause microgrids to experience intermittent energy shortage or surplus. To this end, direct energy trading among microgrids can be a viable solution to balance energy and lower the operational cost.

Direct trading is beneficial to both sellers and buyers compared to the trading with the utility company by reducing the intermediate trading step. However, there are several challenges in designing a direct energy trading mechanism. First, it is difficult to reach an agreement on the *trading price*, which should not be biased toward either sellers or buyers. Second, it is crucial to determine the *power flow* from sellers to buyers while satisfying the distribution network constraints. Third, it would be desirable that direct trading can maximize the *social welfare* (or equivalently, minimize the total cost of energy generation and operation) so that regulators can advocate direct energy trading with legitimate support.

There have been some efforts to tackle the first challenge associated with trading price [2]–[5]. For example, auction mechanism is applied for direct trading among microgrids in [2]. A coalition of sellers and buyers is considered in [3] to collectively trade energy with the utility company and share the revenue using the Shapley value. The concept of peer-to-peer trading between any pair of microgrids using Nash bargaining solution is proposed in [5]. However, [2]–[5] do not consider the distribution network constraints. In resolving the second and third challenges associated with physical constraints and optimal operation, several works study the energy management system of microgrids and/or distribution network, but without a market clearing mechanism [6]–[8]. In these works, microgrids are assumed to cooperate to minimize their aggregate cost in a distributed fashion. Some works have investigated the distributed optimal power flow (OPF) for energy trading in a distribution network using techniques such as the alternative direction method of multipliers (ADMM) [8]. The combination of OPF and direct trading is proposed [9], but a heuristic market clearing may not guarantee social welfare.

In this paper, we provide a framework that can address the above challenges. The proposed framework determines the amount of direct energy trading and the corresponding payment among microgrids, considering the operational constraints imposed by the distribution network. We formulate the problem as a *generalized* Nash bargaining (GNB) problem. We summarize our key contributions as follows.

1) *Direct Trading Framework:* We design a general market mechanism for direct trading among microgrids considering full AC power flow model for the distribution network. Solving the GNB problem can incentivize microgrids to participate in direct trading rather than trading with the utility company. We prove that solving the GNB problem minimizes the total cost, and thereby the proposed framework maximizes the social welfare while each microgrid can maximize its own profit.

2) *Distributed Optimization Methods:* We address the non-convexity and obtain an optimal solution of the GNB problem by first solving the OPF and then clearing the market. To solve the OPF problem in a distributed manner, we leverage ADMM to decouple the optimization variables of the microgrids and the distribution network. This enables us to determine the amount of energy trading while concurrently solving the OPF. Then, the market is cleared by using ADMM in a privacy

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preserving manner. The proposed market mechanism ensures that the profit of each microgrid is proportional to the amount of energy exchange by exploiting the notion of market power.

3) *Significant Cost Reduction:* Simulation results show that the proposed direct trading can reduce the total network cost by 37.2% compared to the case without direct trading. Furthermore, the costs are reduced by 9−42.8%, the revenues are increased by up to 73% depending on microgrids . The power losses are also reduced by 20.6%. Finally, all participating microgrids fairly achieve the same trading profit per kWh.

The rest of this paper is organized as follows. In Section II we describe the overall system model including the distribution network and the components within each microgrid. We formulate our direct trading with power flow problem in Section III. We present a distributed approach for solving the OPF and clearing the market in Section IV. Simulation results are provided in Section V, and conclusion is given in Section VI.

II. System Model

Consider a radial distribution network represented by a graph $\mathcal{G}(\mathcal{N},\mathcal{E})$, where $\mathcal{N}$ is the set of buses and $\mathcal{E}\subseteq\mathcal{N}\times\mathcal{N}$ is the set of branches in the network. We consider a set $\mathcal{M}\subseteq\mathcal{N}$ of $M=|\mathcal{M}|$ microgrids. Let $0\in\mathcal{N}$ denote the slack bus of $\mathcal{G}$, where the utility is connected as an external power source of the distribution network. The direct energy trading framework among microgrids in a distribution network is shown in Fig. 1. Let $\mathcal{T}=\{1,\dots,T\}$ denote the operational horizon, which is divided into T time slots with equal duration (e.g., one hour) denoted by Δt.

For bus $i\in\mathcal{N}$, let $V_i(t)$ denote the complex voltage in time slot $t\in\mathcal{T}$, and let $s_i(t)=p_i(t)+\mathbf{i}q_i(t)$ denote the complex power injection into bus i. For line $(i,j)\in\mathcal{E}$, let $z_{ij}=r_{ij}+\mathbf{i}x_{ij}$ denote the line impedance, and $I_{ij}(t)$ denote complex current from bus i to bus j in time slot t. For branch $(i,j)\in\mathcal{E}$ and time slot $t\in\mathcal{T}$ we have $V_i(t)-V_j(t)=z_{ij}I_{ij}(t)$. Let $I^*_{ij}(t)$ denote the complex conjugate of $I_{ij}(t)$. Then, the complex power flow in line $(i,j)\in\mathcal{E}$ is defined by $S_{ij}(t)=V_i(t)I^*_{ij}(t)$ from which the real power $P_{ij}(t)$ and the reactive power $Q_{ij}(t)$ are determined such that $S_{ij}(t)=P_{ij}(t)+\mathbf{i}Q_{ij}(t)$.

The power balance equation for bus $j\in\mathcal{N}$ is given by $s_j(t)=S_{ij}(t)-z_{ij}\left|I^*_{ij}(t)\right|^2-\sum_{k\neq i:(j,k)\in\mathcal{E}}S_{jk}(t)$. Let $l_{ij}(t)=|I_{ij}(t)|^2$ and $v_i(t)=|V_i(t)|^2$. Using the branch flow model in [10], we have the following equations with real variables for all $(i,j)\in\mathcal{E}$ and $t\in\mathcal{T}$,

$$p_j(t) = P_{ij}(t)-r_{ij}l_{ij}(t)-\sum_{k\neq i:(j,k)\in\mathcal{E}}P_{jk}(t), \quad (1)$$

$$q_j(t) = Q_{ij}(t)-x_{ij}l_{ij}(t)-\sum_{k\neq i:(j,k)\in\mathcal{E}}Q_{jk}(t), \quad (2)$$

$$v_j(t)=v_i(t)-2(r_{ij}P_{ij}(t)+x_{ij}Q_{ij}(t))+(r_{ij}^2+x_{ij}^2)l_{ij}(t), \quad (3)$$

$$l_{ij}(t) = \frac{P_{ij}(t)^2+Q_{ij}(t)^2}{v_i(t)}. \quad (4)$$

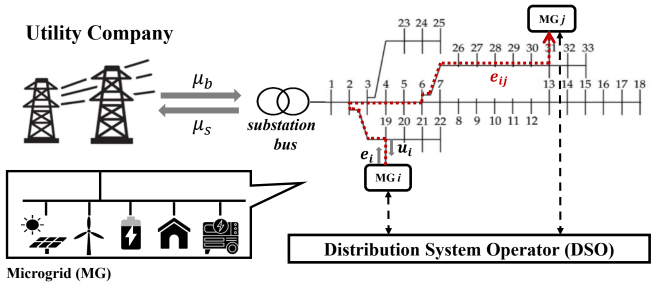


Figure 1. The system schematic of microgrids energy trading through the distribution network.

We consider the following voltage tolerance constraint:

$$v_i^{\min}\le v_i(t)\le v_i^{\max},\ i\in\mathcal{N}\backslash\{0\}. \quad (5)$$

As shown in Fig. 1, the microgrids are interconnected by the distribution network $\mathcal{G}$ through which energy can be traded. Each microgrid $i\in\mathcal{M}$ has its own renewable or fuel-based distributed generator (DG), energy storage, and local loads. The goal of each microgrid is to minimize its total operational cost which includes the cost of purchasing energy from the utility, battery degradation cost, and fuel-based distributed generation operational cost. We assume fixed loads that can be forecasted with reasonably good accuracy.

1) Power trading with the utility: Let $u_{b,i}(t)$ denote the power purchased from the utility company by microgrid i and $\mu_b(t)$ denote the purchasing price (\$/MWh) in time slot t. Due to the physical or contractual power limit, we have

$$0\le u_{b,i}(t)\le u_{b,i}^{\max},\ \ i\in\mathcal{M},\ \ t\in\mathcal{T}, \quad (6)$$

where $u_{b,i}^{\max}$ denotes the maximum purchasing power of microgrid i. Let $d_i(t)$ denote the load of microgrid $i\in\mathcal{M}$ in time slot t. Due to the stochastic nature of the renewable generation, the local generation level of a microgrid can exceed the total local load demands. The microgrid can sell its surplus power to the utility at selling price (\$/MWh) $\mu_s(t)$ in time slot t. The amount of selling power, denoted by $u_{s,i}(t)$, is also subject to the physical or contractual power limit:

$$0\le u_{s,i}(t)\le u_{s,i}^{\max},\ \ i\in\mathcal{M},\ \ t\in\mathcal{T}, \quad (7)$$

where $u_{s,i}^{\max}$ denotes the maximum selling power of microgrid i. Then, the cost of purchasing power by microgrid $i\in\mathcal{M}$ from the utility during time period $\mathcal{T}$ is

$$C_{u,i}(\mathbf{u}_i)=\sum_{t\in\mathcal{T}}[\mu_b(t)u_{b,i}(t)-\mu_s(t)u_{s,i}(t)]\Delta t, \quad (8)$$

where $\mathbf{u}_i=(u_{b,i}(t),\,u_{s,i}(t),\,t\in\mathcal{T})$ is a power trading profile of microgrid $i\in\mathcal{M}$ with the utility.

2) Battery operation: The charging and discharging powers of microgrid $i\in\mathcal{M}$ in time slot t, denoted by $b_{c,i}(t)$ and $b_{d,i}(t)$, are limited by the capacity of power conditioning system such that

$$0\le b_{c,i}(t)\le b_{c,i}^{\max}, \quad (9a)$$

$$0\le b_{d,i}(t)\le b_{d,i}^{\max}, \quad (9b)$$

where $b_{c,i}^{\max}$ and $b_{d,i}^{\max}$ are the maximum of charging power and discharging power of the battery in microgrid i, respectively. The stored energy in the battery $E_{b,i}(t)$ changes according to the following equation:

$$E_{b,i}(t+1) = E_{b,i}(t) + \left(\eta_{c,i} b_{c,i}(t) - \frac{1}{\eta_{d,i}} b_{d,i}(t)\right)\Delta t, \quad (10)$$

where $\eta_{c,i}$ and $\eta_{d,i}$ are the charging and discharging efficiencies of microgrid i. Since battery degradation is known to be severe at both ends of the state-of-charge (SoC), i.e., either empty or full, $E_{b,i}(t)$ should be constrained by [11]

$$\mathrm{SoC}_i^{\min} \le \frac{E_{b,i}(t)}{E_{b,i}^{\max}} \le \mathrm{SoC}_i^{\max}, \quad (11)$$

where $\mathrm{SoC}_i^{\min}$ and $\mathrm{SoC}_i^{\max}$ denote the minimum and maximum SoC of the battery, and $E_{b,i}^{\max}$ denotes the maximum battery capacity in microgrid i.

Although the battery degradation depends on the SoC, the degradation density function of the SoC is almost flat between $\mathrm{SoC}_i^{\min}$ and $\mathrm{SoC}_i^{\max}$ [12]. Thus, the battery degradation cost can be computed by the amount of transferred energy:

$$C_{b,i}(\mathbf{b}_i) = c_{b,i} \sum_{t\in\mathcal{T}} [b_{c,i}(t) + b_{d,i}(t)]\Delta t, \quad (12)$$

where $\mathbf{b}_i = (b_{c,i}(t),\, b_{d,i}(t),\, t \in \mathcal{T})$ and $c_{b,i}$ is the degradation cost coefficient per unit energy.

3) Distributed generation cost: Let $r_i(t)$ denote renewable generation of microgrid $i \in \mathcal{M}$ in time slot t. We assume that $r_i(t)$ can be predicted reasonably well as in [5], [6]. Renewable generation is assumed to have zero marginal cost in the short run [5]. On the other hand, fuel-based generation such as fuel cell, distributed micro turbine or diesel generator has a nonlinear cost function [6]. We use the following quadratic cost function for a fuel-based DG in microgrid i:

$$C_{g,i}(\mathbf{g}_i) = \sum_{t\in\mathcal{T}} \left(\kappa_{2,i} g_i(t)^2 + \kappa_{1,i} g_i(t) + \kappa_{0,i}\right)\Delta t, \quad (13)$$

where $\mathbf{g}_i = (g_i(t),\, t \in \mathcal{T})$, and the positive coefficients of $\kappa_{2,i}$, $\kappa_{1,i}$, and $\kappa_{0,i}$ depend on the type of DG. The output power of DG in microgrid i is bounded by

$$g_i^{\min} \le g_i(t) \le g_i^{\max}, \quad (14)$$

where $g_i^{\min}$ and $g_i^{\max}$ are the minimum and maximum generation capacities in microgrid i, respectively.

4) Total cost of microgrid: The active power balance equation at microgrid $i \in \mathcal{M}$ in time slot $t \in \mathcal{T}$ is

$$r_i(t)+g_i(t)+u_{b,i}(t)+b_{d,i}(t) = d_i(t)+u_{s,i}(t)+b_{c,i}(t), \quad (15)$$

where $d_i(t)$ is the real power demand of microgrid i in time slot t. Then, the left-hand side corresponds to the power generations and the right-hand side corresponds to the power demands. Then, the *internal* cost function of microgrid $i \in \mathcal{M}$ is given by

$$\widetilde{C}_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) = C_{u,i}(\mathbf{u}_i) + C_{b,i}(\mathbf{b}_i) + C_{g,i}(\mathbf{g}_i). \quad (16)$$

5) Microgrid's local optimization problem: If microgrid $i \in \mathcal{M}$ does not participate in direct energy trading with other microgrids, it solves the following optimization problem:

P0: Microgrid's Optimization without Direct Trading

$$\begin{aligned} &\text{minimize} && \widetilde{C}_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) \\ &\text{subject to} && (6), (7), (9a)\text{–}(11), (14), (15), \\ &\text{variables} && \{\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i\}. \end{aligned}$$

Problem **P0** is a convex problem since the objective function and all constraints are convex. Problem **P0** is solved by microgrid $i \in \mathcal{M}$. The optimal value is denoted by $\overline{C}_i$.

III. GNB for Direct Energy Trading

When microgrid $i \in \mathcal{M}$ has energy surplus or deficit, it can trade power through the distribution network $\mathcal{G}$ as shown in Fig. 1. Let $e_{ij}(t)$ denote the *exporting* power from microgrid i to microgrid j. In a lossless power network, we have $e_{ij}(t) + e_{ji}(t) = 0$. However, the power losses may not be negligible. Specifically, in the distribution network, we have $e_{ij}(t) + e_{ji}(t) = r_{ij} l_{ij}(t)$. Note that the power losses depend on $l_{ij}(t) = |I_{ij}(t)|^2$, i.e., the solution of OPF. However, before solving the OPF, we do not know the feasibility of direct trading between microgrids i and j due to physical constraints. To overcome the complexity to trace $e_{ij}(t)$ for all tradable (i, j) pairs of microgrids, we focus on the net *exporting* power $e_i(t)$ to all other microgrids, which is defined as $e_i(t) = \sum_{j\in\mathcal{M}\setminus\{i\}} e_{ij}(t)$. If $e_i(t)$ is negative, then microgrid i will import power from other microgrids through the distribution network. The net power *injection into* microgrid i becomes

$$p_i(t) = u_{b,i}(t) - u_{s,i}(t) - e_i(t), \;\; i \in \mathcal{M}, t \in \mathcal{T}. \quad (17)$$

The power balance equation of (15) becomes

$$\begin{aligned} &r_i(t) + g_i(t) + u_{b,i}(t) + b_{d,i}(t) \quad (18) \\ &= d_i(t) + e_i(t) + u_{s,i}(t) + b_{c,i}(t), \;\; i \in \mathcal{M}, t \in \mathcal{T}. \end{aligned}$$

We also have the following constraint

$$\sum_{i\in\mathcal{M}} e_i(t) = 0, \;\; t \in \mathcal{T}, \quad (19)$$

which implies that the sum of all exporting powers should be equal to the sum of all importing powers in time slot t. Let $\mathbf{e}_i = (e_i(t),\, t \in \mathcal{T})$ denote the trading profile of microgrid $i \in \mathcal{M}$. To incentivize direct trading, the cost after direct trading should be less than or equal to the cost before direct trading $\overline{C}_i$. To determine the cost after direct trading, we consider two other factors: the distribution network access fee of microgrid i, denoted by β_i, and the direct trading payment of microgrid i denoted by π_i. Microgrid i participates in the direct energy trading only if

$$\widetilde{C}_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) + \beta_i + \pi_i \le \overline{C}_i, \quad i \in \mathcal{M}. \quad (20)$$

We consider a non-profit organization called distribution system operator (DSO), which manages and balances the distribution network. The DSO should compensate power losses in the distribution network by purchasing power from the utility company through the slack bus. Thus, DSO imposes an access fee β_i for microgrid $i \in \mathcal{M}$ to cover the *overhead cost* for direct trading. Let $\beta = \sum_{i\in\mathcal{M}} \beta_i$ denote the total overhead cost. Then, we have

$$\beta = \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{E}} r_{ij} l_{ij}(t)\xi(t), \tag{21}$$

where $\xi(t)$ is a coefficient that accounts for the overhead cost from the power losses and network maintenance. For example, when $\xi(t)$ is equal to $\mu_b(t)$, the access fee accounts for the cost from power losses. Note that determining $\xi(t)$ requires the detailed analysis on the operational cost and is beyond the scope of this paper. When the access fee β_i is imposed in proportion to the amount of direct trading, we have

$$\beta_i = \frac{\sum_{t\in\mathcal{T}} |e_i(t)|}{\sum_{j\in\mathcal{M}} \sum_{t\in\mathcal{T}} |e_j(t)|}\beta, \quad i \in \mathcal{M}. \tag{22}$$

Then, the cost for microgrid i including the access fee is defined by

$$C_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) = \widetilde{C}_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) + \beta_i, \quad i \in \mathcal{M}. \tag{23}$$

Finally, the payment of one microgrid becomes the revenue of the other microgrids, and the sum of payments is zero, i.e.,

$$\sum_{i\in\mathcal{M}} \pi_i = 0. \tag{24}$$

Then, the GNB problem is formulated as follows.

P1: Generalized Nash Bargaining (GNB) Problem

$$\begin{aligned} \text{maximize} \quad & \prod_{i\in\mathcal{M}} \left[\,\overline{C}_i - (C_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) + \pi_i)\right]^{\alpha_i} \qquad (25)\\ \text{subject to} \quad & (1)\text{–}(5), (6), (7), (9a)\text{–}(11), (14),\\ & (17)\text{–}(20), (24),\\ \text{variables} \quad & \{\mathbf{e}_i, \mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i, \pi_i, i \in \mathcal{M}, \mathbf{P}, \mathbf{Q}, \mathbf{v}, \mathbf{l}, \mathbf{s}\}, \end{aligned}$$

where the positive parameter α_i denotes the market power of microgrid $i \in \mathcal{M}$, $\mathbf{P} = (P_{ij}(t), (i,j) \in \mathcal{E}, t \in \mathcal{T})$, $\mathbf{Q} = (Q_{ij}(t), (i,j) \in \mathcal{E}, t \in \mathcal{T})$, $\mathbf{v} = (v_i(t), i \in \mathcal{N} \setminus \{0\}, t \in \mathcal{T})$, $\mathbf{l} = (l_{ij}(t), (i,j) \in \mathcal{E}, t \in \mathcal{T})$, $\mathbf{s} = (s_i(t), i \in \mathcal{N}, t \in \mathcal{T})$.

Instead of solving **P1**, we provide Proposition 1 stating that the solution of **P1** also minimizes the total cost of the distribution network, which gives us a way to detour in solving **P1** in two separate steps: solving the OPF and then determining the market clearing.

Proposition 1 (Social Welfare Maximization): Let C_i^* denote the optimal value of $C_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i)$ at the solution of **P1**. Then, the solution of **P1** minimizes the total cost of the distribution network, which is given by $\sum_{i\in\mathcal{M}} C_i^*$.

Proof: For notational simplicity, we omit the variables $(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i)$ in C_i. We prove by contradiction. Let $\{C_i^*, i \in \mathcal{M}\}$ be obtained from the solution of **P1**. Suppose that $\sum_{i\in\mathcal{M}} C_i^*$ does not minimize $\sum_{i\in\mathcal{M}} C_i$. Then, there exists C_i' such that $\sum_{i\in\mathcal{M}} C_i' < \sum_{i\in\mathcal{M}} C_i^*$.

Let $\Delta C_i = C_i' - C_i^*$. Then, we have

$$\sum_{i\in\mathcal{M}} \Delta C_i < 0. \tag{26}$$

We consider another cost C_i and the payment π_i such that $C_i = C_i^* + \Delta C_i$ for $i = 1, \ldots, M$ and $\pi_i = \pi_i^* - \Delta C_i$ for $i = 1, \ldots, M-1$, and $\pi_M = \pi_M^* - \Delta C_M + \epsilon$ where $\epsilon = -\sum_{i\in\mathcal{M}}(\pi_i^* - \Delta C_i)$ to satisfy the constraint (24). Then,

$$\begin{aligned} &\prod_{i=1}^{M} \left[\,\overline{C}_i - (C_i + \pi_i)\right]^{\alpha_i}\\ &\quad = \left[\overline{C}_1 - (C_1^* + \Delta C_1 + \pi_1^* - \Delta C_1)\right]^{\alpha_1} \times \cdots \times\\ &\qquad \left[\overline{C}_M - (C_M^* + \Delta C_M + \pi_M^* - \Delta C_M + \epsilon)\right]^{\alpha_M}. \end{aligned} \tag{27}$$

From (24) and (26), $\epsilon = \sum_{i\in\mathcal{M}} \Delta C_i < 0$. Then, all other terms of (27) remain the same as $\left[\,\overline{C}_i - (C_i^* + \pi_i^*)\right]^{\alpha_i}$, but the last term increases, which makes

$$\prod_{i=1}^{M} \left[\,\overline{C}_i - (C_i + \pi_i)\right]^{\alpha_i} > \prod_{i=1}^{M} \left[\,\overline{C}_i - (C_i^* + \pi_i^*)\right]^{\alpha_i}.$$

This contradicts that C_i^* and π_i^* maximize **P1**. ■

IV. Distributed Solution Approach

The OPF problem is formulated as follows: The cost function is equal to $\sum_{i\in\mathcal{M}} C_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) = \sum_{i\in\mathcal{M}} \left(\widetilde{C}_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) + \beta_i\right)$ according to (21) and (22). The distribution network constraints are given by (1)−(5), the constraints of microgrids are given by (6), (7), (9a)−(11), (14), and the power balancing constraints with trading are given by (17)−(19). The OPF problem is nonconvex due to the quadratic equality constraint of (4). We apply convex relaxation by replacing (4) with the inequality constraint:

$$l_{ij}(t) \geq \frac{P_{ij}(t)^2 + Q_{ij}(t)^2}{v_i(t)}, \tag{28}$$

which gives us the following relaxed OPF problem.

P2: OPF-r Problem with ADMM

$$\begin{aligned} \text{minimize} \quad & \sum_{i\in\mathcal{M}} \widetilde{C}_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) + \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{E}} r_{ij} l_{ij}(t)\xi(t)\\ \text{subject to} \quad & \text{MG: } (6), (7), (9a)\text{–}(11), (14), (18),\\ & \text{NET: } (1)\text{–}(3), (5), (17), (19), (28),\\ & \text{AUX: } \mathbf{e}_i = \hat{\mathbf{e}}_i, \mathbf{u}_i = \hat{\mathbf{u}}_i, i \in \mathcal{M},\\ \text{variables} \quad & \{\mathbf{e}_i, \mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i, \hat{\mathbf{e}}_i, \hat{\mathbf{u}}_i, i \in \mathcal{M}, \mathbf{P}, \mathbf{Q}, \mathbf{v}, \mathbf{l}, \mathbf{s}\}. \end{aligned}$$

The augmented Lagrangian L is

$$\begin{aligned} &L(\mathbf{e}_i, \mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i, \hat{\mathbf{e}}_i, \hat{\mathbf{u}}_i, \boldsymbol{\lambda}_i, i \in \mathcal{M}, \mathbf{P}, \mathbf{Q}, \mathbf{v}, \mathbf{l}, \mathbf{s})\\ &\quad = \sum_{i\in\mathcal{M}} \widetilde{C}_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) + \sum_{t\in\mathcal{T}} \sum_{(i,j)\in\mathcal{E}} r_{ij} l_{ij}(t)\xi(t)\\ &\qquad + \sum_{i\in\mathcal{M}} (\boldsymbol{\lambda}_{e,i}^{\mathrm{T}}(\hat{\mathbf{e}}_i - \mathbf{e}_i) + \boldsymbol{\lambda}_{u,i}^{\mathrm{T}}(\hat{\mathbf{u}}_i - \mathbf{u}_i)\\ &\qquad + \frac{\rho}{2}||\hat{\mathbf{e}}_i - \mathbf{e}_i||^2 + \frac{\rho}{2}||\hat{\mathbf{u}}_i - \mathbf{u}_i||^2), \end{aligned} \tag{29}$$

where $\boldsymbol{\lambda}_i = (\boldsymbol{\lambda}_{e,i}, \boldsymbol{\lambda}_{u,i})$ and $\boldsymbol{\lambda}_{e,i}$, $\boldsymbol{\lambda}_{u,i}$ are dual variable vectors for the constraints of $\mathbf{e}_i = \hat{\mathbf{e}}_i$ and $\mathbf{u}_i = \hat{\mathbf{u}}_i$, respectively.

Then, we now develop the following MG$_i$ update rule, DSO update rule, and the dual variables update rule.

For MG$_i$ update, at the $(m+1)^{\text{th}}$ iteration, each microgrid $i \in \mathcal{M}$ solves the following optimization problem with $\hat{\mathbf{e}}_i = \hat{\mathbf{e}}_i^{(m)}$, $\hat{\mathbf{u}}_i = \hat{\mathbf{u}}_i^{(m)}$, and $\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_i^{(m)}$.

MG$_i$ update

$$\begin{aligned} \text{minimize} \quad & \widetilde{C}_i(\mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i) \\ & + \boldsymbol{\lambda}_{e,i}^{\mathrm{T}}(\hat{\mathbf{e}}_i - \mathbf{e}_i) + \boldsymbol{\lambda}_{u,i}^{\mathrm{T}}(\hat{\mathbf{u}}_i - \mathbf{u}_i) \\ & + \frac{\rho}{2}||\hat{\mathbf{e}}_i - \mathbf{e}_i||^2 + \frac{\rho}{2}||\hat{\mathbf{u}}_i - \mathbf{u}_i||^2 \qquad (30) \\ \text{subject to} \quad & (6), (7), (9a)\text{–}(11), (14), (18), \\ \text{variables} \quad & \{\mathbf{e}_i, \mathbf{u}_i, \mathbf{b}_i, \mathbf{g}_i\}. \end{aligned}$$

The MG$_i$ update is a convex optimization problem. The solution variables of MG$_i$ update are labeled as $\mathbf{e}_i = \mathbf{e}_i^{(m+1)}$ and $\mathbf{u}_i = \mathbf{u}_i^{(m+1)}$, which are used for DSO update below.

DSO update

$$\begin{aligned} \text{minimize} \quad & \sum_{t \in \mathcal{T}} \sum_{(i,j) \in \mathcal{E}} r_{ij} l_{ij}(t) \xi(t) \\ & + \sum_{i \in \mathcal{M}} (\boldsymbol{\lambda}_{e,i}^{\mathrm{T}}(\hat{\mathbf{e}}_i - \mathbf{e}_i) + \boldsymbol{\lambda}_{u,i}^{\mathrm{T}}(\hat{\mathbf{u}}_i - \mathbf{u}_i) \\ & + \frac{\rho}{2}||\hat{\mathbf{e}}_i - \mathbf{e}_i||^2 + \frac{\rho}{2}||\hat{\mathbf{u}}_i - \mathbf{u}_i||^2) \qquad (31) \\ \text{subject to} \quad & (1)\text{–}(3), (5), (17), \sum_{i \in \mathcal{M}} \hat{\mathbf{e}}_i = 0, (28), \\ \text{variables} \quad & \{\hat{\mathbf{e}}_i, \hat{\mathbf{u}}_i, i \in \mathcal{M}, \mathbf{P}, \mathbf{Q}, \mathbf{v}, \mathbf{l}, \mathbf{s}\}. \end{aligned}$$

The DSO update is a convex optimization problem. Let the solution of DSO update be labeled as $\hat{\mathbf{e}}_i^{(m+1)}$ and $\hat{\mathbf{u}}_i^{(m+1)}$. Using the solutions of MG$_i$ update and DSO update, the dual variables are updated as follows.

Dual variable update

$$\boldsymbol{\lambda}_{e,i}^{(m+1)} = \boldsymbol{\lambda}_{e,i}^{(m)} + \rho(\hat{\mathbf{e}}_i^{(m+1)} - \mathbf{e}_i^{(m+1)}), \qquad (32)$$

$$\boldsymbol{\lambda}_{u,i}^{(m+1)} = \boldsymbol{\lambda}_{u,i}^{(m)} + \rho(\hat{\mathbf{u}}_i^{(m+1)} - \mathbf{u}_i^{(m+1)}). \qquad (33)$$

Then, the iteration of the MG$_i$ update, the DSO update, and the dual update converges to an optimal solution [13].

Next, we address the market clearing problem. Let C_i^o denote the *optimal* cost of microgrid $i \in \mathcal{M}$ obtained after solving **P2**. The payment π_i, $i \in \mathcal{M}$ can be determined using the minimum cost C_i^o, $i \in \mathcal{M}$. Note that we use C_i^o instead of C_i^* (which comes from the solutions of **P1**) because they may not necessarily be the same. After substituting C_i^o into **P1**, we have the following market clearing problem.

P3: P2P Market Clearing Problem

$$\begin{aligned} \text{maximize} \quad & \prod_{i \in \mathcal{M}} \left[\overline{C}_i - (C_i^o + \pi_i)\right]^{\alpha_i} \qquad (34) \\ \text{subject to} \quad & \sum_{i \in \mathcal{M}} \pi_i = 0 \\ \text{variables} \quad & \{\pi_i, i \in \mathcal{M}\}, \end{aligned}$$

which is solvable by the exchange ADMM to preserve privacy.

Proposition 2 (Fairness): If the market power α_i is set in proportion to the total traded energy, i.e., $\alpha_i = \frac{\sum_{t \in \mathcal{T}} |e_i^o(t)|}{\sum_{i \in \mathcal{M}} \sum_{t \in \mathcal{T}} |e_i^o(t)|}$ where $e_i^o(t)$ is an optimal solution of **P2**, then each microgrid has the *equal trading profit* per unit energy $\Gamma = \frac{\sum_{i \in \mathcal{M}} \delta_i}{\sum_{i \in \mathcal{M}} \sum_{t \in \mathcal{T}} |e_i^o(t)|}$ where $\delta_i = \overline{C}_i - C_i^o$ upon market clearing.

Proof: By taking log and negating the objective function, we have the following minimization problem

$$\begin{aligned} \text{minimize} \quad & \sum_{i \in \mathcal{M}} -\alpha_i \log(\delta_i - \pi_i) \\ \text{subject to} \quad & \sum_{i \in \mathcal{M}} \pi_i = 0 \\ \text{variables} \quad & \{\pi_i, i \in \mathcal{M}\}. \end{aligned}$$

Then, the Lagrangian is given by $L = \sum_{i \in \mathcal{M}} \left(-\alpha_i \log(\delta_i - \pi_i) + \lambda \pi_i\right)$. From $\frac{\partial L}{\partial \pi_i} = 0$, we have $\pi_i = \delta_i + \frac{\alpha_i}{\lambda}$. From $\sum_{i \in \mathcal{M}} \pi_i = 0$ and $\sum_{i \in \mathcal{M}} \alpha_i = 1$, we have $\frac{1}{\lambda} = -\sum_{i \in \mathcal{M}} \delta_i$. Thus, the payment of microgrid i is simply given by

$$\pi_i = \delta_i - \alpha_i \sum_{j \in \mathcal{M}} \delta_j. \qquad (35)$$

Since the *profit* of microgrid i is defined as the reduced cost after the payment, we have $\gamma_i = \delta_i - \pi_i = \alpha_i \sum_{j \in \mathcal{M}} \delta_j$. Thus, the total profit $\sum_{j \in \mathcal{M}} \delta_j$ of direct trading is allocated to each microgrid i based on its market power α_i. Then, the profit per unit energy is $\frac{\gamma_i}{\sum_{t \in \mathcal{T}} |e_i^o(t)|} = \frac{\sum_{i \in \mathcal{M}} \delta_i}{\sum_{i \in \mathcal{M}} \sum_{t \in \mathcal{T}} |e_i^o(t)|} = \Gamma$. When substituting π_i in (35) into (34), we have the maximum value given by $\prod_{i \in \mathcal{M}} \left[\overline{C}_i - (C_i^o + \pi_i)\right]^{\alpha_i} = \prod_{i \in \mathcal{M}} \left[\alpha_i \sum_{j \in \mathcal{M}} \delta_j\right]^{\alpha_i}$. The maximum value only depends on $\sum_{i \in \mathcal{M}} \delta_i$, and it is uniquely determined by solving **P2**. ■

Proposition 3 (Converse): If the solution of **P1** exists, then the solutions of **P2** and **P3** maximize **P1**.

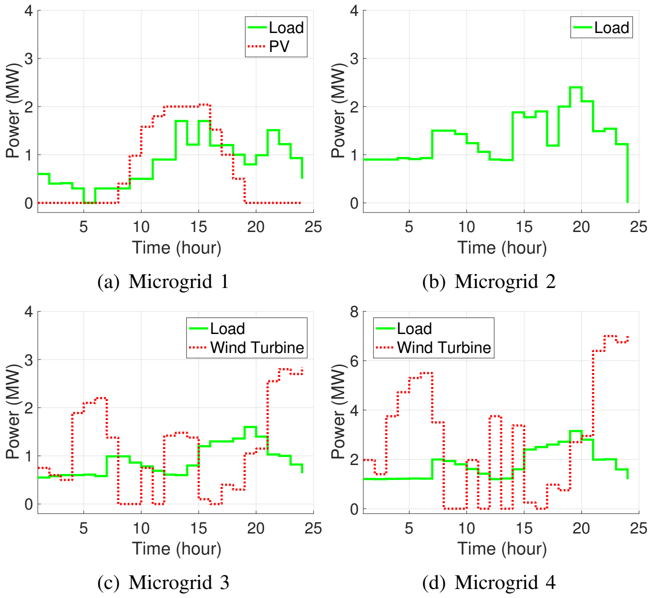
Proof: Let C_i^* and π_i^* be obtained from the solution of **P1**. Let C_i^o be obtained from the solution of **P2**. Recall that we do not claim that $C_i^o = C_i^*$ because we do not know if the solution of **P2** can be a part of the solution of **P1**. Nevertheless, we have the property of $\sum_{i \in \mathcal{M}} C_i^* = \sum_{i \in \mathcal{M}} C_i^o$ by Proposition 1. Then, we replace C_i in (25) with C_i^o, which transforms **P1** into the form of **P3**. Let π_i^o be the solution of **P3**. Then, from the structure of the problem, it can be shown that $\pi_i^o = C_i^* - C_i^o + \pi_i^*$, $i \in \mathcal{M}$. Then, the objective function of **P3** can be equal to that of **P1**. ■

V. Performance Evaluation

In this section, we provide numerical experiments to demonstrate the virtue of the proposed direct trading technique considering four microgrids interconnected in the IEEE 33-bus test system [10]. We use the time-of-use (ToU) pricing provided by California Independent System Operator (CAISO) [6], which serves as the purchasing price from the utility company in our work. The selling price to the utility is set as half of the purchasing price. We consider two cases. In Case 1, each microgrid solves **P0**, i.e., schedules its battery and/or DG to minimize the cost function. In Case 2, microgrids trade energy directly (i.e., solve **P1**) in two steps: solving the OPF-r **P2** and solving the payment problem **P3**. All microgrids

Table I
SIMULATION PARAMETERS.

Parameters	Value / Component
Number of time slots per day	24
Battery size	3 MWh
Maximum battery power	1 MW
Battery charging efficiency	0.9
Battery discharging efficiency	0.9
Battery degradation cost	$10/MWh
Maximum power of DG	3 MW
Maximum SoC	0.9
Minimum SoC	0.1
maximum voltage (p.u.)	1.05
minimum voltage (p.u.)	0.95
κ_2	10
κ_1	61.1



(a) Microgrid 1 (b) Microgrid 2 (c) Microgrid 3 (d) Microgrid 4

Figure 2. Load and renewable generation profiles of different microgrids.

have batteries, loads and DGs. Each microgrid has its own load profile and renewable generation profile as shown in Fig. 2; microgrid 1 has PV generation, microgrid 2 has no renewable generation, and microgrids 3 and 4 have wind power generation. Note that the PV generation is during daytime while the wind power generations are mostly during nighttime. Table I summarizes key parameters in the simulation.

Table II presents the costs before and after direct energy trading (DET). The sum of costs covering all four microgrids is significantly reduced from $2246.74 (**Cost before DET**) to $1588.66 (**Cost after DET**), i.e., 29.3% of reduction. The power loss cost is also reduced from $284 to $225 by 20.6%. The total network cost including power loss cost is then reduced from $2530.74 to $1588.66 by 37.2%.

VI. CONCLUSION

In this paper, we investigated direct energy trading among microgrids considering both the economical and technical aspects of the distribution power market and network constraints. We formulated direct energy trading as a nonconvex generalized Nash bargaining problem and showed that the

Table II
MICROGRID (MG) COSTS ($) USING DIRECT ENERGY TRADING (DET).

Metric	MG1	MG2	MG3	MG4
Cost before DET	**372.37**	**2175.40**	**16.69**	**−317.71**
Cost with OPF	439.42	453.21	386.31	84.15
Access Fee β_i	54.65	68.32	23.15	79.45
GNB Payment π_i	−281.14	1454.53	−460.30	−713.10
Cost after DET	**212.93**	**1976.07**	**−50.84**	**−549.50**
Profit γ_i	159.44	199.33	67.53	231.78
Quantity (MWh)	22.408	28.015	9.491	32.577
Profit per MWh	7.11	7.11	7.11	7.11
Market power	0.242	0.303	0.103	0.352

problem can be solved by decomposing it into two phases: solving the OPF and solving the payment. In both cases, we leveraged ADMM to decouple the optimization variables of the DSO and microgrids, and to preserve the privacy of microgrids. The proposed DSO-based market mechanism is *efficient* in maximizing the social welfare and minimizing the network loss, and also *fair* by guaranteeing the equal trading profit per unit energy among microgrids. Simulation results demonstrated that direct energy trading reduces the total cost including the costs of all microgrids and network loss by 37.2% compared to the case without direct trading.

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