Dynamic User Pairing and Power Allocation for Throughput Maximization in NOMA Systems

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***Abstract*—Non-orthogonal multiple access (NOMA) can enhance the spectral efficiency of the fifth generation (5G) wireless networks. System level optimization on power allocation and user pairing for improving the throughput of NOMA systems has been well studied. However, most of the existing works have not taken into account dynamic traffic arrival and possible packet transmission failure. In this paper, we consider downlink transmission of a power-domain NOMA system with dynamic packet arrival, where the base station supports both NOMA and orthogonal multiple access (OMA). We propose a packet-level scheduling scheme for the base station to decide using either NOMA or OMA, and to determine user pairing and power allocation, with an objective to maximize the aggregate throughput. To tackle the challenges introduced by dynamic packet arrivals and possible transmission failure, we first derive the probability of successful packet transmission with limited feedback on channel state information (CSI). We then formulate a throughput maximization problem as a stochastic network optimization problem taking into account the backlog stabilities. We decompose the problem into two subproblems for NOMA and OMA, respectively, and obtain the optimal user pairing and power allocation. Packet-level simulations show that the proposed scheme obtains a higher throughput compared with the distance-based user pairing scheme and the CSI-based power allocation scheme.**

I. Introduction

Non-orthogonal multiple access (NOMA) has recently been recognized as a promising paradigm to enhance the spectral efficiency and fairness of the fifth generation (5G) wireless networks [1]–[3]. By exploiting the non-orthogonal multiplexing of resources in either power-domain or code-domain, multiple users can be served simultaneously by the base station in the same frequency channel. The potential gains of power-domain NOMA depend on the effectiveness of power allocation and user scheduling [1]. Chen *et al.* in [4] proposed a user pairing policy for massive multiple-input multiple-output (MIMO) NOMA systems and maximized the throughput by mitigating the inter-pair interference. Cui *et al.* in [5] proposed a K-means online user clustering algorithm for millimeter wave NOMA systems, and developed a power allocation scheme for each cluster to maximize the throughput. Liu *et al.* in [6] proposed a power allocation scheme to enhance both the coverage and throughput. However, the aforementioned studies did not take into account packet-level scheduling in a system with dynamic packet arrivals and packet transmission failure. Moreover, as shown in [7], NOMA may not always outperform orthogonal multiple access (OMA). Thus, packet-level scheduling schemes for NOMA systems with dynamic packet arrivals and possible transmission failure should further be investigated.

Some recent works studied dynamic user pairing and power allocation in NOMA systems. The authors in [8] leveraged stochastic network optimization to perform dynamic user scheduling and power allocation for NOMA-based Internet of Things (IoT) networks. However, perfect channel state information (CSI) was assumed to be available at the base station. In addition, the throughput maximization with the backlog stability requirement was also not considered in [8]. In [7], we explored the benefit of full-duplex relaying in NOMA systems with dynamic packet arrivals for spatially random users, where the packet-level scheduling for throughput maximization was not studied. To address the aforementioned issues, in this paper we extend our work in [7] and propose a dynamic packet-level scheduling scheme for downlink power-domain NOMA systems, while taking into account dynamic traffic arrival and limited feedback on channel conditions. In particular, we develop an efficient scheduling scheme to determine the optimal transmission mode (i.e., either NOMA or OMA), user pairing, and power allocation coefficients in the packet-level with an objective to maximize the aggregate throughput. The main contributions of this paper are summarized as follows.

- We derive the closed-form expression of the successful packet transmission probability, which can be expressed as a function of control variables and limited feedback on CSI for downlink power-domain NOMA systems.
- We formulate a user pairing and power allocation problem to maximize the throughput, which is a stochastic network optimization problem. We leverage tools from Lyapunov optimization to characterize the time-varying backlogs.
- We decompose the problem into two subproblems and propose efficient algorithms to solve them. By combining the results of two subproblems, the optimal user pairing and power allocation scheme can be obtained.
- We conduct packet-level simulations to evaluate the performance of the proposed user pairing and power allocation scheme. Simulations show that the proposed scheme can achieve a higher throughput when compared with distance-based power allocation scheme [9] and CSI-based user pairing scheme [10]. Meanwhile, the proposed scheme maintains the stability of backlogs.

This paper is organized as follows. The system model and the derivation of successful packet transmission probability are presented in Section II. The problem formulation and the proposed user pairing and power allocation schemes are described in Section III. Simulation results are given in Section IV. Conclusions are drawn in Section V.

II. System Model

Consider the downlink transmission in a cell with one base station serving multiple users over a single frequency channel. We denote $\mathcal{M} = \{1, 2, \ldots, M\}$ as the set of users within the cell. Time is slotted into constant durations with unit size and the time interval $[t, t+1)$ is referred to as time slot t for $t \in \mathcal{T} = \{0, 1, 2, \ldots\}$. The packets intended for each user arrive at the base station at the beginning of each time slot. The packet arrival for each user is assumed to be independent and identically distributed. To facilitate packet-level scheduling, the base station maintains a separate queue for each user. Specifically, queue Q_i buffers the packets to be transmitted to user $i \in \mathcal{M}$. We denote $a_i(t) \in \{0, 1, 2, \ldots\}$ as the number of packets arrived in queue Q_i at the base station for user i in time slot t, with mean $\lambda_i(t)$. In addition, we denote $Q_i(t)$ as the length of queue Q_i in time slot t.

The channel between user i and the base station suffers from path loss and Rayleigh fading. Let $r_i(t)$ denote the distance between user i and the base station in time slot t. We adopt the non-singular path loss $\ell(r_i(t)) = 1/(1 + (r_i(t))^\beta)$, where β is the path loss exponent. Denote $h_i(t)$ as the channel gain between the base station and user i in time slot t and $|h_i(t)|^2$ follows an exponential distribution.

We assume that the base station only has limited feedback from the users. Specifically, user i sends its distance information (i.e., $r_i(t)$) and a binary indicator, denoted as $s_i(t)$, to the base station at the beginning of time slot t. The binary indicator $s_i(t)$ is equal to 1 if $|h_i(t)|^2 \ell(r_i(t)) \geq \theta$, and is equal to 0 otherwise, where θ is a predefined threshold. Hence, the limited CSI $S_i(t)$ for user i can be expressed as a tuple $S_i(t) = (r_i(t), s_i(t))$. For notational ease, we denote $\boldsymbol{S}(t) = (S_1(t), S_2(t), \ldots, S_M(t))$ as the channel state vector.

The base station supports both NOMA and OMA for downlink transmission. We consider a two-user pairing strategy [7] for NOMA, where two users are allocated different proportions of the total transmit power P. The user allocated with a higher transmit power is denoted as the *high-power user* with subscript H, while the user allocated with a lower transmit power is denoted as the *low-power user* with subscript L. Define $\alpha_{\mathrm{H}}^2(t)P$ and $\alpha_{\mathrm{L}}^2(t)P$ to be the transmit power allocated to the high-power and low-power users in time slot t, respectively. At the receiver side, the packet is received successfully if the signal-to-interference-plus-noise ratio (SINR) exceeds a threshold $\Gamma^{\min}$. Suppose that user $i, i \in \mathcal{M}$, is the high-power user and user $j, j \in \mathcal{M}\backslash\{i\}$, is the low-power user. The SINR observed by user i for its own signal is given by [7], [11]

$$\Gamma_i^{\mathrm{H}}(\alpha_{\mathrm{H}}^2(t), \alpha_{\mathrm{L}}^2(t)) = \frac{\alpha_{\mathrm{H}}^2(t)P|h_i(t)|^2 \ell(r_i(t))}{\alpha_{\mathrm{L}}^2(t)P|h_i(t)|^2 \ell(r_i(t)) + \sigma^2}, \quad (1)$$

where σ^2 is the variance of the additive white Gaussian noise.

Before user j decodes the signal intended for itself, it should successfully decode the signal intended for user i first. The corresponding SINR is given by

$$\Gamma_j^{\mathrm{H}}(\alpha_{\mathrm{H}}^2(t), \alpha_{\mathrm{L}}^2(t)) = \frac{\alpha_{\mathrm{H}}^2(t)P|h_j(t)|^2 \ell(r_j(t))}{\alpha_{\mathrm{L}}^2(t)P|h_j(t)|^2 \ell(r_j(t)) + \sigma^2}. \quad (2)$$

When $\Gamma_j^{\mathrm{H}}(\alpha_{\mathrm{H}}^2(t), \alpha_{\mathrm{L}}^2(t)) > \Gamma^{\min}$, the signal intended for user i is removed by applying successive interference cancellation (SIC), and then user j decodes the signal intended for itself with signal-to-noise ratio (SNR) given by

$$\Gamma_j^{\mathrm{L}}(\alpha_{\mathrm{H}}^2(t), \alpha_{\mathrm{L}}^2(t)) = \alpha_{\mathrm{L}}^2(t)P|h_j(t)|^2 \ell(r_j(t))/\sigma^2. \quad (3)$$

The base station can also transmit a packet using OMA if the channel conditions are not suitable to use NOMA, i.e., the packet transmission using NOMA fails with a high probability. If a packet intended for user k, $k \in \mathcal{M}$, is transmitted using OMA, then the SNR observed at user k is given by

$$\Gamma_k^{\mathrm{OMA}} = P|h_k(t)|^2 \ell(r_k(t))/\sigma^2. \quad (4)$$

We introduce a binary control variable $x_i(t)$ for user i such that $x_i(t) = 1$ if the packet intended for user i is transmitted using NOMA and user i is the high-power user in time slot t, and $x_i(t) = 0$ otherwise. As there can only be at most one high-power user when NOMA is used in time slot t, we have

$$\sum\nolimits_{i \in \mathcal{M}} x_i(t) \leq 1. \quad (5)$$

Similarly, we define a binary control variable $y_i(t)$ for user i such that $y_i(t) = 1$ if the packet intended for user i is transmitted using NOMA and user i is the low-power user in time slot t, and $y_i(t) = 0$ otherwise. Hence, we have

$$\sum\nolimits_{j \in \mathcal{M}} y_j(t) \leq 1. \quad (6)$$

We define a binary control variable $z_i(t)$ such that $z_i(t) = 1$ if the packet intended for user i is transmitted using OMA in time slot t, and $z_i(t) = 0$ otherwise. We have

$$\sum\nolimits_{k \in \mathcal{M}} z_k(t) \leq 1. \quad (7)$$

To ensure that NOMA and OMA cannot be used simultaneously, the following two constraints need to be satisfied

$$\sum\nolimits_{i \in \mathcal{M}} x_i(t) + \sum\nolimits_{k \in \mathcal{M}} z_k(t) \leq 1, \quad (8)$$

$$\sum\nolimits_{j \in \mathcal{M}} y_j(t) + \sum\nolimits_{k \in \mathcal{M}} z_k(t) \leq 1. \quad (9)$$

To ensure that a single packet is transmitted using either NOMA or OMA in a time slot, we have

$$x_i(t) + y_i(t) + z_i(t) \leq 1, \qquad i \in \mathcal{M}. \quad (10)$$

In addition, we have the following constraint

$$x_i(t), y_i(t), z_i(t) \in \{0, 1\}, \qquad i \in \mathcal{M}. \quad (11)$$

For notational ease, we further denote vectors $\boldsymbol{x}(t) = (x_1(t), x_2(t), \ldots, x_M(t))$, $\boldsymbol{y}(t) = (y_1(t), y_2(t), \ldots, y_M(t))$, and $\boldsymbol{z}(t) = (z_1(t), z_2(t), \ldots, z_M(t))$. As $\alpha_{\mathrm{H}}^2(t)$ and $\alpha_{\mathrm{L}}^2(t)$

$$\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))=\begin{cases}1, & \text{if } x_i(t)=1,y_i(t)=z_i(t)=0,s_i(t)=1,Q_i(t)>0,\theta\geq\rho_{\rm H}(t),\\ e^{\frac{\theta-\rho_{\rm H}(t)}{\ell(r_i(t))}}, & \text{if } x_i(t)=1,y_i(t)=z_i(t)=0,s_i(t)=1,Q_i(t)>0,\theta<\rho_{\rm H}(t),\\ \frac{e^{-\rho_{\rm H}(t)/\ell(r_i(t))}-e^{-\theta/\ell(r_i(t))}}{1-e^{-\theta/\ell(r_i(t))}}, & \text{if } x_i(t)=1,y_i(t)=z_i(t)=0,s_i(t)=0,Q_i(t)>0,\theta\geq\rho_{\rm H}(t),\\ 0, & \text{if } x_i(t)=1,y_i(t)=z_i(t)=0,s_i(t)=0,Q_i(t)>0,\theta<\rho_{\rm H}(t),\\ 1, & \text{if } y_i(t)=1,x_i(t)=z_i(t)=0,s_i(t)=1,Q_i(t)>0,\theta\geq\rho^{\max}(t),\\ e^{\frac{\theta-\rho^{\max}(t)}{\ell(r_i(t))}}, & \text{if } y_i(t)=1,x_i(t)=z_i(t)=0,s_i(t)=1,Q_i(t)>0,\theta<\rho^{\max}(t),\\ \frac{e^{-\rho_C/\ell(r_i(t))}-e^{-\theta/\ell(r_i(t))}}{1-e^{-\theta/\ell(r_i(t))}}, & \text{if } y_i(t)=1,x_i(t)=z_i(t)=0,s_i(t)=0,Q_i(t)>0,\theta\geq\rho^{\max}(t),\\ 0, & \text{if } y_i(t)=1,x_i(t)=z_i(t)=0,s_i(t)=0,Q_i(t)>0,\theta<\rho^{\max}(t),\\ 1, & \text{if } z_i(t)=1,x_i(t)=y_i(t)=0,s_i(t)=1,Q_i(t)>0,\\ \frac{e^{-\rho/\ell(r_i(t))}-e^{-\theta/\ell(r_i(t))}}{1-e^{-\theta/\ell(r_i(t))}}, & \text{if } z_i(t)=1,x_i(t)=y_i(t)=0,s_i(t)=0,Q_i(t)>0,\\ 0, & \text{otherwise.}\end{cases} \quad (14)$$

denote the proportions of the transmit power allocated to the high-power and low-power users, respectively, we have

$$\alpha_{\rm H}^2(t)>\alpha_{\rm L}^2(t), \quad (12)$$

$$\alpha_{\rm H}^2(t)+\alpha_{\rm L}^2(t)=1. \quad (13)$$

We define $\Psi_i(t)$ as the event that the packet intended for user i is sent by the base station and also successfully decoded by user i in time slot t. Hence, the probability that event $\Psi_i(t)$ occurs is denoted as $\mathbb{P}[\Psi_i(t)]$. Let $\mathbf{\Pi}(t)=\big(\boldsymbol{x}(t),\boldsymbol{y}(t),\boldsymbol{z}(t),\alpha_{\rm H}^2(t),\alpha_{\rm L}^2(t)\big)$ denote the vector of all control variables. Then, $\mathbb{P}[\Psi_i(t)]$ is a function of $\mathbf{\Pi}(t)$ and $\boldsymbol{S}(t)$, i.e., $\mathbb{P}[\Psi_i(t)]=\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))$, which can be derived as follows.

1) When $Q_i(t)>0$ and user i is the high-power user of NOMA transmission, i.e., $x_i(t)=1$, we have

$$\begin{aligned}\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))&=\mathbb{P}\big[\Gamma_i^{\rm H}(\alpha_{\rm H}^2(t),\alpha_{\rm L}^2(t))>\Gamma^{\min}\big]\\&=\mathbb{P}\left[\frac{\alpha_{\rm H}^2(t)P|h_i(t)|^2\ell(r_i(t))}{\alpha_{\rm L}^2(t)P|h_i(t)|^2\ell(r_i(t))+\sigma^2}>\Gamma^{\min}\right]\\&=\mathbb{P}\left[|h_i(t)|^2>\frac{\rho_{\rm H}(t)}{\ell(r_i(t))}\right],\end{aligned}$$

where $\rho_{\rm H}(t)=\frac{\rho}{\alpha_{\rm H}^2(t)-\alpha_{\rm L}^2(t)\Gamma^{\min}}$ and $\rho=\frac{\Gamma^{\min}\sigma^2}{P}$. Given that $s_i(t)=1$, we have

$$\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))=\begin{cases}1, & \text{if } x_i(t)=1,s_i(t)=1,\theta\geq\rho_{\rm H}(t),\\ e^{\frac{\theta-\rho_{\rm H}(t)}{\ell(r_i(t))}}, & \text{if } x_i(t)=1,s_i(t)=1,\theta<\rho_{\rm H}(t).\end{cases}$$

On the other hand, given that $s_i(t)=0$, we have

$$\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))=\begin{cases}\frac{e^{-\rho_{\rm H}(t)/\ell(r_i(t))}-e^{-\theta/\ell(r_i(t))}}{1-e^{-\theta/\ell(r_i(t))}}, & \text{if } x_i(t)=1,s_i(t)=0,\theta\geq\rho_{\rm H}(t),\\ 0, & \text{if } x_i(t)=1,s_i(t)=0,\theta<\rho_{\rm H}(t).\end{cases}$$

2) When $Q_i(t)>0$ and user i is the low-power user of NOMA transmission, i.e., $y_i(t)=1$. When the binary indicator $s_i(t)=1$, we can show that

$$\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))=\begin{cases}e^{\frac{\theta-\rho^{\max}(t)}{\ell(r_i(t))}}, & \text{if } y_i(t)=1,s_i(t)=1,\theta<\rho^{\max}(t),\\ 1, & \text{if } y_i(t)=1,s_i(t)=1,\theta\geq\rho^{\max}(t),\end{cases}$$

where $\rho_{\rm L}(t)=\frac{\rho}{\alpha_{\rm L}^2(t)}$ and $\rho^{\max}(t)=\max\left[\rho_{\rm H}(t),\rho_{\rm L}(t)\right]$. The detailed proof can be found in the Appendix.

On the other hand, given that $s_i(t)=0$, we have

$$\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))=\begin{cases}\frac{e^{-\rho^{\max}(t)/\ell(r_i(t))}-e^{-\theta/\ell(r_i(t))}}{1-e^{-\theta/\ell(r_i(t))}}, & \text{if } y_i(t)=1,s_i(t)=0,\theta\geq\rho^{\max}(t),\\ 0, & \text{if } y_i(t)=1,s_i(t)=0,\theta<\rho^{\max}(t).\end{cases}$$

The proof is similar to the one presented in the Appendix and hence is omitted due to the page limitation.

3) For OMA transmission, when the binary indicator $s_i(t)$ is equal to 1, we have

$$\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))=\begin{cases}1, & \text{if } \theta\geq\rho,\\ 0, & \text{otherwise.}\end{cases}$$

On the other hand, when $s_i(t)=0$, we have

$$\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))=\frac{e^{-\rho/\ell(r_i(t))}-e^{-\theta/\ell(r_i(t))}}{1-e^{-\theta/\ell(r_i(t))}}.$$

4) If the packet intended for user i is not transmitted in time slot t, then the binary control variables $x_i(t),y_i(t),z_i(t)$ are all equal to zero, and hence $\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))=0$.

Based on the analysis above, the expression of $\hat{\Psi}_i(\mathbf{\Pi}(t),\boldsymbol{S}(t))$ under different settings of the control variables is summarized in (14), shown at the top of the page.

III. Problem Formulation and Proposed Algorithm

We present the problem formulation of maximizing the throughput under the constraint of stable backlogs and then propose an efficient scheduling scheme in this section. We define $d_m(t)$ as the expected number of packets that are successfully sent from queue Q_m in time slot t. Given all the control variables $\mathbf{\Pi}(t)$ and CSI feedback $\boldsymbol{S}(t)$, $d_m(t)$ is given by

$$d_m(t) := d_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) = \hat{\Psi}_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)). \tag{15}$$

We also define $b_m(t)$ as the number of packets that are transmitted from queue Q_m in time slot t. Given $\mathbf{\Pi}(t)$ and $\boldsymbol{S}(t)$, $b_m(t)$ can be expressed as

$$b_m(t) := b_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) = \max\left[x_m(t), y_m(t), z_m(t)\right].$$

We consider the constraint that all queues should be stable and adopt the following definition of backlog stability [12]

$$Q_{\text{av}} := \limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{m\in\mathcal{M}} \mathbb{E}[Q_m(t)] < \infty. \tag{16}$$

We aim to develop a power allocation and user scheduling algorithm that yields $\mathbf{\Pi}(t)$ that maximizes the long-term expected throughput of downlink NOMA systems

$$\begin{aligned} \underset{\mathbf{\Pi}(t)}{\text{maximize}} \quad & \lim_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\left[\sum_{m\in\mathcal{M}} d_m(t)\right] \\ \text{subject to} \quad & \text{constraints (5)} - \text{(13)}, \\ & Q_{\text{av}} < \infty. \end{aligned} \tag{17}$$

For notational ease, we define the backlog vector $\mathbf{Q}(t) = (Q_1(t), \ldots, Q_M(t))$. To address the stability constraint (16) while maximizing the throughput, we use the Lyapunov optimization technique [12] to solve problem (17). We apply the following Lyapunov function to measure the backlogs:

$$L(\boldsymbol{Q}(t)) := \frac{1}{2} \sum_{m\in\mathcal{M}} Q_m(t)^2. \tag{18}$$

The conditional Lyapunov drift in time slot t is given by

$$\Delta(\boldsymbol{Q}(t)) := \mathbb{E}[L(\boldsymbol{Q}(t+1)) - L(\boldsymbol{Q}(t)) \mid \boldsymbol{Q}(t)]. \tag{19}$$

For $Q_m(t+1)$, we have

$$Q_m(t+1) = \max\left[Q_m(t) - b_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)), 0\right] + a_m(t).$$

The bound on the change of Lyapunov function between two consecutive time slots [12] can be derived as

$$\begin{aligned} & L(\boldsymbol{Q}(t+1)) - L(\boldsymbol{Q}(t)) \\ & = \frac{1}{2} \sum_{m\in\mathcal{M}} \left([\max(Q_m(t) - b_i(t), 0) + a_m(t)]^2 - Q_m(t)^2\right) \\ & \leq \frac{1}{2} \sum_{m\in\mathcal{M}} \left(a_m(t)^2 + b_m(t)^2 + 2Q_m(t)[a_m(t) - b_m(t)]\right). \end{aligned}$$

Then, we can derive the bound of the conditional Lyapunov drift as follows [12]

$$\begin{aligned} & \Delta(\boldsymbol{Q}(t)) = \mathbb{E}[L(\boldsymbol{Q}(t+1)) - L(\boldsymbol{Q}(t)) \mid \boldsymbol{Q}(t)] \\ & \leq \mathbb{E}\left[\sum_{m\in\mathcal{M}} \frac{a_m(t)^2 + b_m(t)^2}{2} \,\middle|\, \boldsymbol{Q}(t)\right] + \sum_{m\in\mathcal{M}} Q_m(t)\lambda_m(t) \\ & \qquad - \mathbb{E}\left[\sum_{m\in\mathcal{M}} Q_m(t) b_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) \,\middle|\, \boldsymbol{Q}(t)\right] \\ & \leq N + \sum_{m\in\mathcal{M}} Q_m(t)\lambda_m(t) \\ & \qquad - \mathbb{E}\left[\sum_{m\in\mathcal{M}} Q_m(t) b_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) \,\middle|\, \boldsymbol{Q}(t)\right], \end{aligned}$$

where N is a finite constant.

To take into account the maximization of throughput, we add the term $V\mathbb{E}\left[-\sum_{m\in\mathcal{M}} d_m(\mathbf{\Pi}(t), \boldsymbol{S}(t))\right]$ to both sides of the above inequality, where $V > 0$ is a parameter denoting the importance of throughput. We can derive the following bound on the Lyapunov drift-plus-penalty equation

$$\begin{aligned} & \Delta(\boldsymbol{Q}(t)) + V\mathbb{E}\left[-\sum_{m\in\mathcal{M}} d_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) \,\middle|\, \boldsymbol{Q}(t)\right] \\ & \leq N - \mathbb{E}\left[\sum_{m\in\mathcal{M}} Q_m(t) b_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) \,\middle|\, \boldsymbol{Q}(t)\right] \\ & + \sum_{m\in\mathcal{M}} Q_m(t)\lambda_m(t) + V\mathbb{E}\left[-\sum_{m\in\mathcal{M}} d_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) \,\middle|\, \boldsymbol{Q}(t)\right]. \end{aligned} \tag{20}$$

According to Lyapunov optimization, problem (17) can be solved by using the *drift-plus-penalty algorithm* [Ch. 3, 10]. Specifically, with observations on $\boldsymbol{Q}(t)$ and $\boldsymbol{S}(t)$, the scheme that decides $\mathbf{\Pi}(t)$ such that the bound on the Lyapunov drift-plus-penalty equation, i.e., the right-hand-side of (20), is minimized in each time slot can maintain the stability of backlog given in (16), and meanwhile increase the throughput. Given observations on $\boldsymbol{Q}(t)$ and $\boldsymbol{S}(t)$, minimizing the right-hand-side of (20) can be accomplished by minimizing

$$-V \sum_{m\in\mathcal{M}} d_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) - \sum_{m\in\mathcal{M}} Q_m(t) b_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)).$$

With (15), we further have

$$-\sum_{m\in\mathcal{M}} d_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) = -\sum_{m\in\mathcal{M}} \hat{\Psi}_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)).$$

Therefore, in order to maximize the throughput with constraint on backlog stability, we solve the following problem

$$\begin{aligned} \underset{\mathbf{\Pi}(t)}{\text{maximize}} \quad & \mathcal{G}(\mathbf{\Pi}(t)) := V \sum_{m\in\mathcal{M}} \hat{\Psi}_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) \\ & \qquad + \sum_{m\in\mathcal{M}} Q_m(t) b_m(\mathbf{\Pi}(t), \boldsymbol{S}(t)) \\ \text{subject to} \quad & \text{constraints (5)} - \text{(13)}. \end{aligned} \tag{21}$$

To solve problem (21), we decompose it into two subproblems with respect to the condition that the base station uses either

Algorithm 1 User Pairing and Power Allocation for NOMA

Input: $\mathbf{Q}(t)$, $\boldsymbol{S}(t)$, and $\mathcal{G}_{\text{NOMA}}(t) := 0$.
for $i \in \mathcal{M}$ **do**
  for $j \in \mathcal{M}$ where $i \neq j$ **do**
    Calculate the scalar valued function $\mathcal{G}(\alpha_{\text{H}}(t)) := V(\hat{\Psi}_i(\mathbf{\Pi}(t), \boldsymbol{S}(t)) + \hat{\Psi}_j(\mathbf{\Pi}(t), \boldsymbol{S}(t))) + Q_i(t) + Q_j(t)$ based on (14).
    Find $\alpha_{\text{H}}^*(t)$ that maximizes $\mathcal{G}(\alpha_{\text{H}}(t))$ subject to constraints (12) and (13).
    if $\mathcal{G}(\alpha_{\text{H}}^*(t)) > \mathcal{G}_{\text{NOMA}}(t)$ **then**
      $\mathcal{G}_{\text{NOMA}}(t) := \mathcal{G}(\alpha_{\text{H}}^*(t))$.
      $i^*(t) := i$, $j^*(t) := j$, and $\alpha_{\text{opt}}(t) := \alpha_{\text{H}}^*(t)$.
    end if
  end for
end for
return $i^*(t), j^*(t), \alpha_{\text{opt}}(t), \mathcal{G}_{\text{NOMA}}(t)$.

NOMA or OMA in time slot t. We first solve the following subproblem for NOMA transmission:

$$\begin{aligned} \underset{\mathbf{\Pi}(t)}{\text{maximize}} \quad & \mathcal{G}(\mathbf{\Pi}(t)) \\ \text{subject to} \quad & \sum_{i \in \mathcal{M}} x_i(t) = 1, \sum_{j \in \mathcal{M}} y_j(t) = 1, \\ & x_i(t) + y_i(t) \leq 1, \qquad i \in \mathcal{M}, \\ & x_i(t),\, y_i(t) \in \{0,1\}, \qquad i \in \mathcal{M}, \\ & z_i(t) = 0, \qquad i \in \mathcal{M}, \\ & \text{constraints } (12), (13). \end{aligned} \tag{22}$$

The algorithm proposed for solving the NOMA subproblem is presented in Algorithm 1. Algorithm 1 determines the optimal user pairing, the optimal power allocation, and the optimal value of the objective function in each time slot, denoted as tuple $(i^*(t), j^*(t))$, $\alpha_{\text{opt}}(t)$, and $\mathcal{G}_{\text{NOMA}}(t)$, respectively.

We then consider the second subproblem, which is formulated for OMA transmission and is given by

$$\begin{aligned} \underset{\mathbf{\Pi}(t)}{\text{maximize}} \quad & \mathcal{G}(\mathbf{\Pi}(t)) \\ \text{subject to} \quad & \sum_{i \in \mathcal{M}} z_i(t) = 1, \\ & z_i(t) \in \{0,1\}, \qquad \forall\, i \in \mathcal{M}, \\ & x_i(t) = y_i(t) = 0, \qquad \forall\, i \in \mathcal{M}. \end{aligned} \tag{23}$$

To solve problem (23), we use Algorithm 2 to obtain the optimal user selection $z^*(t)$ and the optimal value of objective function, denoted as $\mathcal{G}_{\text{OMA}}(t)$. After obtaining $\mathcal{G}_{\text{NOMA}}(t)$, and $\mathcal{G}_{\text{OMA}}(t)$, the optimal user pairing and power allocation scheme can be decided as follows. If $\max_{m \in \mathcal{M}} Q_m(t) \geq \max[\mathcal{G}_{\text{OMA}}(t), \mathcal{G}_{\text{NOMA}}(t)]$, then the channel condition is poor and the base station does not transmit a packet. If $\mathcal{G}_{\text{OMA}}(t) > \mathcal{G}_{\text{NOMA}}(t)$, then the base station transmits the packet intended for the user with $z^*(t) = 1$ using OMA in time slot t. Otherwise, if $\mathcal{G}_{\text{NOMA}}(t) \geq \mathcal{G}_{\text{OMA}}(t)$, then the base station assigns user $i^*(t)$ as the high-power user and user $j^*(t)$ as the low-power user with power allocation coefficients $(\alpha_{\text{opt}}^2(t), 1 - \alpha_{\text{opt}}^2(t))$, and transmits the superimposed signals intended for users $i^*(t)$ and $j^*(t)$ using NOMA.

Algorithm 2 User Selection and Power Allocation for OMA

Input: $\mathbf{Q}(t)$, $\boldsymbol{S}(t)$, and $\mathcal{G}_{\text{OMA}}(t) := 0$.
for $z \in \mathcal{M}$ **do**
  Calculate the scalar valued function $\mathcal{G}(z) := V\hat{\Psi}_z(\mathbf{\Pi}(t), \boldsymbol{S}(t)) + Q_z(t)$ based on (14).
  if $\mathcal{G}(z) > \mathcal{G}_{\text{OMA}}(t)$ **then**
    $\mathcal{G}_{\text{OMA}}(t) := \mathcal{G}(z)$.
    $z^*(t) := z$.
  end if
end for
return $z^*(t), \mathcal{G}_{\text{OMA}}(t)$.

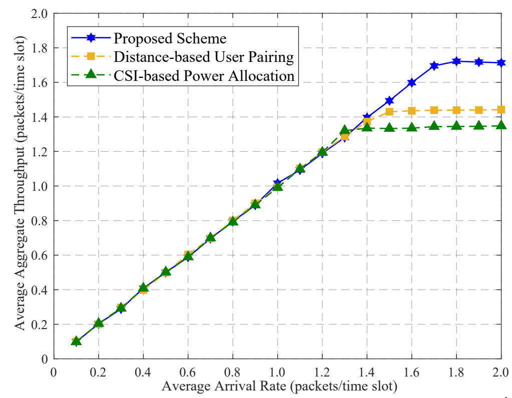

Fig. 1. Average throughput versus average arrival rate when $\Gamma^{\min} = 2$ and $M = 10$.

IV. Performance Evaluation

The packet-level simulation results for the proposed scheme are presented in this section. We compare them with two NOMA schemes [9], [10] in the literature. For NOMA with CSI-based power allocation [9], the transmit power of a user is set to be inversely proportional to its channel gain. For NOMA with distance-based user pairing [10], the random near and random far (RNRF) user pairing scheme, i.e., one user near the base station and one user far from the base station are randomly selected from two different groups of users, is considered. The radius of the circular cell is 800 m, where $M = 10$ users are considered. We set the transmit power P and variance of noise σ^2 to be 1 W and -100 dBm, respectively. The path loss exponent β is set to be 4. The predefined threshold θ is equal to 2ρ. The SINR threshold $\Gamma^{\min}$ is set to be 2. The packet arrivals are modeled as a Poisson random variable.

Fig. 1 illustrates the impact of the packet arrival rate on the throughput. The proposed scheme can achieve a higher throughput gain when the arrival rate is higher than 1.5 packets per second, compared with other schemes. With high arrival rates, the proposed scheme can achieve up to 28% higher throughput than the CSI-based power allocation scheme and 19% higher than the distance-based user pairing.

Figs. 2 and 3 show that the proposed scheme can reduce the average congestion and the packet transmission failure probability, respectively, when the packet arrival rate varies. The proposed algorithm can effectively support stable data transmission at the maximum arrival rate around 1.5 packets per second, which is 200% and 100% higher than the CSI-

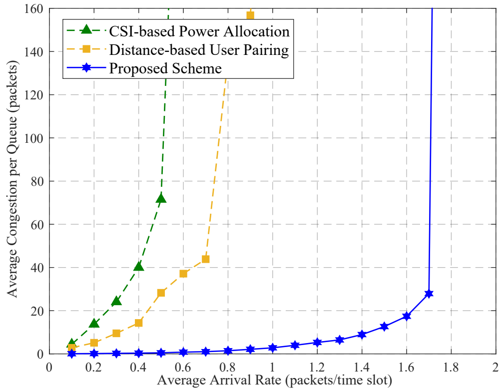

Fig. 2. Average congestion versus average arrival rate when $\Gamma^{\min} = 2$ and $M = 10$.

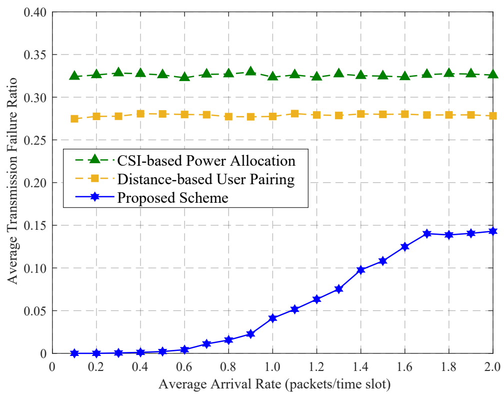

Fig. 3. Average transmission failure ratio versus average arrival rate when $\Gamma^{\min} = 2$ and $M = 10$.

based and distance-based schemes, respectively. Besides, the proposed scheme can achieve a lower packet transmission failure probability, especially when packet arrival rate is relatively low. In the high packet arrival rate regime, the proposed scheme can maintain a 14% packet transmission failure probability, which is 56% and 50% lower than the CSI-based and distance-based NOMA schemes, respectively.

V. Conclusion

In this paper, we proposed a packet-level user pairing and power allocation scheme for NOMA systems to maximize the throughput, while taking into account dynamic packet arrival, limited feedback on CSI, and possible packet transmission failure. We formulated a joint user pairing and power allocation problem under the constraint of backlog stability. By using the tools from Lyapunov optimization, we developed an efficient algorithm to solve the problem via combining the results from two subproblems. Packet-level simulations show that the proposed scheme can achieve a higher throughput, maintain stable backlogs, and achieve a lower transmission failure probability compared to the existing schemes. For future work, we will investigate the throughput maximization for code-domain NOMA with dynamic packet arrival and limited feedback on CSI.

Appendix

Suppose that user j is the low-power NOMA user, we have $y_j(t) = 1$. In order to successfully decode the packet, the inequalities $\Gamma_j^{\mathrm{H}}(\alpha_{\mathrm{H}}^2(t), \alpha_{\mathrm{L}}^2(t)) > \Gamma^{\min}$, and $\Gamma_j^{\mathrm{L}}(\alpha_{\mathrm{H}}^2(t), \alpha_{\mathrm{L}}^2(t)) > \Gamma^{\min}$ should hold, and they are equivalent to $|h_i(t)|^2 > \frac{\rho_{\mathrm{L}}(t)}{\ell(r_i(t))}$, and $|h_i(t)|^2 > \frac{\rho_{\mathrm{H}}(t)}{\ell(r_i(t))}$. Therefore, we have

$$\hat{\Psi}_i(\boldsymbol{\Pi}(t), \boldsymbol{S}(t)) = \mathbb{P}\left[|h_i(t)|^2 > \frac{\rho^{\max}(t)}{\ell(r_i(t))}\right].$$

Conditioning the above probability on $s_i(t) = 1$, we have

$$\hat{\Psi}_i(\boldsymbol{\Pi}(t), \boldsymbol{S}(t)) = \mathbb{P}\left[|h_i(t)|^2 > \frac{\rho^{\max}(t)}{\ell(r_i(t))} \,\middle|\, |h_i(t)|^2 \geq \frac{\theta}{\ell(r_i(t))}\right].$$

If $\theta > \rho^{\max}(t)$, then $\hat{\Psi}_i(\boldsymbol{\Pi}(t), \boldsymbol{S}(t)) = 1$. If $\theta \leq \rho^{\max}(t)$, $\hat{\Psi}_i(\boldsymbol{\Pi}(t), \boldsymbol{S}(t))$ can be obtained by using the memoryless property of an exponentially distributed random variable

$$\hat{\Psi}_i(\boldsymbol{\Pi}(t), \boldsymbol{S}(t)) = \mathbb{P}\left[|h_i(t)|^2 > \frac{\rho^{\max}(t) - \theta}{\ell(r_i(t))}\right] = e^{\frac{\theta - \rho^{\max}(t)}{\ell(r_i(t))}}.$$

Then, we obtain the following result for the low-power user

$$\hat{\Psi}_i(\boldsymbol{\Pi}(t), \boldsymbol{S}(t)) = \begin{cases} e^{\frac{\theta - \rho^{\max}(t)}{\ell(r_i(t))}}, & \text{if } y_i(t) = 1, s_i(t) = 1, \theta < \rho^{\max}(t), \\ 1, & \text{if } y_i(t) = 1, s_i(t) = 1, \theta \geq \rho^{\max}(t). \end{cases}$$

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