Bandwidth Allocation and Pricing for SDN-enabled Home Networks

Homa Eghbali and Vincent W.S. Wong
Department of Electrical and Computer Engineering
The University of British Columbia, Vancouver, Canada
e-mail: {homae, vincentw}@ece.ubc.ca

Abstract—**In this paper, we propose to combine the emerging software defined networking (SDN) paradigm with the existing residential broadband infrastructure to enable home users to have dynamic control over their traffic flows. The SDN centralized control technology enables household devices to have virtualized services with quality of service (QoS) guarantee. SDN-enabled open application programming interfaces (APIs) allow Internet service providers (ISPs) to perform bandwidth slicing in home networks and implement time-dependent hybrid pricing. Given the requests from household devices for virtualized and non-virtualized services, we formulate a Stackelberg game to characterize the pricing strategy of ISP as well as bandwidth allocation strategy in home networks. In the Stackelberg game, the leader is the ISP and the followers are the home networks. We determine the optimal strategies which provide maximal payoff for the ISP. Numerical results show that our proposed SDN-enabled home network technology with the hybrid pricing scheme provides a better performance than a usage-based pricing scheme tailored for best-effort home networks.**

I. INTRODUCTION

Software defined networking (SDN) is an emerging technology which separates the forwarding functions from control functions and enables network-wide programmability [1], [2], [3]. SDN can simplify the development of new mechanisms for network management and provide application developers with better visibility and control over tasks without requiring detailed knowledge of the network state [4].

With the rapid growth in the number of household devices demanding streaming entertainment content such as Netflix and YouTube, it is important to maintain quality of experience (QoE) in online video streaming over the residential best-effort networks. In order to maintain users' QoE and to guarantee the quality of service (QoS) [5], a viable solution is to apply SDN in home networks [6], [7]. In this setting, the control plane of SDN enables the Internet service provider (ISP) to make fine-grained decisions of bandwidth allocation in home networks. The SDN controller provides ISPs with direct control over the subscribers' traffic flows, enabling them to employ QoS mechanisms required by users as well as the content providers.

ISPs can also use SDN to implement innovative time-dependent pricing strategies and monetize their resources based on the timely service requests of home networks. Various time-dependent pricing schemes have been proposed to tackle the congestion control problem in wireline or wireless networks [8]. Flat-rate and usage-based time-dependent pricing schemes combined with traffic cap and user heterogeneity are compared in terms of the profit of ISP and customers' surplus in [9]. SDN allows ISPs to exploit new revenue opportunities by deploying time-dependent service quality mechanisms. To the best of our knowledge, time-dependent pricing model for SDN-enabled broadband networks empowered with differentiated offerings on service quality has not yet been addressed in the literature.

Home-based user provided connectivity (UPC) has emerged for ubiquitous connectivity in urban areas with a large number of wireless access points [8]. Through UPC services, a home user allows other users to occasionally connect through its access point in exchange for reimbursement. The emerging paradigm of SDN can allow ISPs to engage home networks in managing their excess resources and providing incentives via time-dependent reimbursements.

In this paper, we study the bandwidth allocation problem for SDN-enabled home networks with virtualized and non-virtualized service requests. In particular, applications with minimum bandwidth requirement which are granted reserved resources through the application programming interfaces (APIs) are referred to as virtualized services. All other applications that are not scheduled to receive reserved resources are referred to as non-virtualized services. Through the SDN centralized control technology, a home network is empowered to share its excess resources with the neighboring home networks and mobile users and receive proportional time-dependent reimbursement in return. The bandwidth allocation strategy for home networks is based on a hybrid pricing/reimbursement strategy of ISP, both of which can be characterized following a Stackelberg game theoretical model. The main contributions of this work are summarized as follows:

- We propose a centralized low-complexity bandwidth allocation algorithm for real-time and non-real-time service requests in SDN-enabled home networks.
- We use a Stackelberg game model to characterize the interplay between the hybrid pricing/reimbursement strategy of ISP and the traffic consumption of home networks. The model takes into account the required QoS for heterogenous services requested by household devices.
- Numerical results show that the proposed SDN-enabled home network technology provides both ISP and home networks with larger payoffs compared to the usage-based pricing scheme in [9] designed for best-effort residential networks. Moreover, SDN provides both ISP

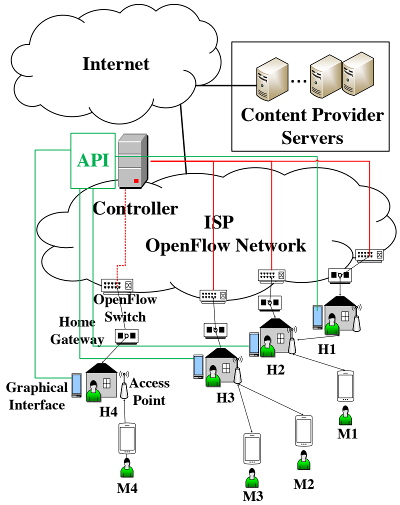

Fig. 1. Illustration of SDN-enabled network topology.

and home networks with payoffs that are resilient to demand fluctuations during peak-time.

The rest of this paper is organized as follows: We present the system model in Section II followed by the Stackelberg game analysis in Section III. Numerical results are presented in Section IV. Conclusions are given in Section V.

II. System Model

Consider a geographical area with an ISP that provides Internet connectivity for a set of home networks. Each household operates a set of household devices running different applications. Each household is equipped with an access point, which provides Internet connectivity for the household devices. With proper reimbursement, it can also provide connectivity to the neighboring home networks' devices and/or roaming mobile users. Fig. 1 illustrates our SDN-enabled network topology. In Fig. 1, H2's access point can provide Internet connectivity for the neighboring household H1, as well as mobile user M1. Each household is equipped with a home gateway that provides high-speed Internet connectivity through a digital subscriber line or broadband cable link. The gateway is connected to ISP's local exchange through an SDN-enabled OpenFlow switch. This switch is controlled by a controller, running locally inside the ISP backhaul. The OpenFlow switch can perform bandwidth slicing and allocate resources to virtualized services in order to guarantee their QoS. ISP can peer with the servers of content providers which provide the services (e.g., multimedia streaming) being requested by household devices.

The proposed SDN-enabled home network architecture requires three different types of APIs. The first type allows the home network administrator to communicate with the network controller of ISP using an ISP exposed simple graphical interface. Using the graphical interface, the home network administrator receives a pricing/reimbursement quota from the ISP for a specific period of time. The administrator can request the amount of resources for virtualized and non-virtualized services. The second type of API is designed to dynamically reserve network path for the virtualized traffic requested by wired-connected household devices. The third type of API is required to accommodate wireless local area network virtualization [10] and provide QoS control and bandwidth guarantee for the wirelessly-connected household devices. The operational flow of the events is as follows: The household content requests go to the ISP controller. Depending on whether the requested data requires virtualization, different actions are taken. In case of non-virtualized traffic requests, the content requests are directly forwarded to the content provider servers without requiring the intervention of the APIs. For virtualized data requests, however, the content requests are first forwarded to the respective APIs for resource allocation. Following that, the requested contents are forwarded from the content provider servers to the home user devices. Home networks with bandwidth requests beyond their broadband capacity limit, send their content requests to the respective APIs for resource allocation via the access point of their neighboring home networks.

In order to characterize the utility function of the household devices' application streams, we consider two types of traffic use cases, namely real-time applications and delay tolerant elastic applications. The utility of the elastic applications can be characterized by a continuously differentiable, strictly concave, and increasing utility function, with decreasing marginal increment. On the other hand, real-time applications are sensitive to the loss caused by the lack of available bandwidth and can be modeled by a sigmoidal utility function.

III. Problem Formulation

Aiming to characterize the traffic consumption of home networks for elastic and real-time services and the hybrid pricing reimbursement strategy of ISP, we formulate a two-stage Stackelberg game model and capture the interaction between ISP and home networks requesting heterogeneous services for their household devices. In the first stage of the game, the ISP determines the hybrid pricing scheme. In the second stage, the home networks need to partition the resources. That is, they need to determine the amount of bandwidth for non-virtualized and virtualized services as well as their traffic exchange with neighboring home networks and remote mobile users. We assume that the ISP is the leader and the consumers are the followers. The followers make their decisions according to the hybrid pricing strategy of ISP.

We denote the set of home networks by $\mathcal{N} = \{1, 2, \ldots, N\}$ consisting of N households. ISP charges the home network $n \in \mathcal{N}$ for requesting services via the SDN a time-dependent usage-based retail price $q_n(t)$ for data consumption at time t. Home network n receives from ISP a reimbursement, which is proportional to the amount of data its access point shares with other neighboring users. Let $p_n(t) \in [0, 1]$ denote the reimbursement weighting factor for network n at time t. The strategy of ISP at time t (in ms) includes the reimbursement and pricing vectors $\mathbf{p}_t = (p_n(t))_{n\in\mathcal{N}}$ and $\mathbf{q}_t = (q_n(t))_{n\in\mathcal{N}}$, respectively. We assume that the payoff of ISP is equal to the total service fee charged from the home networks subtracting the monetary amount home networks received as reimbursement for sharing their resources with others.We can express the payoff of ISP at time t when choosing strategies $(\mathbf{p}_t, \mathbf{q}_t)$ as follows:

$$\pi_{ISP,t}\left(\mathbf{p}_t, \mathbf{q}_t; \mathbf{x}_{n,t}, \hat{\mathbf{x}}_{n,t}, \tilde{x}_n\left(t\right)\right) = \sum_{n\in\mathcal{N}} q_n\left(t\right)\left(\sum_{i\in\mathcal{N}_\mathcal{E}} x_{n,i}\left(t\right) + \sum_{j\in\mathcal{N}_\mathcal{J}} \hat{x}_{n,j}\left(t\right) + \left(1-p_n\left(t\right)\right)\tilde{x}_n\left(t\right)\right), \tag{1}$$

where $\mathcal{N}_\mathcal{E}$ and $\mathcal{N}_\mathcal{J}$ denote the set of elastic and real-time applications, respectively. $\hat{x}_{n,j}\left(t\right)$ and $x_{n,i}\left(t\right)$ denote the amount of broadband bandwidth that household n consumes for real-time application $j\in\mathcal{N}_\mathcal{J}$ and elastic application $i\in\mathcal{N}_\mathcal{E}$ at time t, respectively. $\tilde{x}_n\left(t\right)$ denotes the amount of resources that household n forwards to its neighboring users at time t, $\mathbf{x}_{n,t} = \left(x_{n,i}\left(t\right)\right)_{i\in\mathcal{N}_\mathcal{E}}$, and $\hat{\mathbf{x}}_{n,t} = \left(\hat{x}_{n,j}\left(t\right)\right)_{j\in\mathcal{N}_\mathcal{J}}$. For simplicity, we assume that ISP charges home network n the same usage-based time-dependent price $q_n\left(t\right)$ for consuming virtualized and non-virtualized services. We can formulate the payoff of the home network subscriber n at time t as follows:

$$\begin{aligned}&\pi_{home,\,n,\,t}\left(\mathbf{x}_{n,t},\, \hat{\mathbf{x}}_{n,t},\, \tilde{x}_n\left(t\right);\, \mathbf{p}_t,\, \mathbf{q}_t\right)\\ &= \sum_{i\in\mathcal{N}_\mathcal{E}} U_i\left(x_{n,i}\left(t\right)\right) + \sum_{j\in\mathcal{N}_\mathcal{J}} \hat{U}_j\left(\hat{x}_{n,j}\left(t\right)\right) - q_n\left(t\right)\\ &\times\left(\sum_{i\in\mathcal{N}_\mathcal{E}} x_{n,i}\left(t\right) + \sum_{j\in\mathcal{N}_\mathcal{J}} \hat{x}_{n,j}\left(t\right) - p_n\left(t\right)\tilde{x}_n\left(t\right)\right),\end{aligned} \tag{2}$$

where $U_i\left(x_{n,i}\left(t\right)\right)$ denotes the utility function for elastic service $i\in\mathcal{N}_\mathcal{E}$ which can be modeled by the α-fair utility function [11] as follows:

$$U_i\left(x_{n,i}\left(t\right)\right) = \begin{cases}\ln\left(x_{n,i}\left(t\right)+1\right), & \text{if } \psi_i = 1,\\ \dfrac{\left(x_{n,i}\left(t\right)+1\right)^{\left(1-\psi_i\right)}-1}{1-\psi_i}, & \text{if } \psi_i\in\left(0,1\right)\cup\left(1,\infty\right),\end{cases} \tag{3}$$

where ψ_i is a fixed utility parameter. Let $\hat{U}_j\left(\hat{x}_{n,j}\left(t\right)\right)$ denote the utility function for real-time application $j\in\mathcal{N}_\mathcal{J}$, which can be modeled by a sigmoidal function defined as follows:

$$\hat{U}_j\left(\hat{x}_{n,j}\left(t\right)\right) = \frac{\hat{x}_{n,j}^{a_j}\left(t\right)}{k_j + \hat{x}_{n,j}^{a_j}\left(t\right)}, \tag{4}$$

where $\hat{x}_{n,j}\left(t\right)\geq 0$, $a_j > 1$, $k_j\geq 0$, and $\hat{x}_{n,j}^{in}\left(t\right) = \sqrt[a_j]{\frac{k_j\left(a_j-1\right)}{\left(a_j+1\right)}}$, such that $\hat{U}_j''\left(\hat{x}_{n,j}\left(t\right)\right) > 0$ for $\hat{x}_{n,j}\left(t\right) < \hat{x}_{n,j}^{in}\left(t\right)$ and $\hat{U}_j''\left(\hat{x}_{n,j}\left(t\right)\right) < 0$ for $\hat{x}_{n,j}\left(t\right) > \hat{x}_{n,j}^{in}\left(t\right)$, where $\hat{x}_{n,j}^{in}\left(t\right)$ is called the *point of inflection*.

We first consider the second stage of the game, i.e., given the ISP pricing and reimbursement strategy $(\mathbf{p}_t, \mathbf{q}_t)$, users aim to maximize their payoff function by choosing their traffic consumption for elastic and real-time services as well as the amount of bandwidth they would like to share with others:

$$\begin{aligned}\underset{\mathbf{x}_{n,t},\hat{\mathbf{x}}_{n,t},\tilde{x}_n(t)}{\text{maximize}}\quad & \pi_{home,\,n,\,t}\left(\mathbf{x}_{n,t},\, \hat{\mathbf{x}}_{n,t},\, \tilde{x}_n\left(t\right);\, \mathbf{p}_t,\, \mathbf{q}_t\right)\\ \text{subject to}\quad & 0\leq x_{n,i}\left(t\right)\leq\Delta_{n,i}\left(t\right), \qquad \forall i\in\mathcal{N}_\mathcal{E},\\ & \beta_{n,j}\left(t\right)C_n\left(t\right)\leq\hat{x}_{n,j}\left(t\right)\leq\hat{\Delta}_{n,j}\left(t\right), \qquad \forall j\in\mathcal{N}_\mathcal{J},\\ & 0\leq\tilde{x}_n\left(t\right)\leq p_n\left(t\right)\gamma_n\left(t\right)C_n\left(t\right)\\ & \sum_{i\in\mathcal{N}_\mathcal{E}} x_{n,i}\left(t\right) + \sum_{j\in\mathcal{N}_\mathcal{J}}\hat{x}_{n,j}\left(t\right)\leq\left(\alpha_n\left(t\right) + \beta_n\left(t\right)\right)C_n\left(t\right),\end{aligned} \tag{5}$$

where $C_n\left(t\right)$ denotes the maximum available broadband capacity for home network n at time t, $\beta_{n,j}\left(t\right)\in[0,1]$ denotes the fraction of $C_n\left(t\right)$ that is reserved for real-time service j requested by devices in household n at time t, $\alpha_n\left(t\right)\in[0,1]$ denotes the fraction of $C_n\left(t\right)$ that can be allocated to non-virtualized traffic in household n through the best effort network at time t, $\gamma_n\left(t\right)\in[0,1]$ denotes the fraction of $C_n\left(t\right)$ that home network n administrator decides to share with others via its access point, $\beta_n\left(t\right) = \sum_{j\in\mathcal{N}_\mathcal{J}}\beta_{n,j}\left(t\right)$, and $\Delta_{n,i}\left(t\right)$ and $\hat{\Delta}_{n,j}\left(t\right)$ denote the maximum demand of home network n for elastic service i and real-time service j at time t, respectively. The administrator of home network n is empowered with the ability to tune $\alpha_n\left(t\right)$, $\beta_{n,j}\left(t\right)$, $\forall j\in\mathcal{N}_\mathcal{J}$, and $\gamma_n\left(t\right)$. Note that $\alpha_n\left(t\right)+\beta_n\left(t\right)+\gamma_n\left(t\right)\leq 1$. Upon receiving the requests from the home networks, ISP determines the pricing reimbursement strategy by solving the following optimization problem

$$\begin{aligned}\underset{\mathbf{p}_t,\,\mathbf{q}_t}{\text{maximize}}\quad & \pi_{ISP,t}\left(\mathbf{p}_t, \mathbf{q}_t; \mathbf{x}_{n,t}, \hat{\mathbf{x}}_{n,t}, \tilde{x}_n\left(t\right)\right)\\ \text{subject to}\quad & 0\leq p_n\left(t\right)\leq 1, \qquad \forall n\in\mathcal{N},\\ & 0\leq q_n\left(t\right)\leq q_{max}, \qquad \forall n\in\mathcal{N},\end{aligned} \tag{6}$$

where q_{max} denotes the maximal usage-based price beyond which the home networks will not be interested in consuming ISP resources. In pursuing a solution to the Stackelberg game, our intention is to find the Nash equilibrium (NE) where neither the ISP nor the home networks have any incentives to deviate unilaterally from that point. The NE is defined as follows:

Definition: (Nash Equilibrium) Let $\left(\mathbf{p}_t,\, \mathbf{q}_t\right)$ be a feasible solution of problem (6) and $\left(\mathbf{x}_{n,t},\, \hat{\mathbf{x}}_{n,t},\, \tilde{x}_n\left(t\right)\right)$ be a feasible solution of problem (5). The point $\left(\mathbf{p}_t^*,\, \mathbf{q}_t^*,\, \mathbf{x}_{n,t}^*,\, \hat{\mathbf{x}}_{n,t}^*,\, \tilde{x}_n^*\left(t\right)\right)$ is an NE for the Stackelberg game if for any point $\left(\mathbf{p}_t,\, \mathbf{q}_t,\, \mathbf{x}_{n,t},\, \hat{\mathbf{x}}_{n,t},\, \tilde{x}_n\left(t\right)\right)$, we have

$$\begin{aligned}&\pi_{ISP,\,t}\left(\mathbf{p}_t^*,\, \mathbf{q}_t^*;\, \mathbf{x}_{n,t}^*,\, \hat{\mathbf{x}}_{n,t}^*,\, \tilde{x}_n^*\left(t\right)\right)\\ &\geq\pi_{ISP,\,t}\left(\mathbf{p}_t,\, \mathbf{q}_t;\, \mathbf{x}_{n,t}^*,\, \hat{\mathbf{x}}_{n,t}^*,\, \tilde{x}_n^*\left(t\right)\right),\end{aligned}$$

and

$$\begin{aligned}&\pi_{home,\,n,\,t}\left(\mathbf{x}_{n,t}^*,\, \hat{\mathbf{x}}_{n,t}^*,\, \tilde{x}_n^*\left(t\right);\, \mathbf{p}_t^*,\, \mathbf{q}_t^*\right)\\ &\geq\pi_{home,\,n,\,t}\left(\mathbf{x}_{n,t},\, \hat{\mathbf{x}}_{n,t},\, \tilde{x}_n\left(t\right);\, \mathbf{p}_t^*,\, \mathbf{q}_t^*\right).\end{aligned}$$

A. The Best Response of Home Networks in Stage II

In order to analyze the Stackelberg game, we first consider the second stage of the game, which aims to maximize the payoff of home networks given the pricing and reimbursement strategy of ISP. In problem (5), the objective function is non-concave in general. Therefore, problem (5) is a non-convex optimization problem. By a logarithmic change of variables $z_j \triangleq \ln(x_j)$ and $\widehat{\hat{U}}_j(z_j) \triangleq \hat{U}_j(e^{x_j})$, the concave part of the utility function $\hat{U}_j(x_j)$ in (2) turns convex, while its convex part always remains convex [12]. Thus, we have

$$\begin{aligned}
\underset{\mathbf{z}_{n,t},\, \hat{\mathbf{z}}_{n,t},\, \tilde{x}_n(t)}{\text{maximize}} \quad & \sum_{i\in\mathcal{N}_{\mathcal{E}}} \widehat{U}_i(z_{n,i}(t)) + \sum_{j\in\mathcal{N}_{\mathcal{J}}} \widehat{\hat{U}}_j(\hat{z}_{n,j}(t)) \\
& - q_n(t)\Bigg(\sum_{i\in\mathcal{N}_{\mathcal{E}}} e^{z_{n,i}(t)} - p_n(t)\tilde{x}_n(t) \\
& + \sum_{j\in\mathcal{N}_{\mathcal{J}}} e^{\hat{z}_{n,j}(t)}\Bigg) \\
\text{subject to} \quad & 0 \le z_{n,i}(t) \le \Gamma_{n,i}(t), \qquad \forall i \in \mathcal{N}_{\mathcal{E}}, \\
& \widehat{\beta}_{n,j}(t) \le \hat{z}_{n,j}(t) \le \hat{\Gamma}_{n,j}(t), \quad \forall j \in \mathcal{N}_{\mathcal{J}}, \\
& 0 \le \tilde{x}_n(t) \le p_n(t)\gamma_n(t)C_n(t), \\
& \sum_{i\in\mathcal{N}_{\mathcal{E}}} e^{z_{n,i}(t)} + \sum_{j\in\mathcal{N}_{\mathcal{J}}} e^{\hat{z}_{n,j}(t)} \le (\alpha_n(t) \\
& + \beta_n(t))C_n(t),
\end{aligned}$$

where $z_{n,i}(t) \triangleq \ln(x_{n,i}(t))$, $\hat{z}_{n,j}(t) \triangleq \ln(\hat{x}_{n,j}(t))$, $\widehat{U}_i(z_{n,i}(t)) \triangleq U_i(e^{x_{n,i}(t)})$, $\Gamma_{n,i}(t) \triangleq \ln(\Delta_{n,i}(t))$, $\hat{\Gamma}_{n,j}(t) \triangleq \ln\left(\hat{\Delta}_{n,j}(t)\right)$, $\widehat{\beta}_{n,j}(t) \triangleq \ln(\beta_{n,j}(t)C_n(t))$, $\mathbf{z}_{n,t} = (z_{n,i}(t))_{i\in\mathcal{N}_{\mathcal{E}}}$, and $\hat{\mathbf{z}}_{n,t} = (\hat{z}_{n,j}(t))_{j\in\mathcal{N}_{\mathcal{J}}}$.

The Lagrangian is as follows:

$$\begin{aligned}
&L\left(\mathbf{z}_{n,t}, \hat{\mathbf{z}}_{n,t}, \lambda_{n,t}\right) = \sum_{i\in\mathcal{N}_{\mathcal{E}}} \widehat{U}_i(z_{n,i}(t)) - \eta_{n,t}e^{z_{n,i}(t)} \\
&+ \sum_{j\in\mathcal{N}_{\mathcal{J}}} \widehat{\hat{U}}_j(\hat{z}_{n,j}(t)) - \eta_{n,t}e^{\hat{z}_{n,j}(t)} + \lambda_{n,t}(\alpha_n(t) + \beta_n(t)) \\
&\times C_n(t) + q_n(t)p_n(t)\tilde{x}_n(t),
\end{aligned}$$

where $\lambda_{n,t}$ denotes the Lagrangian multiplier for user n at time t and $\eta_{n,t} = q_n(t) + \lambda_{n,t}$. The dual function is

$$\begin{aligned}
&g(\lambda_{n,t}) = \sum_{i\in\mathcal{N}_{\mathcal{E}}} \sup_{z_{n,i}(t)\in\mathcal{Z}_{n,i}(t)} \widehat{U}_i(z_{n,i}(t)) - \eta_{n,t}e^{z_{n,i}(t)} \\
&+ \sum_{j\in\mathcal{N}_{\mathcal{J}}} \sup_{\hat{z}_{n,j}(t)\in\hat{\mathcal{Z}}_{n,j}(t)} \widehat{\hat{U}}_j(\hat{z}_{n,j}(t)) + \lambda_{n,t}(\alpha_n(t) + \beta_n(t)) \\
&\times C_n(t) - \eta_{n,t}e^{\hat{z}_{n,j}(t)} + \sup_{\tilde{x}_n(t)\in\tilde{\mathcal{X}}_n(t)} q_n(t)p_n(t)\tilde{x}_n(t),
\end{aligned}$$

where $\mathcal{Z}_{n,i}(t) = \{z_{n,i}(t) \mid 0 \le z_{n,i}(t) \le \Gamma_{n,i}(t)\}$, $\hat{\mathcal{Z}}_{n,j}(t) = \left\{\hat{z}_{n,j}(t) \mid \widehat{\beta}_{n,j}(t) \le \hat{z}_{n,j}(t) \le \hat{\Gamma}_{n,j}(t)\right\}$, and $\tilde{\mathcal{X}}_n(t) = \{\tilde{x}_n(t) \mid 0 \le \tilde{x}_n(t) \le p_n(t)\gamma_n(t)C_n(t)\}$. The dual problem is

$$\begin{aligned}
\text{minimize} \quad & g(\lambda_{n,t}) \\
\text{subject to} \quad & \lambda_{n,t} \ge 0.
\end{aligned} \tag{7}$$

For $\psi_i > 1$, $\widehat{U}_i(z_{n,i}(t)) = \frac{\left(e^{z_{n,i}(t)}+1\right)^{(1-\psi_i)}-1}{1-\psi_i}$ is a sigmoidal function, which can be approximated by the standard sigmoidal function $\frac{c_3}{1+c_1e^{-c_2z_{n,i}(t)}}$, where $c_3 = \dfrac{1}{\psi_i - 1}$, $c_1 = \dfrac{1}{2^{\psi_i-1}-1}$, and $c_2 = \frac{-\ln(c_1)}{\ln(\psi_i-1)}$. After setting $\frac{\partial^2\widehat{U}_i(z_{n,i}(t))}{\partial(z_{n,i}(t))^2} = 0$, we obtain the point of inflection $z_{n,i}^{in}(t) = \ln\left(\frac{1}{\psi_i-1}\right) = \frac{\ln(c_1)}{c_2}$. When $0 < \psi_i \le 1$, $\widehat{U}_i(z_{n,i}(t))$ is a convex function. Moreover, for real-time service $j \in \mathcal{N}_{\mathcal{J}}$, $\widehat{\hat{U}}_j(\hat{z}_{n,j}(t)) = \dfrac{1}{1+e^{-(a_j\hat{z}_{n,j}(t)+b_j)}}$, which is in the form of a standard sigmoidal function with the point of inflection $\hat{z}_{n,j}^{in}(t) = -\frac{b_j}{a_j}$ where $k_j = e^{-b_j}$. For $\psi_i > 1$, in order to solve problem (7), we need to solve a number of subproblems. First, we have to solve the following problem for each $k \in \mathcal{K} = \mathcal{N}_{\mathcal{E}} \cup \mathcal{N}_{\mathcal{J}}$

$$\underset{z_{n,k}(t)\in\mathcal{Z}_{n,k}(t)}{\text{maximize}} \; R_{n,k}(t) = \widetilde{\widehat{U}}_k(z_{n,k}(t)) - \eta_{n,t}e^{z_{n,k}(t)}, \tag{8}$$

where

$$\widetilde{\widehat{U}}_k(z_{n,k}(t)) = \frac{c_k}{1+e^{-(a_k\hat{z}_{n,k}(t)+b_k)}} \tag{9}$$

denotes the standard sigmoidal utility function for service $k \in \mathcal{K}$ after the logarithmic change of variables, $x_{n,k}(t)$ denotes the allocated bandwidth to service $k \in \mathcal{K}$ at time t, $z_{n,k}(t) \triangleq \ln(x_{n,k}(t))$, and $\mathcal{Z}_{n,k}(t) = \left\{z_{n,k}(t) \mid z_{n,k}^{min}(t) \le z_{n,k}(t) \le z_{n,k}^{max}(t)\right\}$ where $z_{n,k}^{min}(t)$ and $z_{n,k}^{max}(t)$ denote the minimum reserved resources and maximum demand for service k after a logarithm change of variable, respectively. Moreover, we need to solve the following problem with respect to $\tilde{x}_n(t)$

$$\begin{aligned}
\underset{\tilde{x}_n(t)}{\text{maximize}} \quad & q_n(t)p_n(t)\tilde{x}_n(t) \\
\text{subject to} \quad & 0 \le \tilde{x}_n(t) \le p_n(t)\gamma_n(t)C_n(t),
\end{aligned}$$

which results in $\tilde{x}_n^*(t) = p_n(t)\gamma_n(t)C_n(t)$. Let $\breve{\mathcal{Z}}_{n,k}(t) = \left\{z_{n,k}(t) \mid z_{n,k}^{min}(t) \le z_{n,k}(t) \le z_{n,k}^{in}(t)\right\}$ and $\widehat{\mathcal{Z}}_{n,k}(t) = \left\{z_{n,k}(t) \mid z_{n,k}^{in}(t) \le z_{n,k}(t) \le z_{n,k}^{max}(t)\right\}$. When $z_{n,k}(t) \in \breve{\mathcal{Z}}_{n,k}(t)$ and $z_{n,k}(t) \in \widehat{\mathcal{Z}}_{n,k}(t)$, $R_{n,k}(t)$ is a convex and a concave function in $z_{n,k}(t)$, respectively. Thus, problem (8) can be reformulated as follows:

$$z_{n,k}^*(q_n(t), \lambda_{n,t}) = \arg\max_{\left\{z_{n,k}^{min}(t),\, v_{n,k}^*(q_n(t),\, \lambda_{n,t})\right\}} R_{n,k}(t), \tag{10}$$

where

$$v_{n,k}^*(q_n(t), \lambda_{n,t}) = \arg\max_{z_{n,k}(t)\in\widehat{\mathcal{Z}}_{n,k}(t)} R_{n,k}(t).$$

For $z_{n,k}(t) \in \widehat{\mathcal{Z}}_{n,k}(t)$, considering the sigmoidal function $\widetilde{\widehat{U}}_k(z_{n,k}(t))$ in (9), after taking derivative, we obtain

$$\frac{\partial R_{n,k}(t)}{\partial z_{n,k}(t)} = \frac{a_kc_ke^{-(a_kz_{n,k}(t)+b_k)}}{\left(1+e^{-(a_kz_{n,k}(t)+b_k)}\right)^2} - \eta_{n,t}e^{z_{n,k}(t)} = 0. \tag{11}$$

To solve (11), we can express the equation in terms of $\hat{y}_{n,k}(t) = e^{-z_{n,k}(t)}$ as follows:

$$\begin{aligned}
&\eta_{n,t}e^{-2b_k}(\hat{y}_{n,k}(t))^{2a_k} + 2\eta_{n,t}e^{-b_k}(\hat{y}_{n,k}(t))^{a_k} \\
&-a_kc_ke^{-b_k}(\hat{y}_{n,k}(t))^{a_k+1} + \eta_{n,t} = 0.
\end{aligned} \tag{12}$$

To solve (12), we proceed with the approximation $(\hat{y}_{n,k}(t))^{a_k} \approx (\hat{y}_{n,k}(t))^{a_k+1}$ for $a_k \gg 1$ and $0 \le \hat{y}_{n,k}(t) <$

1. Note that when $a_k \gg 1$, the utility function approaches the step-like utility function corresponding to the real-time applications, which require reserved resources for guaranteed QoS. Following the approximation in (13), we obtain

$$\eta_{n,t} e^{-2b_k} \left(\hat{\chi}_{n,k}(t)\right)^2 + \left(2\eta_{n,t} - a_k c_k\right) e^{-b_k} \hat{\chi}_{n,k}(t) + \eta_{n,t} = 0,$$

where $\hat{\chi}_{n,k}(t) = e^{-a_k z_{n,k}(t)}$. We can consider two cases as follows:

Case I: $a_k c_k \geq 4\eta_{n,t}$

Since $0 \leq \hat{\chi}_{n,k}(t) \leq 1$, we have

$$\begin{aligned} &\arg \max_{z_{n,k}^{in}(t) \leq z_{n,k}(t) \leq z_{n,k}^{max}(t)} R_{n,k}(t) \\ &= \min\left\{ \max\left\{ -\frac{1}{a_k} \ln(\xi),\, z_{n,k}^{in}(t) \right\},\, z_{n,k}^{max}(t) \right\}, \end{aligned}$$

where $\xi = \frac{(a_k c_k - 2\eta_{n,t}) - \sqrt{(a_k c_k)^2 - 4 a_k c_k \eta_{n,t}}}{2\eta_{n,t} e^{-b_k}}$.

Case II: $a_k c_k < 4\eta_{n,t}$

Since $\dfrac{\partial R_{n,k}(t)}{\partial z_{n,k}(t)} < 0$ and $R_{n,k}(t)$ is decreasing in $z_{n,k}(t)$, then

$$\arg \max_{z_{n,k}^{in}(t) \leq z_{n,k}(t) \leq z_{n,k}^{max}(t)} R_{n,k}(t) = z_{n,k}^{in}(t).$$

The Lagrangian multiplier $\lambda_{n,t}$ can be updated using the subgradient projection method, as follows:

$$\begin{aligned} \lambda_{n,t}(s+1) = \lambda_{n,t}(s) - \kappa(s) \Bigg(&\sum_{i \in \mathcal{N}_{\mathcal{E}}} e^{z_{n,i}(t)} \\ &+ \left(\alpha_n(t) + \beta_n(t)\right) C_n(t) + \sum_{j \in \mathcal{N}_{\mathcal{J}}} e^{\hat{z}_{n,j}(t)} \Bigg), \end{aligned}$$

where s is the index of iteration and $\kappa(s) = \frac{m}{s+1}$ denotes the diminishing step size with m being a positive constant.

Note that home network n with service demands higher than $C_n(t)$, provides the SDN controller with the exact amount of excess bandwidth it requires to access from the access points of its neighboring home networks, e.g., $\Delta C_n(t)$. In this case, to perform bandwidth allocation, the SDN controller implements problem (5) with the difference that the available capacity is increased to $C_n(t) + \Delta C_n(t)$.

B. The Best Response of ISP in Stage I

We can decouple the optimization problem (6) to tackle the pricing and reimbursement strategies of ISP separately. Therefore, to characterize the optimal reimbursement strategy, given the ISP's pricing strategy, we need to solve the following optimization problem:

$$\begin{aligned} \underset{\mathbf{p}_t}{\text{maximize}} \quad & \Omega_{ISP}(q_n(t)) \\ & = q_n(t)\left(\left(1 - p_n(q_n(t))\right) \tilde{x}_n^*(p_n(q_n(t)))\right) \\ \text{subject to} \quad & 0 \leq p_n(q_n(t)) \leq 1. \end{aligned}$$

Without loss of generality, we focus on the n^{th} sub-problem, associated with home network n. Note that since $0 \leq p_n(q_n(t)) \leq 1$ and $\tilde{x}_n^*(p_n(q_n(t))) = p_n(t)\gamma_n(t) C_n(t)$, we have

$$\frac{\partial \Omega_{ISP}(q_n(t))}{\partial p_n(q_n(t))} = q_n(t)\gamma_n(t) C_n(t)\left(1 - 2p_n(q_n(t))\right), \tag{13a}$$

$$\frac{\partial^2 \Omega_{ISP}(q_n(t))}{\partial (p_n(q_n(t)))^2} = -2q_n(t)\gamma_n(t) C_n(t) < 0. \tag{13b}$$

Note that $\Omega_{ISP}(q_n(t))$ is a concave function of $p_n(q_n(t))$ and the second derivative given in (13b) is negative. Moreover, by setting (13a) to zero, we obtain $p_n(q_n(t))^* = 1/2$. In order to study the best pricing strategy $q_n(p_n(t))^*$, given the reimbursement strategy $p_n(t)$, we consider the following optimization problem:

$$\begin{aligned} \underset{\mathbf{q}_t}{\text{maximize}} \; & \Psi_{ISP}(p_n(t)) = \sum_{n \in \mathcal{N}} q_n(t)\vartheta_{n,t} \\ \text{subject to} \quad & 0 \leq q_n(t) \leq q_{max}, \end{aligned}$$

where $\vartheta_{n,t} = \sum_{i \in \mathcal{N}_{\mathcal{E}}} x_{n,i}^*(q_n(p_n(t))) + \sum_{j \in \mathcal{N}_{\mathcal{J}}} \hat{x}_{n,j}^*(q_n(p_n(t)))$, and $\mathbf{x}_{n,t}^*(q_n(t))$ and $\hat{\mathbf{x}}_{n,t}^*(q_n(t))$ are obtained from problem (10). Considering home network n, since $0 \leq q_n(t) \leq q_{max}$, we obtain

$$\frac{\partial \Psi_{ISP}(p_n(t))}{\partial q_n(p_n(t))} = \vartheta_{n,t} + q_n(t)\theta_{n,t}, \tag{14a}$$

$$\frac{\partial^2 \Psi_{ISP}(p_n(t))}{\partial (q_n(p_n(t)))^2} = 2\theta_{n,t}, \tag{14b}$$

where $\theta_{n,t} = \left(\sum_{i \in \mathcal{N}_{\mathcal{E}}} \frac{\partial x_{n,i}^*(q_n(p_n(t)))}{\partial q_n(p_n(t))} + \sum_{j \in \mathcal{N}_{\mathcal{J}}} \frac{\partial \hat{x}_{n,j}^*(q_n(p_n(t)))}{\partial q_n(p_n(t))} \right)$. Note that since $x_{n,i}^*(q_n(p_n(t)))$ and $\hat{x}_{n,i}^*(q_n(p_n(t)))$ are either independent or decreasing functions of $q_n(p_n(t))$, (15b) is negative. Moreover, setting (15a) to zero, we obtain

$$q_n(p_n(t))^* = \min\left\{ \max\left\{ -\vartheta_{n,t}/\theta_{n,t}, 0 \right\}, q_{max} \right\}.$$

Note that the optimal ISP pricing-reimbursement strategy occurs at the intersection point of $q_n(p_n(t))^*$ and $p_n(q_n(t))^*$.

IV. Performance Evaluation

In this section, we provide numerical results to characterize the interplay between ISP hybrid pricing strategy and the home networks' traffic consumption. We assume that the number of elastic and real time services is 5 each and $x_{n,k}^{max}(t)$ is randomly selected from $[10, 11]$ Mbps. We further consider elastic and real-time applications with utility functions as is provided in (3) and (4), respectively, where ψ_i and a_j are randomly selected from $[6, 10]$ and k_j is randomly selected from $[20, 50]$. Fig. 2 illustrates the payoff of ISP and the home networks versus the pricing strategy of ISP $q_n(t)$ for different $\beta_{n,j}(t)$. As is illustrated in Fig. 2, $q_n^*(t)$ is in fact the intersection point of the payoff curves for ISP and home networks.

Fig. 3 illustrates the average payoff of ISP per service during different time slots of the day by comparing the hybrid pricing scheme proposed for SDN-enabled home networks with the usage-based pricing scheme proposed in [9]. The distributions of the maximal residential traffic demand during peak time are assumed to be normal distribution with mean 20 Mbps and variance ρ, where ρ is randomly selected from $[0, 1]$. We divide the day into 24 time slots. The number of services is set to 40 with 20 elastic services and 20 real-time services. We set

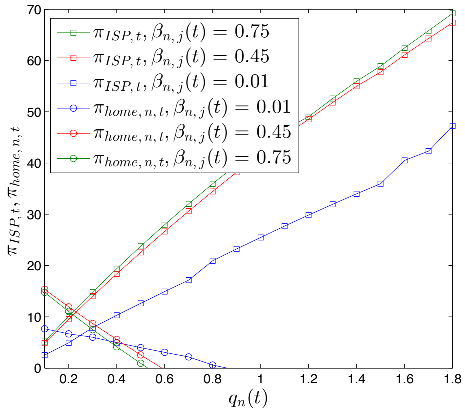

Fig. 2. Payoff of ISP and home networks versus ISP pricing strategy.

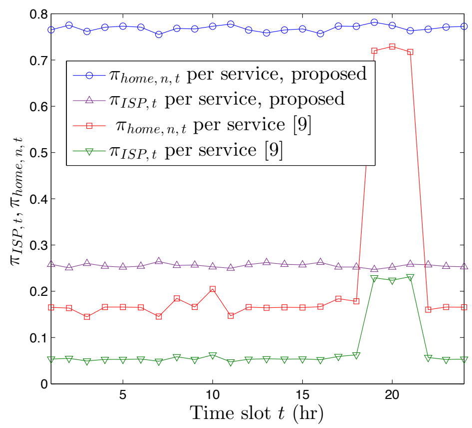

Fig. 3. Average payoff of ISP and home networks per service during different time slots.

8 pm as the peak time of the day. As can be seen from Fig. 3, by employing the SDN technology, both home networks and ISP receive higher payoffs compared to the best-effort residential network. As an example, payoff of SDN-enabled home network is larger than the best-effort home network with usage-based pricing by 400% and 7.1% during regular hours and peak time of the day, respectively. The payoff of ISP empowered with the SDN centralized technology is increased by 400% and 2% during regular hours and peak time of the day, respectively. Moreover, the SDN control technology provides the home networks and ISP with payoffs that are resilient to demand fluctuations during peak-time.

Fig. 4 illustrates the payoff of ISP versus different ISP reimbursement strategies. Results in Fig. 4 show that the optimal value of $p_n^*(t)$ is 0.5, which also agrees with the analytical results provided in (14b).

V. Conclusion

In this paper, we proposed to combine the SDN control paradigm with the existing home network infrastructure. We proposed a novel hybrid pricing scheme for ISPs and formulated a Stackelberg game to analytically characterize ISP's hybrid pricing/reimbursement strategy, as well as the scheduled bandwidth for different applications of household devices. Through comprehensive numerical results, we showed

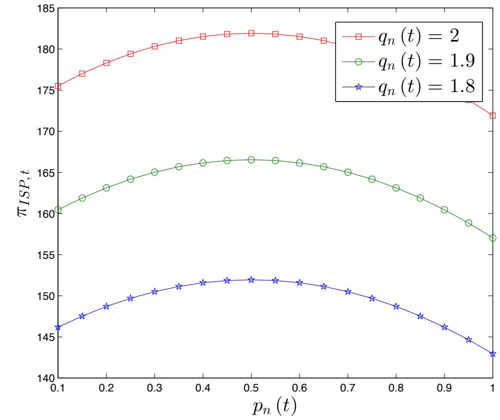

Fig. 4. Payoff of ISP versus ISP reimbursement strategy for different ISP pricing strategy.

the performance gain of the SDN-enabled home network technology over another scheme in [9] on improving the achievable QoS for the virtualized traffic of home network devices. For future work, we will investigate the impact of SDN control technology on the payoff of home networks and ISP by taking into account the limitations of ISP backhaul.

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