Joint Reflection Coefficient Selection and Subcarrier Allocation for Backscatter Systems with NOMA

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***Abstract*—Non-orthogonal multiple access (NOMA) and backscatter communication are two emerging technologies that enable low power communication for the Internet of Things (IoT) devices. In this paper, we consider a multicarrier NOMA (MC-NOMA) backscatter communication system. The objective is to maximize the aggregate data rate of the system by jointly optimizing the reflection coefficients and subcarrier allocation. The formulated problem is nonconvex and exhibits hidden monotonicity structure. To obtain the optimal solution, we propose an algorithm based on discrete monotonic optimization. The proposed algorithm can be considered as a performance benchmark. We also transform the nonconvex problem to another problem by using difference of convex functions and successive convex approximation and propose an algorithm to obtain a suboptimal solution in polynomial time. Simulation results show that the suboptimal scheme achieves an aggregate data rate close to the proposed optimal scheme. Results also show that our proposed schemes provide a higher aggregate data rate than the orthogonal multiple access (OMA) scheme.**

I. Introduction

Power-domain non-orthogonal multiple access (NOMA) is an emerging technology for the fifth generation cellular networks. NOMA can improve the spectral efficiency compared to orthogonal frequency division multiple access (OFDMA). With NOMA, signals from multiple users can be multiplexed on the same resource block. Power and subcarrier allocation to the users are two important design criteria to enhance the performance of NOMA systems. In [1], optimal power allocation to the NOMA users is obtained by maximizing the aggregate throughput. In [2], joint subcarrier and power allocation problem is formulated for uplink NOMA and a suboptimal algorithm is proposed. In [3], joint power and subcarrier allocation is considered in multicarrier (MC) NOMA systems and an optimal solution is obtained by using monotonic optimization. In [4], monotonic optimization is used to solve a joint power and subcarrier allocation problem in full-duplex MC-NOMA systems. In [5], the user pairing problem is considered for uplink NOMA to maximize the total sum rate for all of the subcarriers.

Recently, backscatter communication has gained wide popularity as a low power communication technology for the Internet of Things (IoT). In backscatter communication systems, a backscatter transmitter (or passive tag) does not generate any carrier signal. The device tunes its antenna impedance to generate a reflection coefficient and communicates with the backscatter receiver (or reader) by modulating and reflecting the incident signal from the radio frequency (RF) carrier emitter [6].

Backscatter system with NOMA as a multiple access technique has been considered in some recent works. In [7], power-domain NOMA is used in a backscatter communication system. Simulation results show that backscatter system with NOMA has a better performance compared to backscatter system with time division multiple access (TDMA) in terms of the number of successfully decoded bits in the reader. In [8], a resource allocation problem is formulated for a backscatter system with NOMA and dynamic TDMA. In the aforementioned works, only backscatter system with single carrier NOMA has been investigated.

In this work, we propose a backscatter system with MC-NOMA. Our goal is to maximize the aggregate data rate by jointly optimizing the reflection coefficients and subcarrier allocation to the backscatter devices. The reflection coefficients of backscatter devices are the adjustable parameters in the backscatter system. The contributions in this paper are as follows:

- We formulate the joint reflection coefficient selection and subcarrier allocation problem for backscatter system using MC-NOMA as a nonconvex problem with hidden monotonicity structure.
- We use monotonic optimization technique and propose an optimal scheme for selecting the reflection coefficients of backscatter devices and subcarrier allocation. We design an algorithm based on outer polyblock approximation algorithm to obtain the optimal solution.
- Since the optimal approach has a non-polynomial time complexity, we transform the objective function to another function by using difference of convex functions technique. We use successive convex approximation algorithm to obtain a suboptimal solution in polynomial time. Simulation results show that the aggregate data rate obtained by the proposed suboptimal scheme is very close to the optimal scheme. The proposed schemes using MC-NOMA have a higher aggregate data rate than the orthogonal multiple access (OMA) scheme.

The rest of this paper is organized as follows. In Section II, the system model is described and the resource allocation problem for the proposed system is formulated. Section III presents the optimal and suboptimal schemes for solving the formulated problem. Simulation results are presented in

Section IV. Conclusion is given in Section V.

In this work. we use boldface lower case letters to denote vectors. $\mathbb{R}_+$ denotes the set of non-negative real numbers; $\mathbb{R}^N$ denotes the set of all N dimensional vectors with real entries and $\mathbb{R}^N_+$ denotes the non-negative subset of $\mathbb{R}^N$. $\mathbf{a} \preccurlyeq \mathbf{b}$ indicates that vector $\mathbf{a}$ is component-wise smaller than $\mathbf{b}$. We present the circularly symmetric complex Gaussian distribution with mean γ and variance σ^2 by $\mathcal{CN}(\gamma, \sigma^2)$; $\sim$ stands for "distributed as"; $|.|$ denotes the absolute value of a complex scalar. $\mathbf{e}^d$ is used to denote the vector with 1 in the d-th entry and 0 for all other entries.

II. System Model and Problem Formulation

We consider an MC-NOMA backscatter system with a reader and K single-antenna backscatter devices as shown in Fig. 1. The reader is equipped with successive interference cancellation (SIC) receiver. The frequency bandwidth is divided into M orthogonal subcarriers. In this work, we focus on uplink NOMA communication. We consider that at most two backscatter devices operate on each subcarrier to limit the co-channel interference [4]. Each backscatter device can operate on only one subcarrier. Let $\mathcal{K}$ denote the set of backscatter devices, i.e., $\mathcal{K} = \{1, \ldots, K\}$. Let $\mathcal{M}$ denote the set of subcarriers, i.e., $\mathcal{M} = \{1, \ldots, M\}$. The reader transmits a continuous wave signal with power P as the carrier signal. The backscatter device $k \in \mathcal{K}$ modulates and reflects the incident signal via a reflection coefficient. We denote the magnitude of the reflection coefficient of backscatter device k as η_k, which is a real value between zero and one. In practice, backscatter devices have a set of impedances to generate reflection coefficients with different magnitudes [7]. Let ξ_k^m denote the information symbol of backscatter device k on subcarrier $m \in \mathcal{M}$ with unit average power. Let h_k^m denote the channel gain of backscatter device k on subcarrier m. The channel gain is characterized by both path loss and small-scale fading. We have $h_k^m = \sqrt{r_k^{-\alpha}} d_k^m$, where r_k is the distance between backscatter device k and the reader, α is the path loss exponent, and $d_k^m \sim \mathcal{CN}(0,1)$ is the small-scale fading factor. The reflected signal of backscatter device k on subcarrier m is given by $x_k^m = \sqrt{\eta_k P} h_k^m \xi_k^m$.

Consider that two backscatter devices k and l, where $k, l \in \mathcal{K}$ are selected to perform NOMA on subcarrier $m \in \mathcal{M}$. We assume the global channel state information for all links in the network is available in the reader. The received signal on subcarrier m is as follows:

$$\begin{aligned} y^m &= h_k^m x_k^m + h_l^m x_l^m + n^m \\ &= \sqrt{\eta_k P}(h_k^m)^2 \xi_k^m + \sqrt{\eta_l P}(h_l^m)^2 \xi_l^m + n^m, \end{aligned} \quad (1)$$

where $n^m \sim \mathcal{CN}(0, \sigma^2)$ denotes the additive white Gaussian noise on subcarrier m in the reader. For backscatter devices k and l operating on subcarrier m, consider that backscatter device k experiences a better channel gain than backscatter device l on subcarrier m, i.e., $|h_k^m| > |h_l^m|$. Thus, the reader first decodes the signal of backscatter device k, removes the signal by SIC and then decodes the signal of backscatter device l. We consider binary integer variables $s_{k,l}^m \in \{0,1\}$ as the subcarrier allocation coefficients. The binary variable $s_{k,l}^m$ is equal to 1 when the backscatter devices k and l are selected to perform NOMA on subcarrier m, while backscatter device k has a better channel gain and its signal is decoded first in the reader, otherwise $s_{k,l}^m$ is equal to 0. The aggregate data rate on subcarrier m is obtained by the sum of the data rate of backscatter devices k and l, which is given by

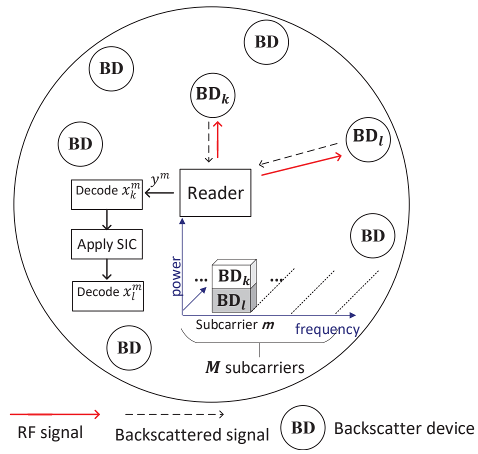

Fig. 1. An MC-NOMA backscatter system consisting of several backscatter devices and a reader equipped with SIC receiver. Signals from two backscatter devices k and l are multiplexed on subcarrier m using NOMA. The reader sends the carrier signal and the backscatter devices send their data to the reader by reflecting the carrier signal.

$$R_{k,l}^m = s_{k,l}^m \left(\log_2 \left(1 + \frac{\eta_k P |h_k^m|^4}{\eta_l P |h_l^m|^4 + \sigma^2} \right) + \log_2 \left(1 + \frac{\eta_l P |h_l^m|^4}{\sigma^2} \right) \right). \quad (2)$$

We further define $g_k^m = \frac{P|h_k^m|^4}{\sigma^2}$ and rewrite equation (2) as follows:

$$R_{k,l}^m = s_{k,l}^m \left(\log_2 \left(1 + \frac{\eta_k g_k^m}{1 + \eta_l g_l^m} \right) + \log_2 \left(1 + \eta_l g_l^m \right) \right). \quad (3)$$

The joint reflection coefficient selection and subcarrier allocation problem for the backscatter communication system is formulated as follows:

$$\underset{\substack{\eta_k, k \in \mathcal{K} \\ s_{k,l}^m, k,l \in \mathcal{K}, m \in \mathcal{M}}}{\text{maximize}} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{l=1}^{K} s_{k,l}^m \log_2 \left(1 + \frac{\eta_k g_k^m}{1 + \eta_l g_l^m} \right)$$

$$+ \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{l=1}^{K} s_{k,l}^m \log_2 \left(1 + \eta_l g_l^m \right) \quad (4a)$$

$$\text{subject to } \; 0 \leq \eta_k \leq 1, \; k \in \mathcal{K} \quad (4b)$$

$$s_{k,l}^m \in \{0,1\}, \; k, l \in \mathcal{K}, m \in \mathcal{M} \quad (4c)$$

$$\sum_{k=1}^{K} \sum_{l=1}^{K} s_{k,l}^m \leq 1, \; m \in \mathcal{M} \quad (4d)$$

$$\sum_{m=1}^{M} \sum_{l=1}^{K} s_{k,l}^m + \sum_{m'=1}^{M} \sum_{l'=1}^{K} s_{l',k}^{m'} \leq 1, \; k \in \mathcal{K} \quad (4e)$$

$$\log_2 \left(1 + \frac{\eta_k g_k^m}{1 + \eta_l g_l^m} \right) \geq s_{k,l}^m R_{\min}, \; k, l \in \mathcal{K}, m \in \mathcal{M} \quad (4f)$$

$$\log_2(1+\eta_l g_l^m)\geq s_{k,l}^m R_{\min},\ \ k,l\in\mathcal{K}, m\in\mathcal{M}. \quad (4g)$$

Constraint (4b) ensures that the magnitude of reflection coefficients is between zero and one. Constraint (4d) ensures that each subcarrier is assigned to at most two backscatter devices. Constraint (4e) guarantees that at most one subcarrier is assigned to each backscatter device. It also ensures that each pair of backscatter devices is operating on at most one subcarrier. The last two constraints guarantee that the data rate of backscatter devices k and l on subcarrier m is greater than the minimum data rate requirement, denoted as R_{min}.

III. Proposed Optimal and Suboptimal Schemes

Problem (4) is a discrete nonconvex problem due to binary variables and nonconvexity of the objective function. However, it has a hidden monotonicity structure. Thus, one approach to obtain the optimal solution of this problem is to apply discrete monotonic optimization.

A. Discrete Monotonic Optimization

First, we present some definitions related to discrete monotonic optimization problem [9]. Given any vector $\mathbf{z}\in\mathbb{R}_+^N$, the *box* $[0,\mathbf{z}]$ is the set of all $\mathbf{x}\in\mathbb{R}_+^N$ satisfying $0\preceq\mathbf{x}\preceq\mathbf{z}$. A set $\mathcal{G}\subset\mathbb{R}_+^N$ is *normal set* if for any point $\mathbf{z}\in\mathcal{G},[0,\mathbf{z}]\subset\mathcal{G}$. A set $\mathcal{H}$ is called *conormal set* if $\mathbf{x}\in\mathcal{H}$ and $\mathbf{x}'\succeq\mathbf{x}$ result $\mathbf{x}'\in\mathcal{H}$. A set $\mathcal{P}\subset\mathbb{R}_+^N$ is called a *polyblock* with vertex set $\mathcal{V}$ if it is the union of a finite number of boxes $[0,\mathbf{z}]$, where $\mathbf{z}\in\mathcal{V}$. Given any non-empty normal set $\mathcal{Z}\subset\mathbb{R}_+^N$ and any vector $\mathbf{z}\in\mathbb{R}_+^N$, the *projection* of $\mathbf{z}$ onto the upper boundary of the normal set $\mathcal{Z}$ is $\Phi(\mathbf{z})=\lambda\mathbf{z}$, where $\lambda=\max\{\alpha\mid\alpha\mathbf{z}\in\mathcal{Z}\}$ and $\alpha\in\mathbb{R}_+$. Given any point $\mathbf{x}\in[0,\mathbf{b}]$, the *lower $\mathcal{A}$-adjustment* of $\mathbf{x}$ is defined as $\lfloor\mathbf{x}\rfloor_{\mathcal{A}}=\max\{\mathbf{x}'\mid\mathbf{x}'\in\mathcal{A}\cup\{\mathbf{0}\},\mathbf{x}'\preceq\mathbf{x}\}$. A *discrete monotonic optimization problem* has the following form:

$$\underset{\mathbf{z}}{\text{maximize}}\quad f(\mathbf{z}) \quad (5a)$$
$$\text{subject to}\quad \mathbf{z}\in\mathcal{G}\cap\mathcal{H}\cap\mathcal{A}, \quad (5b)$$

where $\mathbf{z}$ is the optimization variable. The feasible set is the intersection of a non-empty compact normal set $\mathcal{G}$, a closed conormal set $\mathcal{H}$ and set $\mathcal{A}$ including all possible discrete values of $\mathbf{z}$. Function f is a monotonic increasing function on $\mathbb{R}_+^N$.

B. Optimal Scheme

To transform problem (4) to the standard form of discrete monotonic optimization problem, we rewrite $R_{k,l}^m$ in (3) as follows:

$$\begin{aligned}R_{k,l}^m &= \log_2\left(1+\frac{s_{k,l}^m\eta_k g_k^m}{1+s_{k,l}^m\eta_l g_l^m}\right)+\log_2\left(1+s_{k,l}^m\eta_l g_l^m\right)\\ &= \log_2\left(1+\frac{\tilde{\eta}_{k,l,k}^m g_k^m}{1+\tilde{\eta}_{k,l,l}^m g_l^m}\right)+\log_2\left(1+\tilde{\eta}_{k,l,l}^m g_l^m\right)\\ &= \log_2(u_{k,l}^m)+\log_2(v_{k,l}^m), \end{aligned}\quad (6)$$

where $\tilde{\eta}_{k,l,k}^m=s_{k,l}^m\eta_k$, $\tilde{\eta}_{k,l,l}^m=s_{k,l}^m\eta_l$, $u_{k,l}^m=1+\frac{\tilde{\eta}_{k,l,k}^m g_k^m}{1+\tilde{\eta}_{k,l,l}^m g_l^m}$ and $v_{k,l}^m=1+\tilde{\eta}_{k,l,l}^m g_l^m$. We define vector $\tilde{\boldsymbol{\eta}}\in\mathbb{R}^{2MK^2}$ containing all of the variables $\tilde{\eta}_{k,l,k}^m$ and $\tilde{\eta}_{k,l,l}^m$, and also vector $\mathbf{s}\in\mathbb{R}^{MK^2}$ containing all of the variables $s_{k,l}^m$. According to (6), problem (4) can be rewritten as

$$\begin{aligned}\underset{\tilde{\boldsymbol{\eta}},\mathbf{s}}{\text{maximize}}\ &\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K}\log_2\left(1+\frac{\tilde{\eta}_{k,l,k}^m g_k^m}{1+\tilde{\eta}_{k,l,l}^m g_l^m}\right)\\ &+\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K}\log_2(1+\tilde{\eta}_{k,l,l}^m g_l^m) \quad (7a)\\ \text{subject to}\ &0\leq\tilde{\eta}_{k,l,k}^m\leq 1,\ \ k,l\in\mathcal{K},m\in\mathcal{M} \quad (7b)\\ &0\leq\tilde{\eta}_{k,l,l}^m\leq 1,\ \ k,l\in\mathcal{K},m\in\mathcal{M} \quad (7c)\\ &\log_2\left(1+\frac{\tilde{\eta}_{k,l,k}^m g_k^m}{1+\tilde{\eta}_{k,l,l}^m g_l^m}\right)\geq s_{k,l}^m R_{\min},\\ &\qquad\qquad k,l\in\mathcal{K},m\in\mathcal{M} \quad (7d)\\ &\log_2\left(1+\tilde{\eta}_{k,l,l}^m g_l^m\right)\geq s_{k,l}^m R_{\min}, k,l\in\mathcal{K},m\in\mathcal{M} \quad (7e)\\ &\text{constraints (4c), (4d), (4e).}\end{aligned}$$

We further define $D=2MK^2$ and $\mathbf{z}=(z_1,\ldots,z_D)=(u_{1,1}^1,\ldots,u_{K,K}^M,v_{1,1}^1,\ldots,v_{K,K}^M)$. By defining these new variables, problem (7) can be written as a discrete monotonic optimization problem as follows:

$$\underset{\mathbf{z}}{\text{maximize}}\quad T(\mathbf{z}) \quad (8a)$$
$$\text{subject to}\quad \mathbf{z}\in\mathcal{Z}\cap\mathcal{A}, \quad (8b)$$

where $T(\mathbf{z})=\sum_{d=1}^{D}\log_2(z_d)$ and $\mathcal{Z}=\{\mathbf{z}\mid 1\leq z_d\leq\frac{a_d(\tilde{\boldsymbol{\eta}})}{b_d(\tilde{\boldsymbol{\eta}})},\mathbf{0}\preceq\tilde{\boldsymbol{\eta}}\preceq\mathbf{1},d\in\{1,\ldots,D\}\}$ is the set obtained by removing constraints (4c), (4d), (4e), (7d), and (7e). Functions $a_d(\tilde{\boldsymbol{\eta}})$ and $b_d(\tilde{\boldsymbol{\eta}})$ are defined as follows:

$$a_d(\tilde{\boldsymbol{\eta}})=\begin{cases}1+\tilde{\eta}_{k,l,k}^m g_k^m+\tilde{\eta}_{k,l,l}^m g_l^m, & d=\Delta,\\ 1+\tilde{\eta}_{k,l,l}^m g_l^m, & d=\Delta+\frac{D}{2}.\end{cases} \quad (9)$$

$$b_d(\tilde{\boldsymbol{\eta}})=\begin{cases}1+\tilde{\eta}_{k,l,l}^m g_l^m, & d=\Delta,\\ 1 & d=\Delta+\frac{D}{2},\end{cases} \quad (10)$$

where $\Delta=(m-1)K^2+(k-1)K+l$. For any given $k,l\in\mathcal{K}$ and $m\in\mathcal{M}$, we can find the corresponding $d\in\{1,\ldots,D\}$. Finite set $\mathcal{A}=\{\mathbf{z}\mid 1\leq z_d\leq\frac{a_d(\tilde{\boldsymbol{\eta}})}{b_d(\tilde{\boldsymbol{\eta}})},\tilde{\boldsymbol{\eta}}\in H,\mathbf{s}\in\mathcal{S},d\in\{1,\ldots,D\}\}$ is the set containing the vectors $\mathbf{z}$ which satisfy constraints (4c), (4d), (4e), and (7b)−(7e). Sets H and $\mathcal{S}$ are the sets spanned by constraints (4c), (4d), (4e), and (7b)−(7e). The function $T(\mathbf{z})$ in (8a) is a monotonic increasing function with respect to the variable $\mathbf{z}$. Problem (8) is in standard form of discrete monotonic optimization problem. We propose an optimal algorithm based on outer polyblock approximation algorithm. In this algorithm, we first construct the polyblock $\mathcal{P}^{(1)}$ with the vertex set $\mathcal{V}^{(1)}$ including the vertex $\mathbf{z}^{(1)}$. To obtain $\mathbf{z}^{(1)}$, we choose the maximum possible values for $u_{k,l}^{m^{(1)}}$ and $v_{k,l}^{m^{(1)}}$, $k,l\in\mathcal{K}$ and $m\in\mathcal{M}$ by selecting the value of one for the reflection coefficients and removing the co-channel interference in each subcarrier. Then the vertex $\mathbf{z}^{(1)}$ is projected onto the boundary of set $\mathcal{Z}$. The resulting vertex, $\Phi(\mathbf{z}^{(1)})$, may not be a valid vertex, i.e., it may not satisfy constraints (4c), (4d), (4e), and (7b)−(7e). We convert it to a valid vertex by lower $\mathcal{A}$-adjustment operation and obtain the new vertex $\pi^{\mathcal{A}}(\mathbf{z}^{(1)}) = \lfloor \Phi(\mathbf{z}^{(1)}) \rfloor_{\mathcal{A}}$. Before constructing a smaller polyblock, D new vertices are generated as $\mathcal{V}_{\text{new}}^{(1)} = \{\mathbf{v}_1^{(1)}, \ldots, \mathbf{v}_D^{(1)}\}$. The new vertex $\mathbf{v}_d^{(1)}$ is obtained as $\mathbf{v}_d^{(1)} = \mathbf{z}^{(1)} - \left(z_d^{(1)} - \pi_d^{\mathcal{A}}(\mathbf{z}^{(1)})\right)\mathbf{e}^d$, where $\pi_d^{\mathcal{A}}(\mathbf{z}^{(1)})$ is the d-th entry of $\pi^{\mathcal{A}}(\mathbf{z}^{(1)})$. To obtain the new vertex set $\mathcal{V}^{(2)}$, the vertices whose lower $\mathcal{A}$-adjustment have an objective value less than the current best value ($\text{CBV}^{(1)}$) are removed from $\mathcal{V}_{\text{new}}^{(1)}$. New polyblock $\mathcal{P}^{(2)}$ is constructed by the new vertex set. This polyblock still contains the feasible set of problem (8). From $\mathcal{V}^{(2)}$, we choose the vertex whose lower $\mathcal{A}$-adjustment has the maximum objective value as the vertex whose projection is obtained in the next iteration, i.e., $\mathbf{z}^{(2)} = \arg\max\{T\left(\pi^{\mathcal{A}}(\mathbf{z})\right) \mid \mathbf{z} \in \mathcal{V}^{(2)}\}$. We continue this procedure until the new vertex set generated in the current iteration is empty, i.e., $\mathcal{V}^{(i+1)} = \emptyset$. In other words, there is no vertex whose lower $\mathcal{A}$-adjustment has an objective value greater than the current best value. Note that in each iteration, $\text{CBV}^{(i)}$ and the current best solution ($\text{CBS}^{(i)}$) are updated if the objective value of $\pi^{\mathcal{A}}(\mathbf{z}^{(i)})$ is greater than $\text{CBV}^{(i-1)}$. The outer polyblock approximation algorithm for solving problem (8) is summarized in Algorithm 1. In each iteration of the algorithm, the projection of $\mathbf{z}^{(i)}$ onto the boundary of the set $\mathcal{Z}$ is calculated as $\Phi(\mathbf{z}^{(i)}) = \lambda \mathbf{z}^{(i)}$, where

$$\begin{aligned}
\lambda &= \max\{\alpha \mid \alpha\mathbf{z}^{(i)} \in \mathcal{Z}\} \\
&= \max\left\{\alpha \mid \alpha z_d^{(i)} \leq \frac{a_d(\tilde{\boldsymbol{\eta}})}{b_d(\tilde{\boldsymbol{\eta}})}, \mathbf{0} \preceq \tilde{\boldsymbol{\eta}} \preceq \mathbf{1}\right\} \\
&= \max_{\mathbf{0} \preceq \tilde{\boldsymbol{\eta}} \preceq \mathbf{1}} \min_{1 \leq d \leq D} \frac{a_d(\tilde{\boldsymbol{\eta}})}{z_d^{(i)} b_d(\tilde{\boldsymbol{\eta}})}. \qquad (11)
\end{aligned}$$

Problem (11) is a fractional programming problem. It can be solved by the Dinkelbach algorithm [10]. The projection algorithm is summarized in Algorithm 2. When the optimal solution ($\mathbf{z}^*$) is obtained by the outer polyblock approximation algorithm, we have the optimal reflection coefficients from the projection algorithm. We also obtain the values $u_{k,l}^m$ and $v_{k,l}^m$ from the optimal solution $\mathbf{z}^*$. Thus, we can determine the optimal subcarrier allocation coefficients as follows:

$$s_{k,l}^m = \begin{cases} 1, & u_{k,l}^m > 1 \text{ and } v_{k,l}^m > 1, \\ 0, & \text{otherwise.} \end{cases} \qquad (12)$$

The proposed algorithm achieves the global optimal solution of problem (8). However, this algorithm is computationally complex and its complexity relies on the computation complexity of the projection algorithm [9]. The complexity of this algorithm increases exponentially with the number of backscatter devices and the number of subcarriers. In the next subsection, we propose a suboptimal scheme to obtain a local suboptimal solution for problem (8) in polynomial time.

C. Suboptimal Scheme

In this subsection, we transform problem (4) to a convex optimization problem by using difference of convex functions and successive convex approximation. Since problems (4) and (7) are equivalent, we consider problem (7) to design an algorithm to obtain a suboptimal solution. We note that the variable $\tilde{\eta}_{k,l,k}^m = s_{k,l}^m \eta_k$ is the product of two variables. To make the product terms decomposable, we introduce additional constraints to our problem as follows [3]:

$$\begin{aligned}
&\tilde{\eta}_{k,l,k}^m \leq s_{k,l}^m,\ k,l \in \mathcal{K}, m \in \mathcal{M} && \text{(13a)} \\
&\tilde{\eta}_{k,l,k}^m \leq \eta_k,\ k,l \in \mathcal{K}, m \in \mathcal{M} && \text{(13b)} \\
&\tilde{\eta}_{k,l,k}^m \geq \eta_k - (1 - s_{k,l}),\ k,l \in \mathcal{K}, m \in \mathcal{M} && \text{(13c)} \\
&\tilde{\eta}_{k,l,k}^m \geq 0,\ k,l \in \mathcal{K}, m \in \mathcal{M}. && \text{(13d)}
\end{aligned}$$

Algorithm 1: Outer Polyblock Approximation Algorithm

```
Initialize polyblock P^(1) with the vertex set V^(1) := {z^(1)} and
    set the entries of z^(1) as
    u^{m(1)}_{k,l} := 1 + g^m_k, v^{m(1)}_{k,l} := 1 + g^m_l, k,l ∈ K, m ∈ M
Set the iteration index i := 0, CBV^(0) := −∞ and CBS^(0) := 0
while V^(i+1) ≠ ∅ do
    i := i + 1
    Calculate the projection of z^(i) onto the boundary of set Z,
        Φ(z^(i)), by Algorithm 2
    Obtain the lower A-adjustment of Φ(z^(i)) as
        π^A(z^(i)) := ⌊Φ(z^(i))⌋_A
    if T(π^A(z^(i))) ≥ CBV^(i−1) then
        CBS^(i) := π^A(z^(i))
        CBV^(i) := T(π^A(z^(i)))
    else
        CBS^(i) := CBS^(i−1)
        CBV^(i) := CBV^(i−1)
    end
    Obtain D new vertices as V^(i)_new := {v^(i)_1, ..., v^(i)_D}, where
        v^(i)_d := z^(i) − (z^(i)_d − π^A_d(z^(i))) e^d
    Generate V^(i+1) := {v | v ∈ V^(i)_new, T(π^A(v)) > CBV^(i)}
        and construct new polyblock
    From V^(i+1), select
        z^(i+1) := arg max{T(π^A(z)) | z ∈ V^(i+1)}
end
z* := CBS^(i)
```

Algorithm 2: Projection Algorithm

```
Input: z^(i), set Z
Output: Φ(z^(i)), η̃*
Initialize λ_1 := 0
Set iteration index n := 1 and error tolerance δ ≪ 1
while True do
    η̃*_n := arg max_{0 ⪯ η̃ ⪯ 1} { min_{1≤d≤D} {a_d(η̃*) − λ_n z^(i)_d b_d(η̃*)} }
    λ_{n+1} := min_{1≤d≤D} a_d(η̃*_n) / (z^(i)_d b_d(η̃*_n))
    n := n + 1
    if min_{1≤d≤D} (a_d(η̃*_{n−1}) − λ_n z^(i)_d b_d(η̃*_{n−1})) ≤ δ then
        terminate the while loop
    else
        continue
    end
end
The projection of z^(i) onto the boundary of set Z is
    Φ(z^(i)) = λ_n z^(i) and η̃* = ⌊η̃*_{n−1}⌋_H
```

Besides, constraint (4c) is a nonconvex constraint. We rewrite this constraint as follows [3]:

$$0 \leq s_{k,l}^m \leq 1,\ k,l \in \mathcal{K}, m \in \mathcal{M} \tag{14a}$$

$$\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K} s_{k,l}^m - \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K}(s_{k,l}^m)^2 \leq 0. \tag{14b}$$

Constraint (14b) is the difference of two convex functions and is nonconvex. Constraints (7d) and (7e) are also nonconvex. To handle these constraints, we add these constraints as penalty terms to the objective function in the following form.

$$\begin{aligned}\underset{\tilde{\boldsymbol{\eta}},\mathbf{s}}{\text{minimize}}\ & \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K}\left(-\log_2(u_{k,l}^m) - \log_2(v_{k,l}^m)\right) \\ & +\beta_1\left(\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K} s_{k,l}^m - \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K}(s_{k,l}^m)^2\right) \\ & -\beta_2\left(\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K}\log_2\left(1+\frac{\tilde{\eta}_{k,l,k}^m g_k^m}{1+\tilde{\eta}_{k,l,l}^m g_l^m}\right)-s_{k,l}^m R_{\text{min}}\right) \\ & -\beta_3\left(\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K}\log_2\left(1+\tilde{\eta}_{k,l,l}^m g_l^m\right)-s_{k,l}^m R_{\text{min}}\right) \\ \text{subject to}\ & \text{(4d), (4e), (7b), (7c), (13a)}-\text{(13d), (14a),}\end{aligned} \tag{15}$$

where $\beta_1 \gg 1, \beta_2 \gg 1$ and $\beta_3 \gg 1$ are the penalty factors. In problem (15), the term multiplied by β_1 penalizes the objective function for any $s_{k,l}^m$ that is not equal to 0 or 1. The terms multiplied by β_2 and β_3 penalize the objective function for any backscatter device that has a data rate less than R_{min}. We introduce additional functions and rewrite problem (15) in the following form.

$$\begin{aligned}\underset{\tilde{\boldsymbol{\eta}},\mathbf{s}}{\text{minimize}}\ & F(\tilde{\boldsymbol{\eta}}) + \beta_1(G(\mathbf{s}) - Q(\mathbf{s})) \\ & + \beta_2\left(J(\tilde{\boldsymbol{\eta}},\mathbf{s}) - U(\tilde{\boldsymbol{\eta}})\right) + \beta_3 L(\tilde{\boldsymbol{\eta}},\mathbf{s}) \\ \text{subject to}\ & \text{(4d), (4e), (7b), (7c), (13a)}-\text{(13d), (14a),}\end{aligned} \tag{16}$$

where

$$F(\tilde{\boldsymbol{\eta}}) = \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K} -\log_2(1+\tilde{\eta}_{k,l,k}^m g_k^m + \tilde{\eta}_{k,l,l}^m g_l^m) \tag{17}$$

$$G(\mathbf{s}) = \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K} s_{k,l}^m \tag{18}$$

$$Q(\mathbf{s}) = \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K}(s_{k,l}^m)^2 \tag{19}$$

$$J(\tilde{\boldsymbol{\eta}},\mathbf{s}) = \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K} -\log_2\left(1+\tilde{\eta}_{k,l,k}^m g_k^m + \tilde{\eta}_{k,l,l}^m g_l^m\right) + s_{k,l}^m R_{\text{min}} \tag{20}$$

$$U(\tilde{\boldsymbol{\eta}}) = \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K} -\log_2\left(1+\tilde{\eta}_{k,l,l}^m g_l^m\right) \tag{21}$$

$$L(\tilde{\boldsymbol{\eta}},\mathbf{s}) = \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K} -\log_2\left(1+\tilde{\eta}_{k,l,l}^m g_l^m\right) + s_{k,l}^m R_{\text{min}}. \tag{22}$$

All of the above functions are convex and problem (16) is considered as difference of convex functions programming. Thus, we can use successive convex approximation to obtain a local optimal solution. Functions $-Q(\mathbf{s})$ and $-U(\tilde{\boldsymbol{\eta}})$ in the objective function in (16) make the problem nonconvex. These two functions are differentiable. Thus, we can approximate them by an affine function using first-order condition for convex functions. Consider j as an iteration index starting from one. For any given $\mathbf{s}^{(j)}$ and $\tilde{\boldsymbol{\eta}}^{(j)}$, we have

$$Q(\mathbf{s}) \geq Q(\mathbf{s}^{(j)}) + \nabla_{\mathbf{s}} Q(\mathbf{s}^{(j)})^T(\mathbf{s}-\mathbf{s}^{(j)}) \tag{23}$$

$$U(\tilde{\boldsymbol{\eta}}) \geq U(\tilde{\boldsymbol{\eta}}^{(j)}) + \nabla_{\tilde{\boldsymbol{\eta}}} U(\tilde{\boldsymbol{\eta}}^{(j)})^T(\tilde{\boldsymbol{\eta}}-\tilde{\boldsymbol{\eta}}^{(j)}), \tag{24}$$

where

$$\nabla_{\mathbf{s}} Q(\mathbf{s}^{(j)})^T(\mathbf{s}-\mathbf{s}^{(j)}) = \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K} 2s_{k,l}^{m(j)}(s_{k,l}^m - s_{k,l}^{m(j)}) \tag{25}$$

$$\nabla_{\tilde{\boldsymbol{\eta}}} U(\tilde{\boldsymbol{\eta}}^{(j)})^T(\tilde{\boldsymbol{\eta}}-\tilde{\boldsymbol{\eta}}^{(j)}) = \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{l=1}^{K} -\frac{g_l^m(\tilde{\eta}_{k,l,l}^m - \tilde{\eta}_{k,l,l}^{m(j)})}{(1+\tilde{\eta}_{k,l,l}^{m(j)} g_l^m)\ln 2}. \tag{26}$$

We replace $Q(\mathbf{s})$ and $U(\tilde{\boldsymbol{\eta}})$ in (16) by the right hand side of (23) and (24) and obtain the following problem.

$$\begin{aligned}\underset{\tilde{\boldsymbol{\eta}},\mathbf{s}}{\text{minimize}}\ & F(\tilde{\boldsymbol{\eta}}) + \beta_1(G(\mathbf{s}) - Q(\mathbf{s}^{(j)}) - \nabla_{\mathbf{s}} Q(\mathbf{s}^{(j)})^T(\mathbf{s}-\mathbf{s}^{(j)})) \\ & + \beta_2(J(\tilde{\boldsymbol{\eta}},\mathbf{s}) - U(\tilde{\boldsymbol{\eta}}^{(j)}) - \nabla_{\tilde{\boldsymbol{\eta}}} U(\tilde{\boldsymbol{\eta}}^{(j)})^T(\tilde{\boldsymbol{\eta}}-\tilde{\boldsymbol{\eta}}^{(j)})) \\ & + \beta_3 L(\tilde{\boldsymbol{\eta}},\mathbf{s}) \\ \text{subject to}\ & \text{(4d), (4e), (7b), (7c), (13a)}-\text{(13d), (14a).}\end{aligned} \tag{27}$$

Problem (27) is a convex problem. Solving this problem for any given $\mathbf{s}^{(j)}$ and $\tilde{\boldsymbol{\eta}}^{(j)}$ gives an upper bound for the solution of problem (16). By using successive convex approximation, we can tighten the upper bound. The iterative algorithm to obtain a local optimal solution for problem (27) is summarized in Algorithm 3. This algorithm finds a tightened upper bound for the solution of problem (16) within polynomial time.

Algorithm 3: Successive Convex Approximation Algorithm

1. Initialize penalty factors $\beta_1 \gg 1, \beta_2 \gg 1$ and $\beta_3 \gg 1$, maximum number of iterations I_{max}, iteration index $j := 1$ and starting points $\mathbf{s}^{(1)}$ and $\tilde{\boldsymbol{\eta}}^{(1)}$
2. **while** $j \leq I_{\text{max}}$ **do**
3. Solve problem (27) for given $\mathbf{s}^{(j)}$ and $\tilde{\boldsymbol{\eta}}^{(j)}$ and store the subcarrier allocation and reflection coeffients as $\mathbf{s}$ and $\tilde{\boldsymbol{\eta}}$
4. Set $j := j+1$, $\mathbf{s}^{(j)} := \mathbf{s}$ and $\tilde{\boldsymbol{\eta}}^{(j)} := \tilde{\boldsymbol{\eta}}$
5. **end**
6. $\mathbf{s}^* := \mathbf{s}^{(j)}$ and $\tilde{\boldsymbol{\eta}}^* := \tilde{\boldsymbol{\eta}}^{(j)}$

IV. Performance Evaluation

In this section, the performance of the proposed schemes is evaluated through simulations. The coverage area of the reader is considered as a cell with two ring-shaped boundary. The radii of inner and outer boundary are 20 m and 100 m, respectively. The backscatter devices are distributed randomly and uniformly in the coverage area of the reader. The path loss exponent α is equal to 2.5. The variance of the noise σ^2 is set to -100 dBm. We set the number of subcarriers and the minimum data rate requirement of backscatter devices to

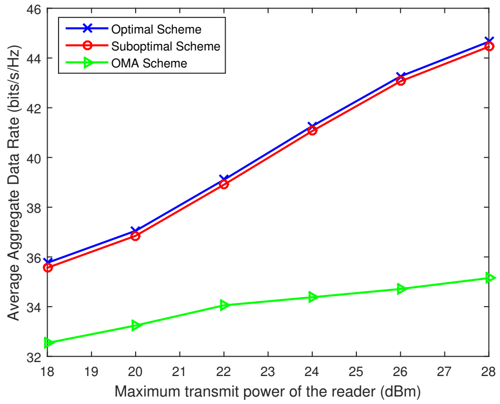

Fig. 2. Average aggregate data rate versus the maximum transmit power of the reader for $K = 6$.

$M = 3$ and $R_{\min} = 3$ bits/s/Hz, respectively. The results are obtained by averaging over different channel and path loss realizations.

Fig. 2 shows the average aggregate data rate versus the maximum transmit power of the reader for $K = 6$ backscatter devices. Results show that the average aggregate data rate increases monotonically with the transmit power of the reader. This is because the received signal-to-interference-plus-noise ratio (SINR) of the backscatter devices is improved when the maximum transmit power of the reader increases. The suboptimal algorithm achieves an average aggregate data rate very close to the optimal scheme. Fig. 2 also shows the average aggregate data rate of a backscatter system using OMA where at most one backscatter device operates on each subcarrier. In this system, the optimal reflection coefficient of a backscatter device operating on a subcarrier has the value of one. We optimize the subcarrier allocation to the backscatter devices. The average aggregate data rate of backscatter system with MC-NOMA is higher than that of backscatter system using OMA. In a backscatter system using OMA, the spectrum resource is not utilized efficiently due to orthogonal subcarrier allocation. As Fig. 2 shows, backscatter system with MC-NOMA achieves a target aggregate data rate with lower maximum transmit power of the reader compared to the backscatter system using OMA.

Fig. 3 shows the average aggregate data rate versus the number of backscatter devices for our proposed optimal and suboptimal schemes as well as the OMA scheme. The average aggregate data rate increases with the number of backscatter devices. The MC-NOMA backscatter system achieves a higher average aggregate data rate than a backscatter system using OMA. This is because in an MC-NOMA backscatter system, both frequency and power domain are exploited for multiple access. Fig. 3 shows that the average aggregate data rate of an MC-NOMA backscatter system increases faster than that of an OMA backscatter system. This is because in an MC-NOMA backscatter system, signals from multiple backscatter devices are multiplexed on each subcarrier. Furthermore, the average aggregate data rate obtained by the suboptimal scheme is very close to the optimal scheme.

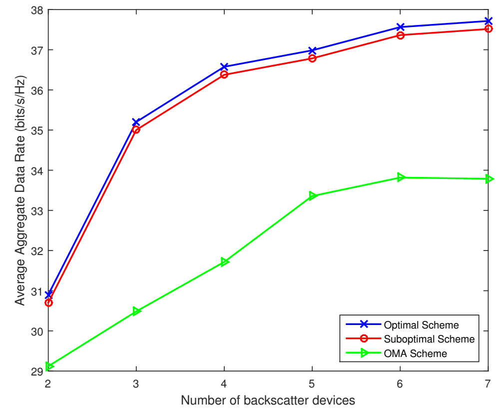

Fig. 3. Average aggregate data rate versus the number of backscatter devices for $P = 22$ dBm.

V. Conclusion

In this paper, we investigated an MC-NOMA backscatter system. We formulated an aggregate data rate maximization problem by jointly optimizing the reflection coefficients and subcarrier allocation. The formulated problem is nonconvex and exhibits hidden monotonicity structure. By using discrete monotonic optimization, we developed an optimal scheme. We also proposed a suboptimal scheme, which is based on successive convex approximation and has polynomial time complexity. Simulation results show that the proposed suboptimal scheme achieves an aggregate data rate close to the optimal scheme. Our proposed schemes have a higher aggregate data rate than the OMA scheme.

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