Game Theoretic Cross Layer Transmission Policies in Multipacket Reception Wireless Networks

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Abstract— We study the structure of the optimal transmission policies for non-cooperating nodes in a finite-size random access wireless network, where the medium access control (MAC) protocol is a variant of the time slotted ALOHA protocol. It is assumed that the network has the multipacket reception capability and every node knows its channel state information (CSI), which is continuously distributed, perfectly at the beginning of each transmission time slot. The objective of each node in the network is to find a transmission policy mapping CSI to transmission probabilities to maximize its individual utility. The problem is formulated as a non-cooperative game of a finite number of rational players and actions with a continuous channel state space. We prove that if the probability of success of a node is a non-decreasing function of its CSI, there exists a threshold transmission policy that maximizes its utility. It is then showed that there exists a Nash equilibrium at which every node adopts a threshold policy. The optimality of threshold policies strongly simplifies the problem of optimizing the transmission policy for a node. We propose a stochastic gradient based algorithm that exhibits the best response dynamic adjustment process for the transmission game. The theoretical results of the paper as well as the performance of the proposed algorithm are illustrated via numerical examples.

Index Terms— Multipacket reception, channel state information, threshold transmission policy, Nash equilibrium, best response dynamics.

I. INTRODUCTION

Classical random access protocols including ALOHA are designed based on the idealized collision channel model: if only one node transmits, its packet is received correctly with certainty but if more than one node transmit at the same time, all packets are lost due to collision. However, the collision channel model does not hold in many important practical communication systems. For example, CDMA systems or systems with multiple antennas at the base station allow one or more packets to be received correctly in the presence of simultaneous transmissions [1].

In [2], the Multipacket Reception (MPR) model was proposed. The MPR model allows modelling systems where one or more packets can be received correctly with fixed probabilities when multiple nodes transmit simultaneously. A limitation of the MPR model is that channel states do not affect the reception of packets directly and all nodes are indistinguishable. In [3], [4] and [5] it is shown that using the MPR model, a non-zero asymptotic system throughput can be obtained. In addition, a decentralized transmission control algorithm that achieves the best asymptotic system throughput was proposed in [3]. A generalization of MPR to the asymmetrical model is given in [6].
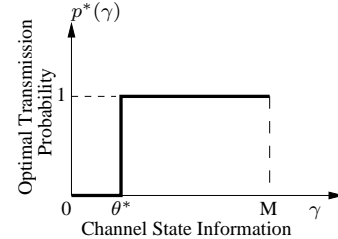

Fig. 1. Threshold structure of the optimal transmission policy proved in this paper: $p^*(\gamma) = \mathbf{I}(\gamma > \theta^*)$. This optimal threshold θ^* can be estimated using (34).

The focus of [7], [8], [9] is on time slotted ALOHA systems with selfish nodes that are allowed to select their own transmission policies, which map the numbers of nodes contending the channel to transmission probabilities, to maximize their individual utilities using a game theoretic approach. The existence of a symmetric Nash equilibrium is proved for the collision channel model in [7], [10] and the MPR reception model in [9].

In [11], the Generalized Multipacket Reception (G-MPR) model was proposed. In the G-MPR model, the probability of receiving a packet correctly depends on the channel states of the transmitting nodes. Hence, the G-MPR model includes the MPR model of [3] as a special case. The G-MPR model provides a framework for exploiting *channel state information* (CSI) for optimal power or transmission control. In other words, the G-MPR model is a reception model that is suitable for exploiting information from the physical (PHY) layer for Medium Access Control (MAC) layer protocol design.

Early work on exploiting CSI includes [12], [13] and [14], which considered exploiting multi-user diversity in the collision channel model via a variant of the ALOHA protocol namely the *channel-aware* ALOHA protocol. Using the G-MPR model, [11] proposed a variant of the ALOHA protocol where transmission probability is allowed to be a function of CSI. This is the MAC protocol we consider in this paper. In [11], the problem of optimal transmission control for the spatially homogeneous slotted ALOHA network where all nodes deploy the same transmit probability function is formulated. The structure of the optimal transmission policies for spatially heterogeneous and homogeneous slotted ALOHA networks is studied in [15].

In this paper, the problem of optimal decentralized transmission control is formulated as a non-cooperative transmission game and the structure of the optimal transmission policy is studied. The main difference between the work of this paper and early work on the application of game theory to ALOHA networks [7], [8], [10], [9] is the exploitation of CSI via the G-

MPR model and the relaxation of the assumption that all nodes are symmetric. In comparison, the main difference between this paper and other work on exploiting CSI for optimal transmission control [11], [15] is the formulation of the problem as a non-cooperative game as well as the introduction of the transmission and waiting costs. The transmission and waiting costs are necessary for the formulation of the game but they also offer a means to take into account factors such as battery constraints and performance requirements. The waiting cost can be adjusted to enhance long-term fairness in the network and can be increased to reduce transmission delays.

The main results of this paper include:

1) We prove that if the probability of correct reception of a packet from a node given the transmission policies of other nodes is a non-decreasing Lebesgue measurable function of its CSI, there exists a *threshold transmission policy* that maximizes the expected reward of the node. See (1) for the definition of a transmission policy and see Fig. 1 for the structure of the optimal policy.

2) Assuming the nodes select their transmission policies in the set of Lebesgue measurable functions, we proved that the non-cooperative transmission game has at least a Nash equilibrium at which every player (node) deploys a threshold transmission policy. See Fig. 1.

3) The symmetric network model where the nodes are equi-distant from the base station and have the same transmission and waiting costs is also considered. For this special case we prove that under verifiable, mild conditions, there exists a symmetric Nash equilibrium profile at which all nodes deploy the same threshold transmission policy. In addition, the symmetric Nash equilibrium transmission threshold is a non-decreasing function of the number of active nodes.

4) We study the best response dynamics algorithm and prove its convergence for two-node transmission games. We explicitly characterize the best response functions for the general multi-node transmission game, where channel state is exponentially distributed. This characterization allows us to verify *deterministically* (but *numerically*) local asymptotic stability of a Nash equilibrium.

5) At each iteration of the best response dynamics algorithm, a node has to solve the stochastic optimization problem (see (23)) for its best response transmission policy. We propose an algorithm that converges to the best response dynamic adjustment process for the transmission game, where each player updates its policy while keeping fixed the strategies of other players. The core of this algorithm is a stochastic gradient algorithm with a constant step size (see (34)), which can be deployed by any node to adaptively estimate its best response transmission policy without knowing the policies, or the channel distribution functions of other nodes.

The paper is organized as follows: Section II describes the wireless network model, Section III is the formulation of the decentralized transmission control problem as a non-cooperative transmission game. Section IV presents the theoretical results on the structure of the optimal transmission

policies as well as the existence of a Nash equilibrium. In Section V, the existence of a symmetric Nash equilibrium for the symmetric game is proved. An algorithm for estimating the optimal transmission policy and Nash equilibrium is proposed in Section VI. Section VII contains numerical examples.

II. THE MULTIPACKET RECEPTION NETWORK MODEL

In this section we define the wireless network and the reception model that are considered in the paper. Section II-A describes the wireless random access network model with the MAC protocol being a variant of the time slotted ALOHA protocol, where instead of fixed transmission probabilities, each node in the network has a transmission policy mapping its CSI to transmission probabilities (see (1) for the definition of a transmission policy). Our network model is similar to the model considered in [11]. The G-MPR model, which was proposed in [11], is mathematically described in the Section II-B.

A. The Network Model

Consider a time slotted wireless network of K nodes (e.g., sensors in a wireless sensor network), where K is a finite positive integer. Transmission is synchronized at the beginning of each time slot. We consider the uplink communication channel where the nodes communicate with a common base station. Let $i = 1, 2, \ldots, K$ index the nodes in the network and the random variable γ_i denote the channel state of node i. Assume that $\gamma_i \in [0, M]$, for some finite $M \in \mathbb{R}_+$, which is the set of non-negative reals. M can be arbitrarily large so that $[0, M]$ includes the entire range of CSI that is of practical interest. Denote the probability distribution function of γ_i by $F_i(\cdot)$. Assume that $F_i(\cdot)$ is continuous for all $i = 1, 2, \ldots, K$. Furthermore, it is assumed that at the beginning of each transmission time slot, node i knows its instantaneous CSI, γ_i, perfectly. Any parameter that influences the reception of packets can be chosen as channel state, for example, channel gain or position of a node with respect to the base station can represent channel state. If channel gain represents channel state, a node can estimate its individual CSI by measure the strength of a beacon signal, which is broadcasted by the base station to all nodes.

In the network, a node that does not have any packet to transmit is referred to as an inactive node. In contrast, a node with at least one packet to transmit is referred to as an active node. At each time slot, inactive nodes perform no action while active nodes must either transmit or wait, i.e. not transmit. The probability with which an active node transmits is determined by its instantaneous CSI and its transmission policy, which is a function that maps CSI to transmit probabilities.

The reception model of the system, which is the G-MPR model, is described in the next subsection. The key difference between the G-MPR model and the conventional collision model is the fact that in the G-MPR model, the reception of packets depends on the current channel states (e.g., Signal to Noise Ratios, distances from the base station) of all transmitting nodes. This is usually the case in CDMA wireless networks. In some cases, it is also possible to abstract the

reception of an uplink, where the base station uses multiple antennas, into the G-MPR model [11]. In the G-MPR model, the concept of collision is redundant, even though it includes the collision channel model as a special case.

The objective of each node in the network is to find a transmission policy mapping channel states to transmit probabilities to maximize its individual utility. In other words, each node has to solve a function optimization problem, i.e. an infinite dimensional optimization problem. A function optimization problem needs to be defined over a function space. In this paper we consider transmission policies that are Lebesgue measurable and the definition of a transmission policy is given below.

Consider the normed linear function space $L_\infty[0, M]$, which is the space of all Lebesgue measurable functions defined on $[0, M]$ that are bounded almost everywhere (a.e.). The norm of a function in $L_\infty[0, M]$ is its essential supremum [16]. Let $B_{L_\infty[0,M]}[0, 1]$ be the set of all functions in $L_\infty[0, M]$ that have norms in the range $[0, 1]$. Define a transmission policy to be a function mapping channel states of a node to its transmission probabilities:

$$p_i(.) : [0, M] \to [0, 1] \tag{1}$$

for $i \in \{1, \ldots, K\}$. A *transmission policy* is sometimes referred to as a *transmit probability function*. We only consider transmission policies that are in $B_{L_\infty[0,M]}[0, 1]$. We now define pure, randomized and threshold transmission policies.

Definition 1: A *pure* transmission policy is a transmit probability function $p(.) : [0, M] \to [0, 1]$ such that $p(\gamma) \in \{0, 1\}$ for all $\gamma \in [0, M]$ except for possibly a zero measure set (with respect to the probability measure $F(\cdot)$ of the channel state γ) of values of γ.

Definition 2: A *randomized* transmission policy is a transmission policy that is not pure. Equivalently, a *randomized* policy is a transmit probability function $p(.) : [0, M] \to [0, 1]$ such that $0 < p(\gamma) < 1$ for some non-zero measure set (with respect to the probability measure $F(\cdot)$ of the channel state) of values of γ.

Definition 3: A *threshold* transmission policy is a transmission policy $p(\cdot) : [0, M] \to [0, 1]$ such that

$$p(\gamma) \stackrel{\text{a.e.}}{=} \begin{cases} 0 & \gamma < \theta \\ 1 & \text{Otherwise} \end{cases} \tag{2}$$

for some $\theta \in \mathbb{R}_+$, $0 < \theta \le M$.

Notation: We now define the notation that is used in the paper.

A superscript or a subscript i indicates that the node being referred to is node i. In comparison $-i$ is used to refer to the set of nodes indexed by $\{1, 2, \ldots, K\} - \{i\}$. This notation is standard in game theory [17], [18].

A_k^K is any unordered set of k integers selected from $1, 2, \ldots, K$, $A_k^K \subseteq \{1, 2, \ldots, K\}$. In the paper, A_k^K is used to specify the set of all transmitting nodes.

$\vec{\gamma}_{A_k^K} = (\gamma_i : i \in A_k^K)$ is a vector representing channel states of the group of nodes indexed by A_k^K.

The expected reward (or utility) of node i is denoted by $T_i(p_i(\cdot), \{p_{-i}(\cdot)\})$ where $p_i(\cdot)$ is the policy played by node i and $\{p_{-i}(\cdot)\}$ denotes the set of transmission policies of all other nodes. Throughout the paper, $\mathbf{I}(\cdot)$ is the indicator

function and $\mathbb{E}_F[\cdot]$ represents the expected value of a random variable with respect to some distribution $F(\cdot)$.

B. The Generalized Multi-Packet Reception Model

The G-MPR model, proposed in [11], provides explicit incorporation of CSI into the reception of packets. It is also the reception model considered in [19], [15], [20].

In the G-MPR model, the outcome of a transmission time slot where k nodes indexed by A_k^K transmit belongs to an event space where each elementary event is represented by a binary k-tuple $\Theta_{A_k^K} = (\vartheta_i : i \in A_k^K)$, where $\vartheta_i = \{0, 1\}$ for each $i \in A_k^K$. $\vartheta_i = 1$ indicates that the packet sent by node i is correctly received and $\vartheta_i = 0$ indicates otherwise.

The reception capability of the system is described by a set of K functions, where the k-th function $\Phi(\vec{\gamma}_{A_k^K}; \Theta_{A_k^K})$ assigns a probability to the outcome $\Theta_{A_k^K}$ when k nodes indexed by A_k^K with channel state $\vec{\gamma}_{A_k^K}$ transmit:

$$\Phi(\vec{\gamma}_{A_k^K}; \Theta_{A_k^K}) = \mathbb{P}(\Theta_{A_k^K}|k \text{ nodes transmit}, \vec{\gamma}_{A_k^K}) \tag{3}$$

Equation (3) means that the distribution of the possible outcomes $\{\Theta_{A_k^K}\}$ is determined by the channel states of the transmitting nodes. Consider the CDMA time slotted system with matched filter receivers and the Signal to Interference Noise Ratio (SINR) threshold reception model as an example. Assuming Signal to Noise Ratio (SNR) represents the channel state, the k-th function $\Phi(\vec{\gamma}_{A_k^K}; \Theta_{A_k^K})$ is given by:

$$\Phi(\vec{\gamma}_{A_k^K}, \Theta_{A_k^K}) = \begin{cases} 1 & \text{if } \Theta_{A_k^K} = \overline{\Theta} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where $\overline{\Theta} = (\overline{\vartheta_i} : i \in A_k^K)$ and

$$\overline{\vartheta_i} = \mathbf{I}(\frac{\gamma_i}{1 + \frac{\sum_{j \neq i, j \in A_k^K} \gamma_j}{N}} > \beta),$$

where γ_j is the SNR of node j, N is the spreading gain and β is the quality of service requirement (QoS) parameter. The derivation of the SINR threshold reception model for CDMA systems with linear multiuser detectors is given in [21].

It is assumed that the reception model (3) is symmetric. Mathematically, this can be expressed as:

$$\Phi(\vec{\gamma}_{A_k^K}; \Theta_{A_k^K}) = \Phi(P_k(\vec{\gamma}_{A_k^K}), P_k(\Theta_{A_k^K})) \tag{5}$$

for any permutation P_k of a k-element vector. This symmetric property is satisfied by the SINR threshold reception model (4) as well as most non-trivial system models. It is also an assumption in [11], [19].

In the network, the nodes do not cooperate and each node is only concerned about its individual utility. Therefore, given the reception model (3), the only bit of information that can be used by node i in the process of estimating its optimal transmission policy is the probability of correct reception of its packet during a time slot, which is determined by the following set of functions:

$$\psi_i(\gamma_i, \vec{\gamma}_{A_k^K - i}) = \mathbb{E}_\Phi[\vartheta_i|k \text{ nodes transmit}, \gamma_i, \vec{\gamma}_{A_k^K - i}] \tag{6}$$

for some $A_k^K \in \Omega_K$ and $i \in A_k^K$. For the CDMA time slotted system with matched filter receivers and the SINR threshold reception model, (6) can be rewritten as:

$$\psi_i(\gamma_i, \vec{\gamma}_{A_k^K - i}) = \mathbf{I}\left(\frac{\gamma_i}{1 + \frac{\sum_{j \neq i, j \in A_k^K} \gamma_j}{N}} > \beta\right). \tag{7}$$

Throughout the paper it is assumed that $\psi_i(\gamma_i, \vec{\gamma}_{A_k^K - i})$ defined by (6) is a Lebesgue measurable function of γ_i. Besides Lebesgue measurability, the assumptions listed below are used throughout the paper. These assumptions are satisfied by most non-trivial systems, e.g., the SINR threshold reception model (7).

1) The probability of success of node i defined by (6) is a non-decreasing in its channel state γ_i:

$$\psi_i(\gamma_i, \vec{\gamma}_{A_k^K - i}) \geq \psi_i(\overline{\gamma}_i, \vec{\gamma}_{A_k^K - i}) \ \ \forall \gamma_i > \overline{\gamma}_i$$
$$\Leftrightarrow \mathbb{E}[\vartheta_i | k \text{ nodes transmit}, \gamma_i, \vec{\gamma}_{A_k^K - i}]$$
$$\geq \mathbb{E}[\vartheta_i | k \text{ nodes transmit}, \overline{\gamma}_i, \vec{\gamma}_{A_k^K - i}] \ \ \forall \gamma_i > \overline{\gamma}_i. \tag{8}$$

Some of the analytical results in this paper can be strengthened if the inequality in (8) is strict, i.e.

$$\mathbb{E}[\vartheta_i | k \text{ nodes transmit}, \gamma_i, \vec{\gamma}_{A_k^K - i}]$$
$$> \mathbb{E}[\vartheta_i | k \text{ nodes transmit}, \overline{\gamma}_i, \vec{\gamma}_{A_k^K - i}] \ \ \forall \gamma_i > \overline{\gamma}_i. \tag{9}$$

Unless it is stated otherwise, in the paper we assume that (8) holds, but not (9).

2) The success probability of a node is lowered when one more node transmits. This is also a condition in [15].

$$\psi_i(\gamma_i, \vec{\gamma}_{A_k^K - i}) \geq \psi_i(\gamma_i, (\vec{\gamma}_{A_k^K - i}, a)) \ \ \forall a > 0$$
$$\Leftrightarrow \mathbb{E}[\vartheta_i | k \text{ nodes transmit}, \gamma_i, \vec{\gamma}_{A_k^K - i}]$$
$$\geq \mathbb{E}[\vartheta_i | k+1 \text{ nodes transmit}, \gamma_i, (\vec{\gamma}_{A_k^K - i}, a)], \tag{10}$$

for all channel state $a > 0$.

3) When the channel state of a node is 0, its success probability is 0:

$$\psi_i(0, \vec{\gamma}_{A_k^K - i}) = 0 \tag{11}$$

III. FORMULATION OF THE DECENTRALIZED OPTIMAL TRANSMISSION CONTROL PROBLEM AS A NON-COOPERATIVE GAME

In this section we formulate the problem of decentralized optimal transmission control for a random access network of a fixed number of active nodes as a non-cooperative transmission game, where each node is selfish and rational. In Section III-A, we define the non-cooperative game. The utility function and the optimization problem that must be solved by each node are derived in Section III-B.

A. Formulation of the non-cooperative optimal transmission game

Formally, the problem of optimal decentralized transmission control for the network model defined in Section II can be formulated as a non-cooperative game with a continuous state space as follows:

- The set of players I is the set of active nodes indexed by $i = 1, 2, \ldots, K$.
- At each time slot node i can choose an action $a_i \in A_i = \{W, T\}$, where W means to wait, T means to transmit. A node can also choose to transmit with some probability.
- A strategy is a transmission policy, defined by (1). Pure and randomized transmission policies are defined in Definitions 1 and 2 respectively. Since the space of pure policies is not finite, the existence of a Nash equilibrium is not straightforward.
- Define a profile to be a set of strategies deployed by all nodes in the network: $\sigma = \{p_1(\cdot), \ldots, p_K(\cdot)\}$
- A mathematical expression for the utility function (expected reward) of a node given the policies deployed by other nodes is derived in Section III-B.

B. Utility Function and the Decentralized Optimization Problem

In a transmission time slot, if node i does not transmit a waiting cost $c_w^{(i)}$ is recorded, if it transmits it has to pay a transmission cost $c_t^{(i)}$. At the end of a transmission time slot, if a packet is received correctly the node receives a reward of 1 unit. The instantaneous reward of node i is then determined as follows:

$$r_i = \begin{cases} 1 - c_t^{(i)} & \text{If } a_i = T, \ \vartheta_i = 1 \\ -c_t^{(i)} & \text{If } a_i = T, \ \vartheta_i = 0 \\ -c_w^{(i)} & \text{If } a_i = W, \text{ i.e. node } i \text{ did not transmit.} \end{cases} \tag{12}$$

The condition that ensures a successful transmission is more preferable than no transmission, and no transmission is more preferable than an unsuccessful transmission is

$$1 > c_t^{(i)} > c_w^{(i)} > 0 \quad \forall \ i = 1, 2, \ldots, K. \tag{13}$$

By the symmetric property of the reception model (6), the expected reward of node i, denoted by $T_i(p_i(\cdot), \{p_{-i}(\cdot)\})$, can be easily derived:

$$T_i(p_i(\cdot), \{p_{-i}(\cdot)\}) = \int_0^M p_i(\gamma_i) \left[\int_0^M \ldots \int_0^M \sum_{k=1}^K \sum_{A_k^K : i \in A_k^K} \right.$$
$$\prod_{l \in A_k^K, l \neq i} p_l(\gamma_l) \prod_{j \notin A_k^K} (1 - p_j(\gamma_j)) \left((1 - c_t^{(i)}) \psi_i(\gamma_i, \vec{\gamma}_{A_k^K - i}) \right.$$
$$\left. - c_t^{(i)}(1 - \psi_i(\gamma_i, \vec{\gamma}_{A_k^K - i})) \right) \prod_{\substack{j=1 \\ j \neq i}}^K dF_j(\gamma_j) \right] - (1 - p_i(\gamma_i)) c_w^{(i)} \ dF_i(\gamma_i)$$
$$= \int_0^M p_i(\gamma_i) \left[\int_0^M \ldots \int_0^M \sum_{k=1}^K \sum_{A_k^K : i \in A_k^K} \prod_{\substack{l \in A_k^K \\ l \neq i}} p_l(\gamma_l) \prod_{j \notin A_k^K} (1 - p_j(\gamma_j)) \right.$$
$$\left. \psi_i(\gamma_i, \vec{\gamma}_{A_k^K - i}) \prod_{\substack{j \neq i \\ j = 1}}^K dF_j(\gamma_j) - c_t^{(i)} + c_w^{(i)} \right] dF_i(\gamma_i) - c_w^{(i)}. \tag{14}$$

Let $\Psi_i(\gamma_i, \{p_{-i}(\cdot)\}) : [0, M] \to [0, 1], \ i = 1, 2, \ldots, K$ be a function mapping channel states of node i to the (average)

probability of receiving its packet correctly given the policies of other nodes. $\Psi_i(\gamma_i, \{p_{-i}(\cdot)\})$ can be calculated from (6) as

$$\Psi_i(\gamma_i, \{p_{-i}(\cdot)\}) = \mathbb{E}_{\{F_{-i}\}}\left[\sum_{k=1}^{K}\sum_{\substack{A_k^K \supset i \\ l \neq i}}\prod_{l \in A_k^K}p_l(\gamma_l)\right.$$

$$\left.\prod_{j \notin A_k^K}(1 - p_j(\gamma_j))\psi_i(\gamma_i, \vec{\gamma}_{A_k^K - \{i\}})\right]. \quad (15)$$

For example, for the SINR threshold reception model (7) for CDMA systems we have

$$\Psi_i(\gamma_i, \{p_{-i}(\cdot)\}) = \sum_{k=1}^{K}\sum_{A_k^K \supset i}\int_0^M \cdots \int_0^M \prod_{j \in A_k^K, j \neq i}p_j(\gamma_j)$$

$$\prod_{l \notin A_k^K}(1 - p_l(\gamma_l))\mathbf{I}\left(\frac{\gamma_i}{1 + \frac{1}{N}\sum\limits_{j \in A_k^K, j \neq i}(\gamma_j)} > \beta\right)\prod_{j \neq i}dF_j(\gamma_j).$$

$$(16)$$

Using the expression (15) for the success probability of node i, (14) can be rewritten as

$$T_i(p_i(\cdot), \{p_{-i}(\cdot)\}) = \int_0^M p_i(\gamma_i)\Big(\Psi_i(\gamma_i, \{p_{-i}(\cdot)\})$$

$$- c_t^{(i)} + c_w^{(i)}\Big)dF_i(\gamma_i) - c_w^{(i)} \quad (17)$$

In the paper, (14) and (17) are used interchangeably as the utility function of node i. The problem of maximizing the utility for node i can be formulated as

$$\sup_{p_i(.) \in B_{L_\infty[0,M]}[0,1]} T_i(p_i(\cdot), \{p_{-i}(\cdot)\}), \quad (18)$$

where $T_i(p_i(\cdot), \{p_{-i}(\cdot)\})$ is given by (14) and $B_{L_\infty[0,M]}[0,1]$ is defined in Section II-A.

In the remaining of the paper, we focus on Nash equilibrium profiles. A Nash equilibrium profile is a profile at which no player can benefit by unilaterally deviating from its current policy [18], [17].

Definition 4: A profile $\sigma^* = \{p_1^*(\cdot), \ldots, p_K^*(\cdot)\}$ is a Nash equilibrium if and only if for all players $i = 1, 2, \ldots, K$ we have $T_i(p_i^*(\cdot), \{p_{-i}^*(\cdot)\}) \geq T_i(p_i(\cdot), \{p_{-i}^*(\cdot)\}) \ \forall p_i(\cdot) \in B_{L_\infty[0,M]}[0,1]$.

From the above definition of a Nash Equilibrium profile it is clear that at a Nash equilibrium point (18) must hold for all nodes. One observation that can be made at this point is that the allowance for different channel state distributions, $F_i(\cdot)$, can lead to unfairness in resource allocation unless the transmission and waiting costs are designed to meet some fairness requirement. The designing of transmission and waiting costs is beyond the scope of our paper. In the next section, we focus on proving the existence of a Nash equilibrium at which every player adopts a threshold transmission policy.

IV. OPTIMALITY OF THRESHOLD POLICIES AND EXISTENCE OF A NASH EQUILIBRIUM

Having formulated the problem of optimal decentralized transmission control as a non-cooperative transmission game in the previous section, in this section we study the structure of the Nash equilibrium transmission policies. We follow a common technique in game theory to prove the existence of a structured Nash equilibrium profile. This technique consists of three steps [1]:

1) Showing that a particular class of policies is optimal (Theorem 1)
2) Proving the existence of a Nash Equilibrium when the policy space is restricted to this class of policies (Theorem 2)
3) Proving the existence of a Nash Equilibrium in the original game by showing that a Nash Equilibrium in the game with the restricted policy space is also a Nash equilibrium in the original game (Corollary 2).

Readers are referred to [18], [17] for examples of the early use of this technique in game theory.

A. Optimality of Threshold Policies

We prove that the utility of a node can always be maximized by a threshold transmission policy.

Theorem 1: Consider a multipacket reception random access network of $K < \infty$ active nodes where the network and reception models are described in Section II. Consider the non-cooperative transmission game formulated in Section III, where the problem of optimizing the utility for node $i = 1, 2, \ldots, K$ is given by (18). Assume the reception model (6) of the network satisfies (8) and (11). There exists a transmit probability function that maximizes node i's expected reward (14) and is a threshold policy:

$$p_i^*(\gamma) \stackrel{a.e}{=} \mathbf{I}(\gamma > \theta) \quad (19)$$

for some $\theta \in [0, M]$.

Proof:

The proof of this theorem is an application of the *bang-bang* principle, presented in [22].

The objective of node i is to maximize its utility, which is given by (17):

$$T_i(p_i(\cdot), \{p_{-i}(\cdot)\}) = \int_0^M p_i(\gamma_i)\Big(\Psi_i(\gamma_i, \{p_{-i}(\cdot)\})$$

$$- c_t^{(i)} + c_w^{(i)}\Big)dF_i(\gamma_i) - c_w^{(i)}$$

It can easily be seen that if $\Psi_i(\cdot, \{p_{-i}(\cdot)\})$, defined by (15), is Lebesgue measurable then

$$p_i^*(\gamma_i) = \begin{cases} 1 & \text{if } \Psi_i(\gamma_i, \{p_{-i}(\cdot)\}) - c_t^{(i)} + c_w^{(i)} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

is a function in $B_{L_\infty[0,M]}[0,1]$, and

$$T(p_i^*(.), \{p_{-i}(\cdot)\}) = \sup_{p_i(.) \in B_{L_\infty[0,M]}[0,1]} T(p_i(\cdot), \{p_{-i}(\cdot)\}).$$

In other words, if $\Psi_i(\cdot, \{p_{-i}(\cdot)\})$ is Lebesgue measurable then the supremum of $T(p_i(\cdot), \{p_{-i}(\cdot)\})$ is attained in $B_{L_\infty[0,M]}[0,1]$ at $p_i^*(\gamma_i)$, which is defined by (20). We now prove that $\Psi_i(\cdot, \{p_{-i}(\cdot)\})$ is Lebesgue measurable and that $p_i^*(\gamma_i)$, defined by (20), belongs to the class of threshold policies.

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[1] We thank anonymous reviewers for very detailed comments on this point.