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# EECE251

## Circuit Analysis I

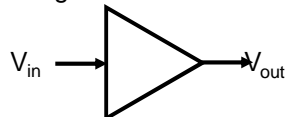
### Set 5: Operational Amplifiers

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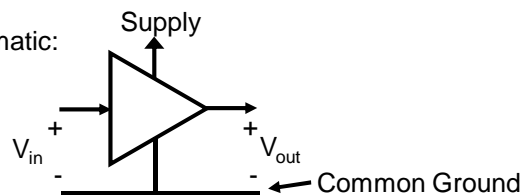
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## Amplifiers

- There are various types of amplifiers.
- Perhaps the most common type is a voltage amplifier (usually referred to simply as an amplifier!) where both input and output of the amplifier are voltages:

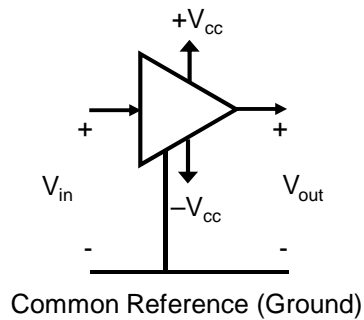


- A bit more realistic schematic:



## Amplifiers

- Typically, amplifiers have two supplies: One is positive ( $+V_{cc}$ ) and one is negative ( $-V_{cc}$ ).



## Saturation

- A practical limitation for amplifiers (at least the ones that we will see in this course) is that the magnitude of their output voltage cannot exceed the supply, that is:

$$-V_{cc} \leq V_{out} \leq +V_{cc}$$

- If the output wants to go beyond the supplies (for example when the input is positive and large) then it will be clipped at  $+V_{cc}$ :

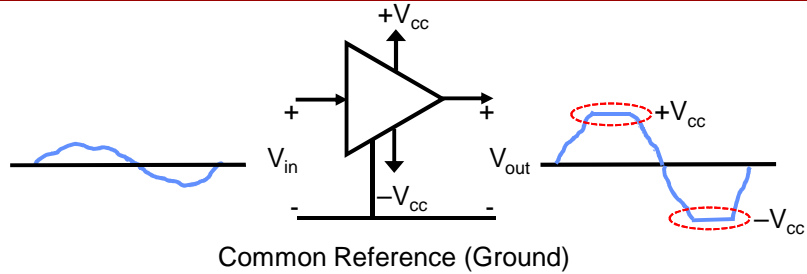
$$V_{out} = +V_{cc}$$

- If the output is so negative then it will be limited by  $-V_{cc}$ :

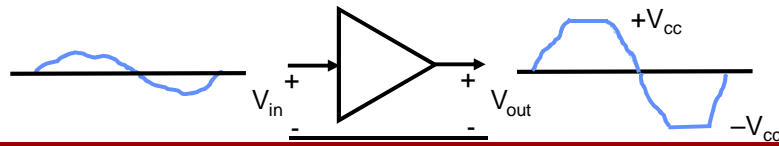
$$V_{out} = -V_{cc}$$

- In these cases we say that the amplifier is saturated

## Saturation



- Often for simplifying the drawings, the supplies and ground connections of the amplifiers will not be explicitly shown



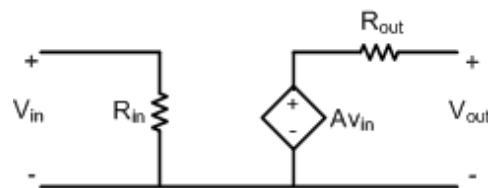
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## Equivalent Model of a (Voltage) Amplifier

- A voltage amplifier can be modeled with voltage-controlled voltage source:



$$V_{out} = AV_{in}$$

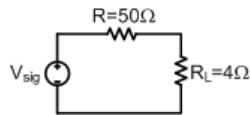
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## Ideal Amplifier

- What do you think the input and output resistance of an ideal amplifier should be?
- Let's look at a example:
- Let's assume you have a signal source with a  $50\Omega$  resistance (its Thevenin equivalent is the signal voltage source in series with a  $50\Omega$ ) and a load of  $4\Omega$ .
- What happens if we naively connect the output of the signal source to the load?



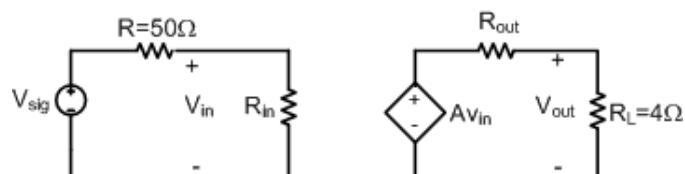
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## Ideal Amplifier

- What if we insert an amplifier between the source and the load.
- In order to maximize the signal at the output can you guess what should be the values for  $R_{in}$  and  $R_{out}$ ?

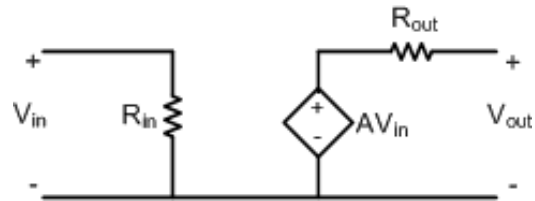


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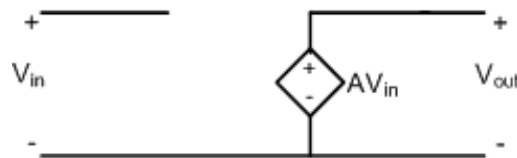
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## Ideal Amplifier



$$R_{in} \rightarrow \infty \text{ and } R_{out} \rightarrow 0$$



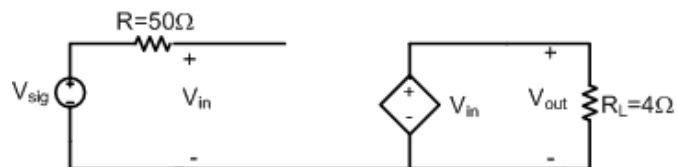
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## Buffer

- An amplifier with the gain of one is typically referred to as “buffer”.
- Buffers are very useful when one wants to “drive” a low resistance with a signal source that has a large series resistance.



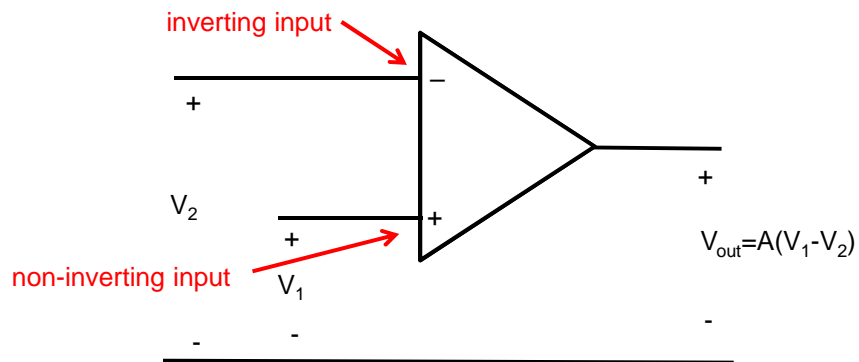
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## Differential Amplifier

- An amplifier that has two inputs and amplifies the potential difference between its input nodes (that is, it amplifies the difference between the voltages of its two inputs)

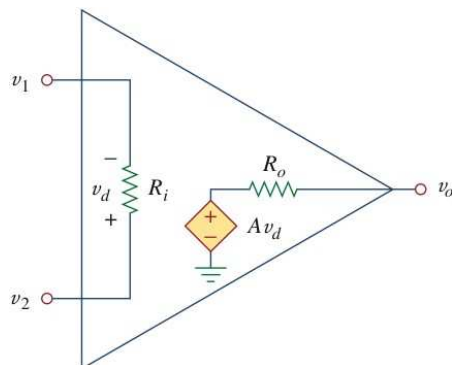


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## Equivalent Model for a Differential Amplifier



$$v_d = v_2 - v_1 \text{ and } v_o = Av_d = A(v_2 - v_1)$$

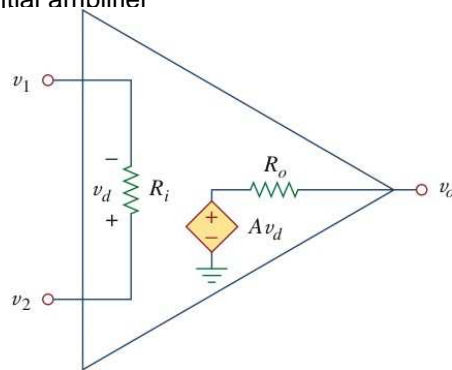
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## Differential Amplifier

For ideal differential amplifier



$$R_i \rightarrow \infty \text{ and } R_o \rightarrow 0$$

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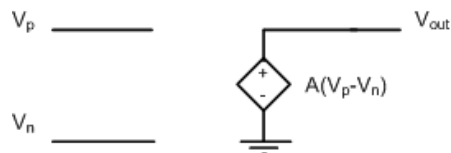
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## Operational Amplifier

- Operational amplifier (or op amp for short) is a differential amplifier whose gain is very large.
- Ideal op amp is an ideal differential amplifier with infinite gain!

$$\text{With } R_i \rightarrow \infty \text{ and } R_o \rightarrow 0$$

$$\text{and } A \rightarrow \infty$$

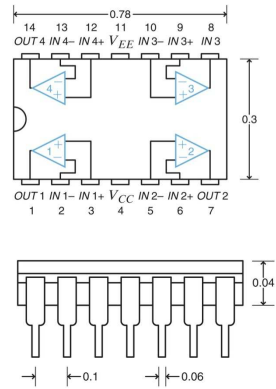
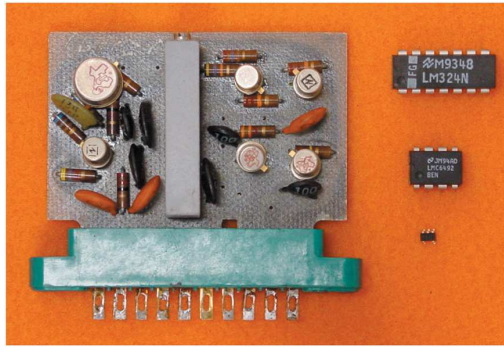


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## Operational Amplifiers

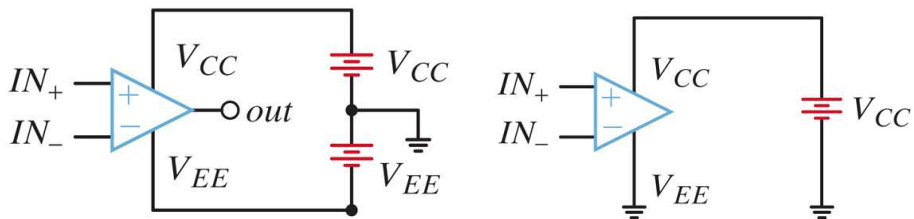


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## Operational Amplifiers



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## Operational Amplifiers (Op Amps)

- In light of their large gain, op amps are usually used in a negative feedback configuration where their output is somehow (usually through a passive component) is connected to their negative (inverting) input.
- If there is no feedback, what do you expect the output will be?
- In practice, If  $V_p > V_n$  then the output will be saturated to the positive supply. Why?
- And, if  $V_p < V_n$  then the output will be saturated to the negative supply

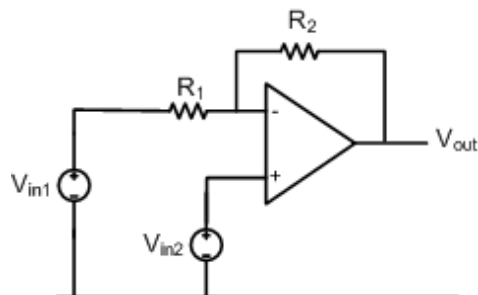
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## Op Amp

- What is the relationship between  $V_p$  and  $V_n$  in an op amp with negative feedback?



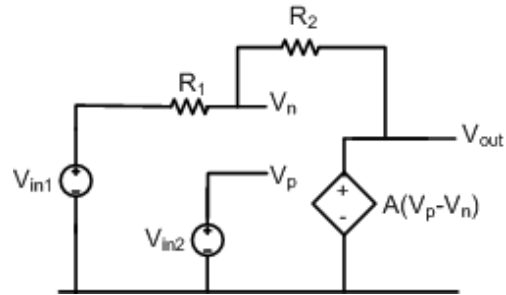
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## Op Amp

- What is the relationship between  $V_p$  and  $V_n$  in an op amp with negative feedback?



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## Op Amps

- Op amps were designed to perform mathematical operations such as subtraction, addition, multiplication, division, integration, and differentiation (therefore the name operational amplifier!).
- So let's have a look at how we can perform these operations using op amps.
- Note that in all these cases we should make sure that we have a negative feedback. Why?

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## Op Amps

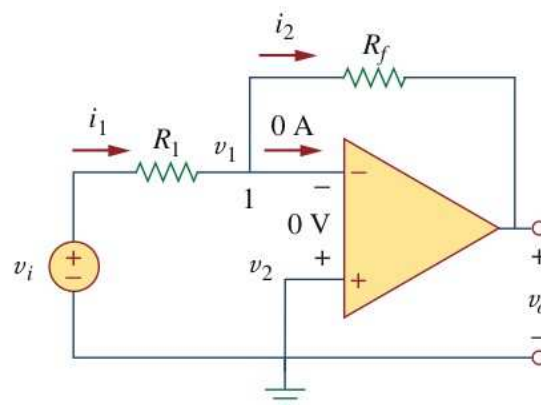
- Strategy to analyze op-amp circuits (assuming ideal op amps):
  - Check to see if there is a negative feedback
    - If so, then use:  $V_p = V_n$ . If there is no negative feedback then we can't assume anything about  $V_p$  and  $V_n$ .
  - Input currents  $I_n$  and  $I_p$  are both zero.
  - Apply nodal analysis
  - Solve nodal equations to express output voltage in terms of input signals.

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## Inverting Amplifier



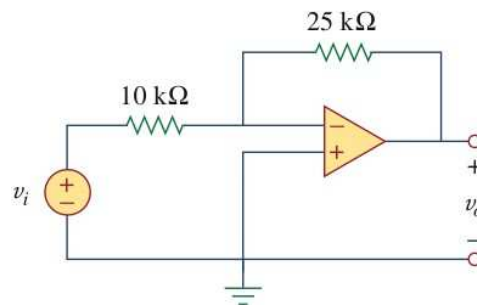
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## Example

- What is the gain of this circuit, that is, what is  $v_o/v_i$ ?
- If  $v_i=0.5V$  what is the output voltage? What is the current in the  $10k\Omega$  resistor?



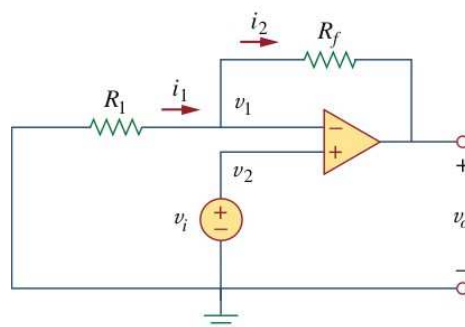
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## Non-inverting Amplifier

- Find the gain ( $v_o/v_i$ ) of the following circuit?



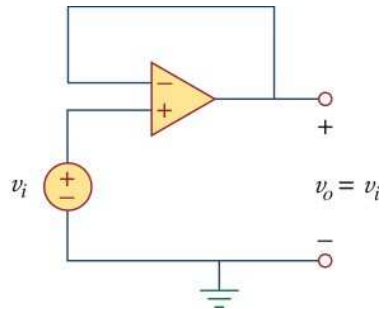
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## Buffer

- Also known as *voltage follower* or *unity gain amplifier*



- What is the use of such amplifier?

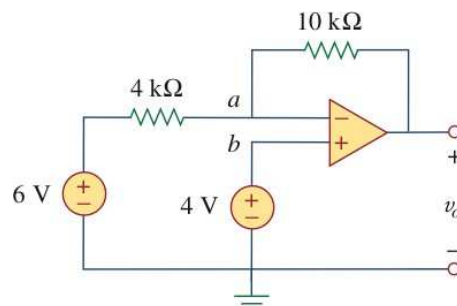
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## Example

- In the following circuit, find the output voltage.

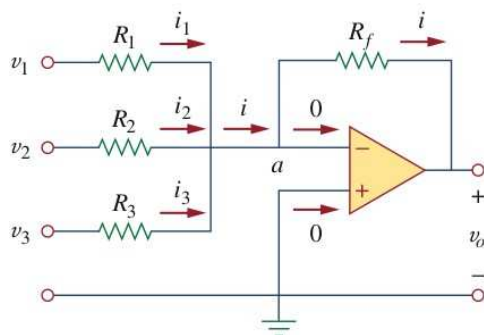


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## Summing Amplifier

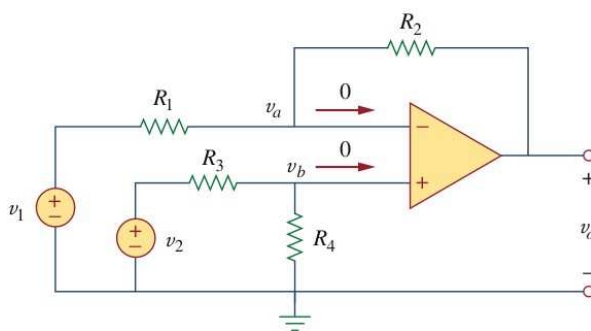


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## Difference Amplifier



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## Example

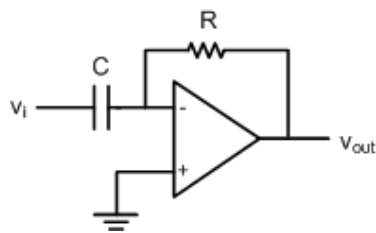
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- Design an op amp circuit with inputs  $v_1$  and  $v_2$  such that

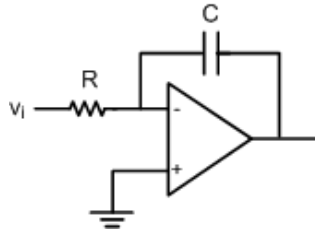
$$v_o = -2v_1 + 1.5v_2$$

## Differentiator

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## Integrator



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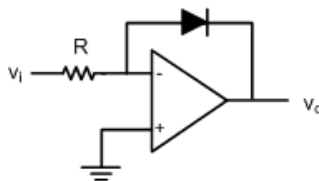
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## Logarithm!

- Let's assume that for a diode (a component that we will see later) we have

$$i_D \approx I_s e^{\frac{v_D}{V_T}}$$

- In the following circuit, find the relationship between  $v_o$  and  $v_i$ .



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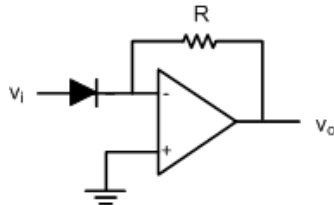
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## Exponential

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- In the following circuit, find the relationship between  $v_o$  and  $v_i$ .



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## Multiplication and Division

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- Can you think of a circuit that can be used to multiply two voltages?
  
  
- How about a circuit that can be used to divide two voltages?

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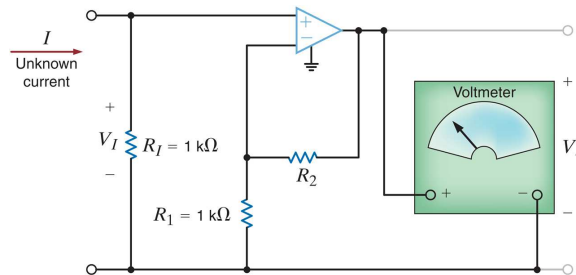
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## Example

- The following circuit is an electronic ammeter. It operates as follows: the unknown current,  $I$ , through  $R_I$  produces a voltage,  $V_I$ .  $V_I$  is amplified by the op-amp to produce a voltage,  $V_o$ , which is proportional to  $I$ . The output voltage is measured with a simple voltmeter. We want to find the value of  $R_2$  such that 10 V appears at  $V_o$  for each milliamp of unknown current.



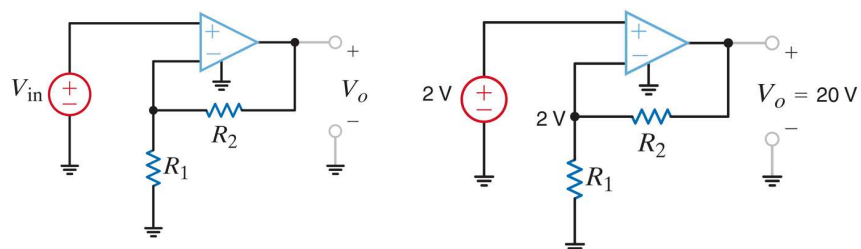
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## Example

- There is a requirement to design a noninverting op-amp configuration with two resistors under the following conditions: the gain must be +10, the input range is  $\pm 2$  V, and the total power consumed by the resistors must be less than 100 mW.



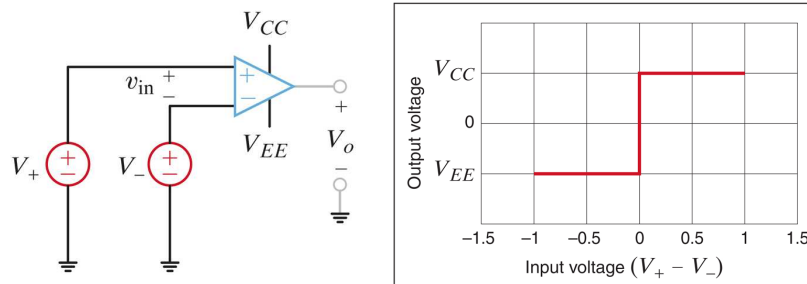
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## Comparators

- A comparator, a variant of the op-amp, is designed to compare the non-inverting and inverting input voltages. When the non-inverting input voltage is greater, the output goes as high as possible, at or near  $V_{CC}$ . On the other hand, if the inverting input voltage is greater, the output goes as low as possible, at or near  $V_{EE}$ .



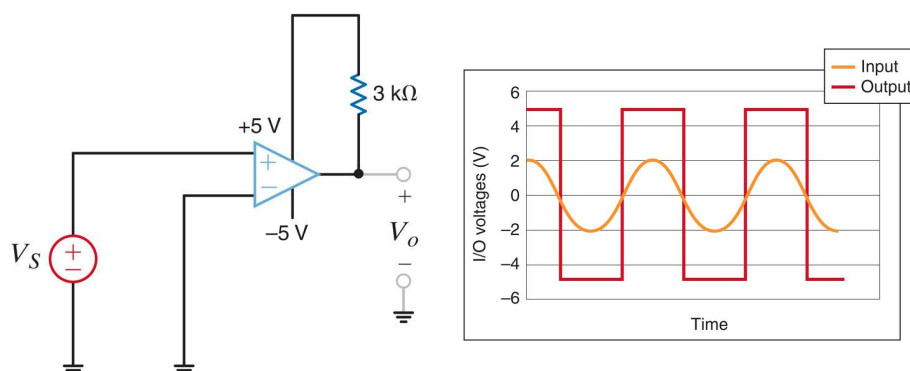
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## Comparators

- A common comparator application is the zero-crossing detector, as shown here:



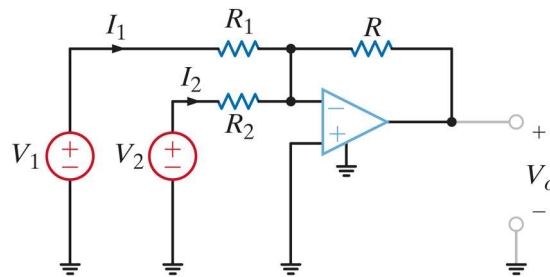
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## Design Example

- We wish to design a weighted-summer circuit that will produce the output. The design specifications call for use of one op-amp and no more than three resistors. Furthermore, we wish to minimize power while using resistors no larger than 10 kΩ.



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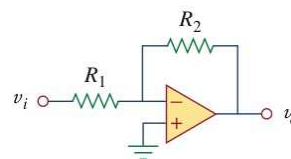
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## Summary

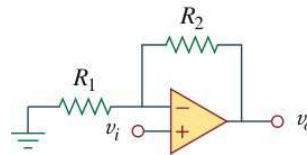
- Inverting amplifier

$$v_o = -\frac{R_2}{R_1} v_i$$



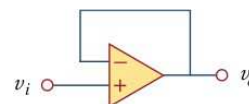
- Non-inverting amplifier

$$v_o = \left(1 + \frac{R_2}{R_1}\right) v_i$$



- Buffer (voltage follower)

$$v_o = v_i$$



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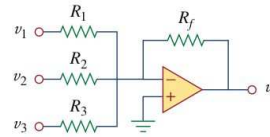
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## Summary

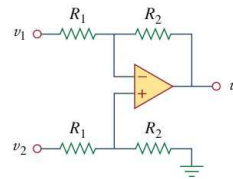
- Summer (adder)

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)$$



- Difference Amplifier

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$



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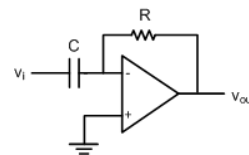
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## Summary

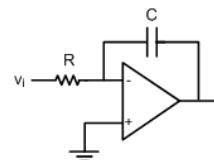
- Differentiator

$$v_o = -RC \frac{dv_i}{dt}$$



- Integrator

$$v_o = -\frac{1}{RC} \int v_i dt$$



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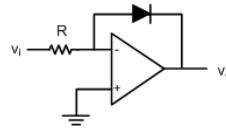
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## Summary

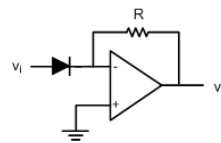
- Taking natural logarithm

$$v_o = -V_T \ln\left(\frac{v_i}{R \cdot I_s}\right)$$



- Raising to the power of e

$$v_o = -R \cdot I_s e^{\frac{v_i}{V_T}}$$

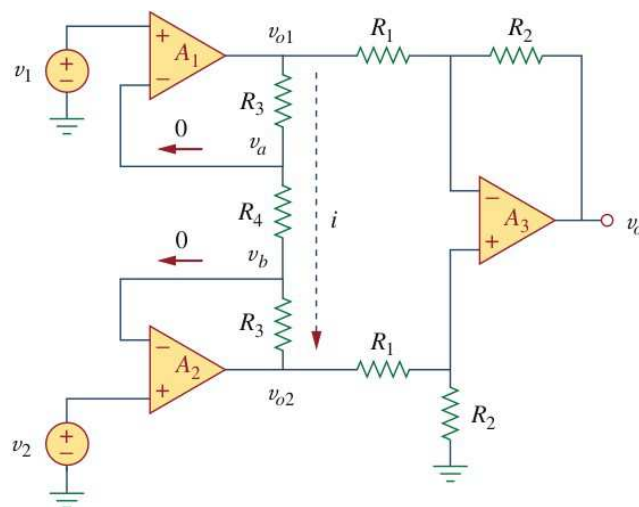


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## Instrumentation Amplifier



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