EECE251

Circuit Analysis

Set 2: Methods of Circuit Analysis

Shahriar Mirabbasi
Department of Electrical and Computer Engineering
University of British Columbia
shahriar@ece.ubc.ca

Reading Material

- Chapter 3 of the textbook
  - Section 3.1: Nodal analysis
  - Section 3.2: Loop analysis (and mesh analysis which is a special case of loop analysis)

Methods of Circuit Analysis

- Two popular and powerful techniques for analyzing circuits are:
  - Nodal analysis: a general procedure to find all the node voltages in a circuit. It is based on KCL and Ohm's Law.
  - Mesh analysis: another general approach to find mesh currents which circulate around closed paths in the circuit. It is based on KVL and Ohm's Law.
- Yet there is another more general! and powerful! technique which we call:
  - Modified Nodal Analysis (MNA)
  - Though more powerful it is not as popular of the first two (Almost all books don't even have it!)

Definitions (Recall)

- These are standard textbook definitions (for node the definition is slightly different from what we saw in slide set 1!)
  - **Node:** A point of connection of two or more circuit elements. A node can be spread out with perfect conductors (wires)
  - **Branch:** A portion of the circuit containing only a single element and the nodes at each end of the element (not that we are assuming that the elements have two terminals!)
  - **Loop:** Any closed path through the circuit in which no node is encountered more than once.

More Terminology

- **Reference node or ground:** a node that is assumed to have a zero potential.
  - If the reference node is not explicitly indicated on the circuit one can arbitrarily choose any node as the ground. We will soon see how to choose a good ground node.

- **Node voltage** is the voltage difference/drop from a given node to the reference node.

Regular Nodal Analysis

- Steps to determine the node voltages for a circuit with no floating voltage source:

1. Select a reference node. A floating voltage source is a voltage source that neither of its terminals is connected to the reference node.

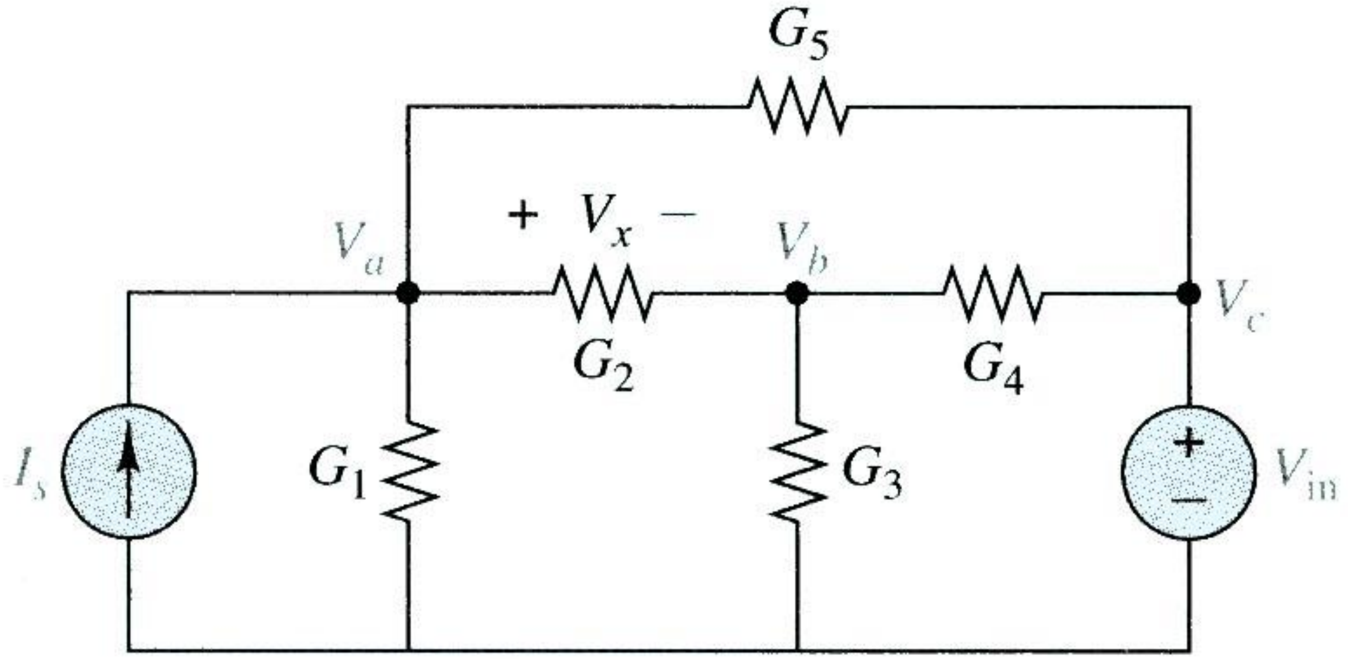
2. Assign voltages to other nodes. These node voltages are referenced to the reference node.

3. Write KCL for all unknown non-reference nodes. When possible use Ohm's law to relate the branch currents to node voltages

4. Solve the resulting system of equations for unknown node voltages.

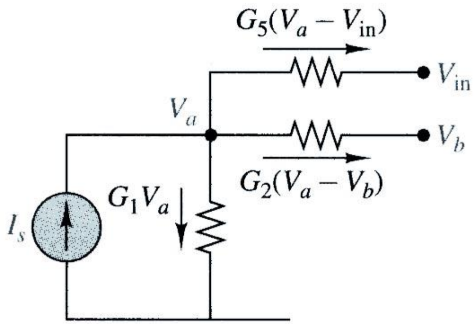
Nodal Analysis Example

- Let’s analyze the following circuit using nodal analysis:



Nodal Analysis

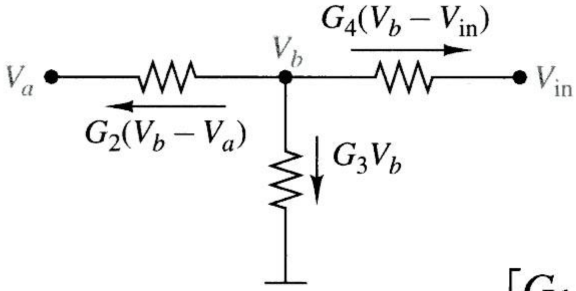
- Consider node a:



$$G_1V_a + G_2(V_a - V_b) + G_5(V_a - V_{\text{in}}) - I_s = 0$$

$$(G_1 + G_2 + G_5)V_a - G_2V_b = I_s + G_5V_{\text{in}}$$

- Consider node b:



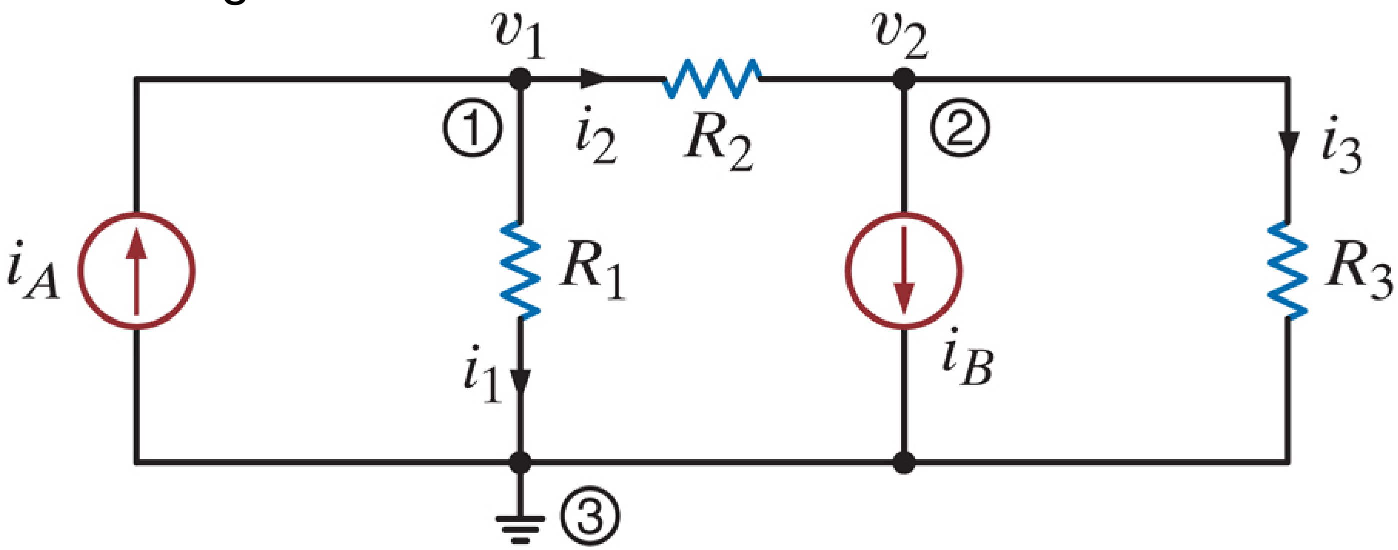
$$G_2(V_b - V_a) + G_3V_b + G_4(V_b - V_{\text{in}}) = 0$$

$$-G_2V_a + (G_2 + G_3 + G_4)V_b = G_4V_{\text{in}}$$

$$\begin{bmatrix} G_1 + G_2 + G_5 & -G_2 \\ -G_2 & G_2 + G_3 + G_4 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} I_s + G_5V_{\text{in}} \\ G_4V_{\text{in}} \end{bmatrix}$$

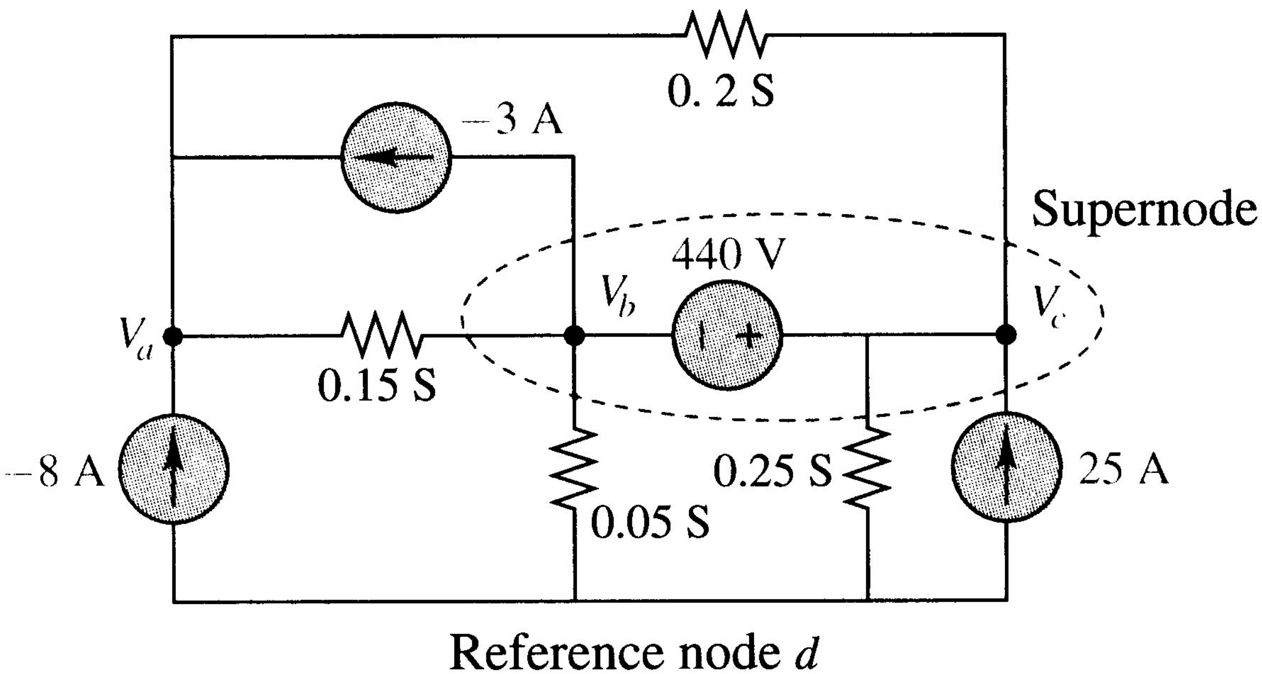
Example

- Write the nodal analysis equations that lead to solve the following circuit.



Floating Voltage Sources

- Problem: The current through the floating voltage source cannot be written as function of its two terminal voltages!



- Solution: Form a supernode which is formed by enclosing the floating voltage source (independent or dependent) and any elements in parallel with it in a closed boundary.

Floating Voltage Sources

- Since there are two nodes (two terminals of the floating voltage source) are enclosed in the supernode, two equations are needed for each supernode:

  – KCL at supernode gives one equation

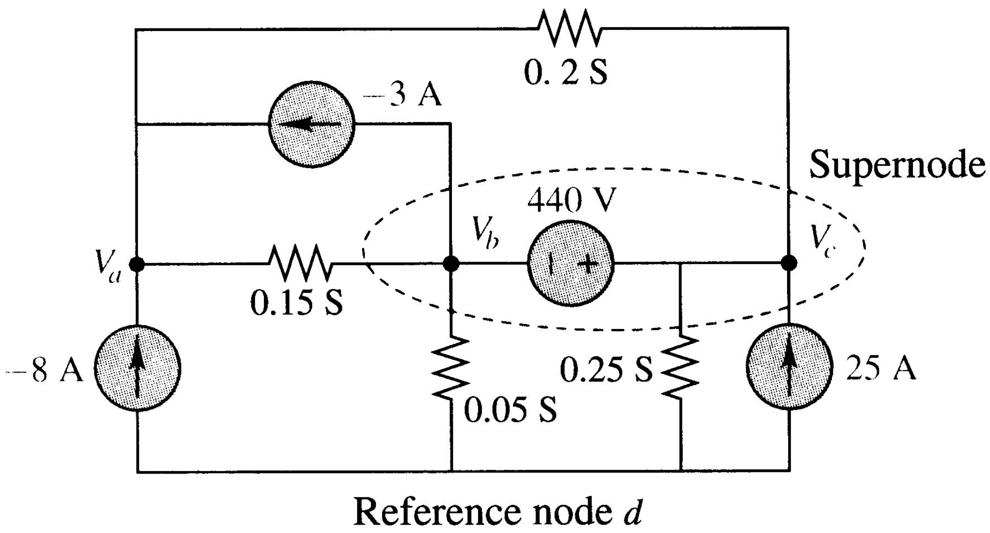
  – The other equation is the relationship between the voltages of the two nodes enclosed in the supernode

- For example for the supernode in the previous slide we can write the following two equations:

$$-3 + 0.15(V_b - V_a) + 0.05V_b + 0.25V_c - 25 + 0.2(V_c - V_a) = 0$$

$$-V_b + V_c = 440$$

Nodal Analysis Example



- KCL at Node a:

$$8 + 0.15(V_a - V_b) + 3 + 0.2(V_a - V_c) = 0$$

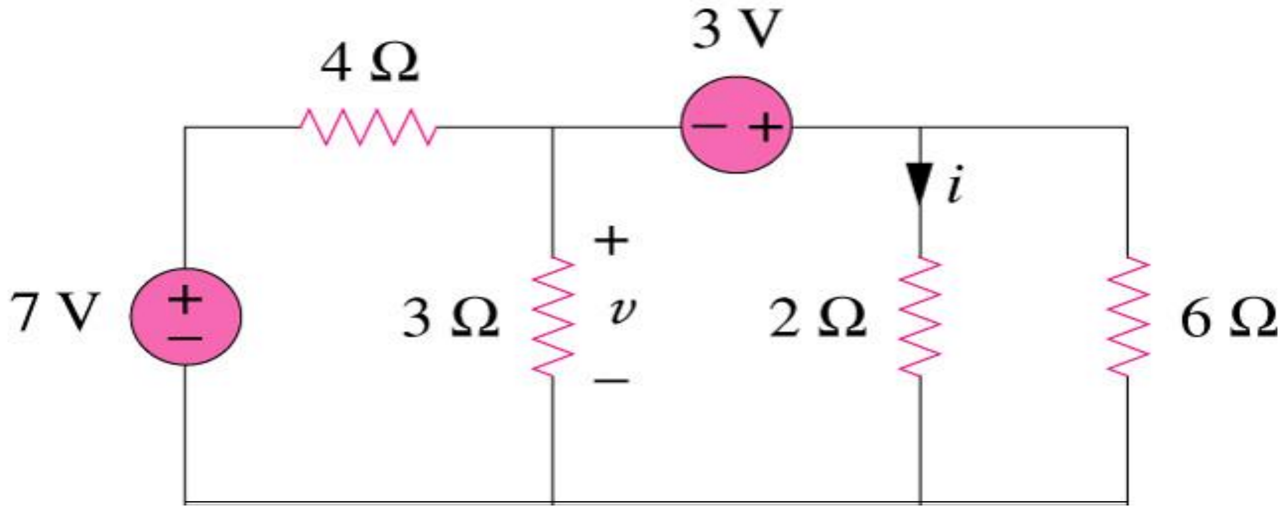
- For the supernode we have:

$$-3 + 0.15(V_b - V_a) + 0.05V_b + 0.25V_c - 25 + 0.2(V_c - V_a) = 0$$

$$-V_b + V_c = 440$$

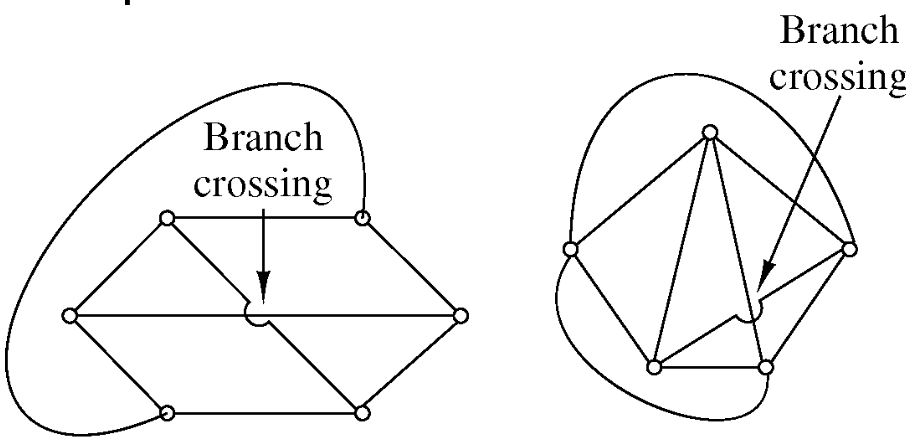
Example

- Find v and i in the circuit below:



Loop and Mesh Analysis

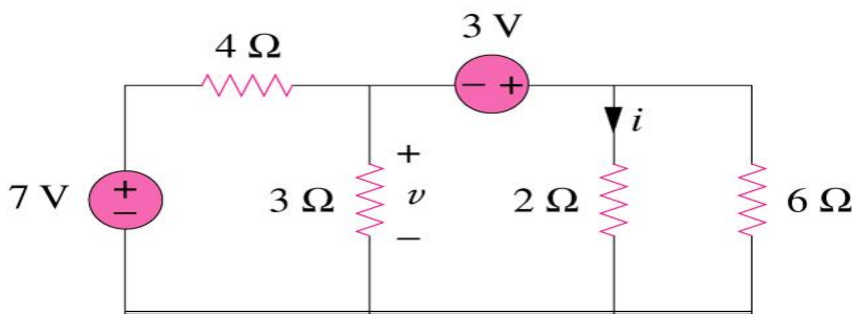
- Mesh analysis is a special case of a more general technique called loop analysis.
- A mesh is a loop that does not contain any other loops within it.
- Mesh analysis is not quite as general as nodal analysis since it can only be applied to planar circuits
- A planar circuit is a circuit that can be drawn in a plane with no branches crossing one another.
- Example of non-planar circuits:



(a) (b)

Note

- Number of linearly independent loops:
  - In a circuit with B branches and N nodes there are B-N+1 linearly independent loops!
  - Number of linearly independent loops is equal to the number of meshes!
- Find the number of linearly independent loops in the following circuit:



Mesh (Loop) Analysis

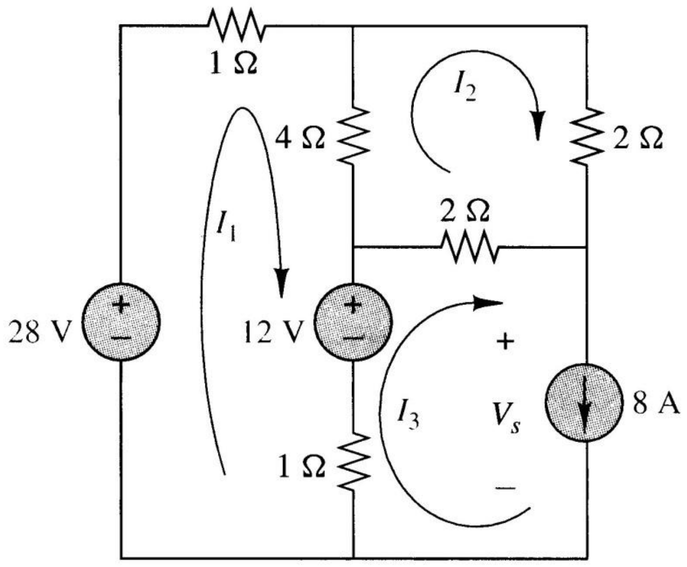
- Steps to calculate mesh (loop) currents for a given circuit in which no current source is shared between two meshes:

1. Assign mesh (loop) currents to each mesh (loop)

2. Write KVL for each of the meshes (loops) and use Ohm's law to express the voltages of the elements in the mesh (loop) in terms of mesh (loop) currents

3. Solve the resulting systems of linear equations for unknown mesh (loop) currents

Mesh Analysis Example



- For mesh 1:

$$28 = I_1 + 4(I_1 - I_2) + 12 + (I_1 - 8) = 6I_1 - 4I_2 + 4 \quad \text{or} \quad 6I_1 - 4I_2 = 24$$

- For mesh 2:

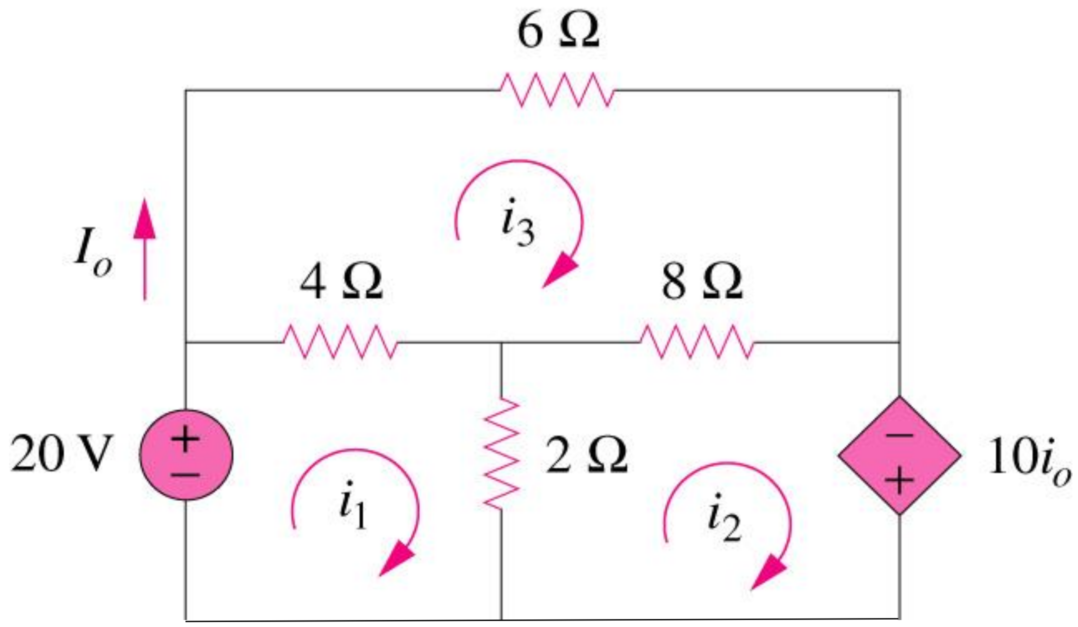
$$0 = 4(I_2 - I_1) + 2I_2 + 2(I_2 - 8) = -4I_1 + 8I_2 - 16 \quad \text{or} \quad -4I_1 + 8I_2 = 16$$

- Mesh 3?

$$\begin{bmatrix} 6 & -4 \\ -4 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 24 \\ 16 \end{bmatrix}$$

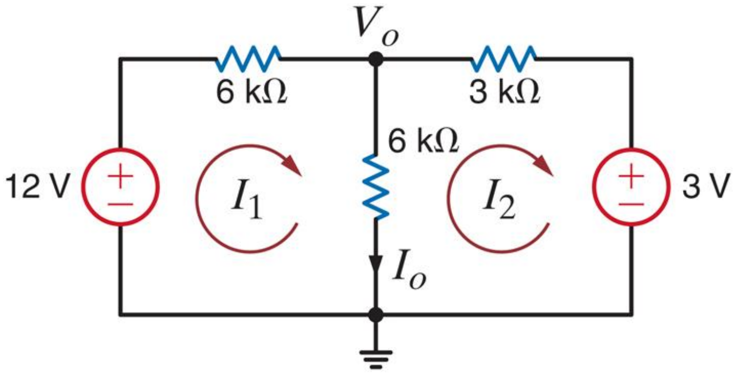
Example

- In the following circuit, use mesh analysis to find I_o:

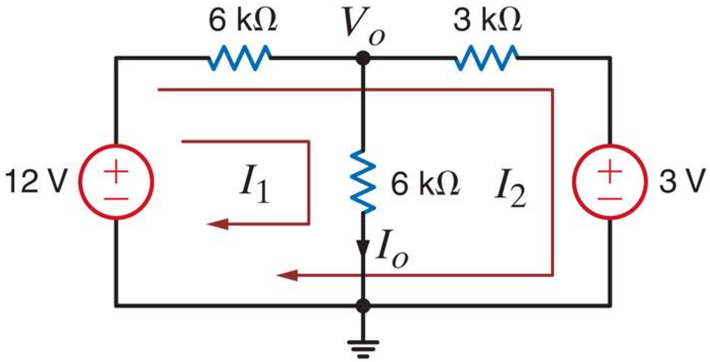


Example

- In the following circuit, find I_o.



Mesh analysis



Loop analysis!

Mesh Analysis

- Steps to calculate mesh (loop) currents for a given circuit in which some current sources are shared between two meshes:

1. Assign mesh currents to every mesh in the circuit.

2. Define a supermesh when two (or more) meshes have a (dependent or independent) current source(s) in common.

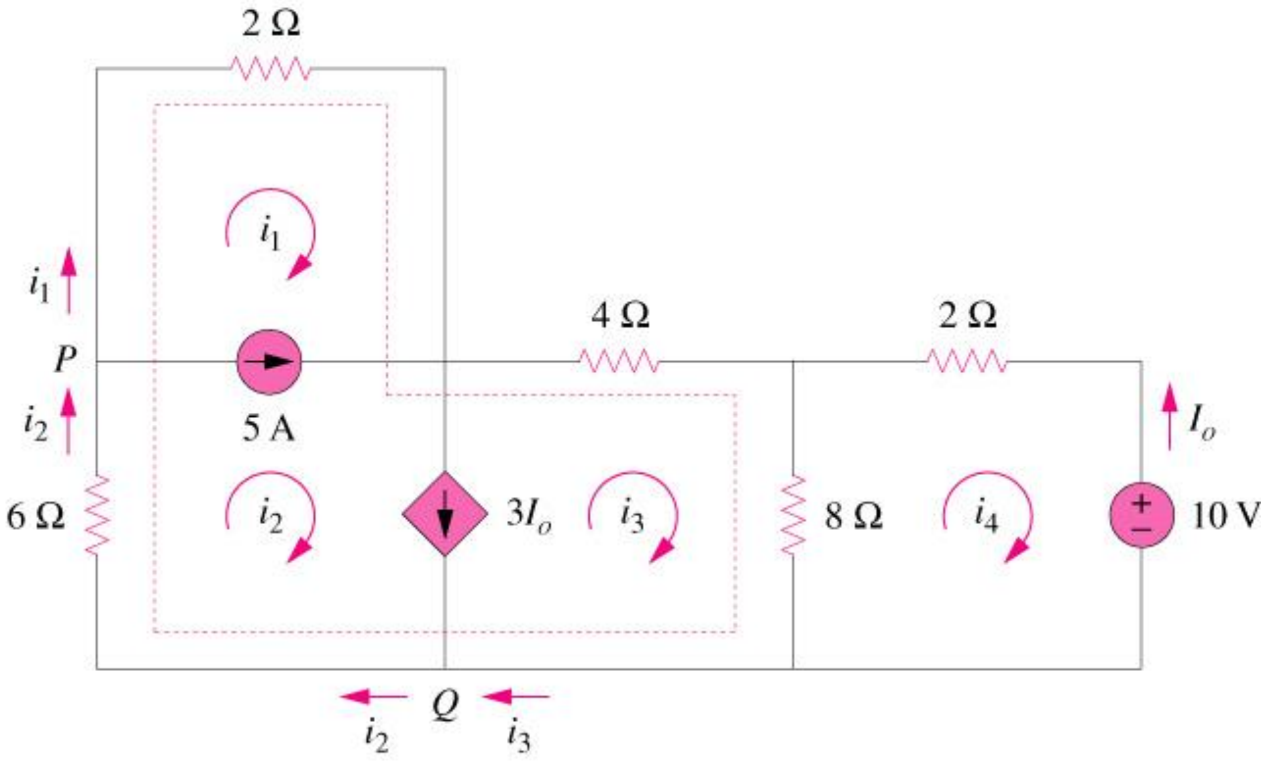
3. Write KVL for each regular mesh.

4. Apply both KVL and KCL to suppermeshes.

5. Solve the resulting system of equations.

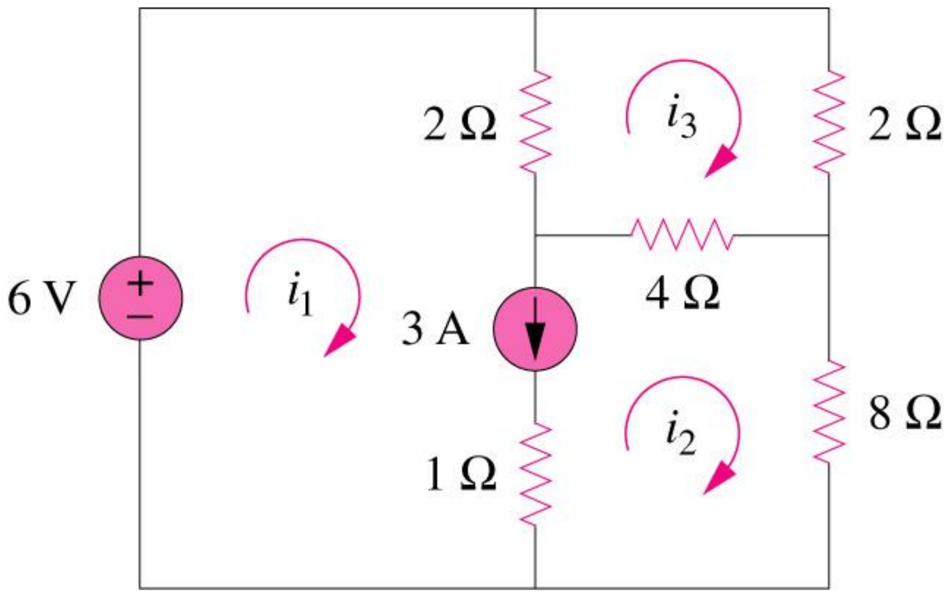
Example

- In the following circuit, find the mesh currents:



Example

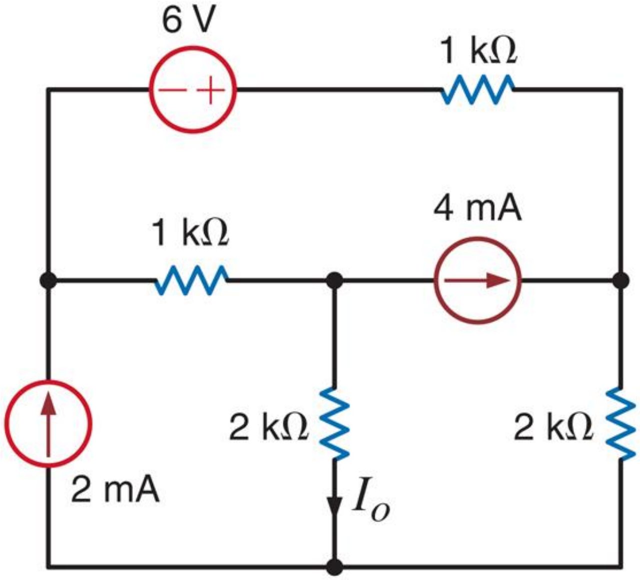
- Write the mesh equations for the following circuit:



Notes

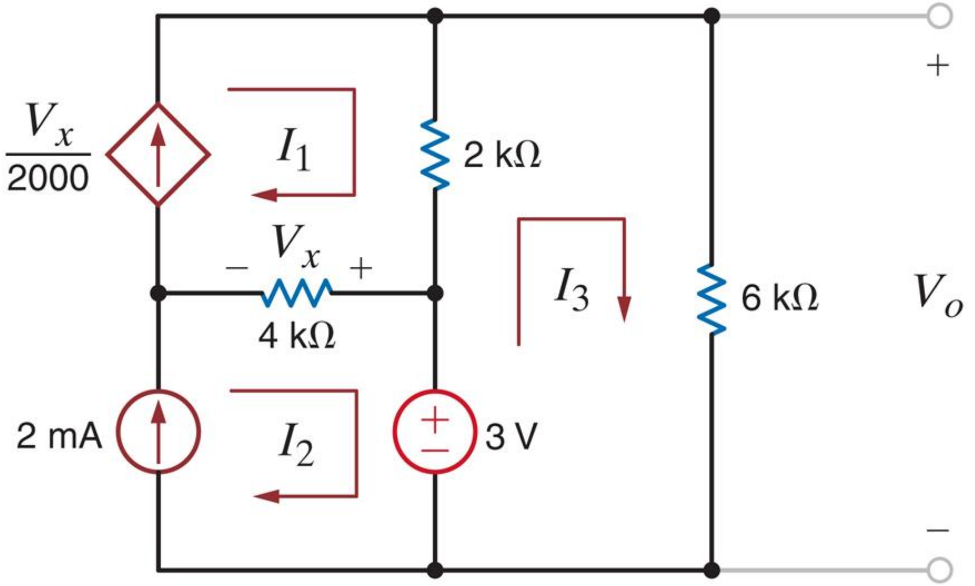
Example

- In the following circuit, find I_o. Use two techniqiues: 1) mesh analysis and 2) loop analysis.



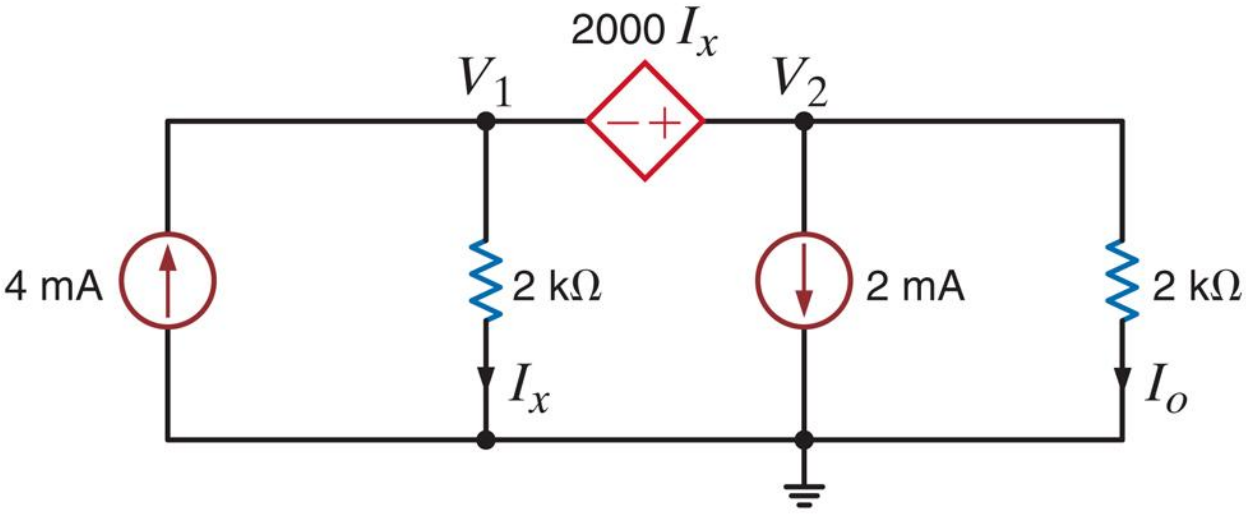
Example

- In the following circuit, use mesh analysis to ginf V_o.



Example

- Use a technique of your choice to solve for



Modified Nodal Analysis

- **Recall:** The following definitions of nodes (and branches) slightly differ from traditional textbook definitions! Please note that almost all components that we deal with in this course are two-terminal components (resistors, sources, …)

- A "**true node**" (or node for short) is the point of connection of three or more circuit elements. (The node includes the interconnection wires.)

- A "**binary node**" (or b-node for short) has only two components connected to it.

- A "**branch**" is a collection of elements that are connected between two "**true nodes**" that includes only those two true nodes (and does not include any other true nodes).

Modified Nodal Analysis

- For modified nodal analysis (MNA) we need some more definitions!
- We identify four general types for branches as follows:

  (we have seen these before, and here we are just formalizing them by giving them proper names!):

R branch

RV branch

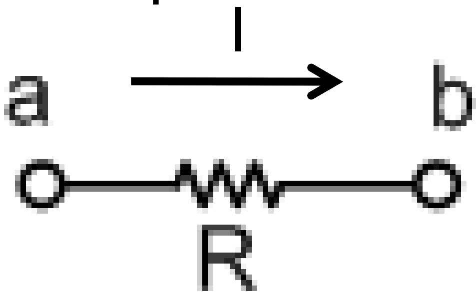
I branch (this also include any branch that consist of a current source in series with other components)

V branch (also known as evil branch!)

Let's see if we can calculate the current of these branches based on the end-point node voltages!

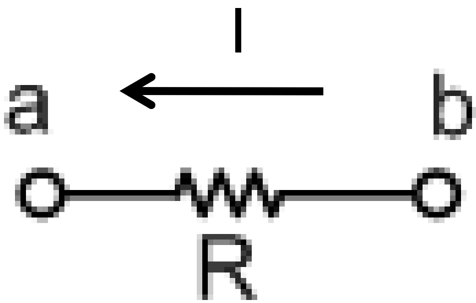
R Branch

- A branch that consist of only a resistor (or series combination of resistors that can be represented by their equivalent resistors)



$$I = \frac{V_a - V_b}{R}$$

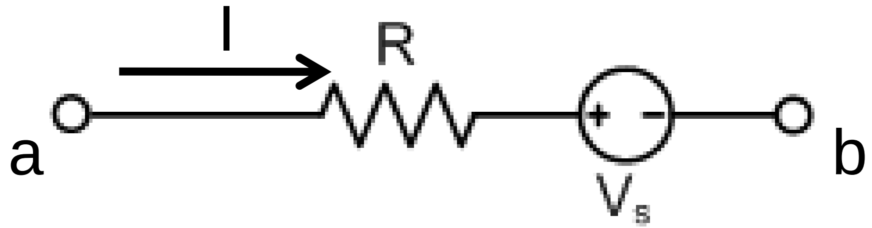
- How about the current in the other direction!



$$I = \frac{V_b - V_a}{R}$$

RV Branch

- A branch that consist of a resistor (or series combination of resistors that can be represented by their equivalent resistors) in series with a voltage source (or a series combination of voltage coursed that sources that can be represented by their equivalent voltage source)



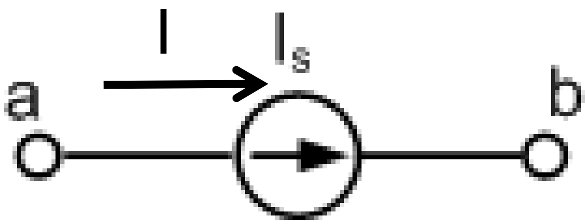
$$I = \frac{V_a - (V_b + V_s)}{R} = \frac{V_a - V_b - V_s}{R}$$

- How about the current in the other direction?
- What if the polarity of the voltage source is reversed?

Board Notes

I Branch

- I branch: A branch that consists of only a current source!



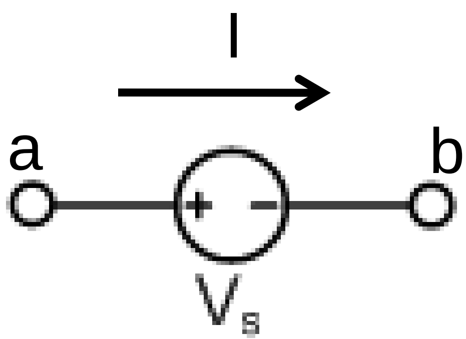
$$I = I_s$$

- Another example of I branch (some times called IR branch): A branch that consists of a resistor (or equivalent resistor) in series with a current source:

$$I = I_s$$

V Branch (Evil Branch)

- A branch that consists of only one voltage source:



$$I = ?$$

- However, the good news is:

$$V_s = V_a - V_b$$

- **Note:** The sources in V, RV, I, and RI branch can be either dependent (controlled) or independent sources.

Modified Nodal Analysis (MNA)

- A general technique to solve a circuit (i.e., to find voltage, current and power of every element in the circuit).
- Unknowns:

  1) controlling variables (for dependent sources)

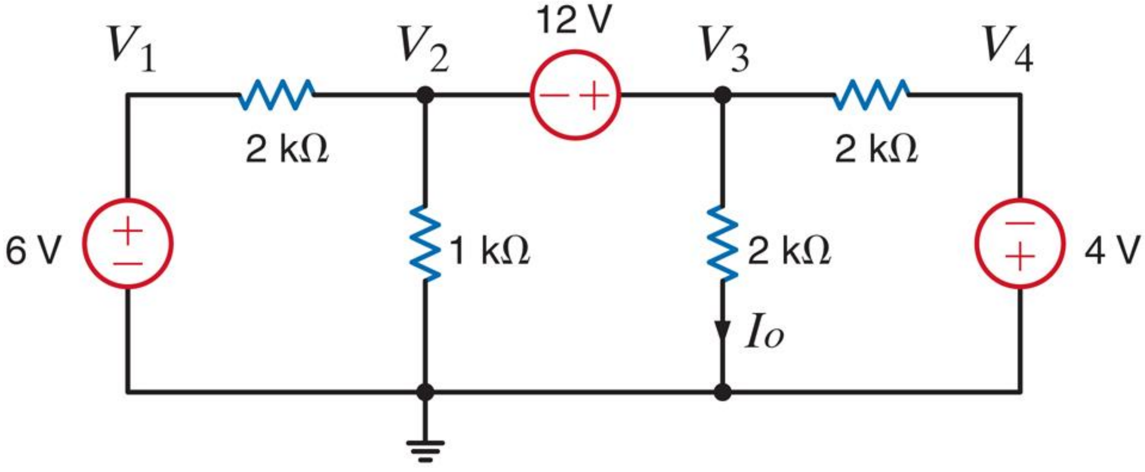
  2) current in V branches (evil branches)

  3) voltage of each true node
- MNA steps:

1. Identify every true node of the circuit.
2. Choose one of them as a reference node (node whose voltage is zero).
3. Write one equation per controlling current or voltage of dependant sources.
4. Write the relationship between the two nodes of the V branch .
5. Write one KCL per true node.

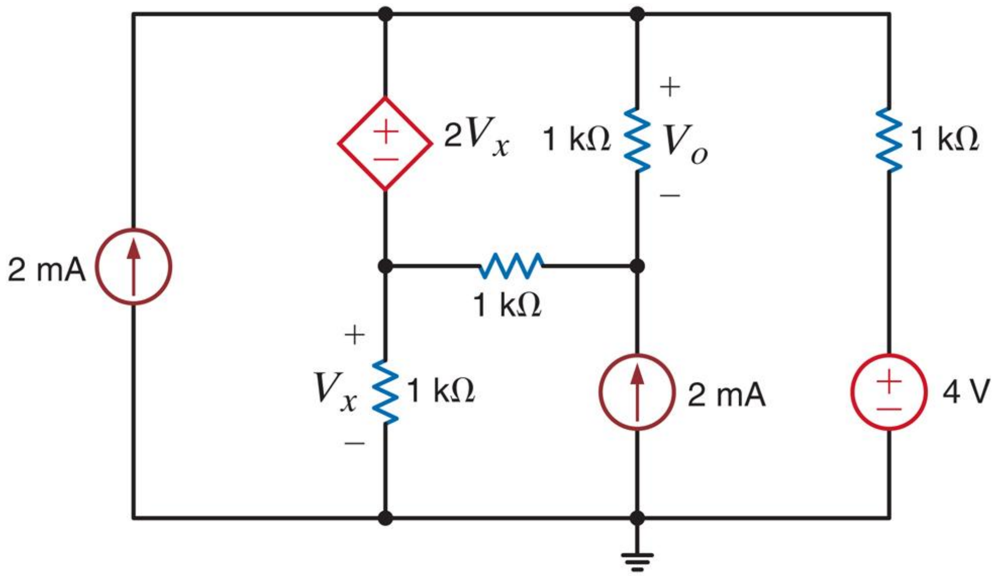
Example

- Use modified nodal analysis to find I_o in the following circuit:



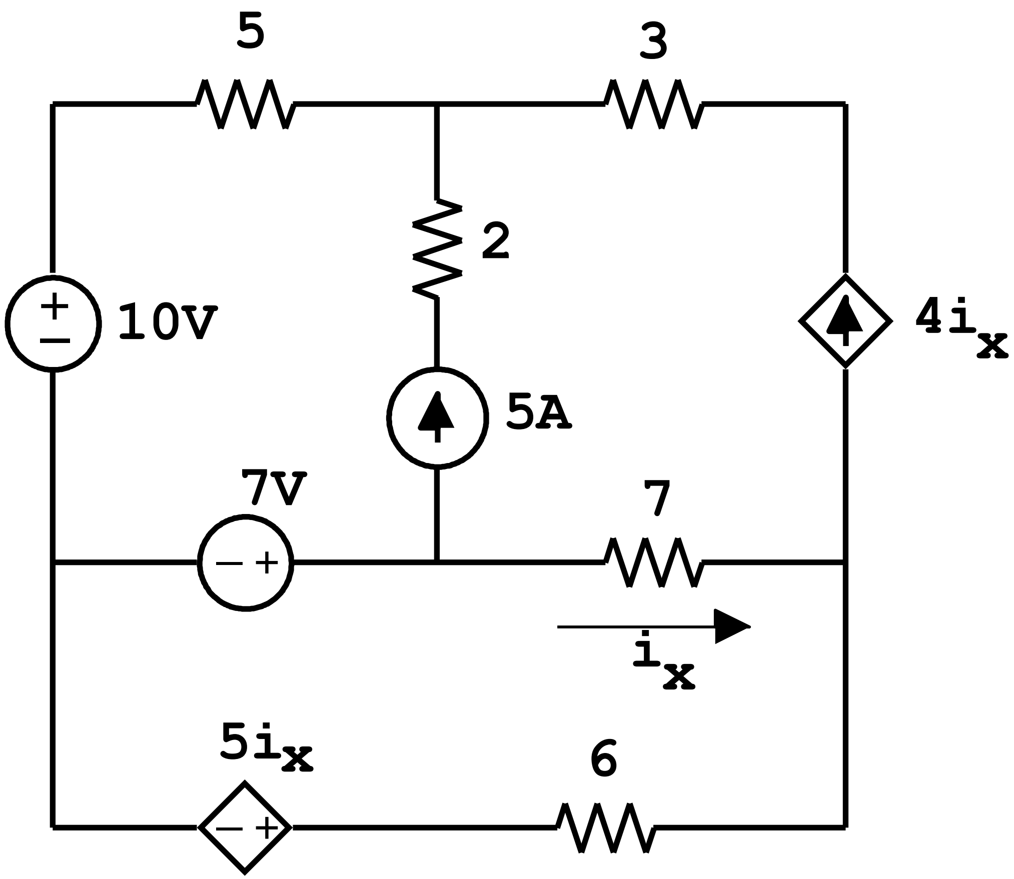
Example

- In the following circuit, find V_o.



Example (MNA)

- Solve the following circuit using MNA.



Notes