Convolutional Neural Networks: Back Propagation

Deep Learning

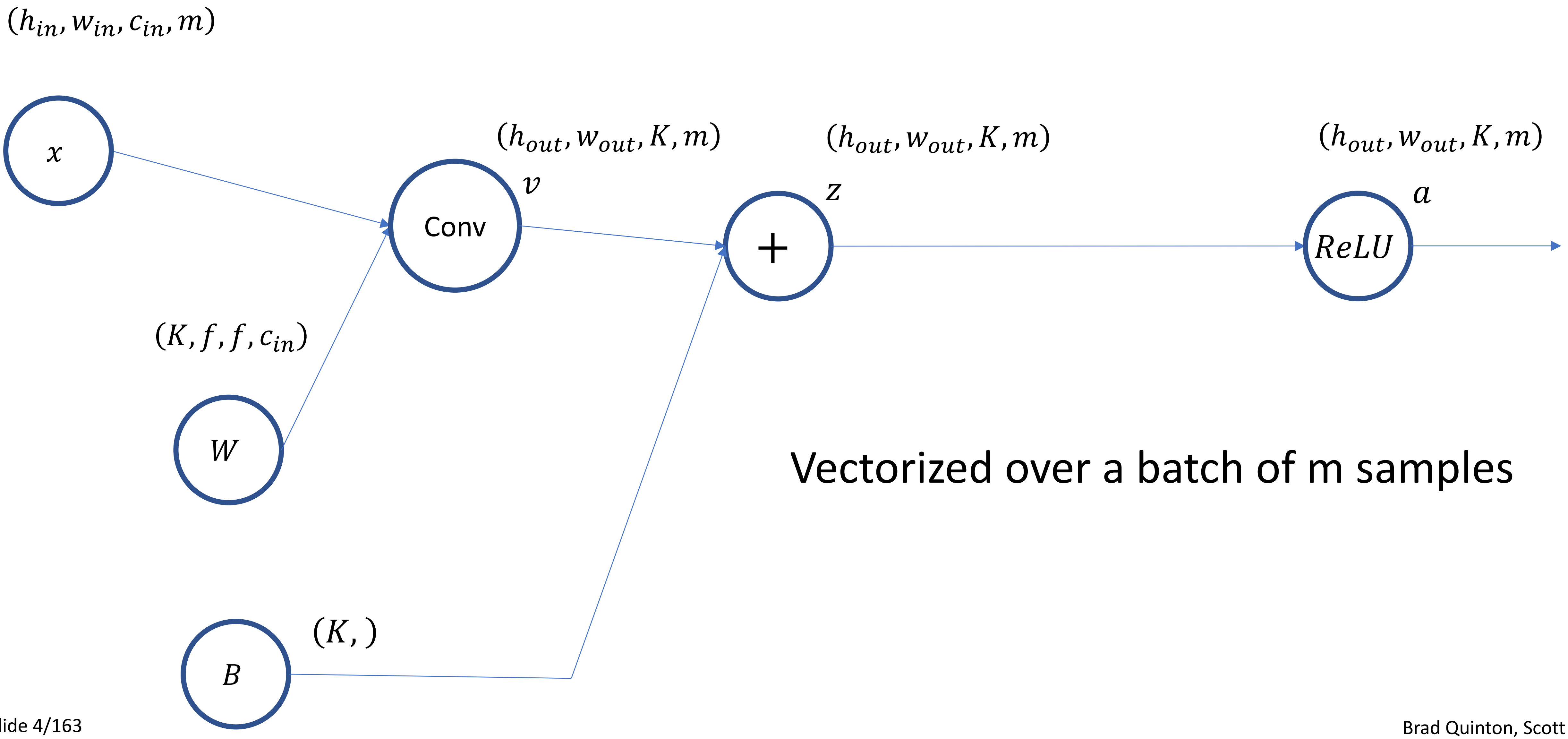
[Brad Quinton](), [Scott Chin]()

Learning Objectives

- More practice with backpropagation
- Understand backprop being applied to convolutional layer
- See some of the “shortcuts” in convolutional layer backprop
- Understand use of numerical gradient

BackPropagation for CNNs

A compute graph for a convolutional layer



Vectorized over a batch of m samples

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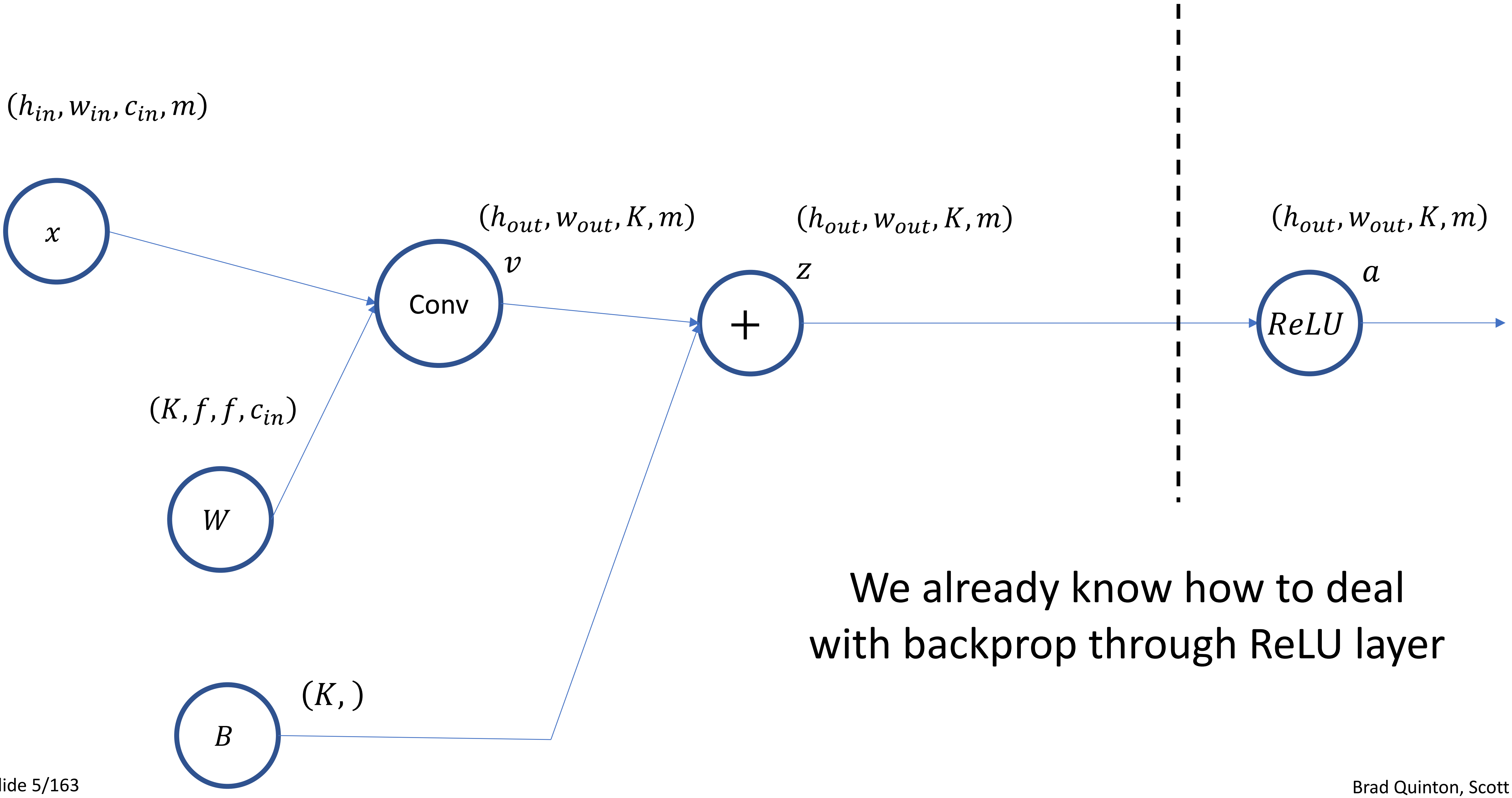
A compute graph for a convolutional layer



We already know how to deal with backprop through ReLU layer

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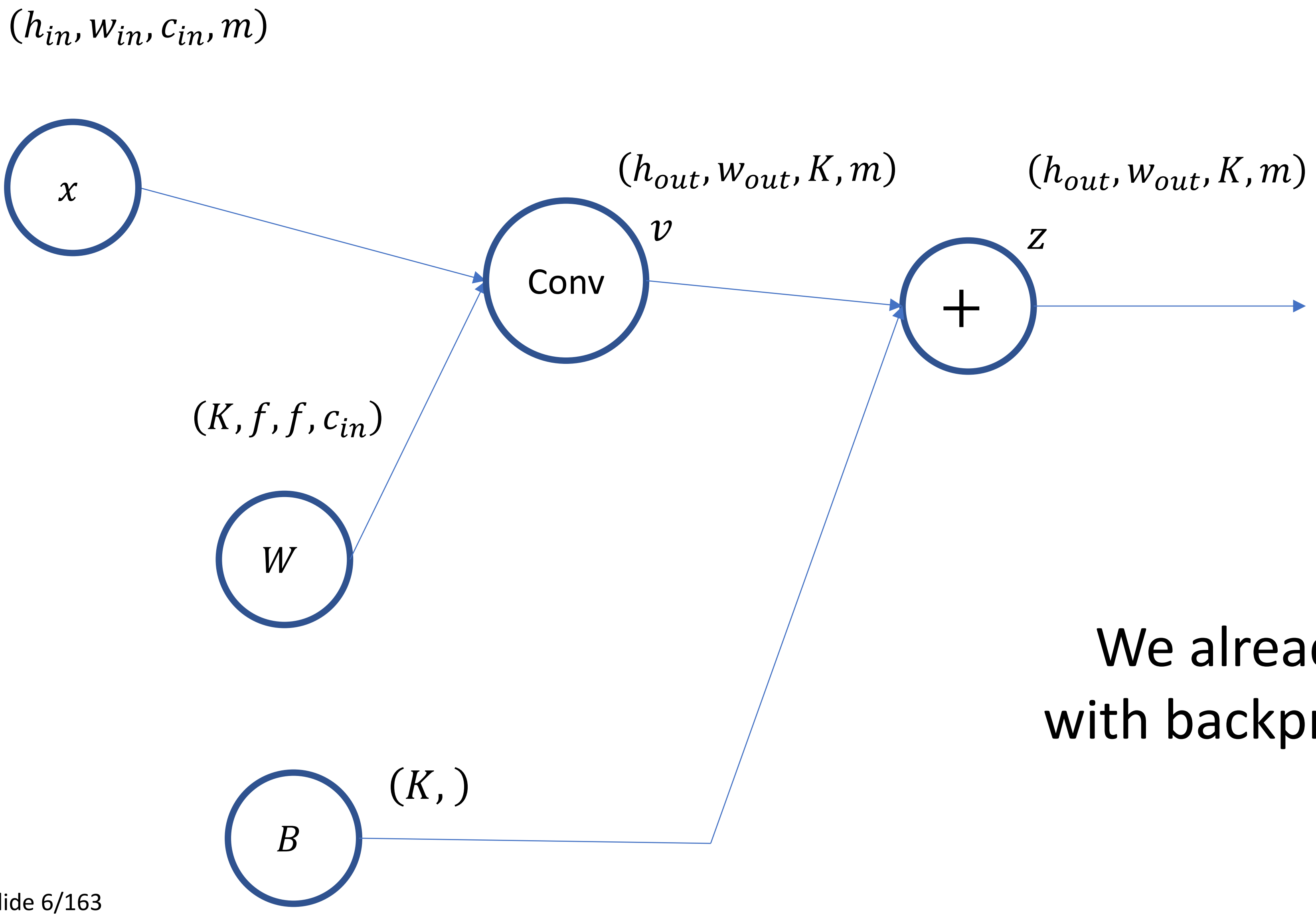
A compute graph for a convolutional layer



We already know how to deal with backprop through ReLU layer

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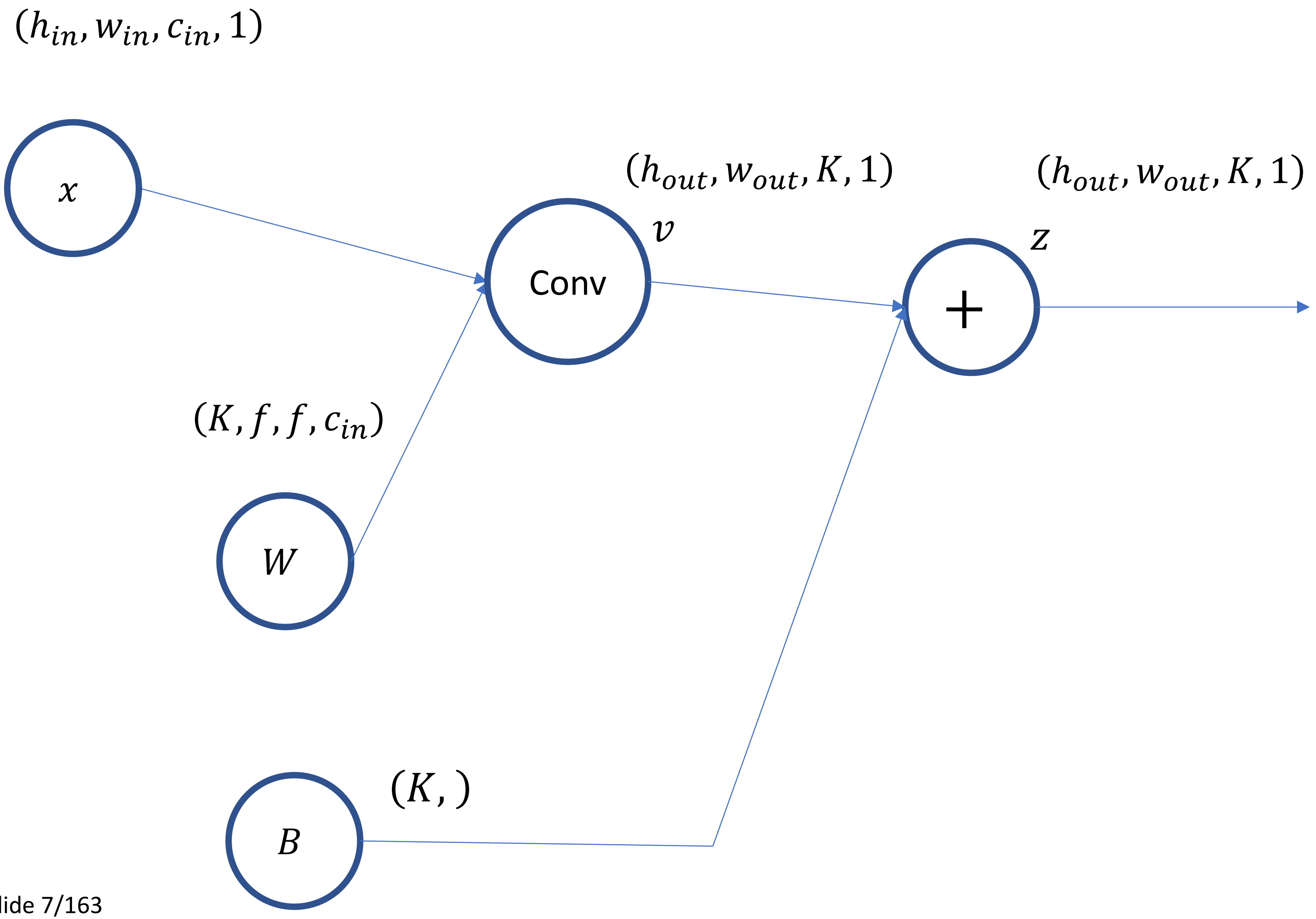
A compute graph for a convolutional layer



Start by considering:

- One sample (m=1)

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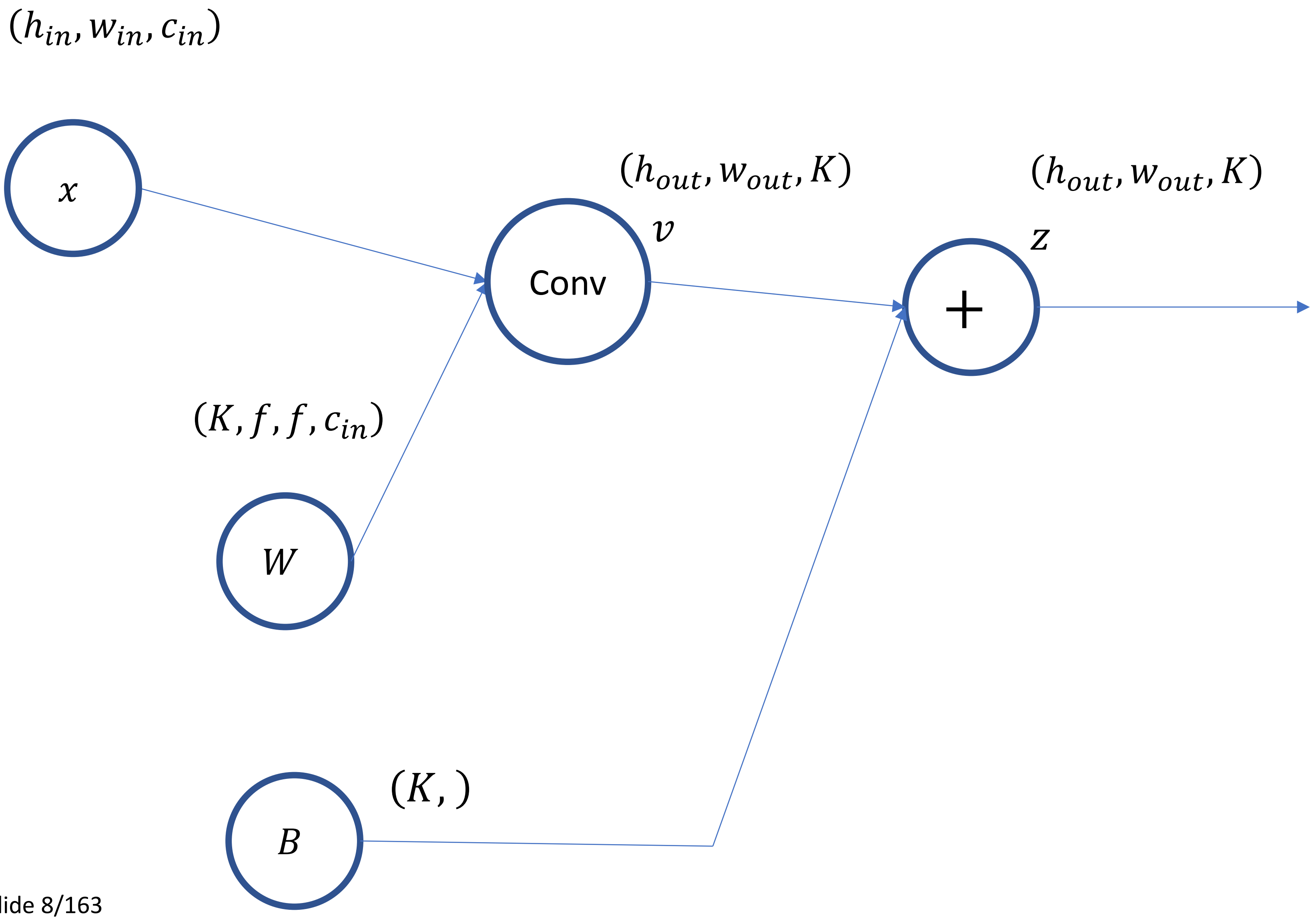
A compute graph for a convolutional layer



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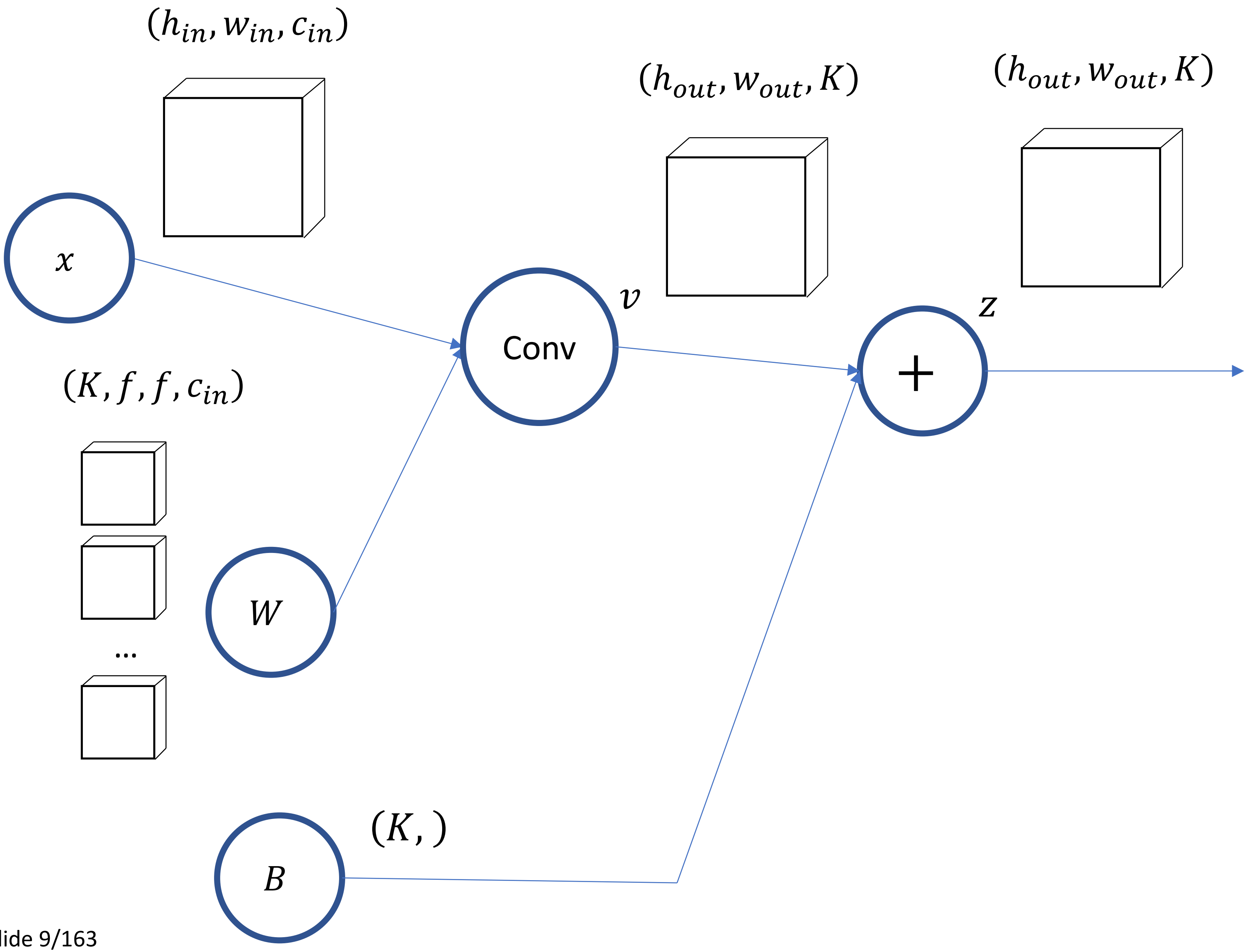
A compute graph for a convolutional layer



Start by considering:

- One sample (m=1)

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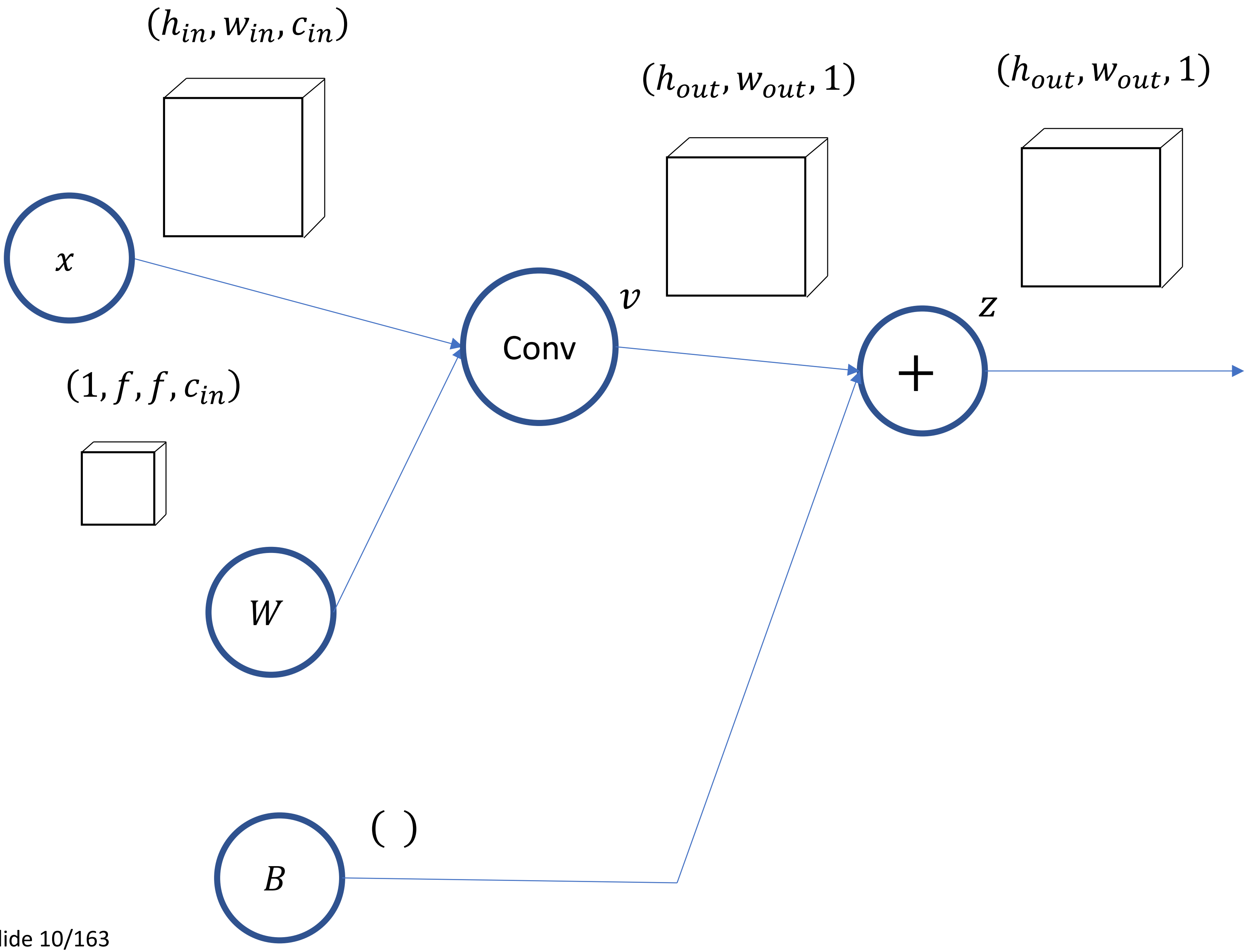
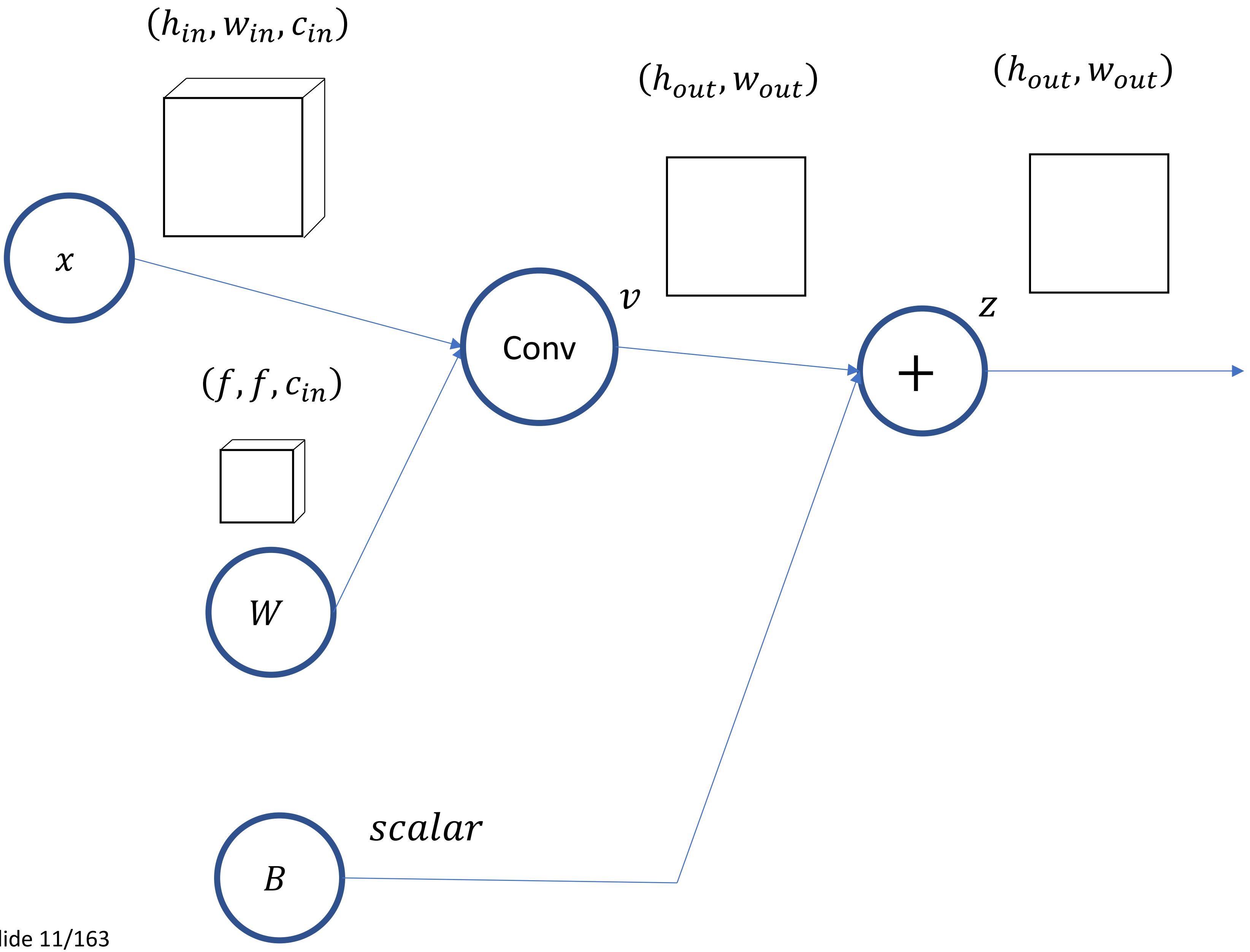
A compute graph for a convolutional layer



Start by considering:
- One sample (m=1)
- One filter (K=1)

A compute graph for a convolutional layer



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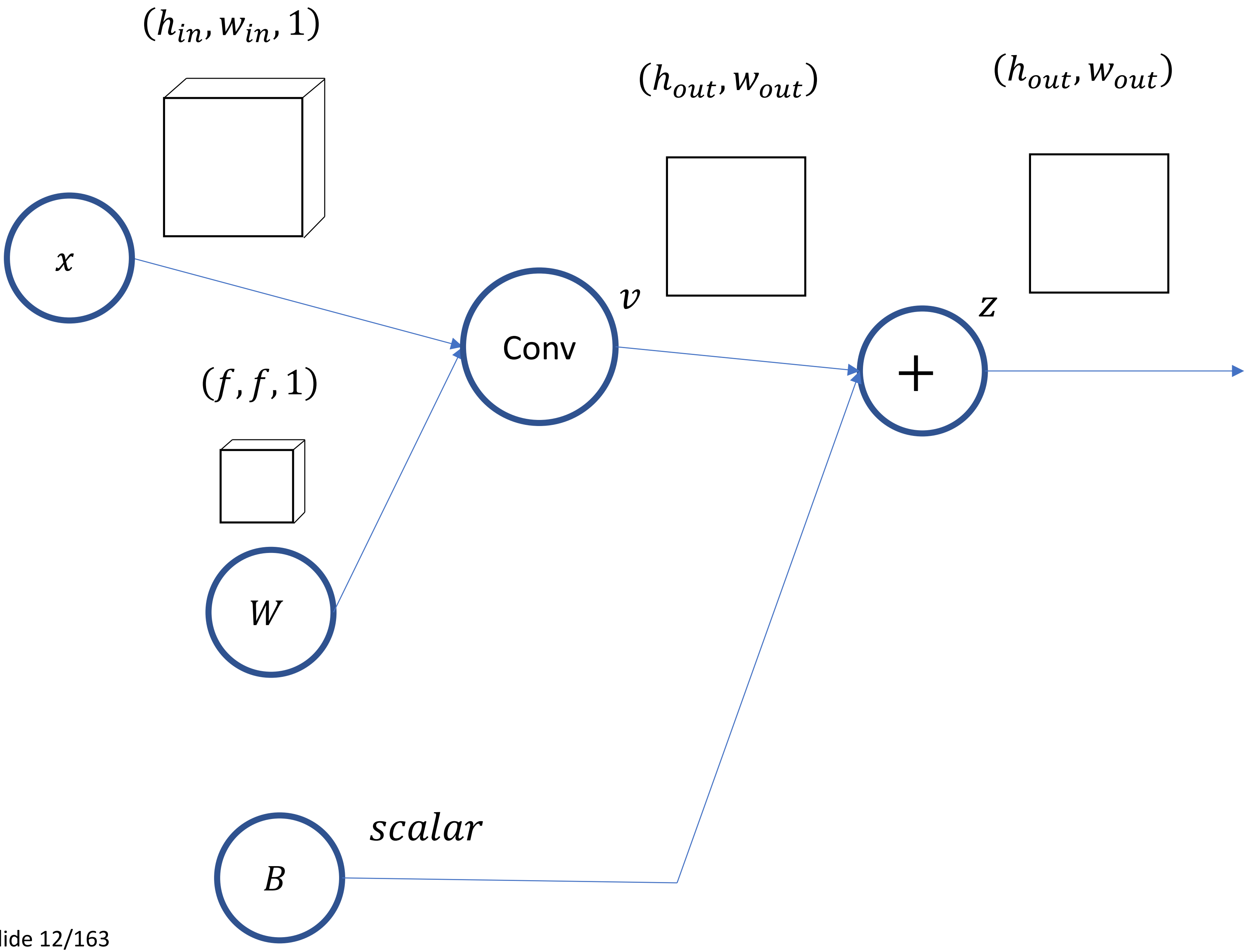
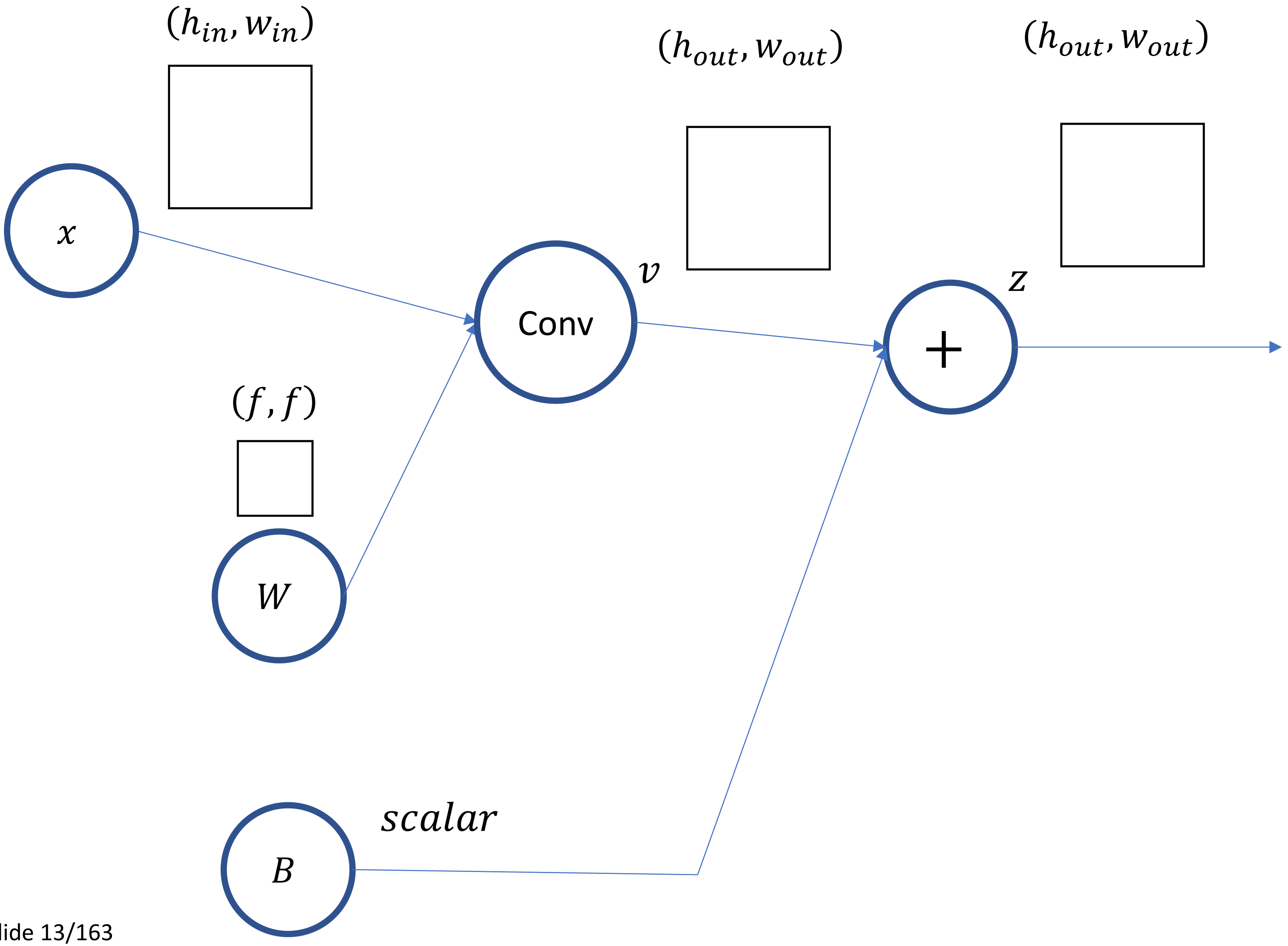
A compute graph for a convolutional layer



Start by considering:

- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)

A compute graph for a convolutional layer



Start by considering:

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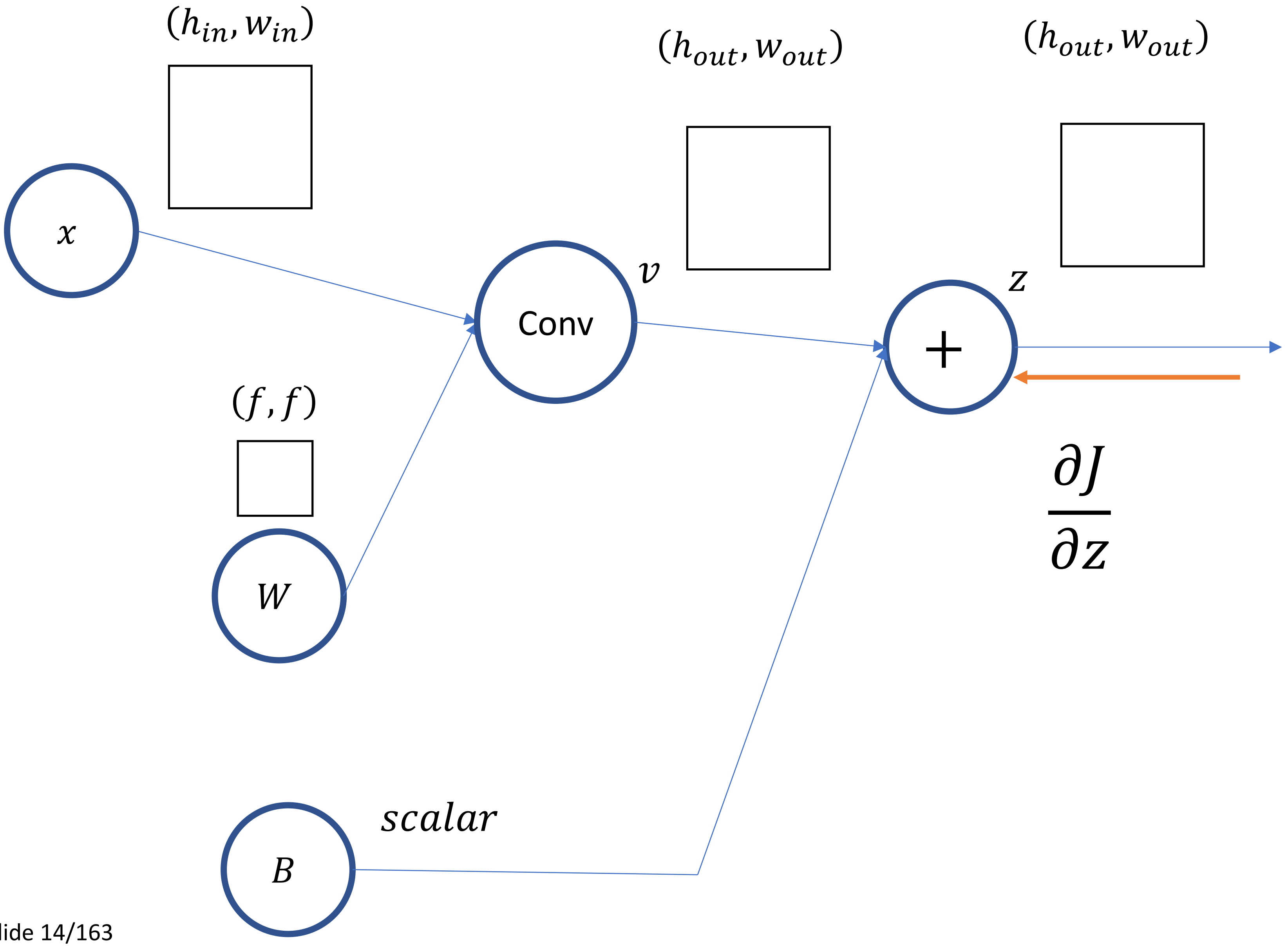
A compute graph for a convolutional layer



Start by considering:

- One sample (m=1)
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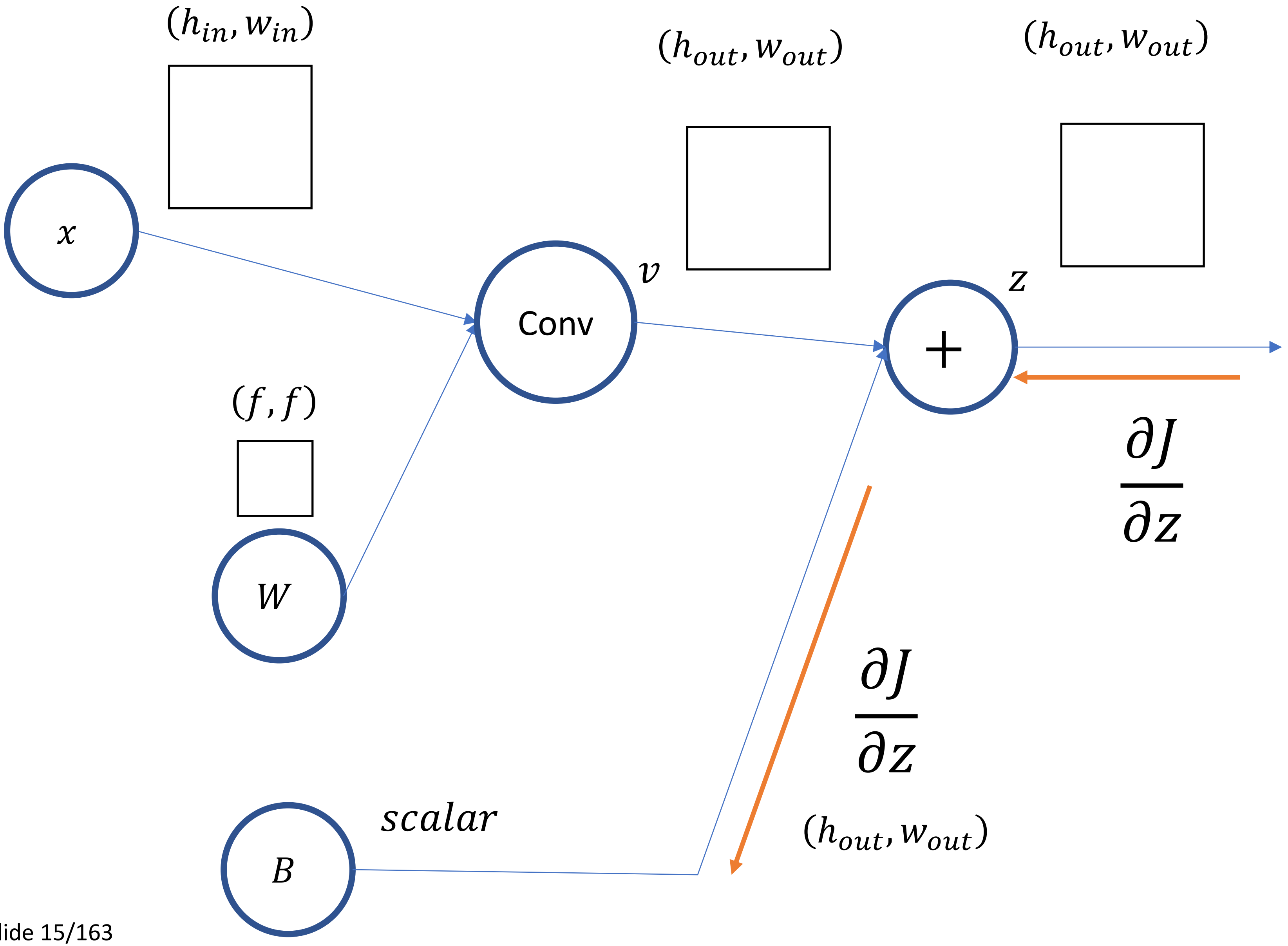
A compute graph for a convolutional layer



Start by considering:

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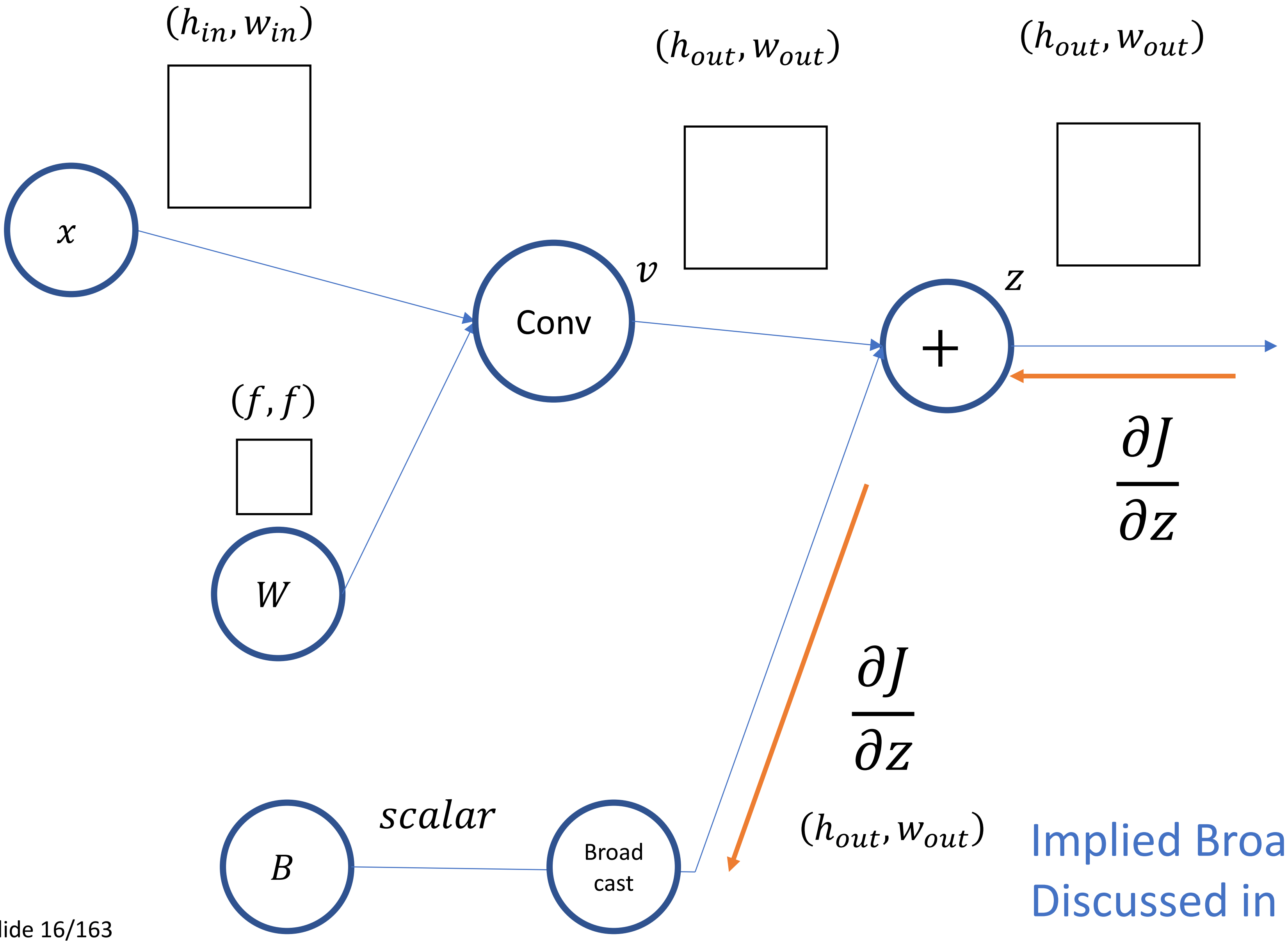
A compute graph for a convolutional layer

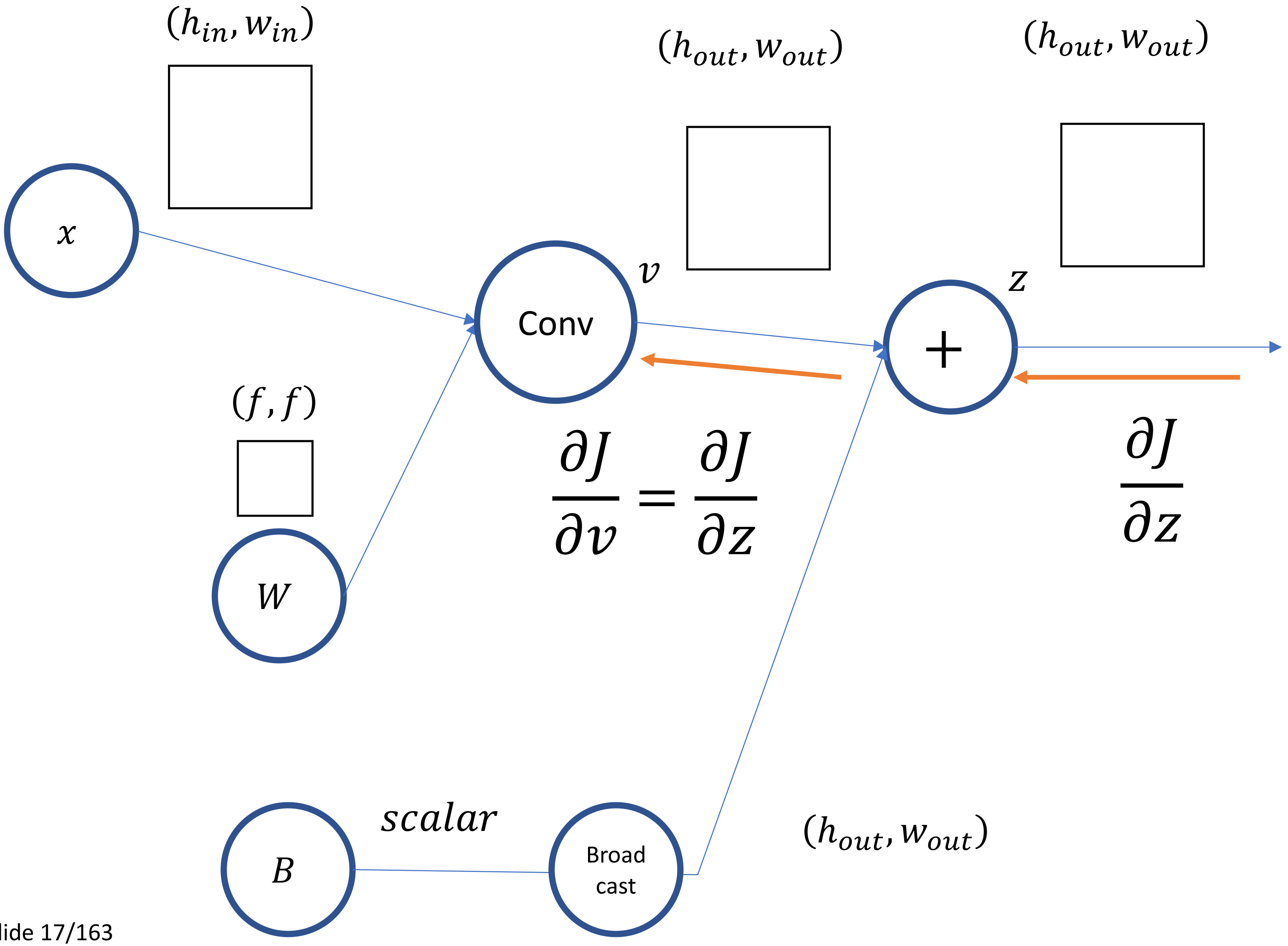


Start by considering:

- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)

Implied Broadcasting here.
Discussed in Lecture 12

A compute graph for a convolutional layer



Start by considering:

- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)

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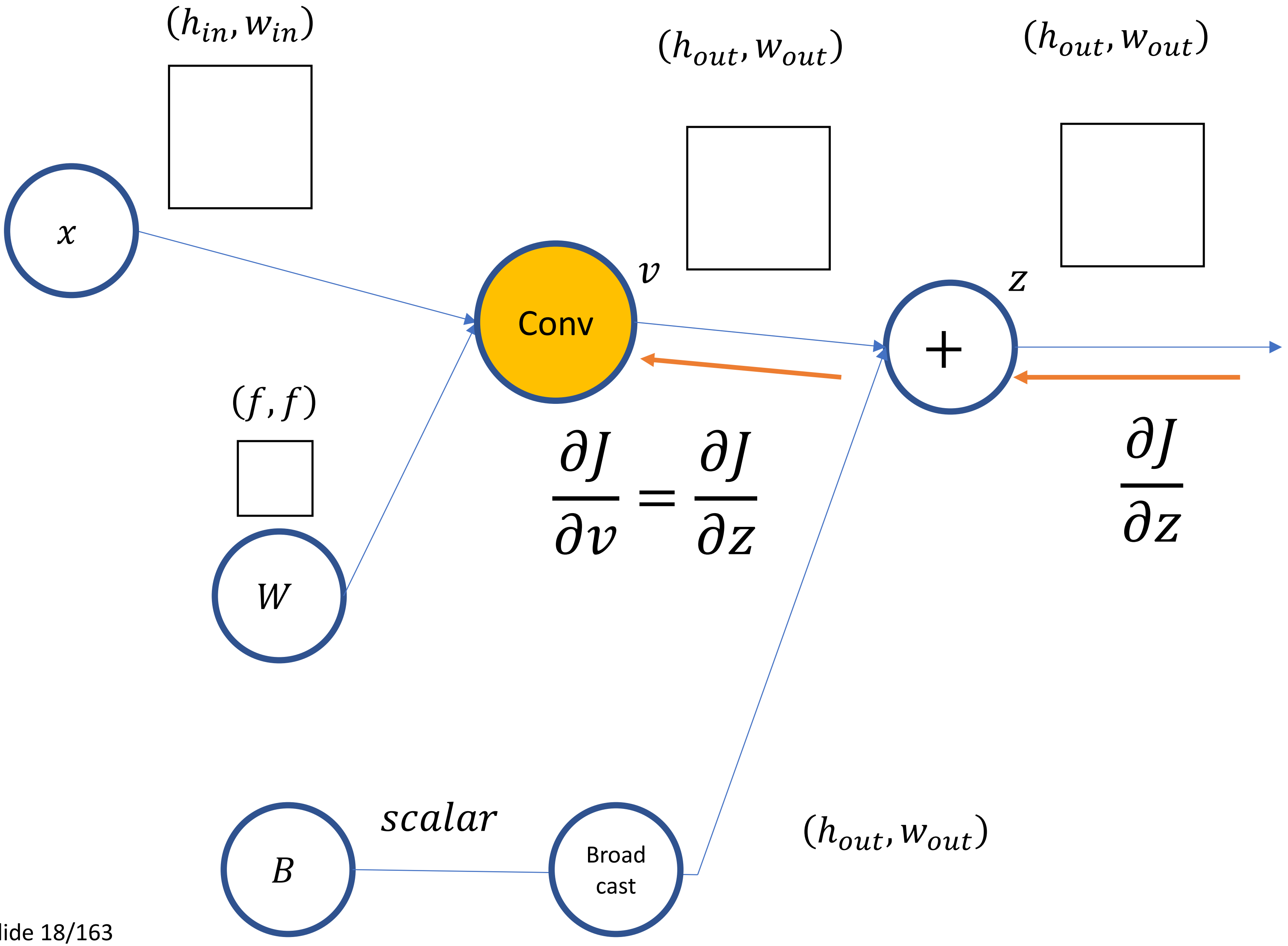
A compute graph for a convolutional layer

How to compute backprop through convolution operation?



Start by considering:

- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)

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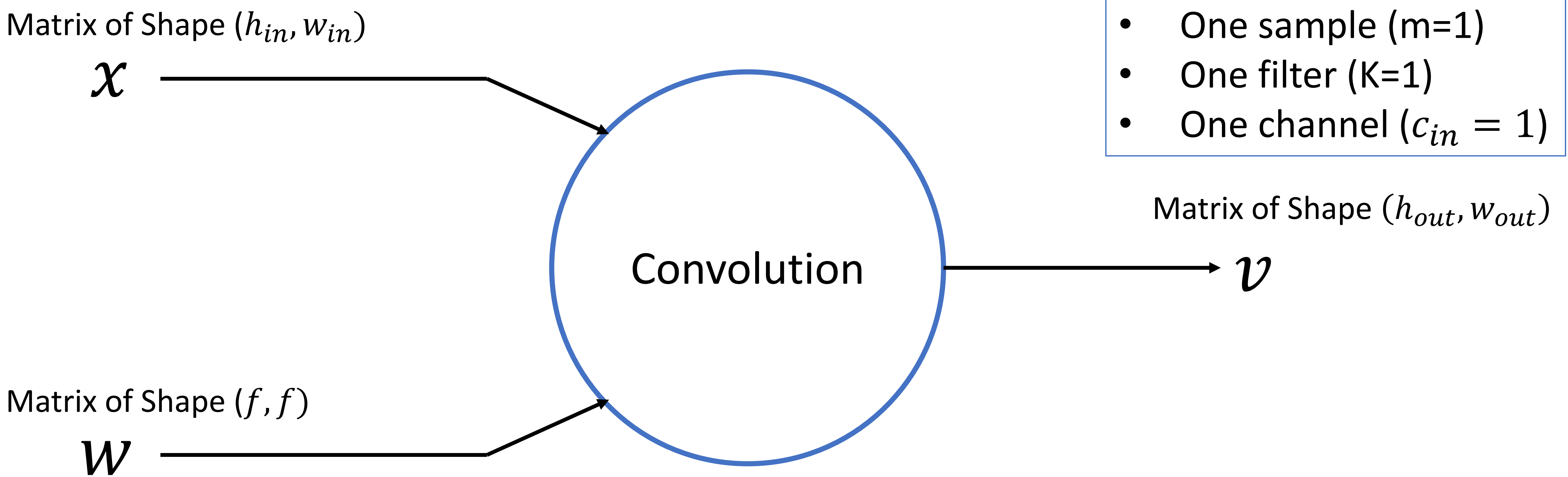
Still only considering:

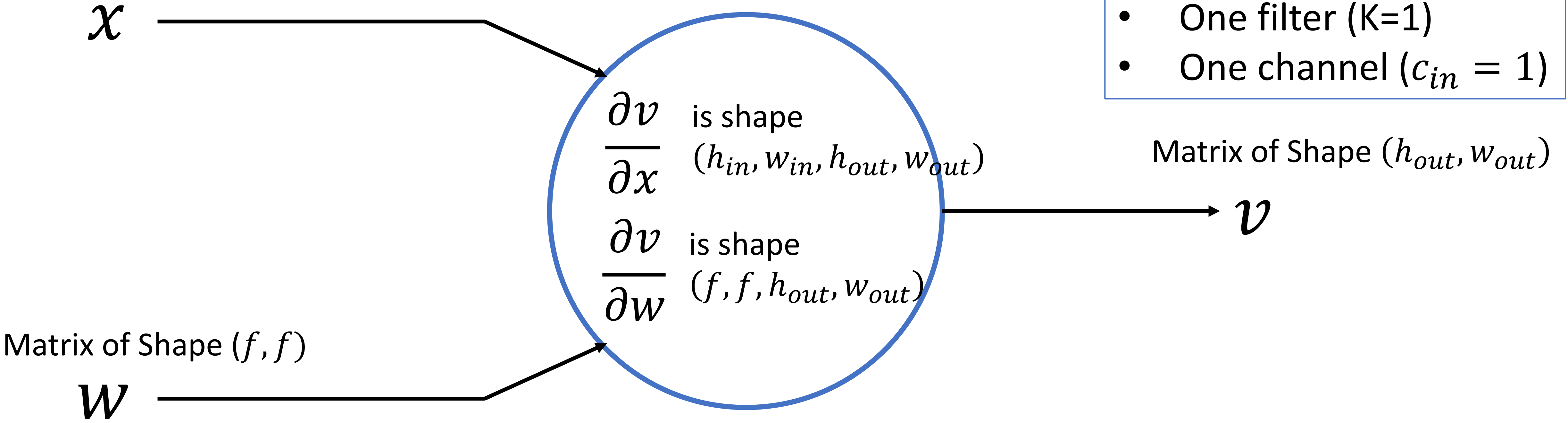
- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)



Local Derivatives are 4D Tensors

Still only considering:

- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)



Local Derivatives are 4D Tensors

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- One sample (m=1)
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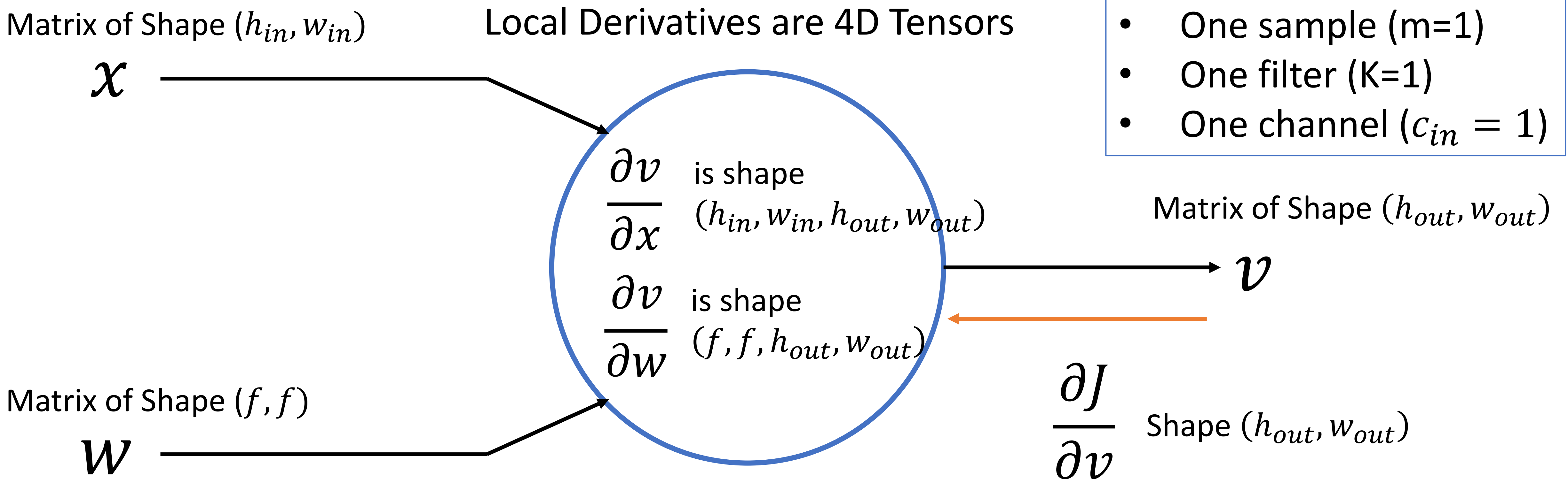

Upstream gradient is with respect to Cost J (a scalar)

i.e. How does each of the (h_{out}, w_{out}) outputs affect Cost

Local Derivatives are 4D Tensors

Still only considering:
- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)

Matrix of Shape (h_{in}, w_{in})

x

$$\frac{\partial J}{\partial x} = \frac{\partial v}{\partial x}\frac{\partial J}{\partial v}$$

$$(h_{in}, w_{in}, h_{out}, w_{out}) * (h_{out}, w_{out})$$
$$= (h_{in}, w_{in})$$

Matrix of Shape (f, f)

w

$\frac{\partial v}{\partial x}$ is shape $(h_{in}, w_{in}, h_{out}, w_{out})$

$\frac{\partial v}{\partial w}$ is shape (f, f, h_{out}, w_{out})

Matrix of Shape (h_{out}, w_{out})

v

$\frac{\partial J}{\partial v}$ Shape (h_{out}, w_{out})

Upstream gradient is with respect to Cost J (a scalar)

i.e. How does each of the (h_{out}, w_{out}) outputs affect Cost

Chain Rule application is **Tensor-Matrix Multiply**

Local Derivatives are 4D Tensors

Still only considering:
- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)

Matrix of Shape (h_{in}, w_{in})

x

$$\frac{\partial J}{\partial x} = \frac{\partial v}{\partial x}\frac{\partial J}{\partial v}$$

$$(h_{in}, w_{in}, h_{out}, w_{out}) * (h_{out}, w_{out}) = (h_{in}, w_{in})$$

Matrix of Shape (f, f)

w

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v}$$

$$(f, f, h_{out}, w_{out}) * (h_{out}, w_{out}) = (f, f)$$

$\frac{\partial v}{\partial x}$ is shape $(h_{in}, w_{in}, h_{out}, w_{out})$

$\frac{\partial v}{\partial w}$ is shape (f, f, h_{out}, w_{out})

Matrix of Shape (h_{out}, w_{out})

v

$\frac{\partial J}{\partial v}$ Shape (h_{out}, w_{out})

Upstream gradient is with respect to Cost J (a scalar)

i.e. How does each of the (h_{out}, w_{out}) outputs affect Cost

Chain Rule application is **Tensor-Matrix Multiply**

$(h_{in}, w_{in}) = (3,3)$ $\quad$ $(f, f) = (2,2)$ $\quad$ $(h_{out}, w_{out}) = (2,2)$

$$\begin{array}{|c|c|c|}\hline x_{11} & x_{12} & x_{13} \\ \hline x_{21} & x_{22} & x_{23} \\ \hline x_{31} & x_{32} & x_{33} \\ \hline \end{array} \quad \text{conv} \quad \begin{array}{|c|c|}\hline w_{11} & w_{12} \\ \hline w_{21} & w_{22} \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|}\hline v_{11} & v_{12} \\ \hline v_{21} & v_{22} \\ \hline \end{array}$$

$$(h_{in}, w_{in}) = (3,3)$$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

conv

$$(f, f) = (2,2)$$

$$\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

=

$$(h_{out}, w_{out}) = (2,2)$$

$$\begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

$$(h_{in}, w_{in}) = (3,3) \qquad (f, f) = (2,2) \qquad (h_{out}, w_{out}) = (2,2)$$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \ \text{conv} \ \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

$(h_{in}, w_{in}) = (3,3)$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

conv

$(f, f) = (2,2)$

$$\begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

=

$(h_{out}, w_{out}) = (2,2)$

$$\begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

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$(h_{in}, w_{in}) = (3,3)$ $\quad$ $(f, f) = (2,2)$ $\quad$ $(h_{out}, w_{out}) = (2,2)$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \text{ conv } \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

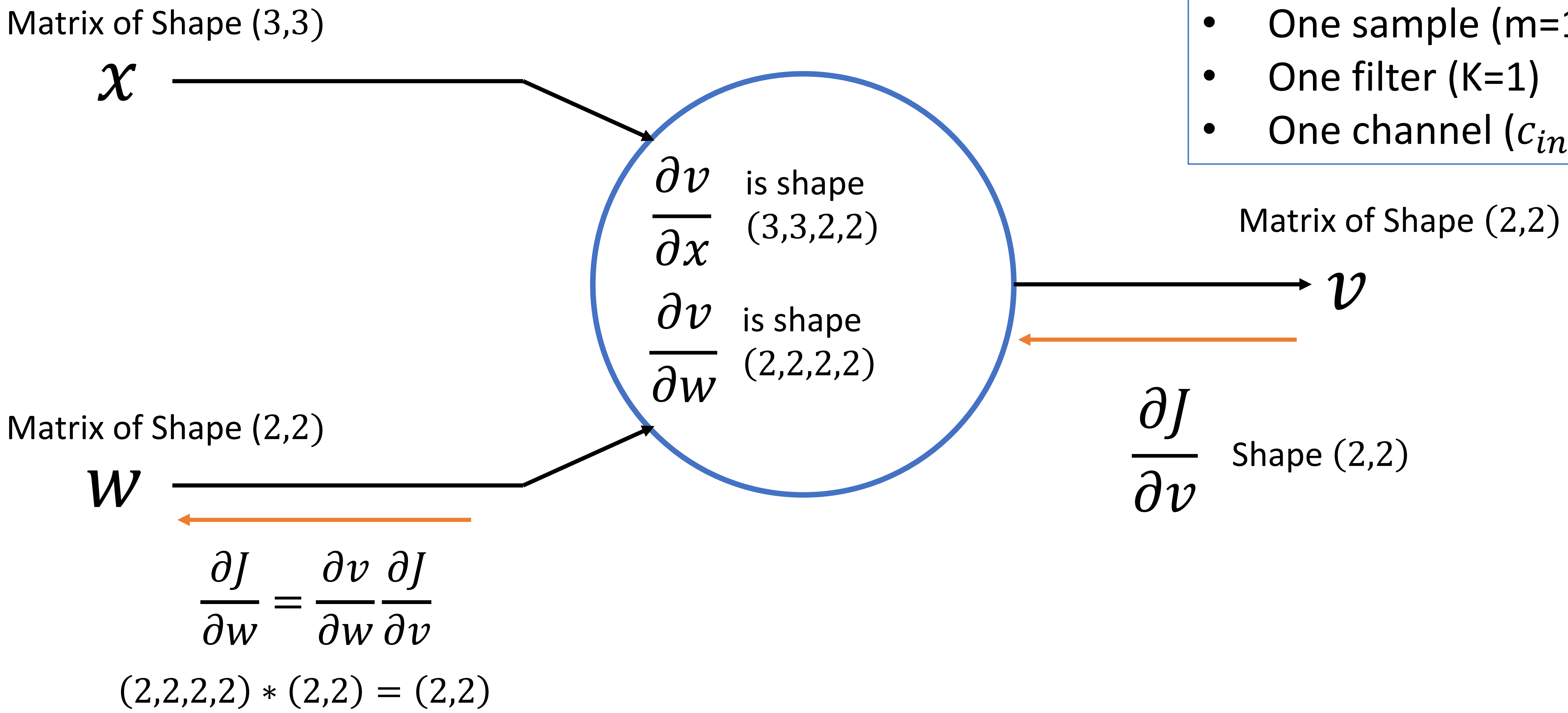
$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

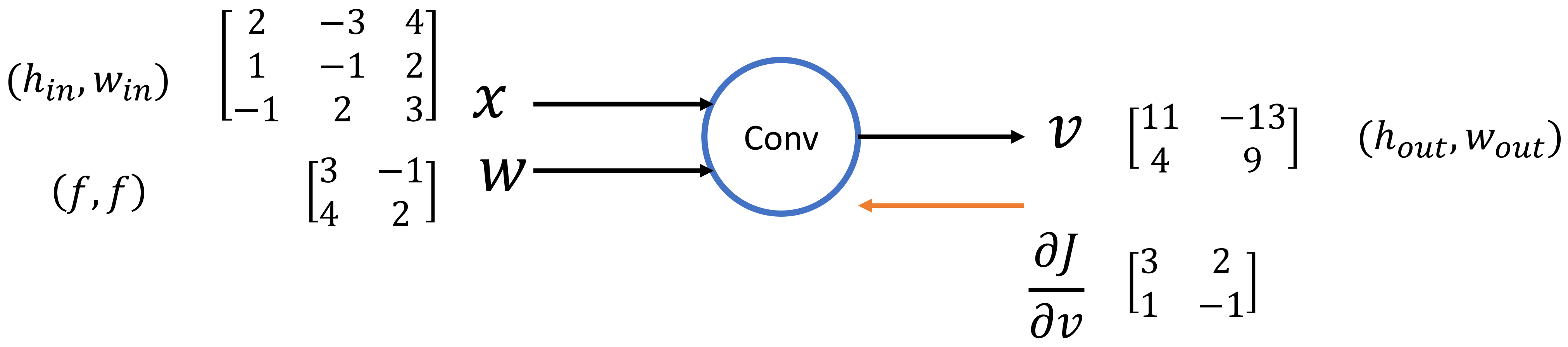
$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$

$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

Still only considering:
- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)




(h_{in}, w_{in})
$\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$
x
(f, f)
$\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$
w
Conv
v
$\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$
(h_{out}, w_{out})
$\frac{\partial J}{\partial v}$
$\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$$\frac{\partial J}{\partial w} = \boxed{\frac{\partial v}{\partial w}} \frac{\partial J}{\partial v}$$

Jacobian

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\dfrac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\dfrac{\partial v}{\partial w}$ is shape $\big((f, f), (h_{out}, w_{out})\big)$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \dfrac{\partial v}{\partial w_{11}} & \dfrac{\partial v}{\partial w_{12}} \\ \dfrac{\partial v}{\partial w_{21}} & \dfrac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $\big((f, f), (h_{out}, w_{out})\big)$

Interpretation: How does each filter weight affect each feature map element

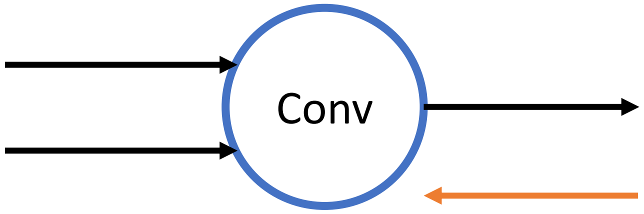
$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial w_{11}}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix} \qquad \frac{\partial v}{\partial w_{11}} = \begin{bmatrix} \frac{\partial v_{11}}{\partial w_{11}} & \frac{\partial v_{12}}{\partial w_{11}} \\ \frac{\partial v_{21}}{\partial w_{11}} & \frac{\partial v_{22}}{\partial w_{11}} \end{bmatrix}$$

(f, f) $\qquad$ (h_{out}, w_{out})

Interpretation: How does this one weight weight affect each feature map element

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial w_{11}}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} \frac{\partial v_{11}}{\partial w_{11}} & \frac{\partial v_{12}}{\partial w_{11}} \\ \frac{\partial v_{21}}{\partial w_{11}} & \frac{\partial v_{22}}{\partial w_{11}} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$
$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$
$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$
$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

Interpretation: How does this one weight weight affect each feature map element

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\dfrac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\dfrac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\dfrac{\partial v}{\partial w_{11}}} & \dfrac{\partial v}{\partial w_{12}} \\ \dfrac{\partial v}{\partial w_{21}} & \dfrac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} \boxed{\dfrac{\partial v_{11}}{\partial w_{11}}} & \dfrac{\partial v_{12}}{\partial w_{11}} \\ \dfrac{\partial v_{21}}{\partial w_{11}} & \dfrac{\partial v_{22}}{\partial w_{11}} \end{bmatrix}$$

(h_{out}, w_{out})

$$\boxed{v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}}$$

$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

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Interpretation: How does this one weight weight affect each feature map element

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial w_{11}}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} \boxed{x_{11}} & \frac{\partial v_{12}}{\partial w_{11}} \\ \frac{\partial v_{21}}{\partial w_{11}} & \frac{\partial v_{22}}{\partial w_{11}} \end{bmatrix}$$

(h_{out}, w_{out})

$$\boxed{v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}}$$

$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$

$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

Interpretation: How does this one weight weight affect each feature map element

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial w_{11}}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} x_{11} & \boxed{\frac{\partial v_{12}}{\partial w_{11}}} \\ \frac{\partial v_{21}}{\partial w_{11}} & \frac{\partial v_{22}}{\partial w_{11}} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

$$\boxed{v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}}$$

$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$

$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

$x, w \rightarrow$ Conv $\rightarrow v$

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} x_{11} & x_{12} \\ \frac{\partial v_{21}}{\partial w_{11}} & \frac{\partial v_{22}}{\partial w_{11}} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$

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Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

$x, w \xrightarrow{\text{Conv}} v$ $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial w_{11}}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} x_{11} & x_{12} \\ \boxed{\frac{\partial v_{21}}{\partial w_{11}}} & \frac{\partial v_{22}}{\partial w_{11}} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

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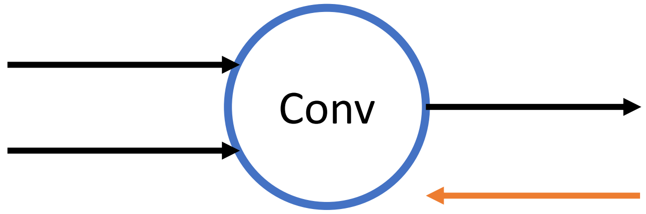
Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial w_{11}}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} x_{11} & x_{12} \\ \boxed{x_{21}} & \frac{\partial v_{22}}{\partial w_{11}} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

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Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\dfrac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\dfrac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\dfrac{\partial v}{\partial w_{11}}} & \dfrac{\partial v}{\partial w_{12}} \\ \dfrac{\partial v}{\partial w_{21}} & \dfrac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & \boxed{\dfrac{\partial v_{22}}{\partial w_{11}}} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

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Interpretation: How does this one weight weight affect each feature map element

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial w_{11}}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & \boxed{x_{22}} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

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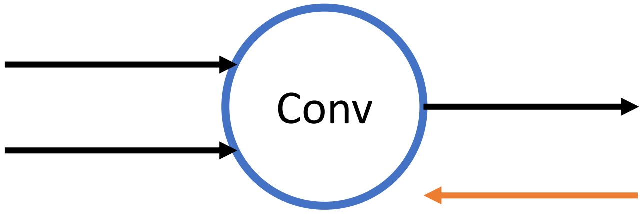
Interpretation: How does this one weight weight affect each feature map element

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial w_{11}}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

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$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x → Conv → v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w → Conv

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial w_{11}}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$

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Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \boxed{\frac{\partial v}{\partial w_{12}}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{12}} = \begin{bmatrix} \frac{\partial v_{11}}{\partial w_{12}} & \frac{\partial v_{12}}{\partial w_{12}} \\ \frac{\partial v_{21}}{\partial w_{12}} & \frac{\partial v_{22}}{\partial w_{12}} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$

$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$

$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

$x, w \longrightarrow$ Conv $\longrightarrow v$ $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\dfrac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\dfrac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \boxed{\frac{\partial v}{\partial w_{12}}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{12}} = \begin{bmatrix} \frac{\partial v_{11}}{\partial w_{12}} & \frac{\partial v_{12}}{\partial w_{12}} \\ \frac{\partial v_{21}}{\partial w_{12}} & \frac{\partial v_{22}}{\partial w_{12}} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$
$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$
$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$
$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \boxed{\frac{\partial v}{\partial w_{12}}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{12}} = \begin{bmatrix} x_{12} & x_{13} \\ x_{22} & x_{23} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + \boxed{w_{12}x_{12}} + w_{21}x_{21} + w_{22}x_{22}$$
$$v_{12} = w_{11}x_{12} + \boxed{w_{12}x_{13}} + w_{21}x_{22} + w_{22}x_{23}$$
$$v_{21} = w_{11}x_{21} + \boxed{w_{12}x_{22}} + w_{21}x_{31} + w_{22}x_{31}$$
$$v_{22} = w_{11}x_{22} + \boxed{w_{12}x_{23}} + w_{21}x_{32} + w_{22}x_{33}$$

Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x $\longrightarrow$ Conv $\longrightarrow$ v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w $\longrightarrow$ Conv

$\dfrac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\dfrac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \boxed{\frac{\partial v}{\partial w_{12}}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{12}} = \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + \boxed{w_{12}x_{12}} + w_{21}x_{21} + w_{22}x_{22}$$
$$v_{12} = w_{11}x_{12} + \boxed{w_{12}x_{13}} + w_{21}x_{22} + w_{22}x_{23}$$
$$v_{21} = w_{11}x_{21} + \boxed{w_{12}x_{22}} + w_{21}x_{31} + w_{22}x_{31}$$
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Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $\big((f, f), (h_{out}, w_{out})\big)$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \frac{\partial v}{\partial w_{12}} \\ \boxed{\frac{\partial v}{\partial w_{21}}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{12}} = \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{21}} = \begin{bmatrix} x_{21} & x_{22} \\ x_{31} & x_{32} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + \boxed{w_{21}x_{21}} + w_{22}x_{22}$$

$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + \boxed{w_{21}x_{22}} + w_{22}x_{23}$$

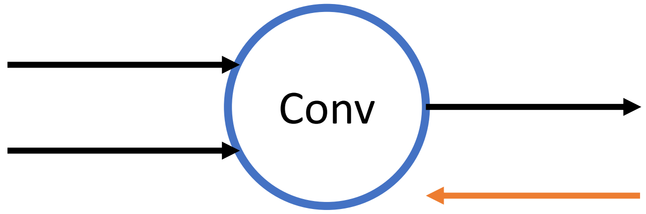
$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + \boxed{w_{21}x_{31}} + w_{22}x_{31}$$

$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + \boxed{w_{21}x_{32}} + w_{22}x_{33}$$

Interpretation: How does this one weight weight affect each feature map element

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $\left((f, f), (h_{out}, w_{out})\right)$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \frac{\partial v}{\partial w_{12}} \\ \boxed{\frac{\partial v}{\partial w_{21}}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{12}} = \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{21}} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

(h_{out}, w_{out})

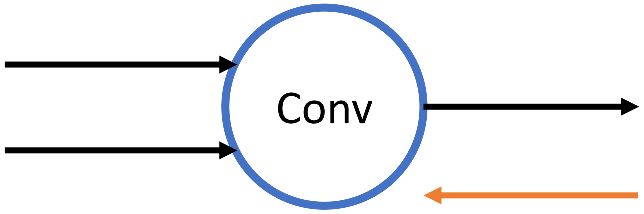
$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$
$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$
$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$
$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

Interpretation: How does this one weight weight affect each feature map element

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \boxed{\frac{\partial v}{\partial w_{22}}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{12}} = \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{21}} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{22}} = \begin{bmatrix} x_{22} & x_{23} \\ x_{32} & x_{33} \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$
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$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

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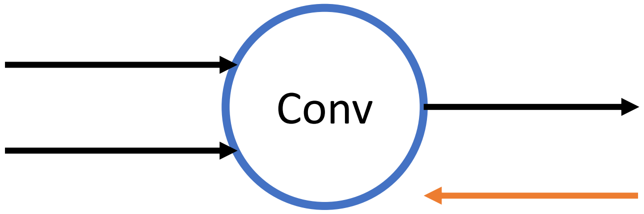
(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \boxed{\frac{\partial v}{\partial w_{22}}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{12}} = \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{21}} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{22}} = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$
$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$
$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$
$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

Brad Quinton, Scott Chin

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

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$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{12}} = \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{21}} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{22}} = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$$

(h_{out}, w_{out})

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $\big((f, f), (h_{out}, w_{out})\big)$

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$$\frac{\partial v}{\partial w} = \begin{bmatrix} \frac{\partial v}{\partial w_{11}} & \frac{\partial v}{\partial w_{12}} \\ \frac{\partial v}{\partial w_{21}} & \frac{\partial v}{\partial w_{22}} \end{bmatrix}$$

(f, f)

$$\frac{\partial v}{\partial w_{11}} = \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$$

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$$\frac{\partial v}{\partial w_{21}} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\frac{\partial v}{\partial w_{22}} = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$$

(h_{out}, w_{out})

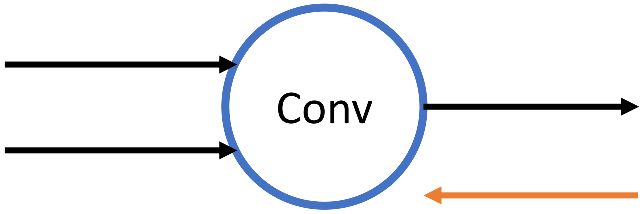
Can write the Jacobian this way

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix}$$

$\big((f, f), (h_{out}, w_{out})\big)$

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix}$$

$((f, f), (h_{out}, w_{out}))$

Now we have the full Jacobian

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f, f), (h_{out}, w_{out}))$

Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix}$$

$((f, f), (h_{out}, w_{out}))$

As expected, Jacobian can get big.
E.g. Second Conv layer of AlexNet
(5,5,256,27,27) ~4.7million numbers

Now we have the full Jacobian

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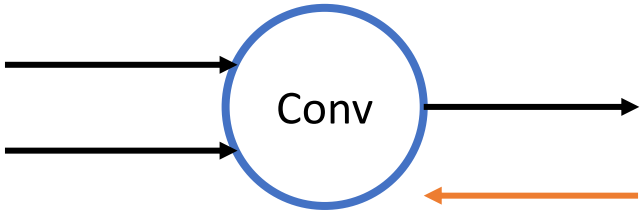
(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial w}$ is shape $((f,f),(h_{out},w_{out}))$

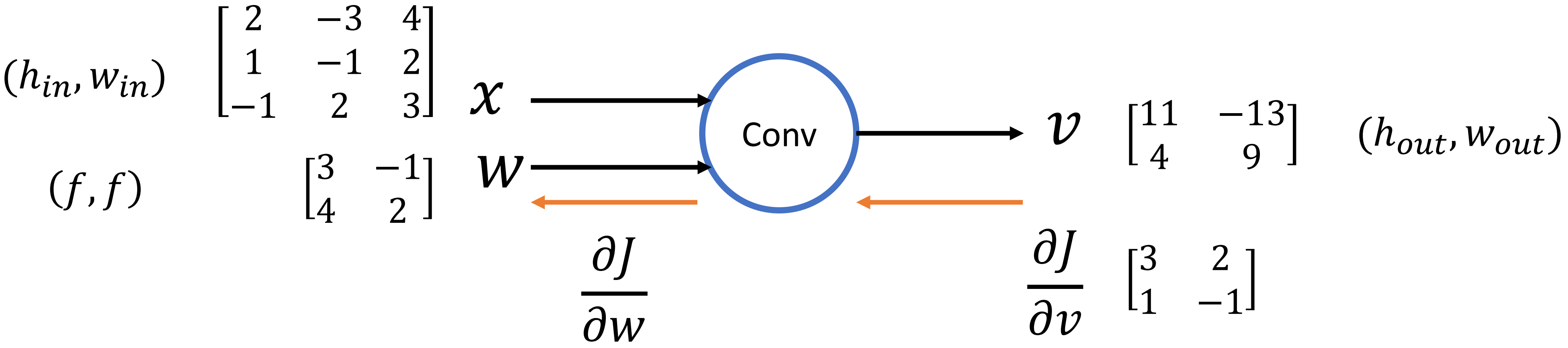
Interpretation: How does each filter weight affect each feature map element

$$\frac{\partial v}{\partial w} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix}$$

$((f,f),(h_{out},w_{out}))$

- For fully-connected layer, Jacobian had a lot of 0's. (Each neuron has own set of weights. Therefore they do not affect the output of other neurons)
- For conv layer every weight affects every output. Parameter sharing)

Now we have the full Jacobian



Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

$((f, f), (h_{out}, w_{out}))$ (h_{out}, w_{out}) (f, f)

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial w}$

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

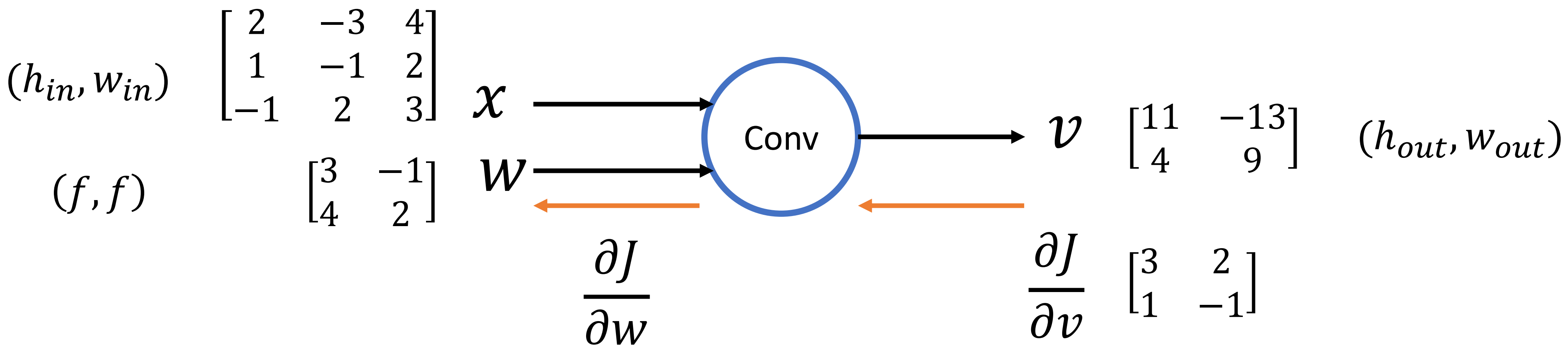
Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \boxed{12} & ? \\ ? & ? \end{bmatrix}$$

$((f, f), (h_{out}, w_{out}))$ (h_{out}, w_{out}) (f, f)

$$\frac{\partial J}{\partial w_{11}} = 2 * 3 + (-3) * (-2) + 1 * 1 + (-1)(-1) = 12$$

Recall: Computing an Inner product
(i.e. sum of elementwise multiply)



Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \underbrace{\begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix}}_{((f,f),(h_{out}, w_{out}))} \underbrace{\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}}_{(h_{out}, w_{out})} = \underbrace{\begin{bmatrix} 12 & \boxed{-20} \\ ? & ? \end{bmatrix}}_{(f,f)}$$

$$\frac{\partial J}{\partial w_{12}} = -3 * 3 + 4 * (-2) + (-1) * 1 + (2)(-1) = -20$$

Recall: Computing an Inner product
(i.e. sum of elementwise multiply)

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial w}$

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ \boxed{2} & ? \end{bmatrix}$$

$((f, f), (h_{out}, w_{out}))$ (h_{out}, w_{out}) (f, f)

$$\frac{\partial J}{\partial w_{21}} = 1 * 3 + (-1) * (-2) + (-1) * 1 + (2)(-1) = 2$$

Recall: Computing an Inner product
(i.e. sum of elementwise multiply)

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ $x \rightarrow$ Conv $\rightarrow v$ $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ $w \rightarrow$ Conv

$\frac{\partial J}{\partial w} \leftarrow$ Conv $\leftarrow \frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ 2 & \boxed{-8} \end{bmatrix}$$

$((f, f), (h_{out}, w_{out}))$ (h_{out}, w_{out}) (f, f)

$$\frac{\partial J}{\partial w_{22}} = -1 * 3 + 2 * (-2) + 2 * 1 + 3 * (-1) = -8$$

Recall: Computing an Inner product
(i.e. sum of elementwise multiply)

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial w}$

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \underbrace{\begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix}}_{((f,f),(h_{out}, w_{out}))} \underbrace{\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}}_{(h_{out}, w_{out})} = \underbrace{\begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}}_{(f,f)}$$

Notice any patterns to this Jacobian?

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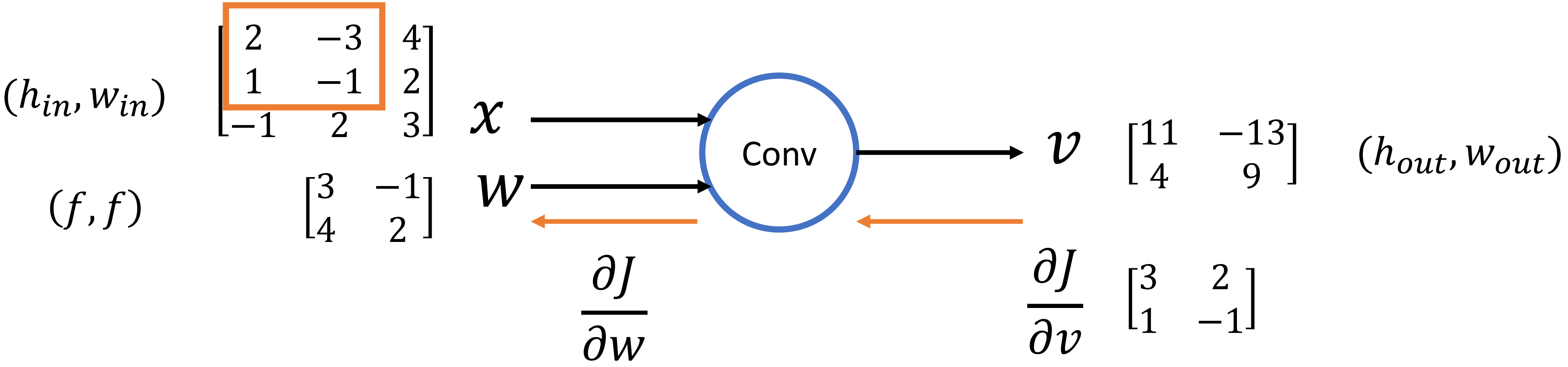

Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \underbrace{\begin{bmatrix} \boxed{\begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix}}_{((f,f),(h_{out},w_{out}))} \underbrace{\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}}_{(h_{out},w_{out})} = \underbrace{\begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}}_{(f,f)}$$

Notice any patterns to this Jacobian?

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ $x \longrightarrow$ Conv $\longrightarrow v$ $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ $w \longrightarrow$ Conv

$\dfrac{\partial J}{\partial w} \longleftarrow$ Conv $\longleftarrow \dfrac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \boxed{\begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix}} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}$$

$$\big((f, f), (h_{out}, w_{out})\big) \quad (h_{out}, w_{out}) \quad (f, f)$$

Notice any patterns to this Jacobian?

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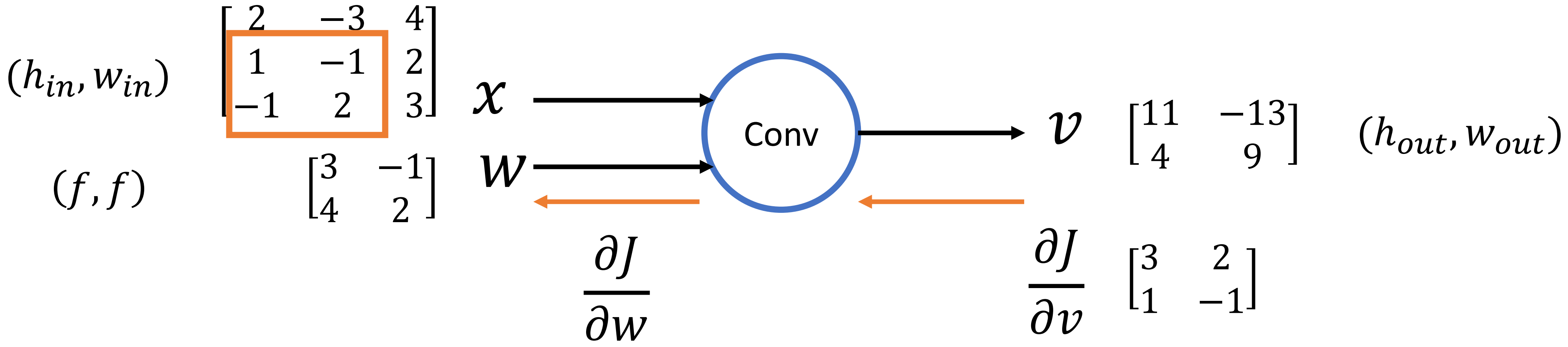

Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \boxed{\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}$$

$\big((f, f), (h_{out}, w_{out})\big)$ (h_{out}, w_{out}) (f, f)

Notice any patterns to this Jacobian?

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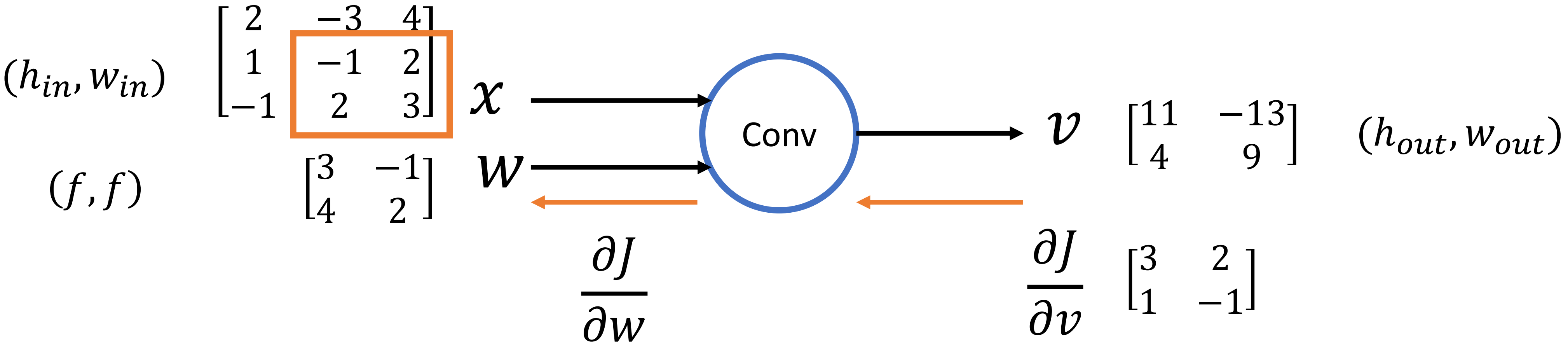

Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \boxed{\begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}$$

$((f, f), (h_{out}, w_{out}))$ $\quad (h_{out}, w_{out})$ $\quad (f, f)$

Notice any patterns to this Jacobian?

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(h_{in}, w_{in}) $\begin{bmatrix}2 & -3 & 4\\ 1 & -1 & 2\\ -1 & 2 & 3\end{bmatrix}$ x

(f, f) $\begin{bmatrix}3 & -1\\ 4 & 2\end{bmatrix}$ w

Conv

v $\begin{bmatrix}11 & -13\\ 4 & 9\end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial w}$

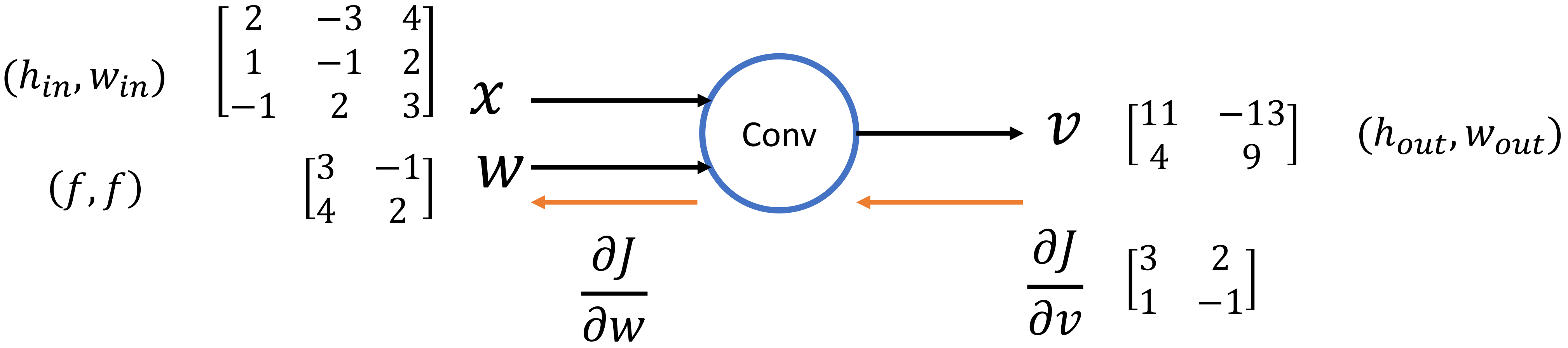
$\frac{\partial J}{\partial v}$ $\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix}$

Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix}\begin{bmatrix}2 & -3\\ 1 & -1\end{bmatrix} & \begin{bmatrix}-3 & 4\\ -1 & 2\end{bmatrix}\\ \begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix} & \boxed{\begin{bmatrix}-1 & 2\\ 2 & 3\end{bmatrix}}\end{bmatrix}\begin{bmatrix}3 & -2\\ 1 & -1\end{bmatrix} = \begin{bmatrix}12 & -20\\ 2 & -8\end{bmatrix}$$

$((f, f), (h_{out}, w_{out}))$ (h_{out}, w_{out}) (f, f)

Each Jacobian slice is a sliding window over the input x



Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}$$

$((f, f), (h_{out}, w_{out}))$ $\quad (h_{out}, w_{out})$ $\quad (f, f)$

We then perform elementwise multiply and sum of each jacobian slice with upstream gradient. What is this operation?

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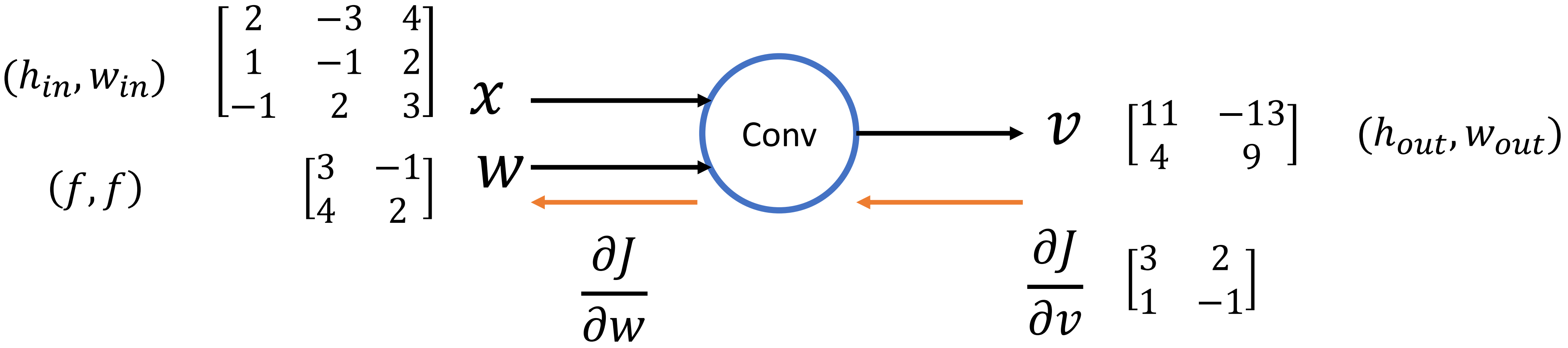

Apply chain rule to compute weight gradients

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \underbrace{\begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix}}_{((f,f),(h_{out},w_{out}))} \underbrace{\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}}_{(h_{out},w_{out})} = \underbrace{\begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}}_{(f,f)}$$

It’s a convolution operation!

$$\frac{\partial J}{\partial w} = x \text{ conv} \frac{\partial J}{\partial v}$$

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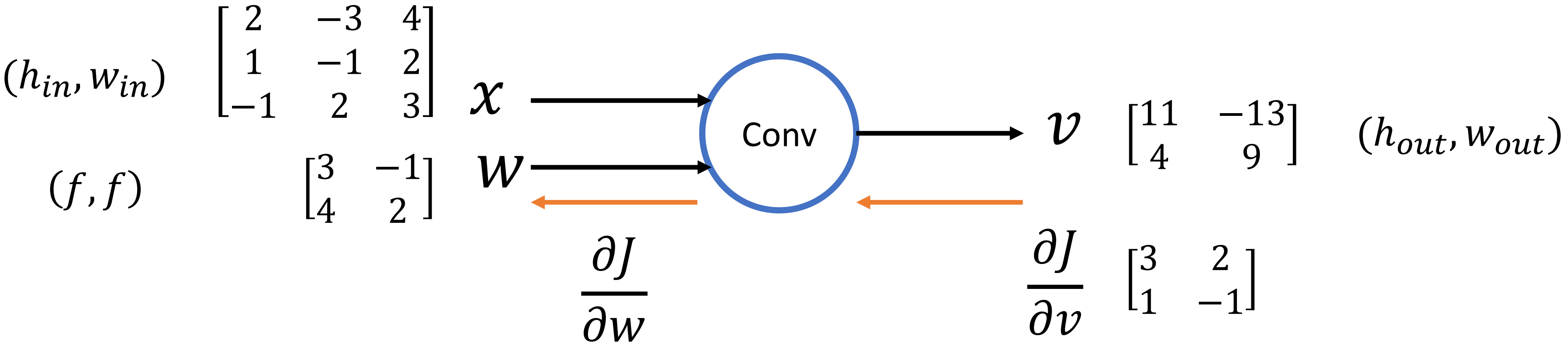

Instead of using Jacobian

$$\frac{\partial J}{\partial w} = x \text{ conv} \frac{\partial J}{\partial v} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}$$

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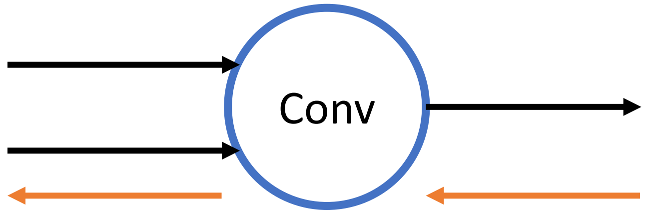
(h_{in}, w_{in}) $\begin{bmatrix} \boxed{\begin{matrix} 2 & -3 \\ 1 & -1 \end{matrix}} & \begin{matrix} 4 \\ 2 \end{matrix} \\ \begin{matrix} -1 & 2 \end{matrix} & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



$\dfrac{\partial J}{\partial w}$

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\dfrac{\partial J}{\partial v}$ $\boxed{\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}}$

Instead of using Jacobian

$$\frac{\partial J}{\partial w} = x \text{ conv} \frac{\partial J}{\partial v} = \begin{bmatrix} \boxed{12} & ? \\ ? & ? \end{bmatrix}$$

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}$$

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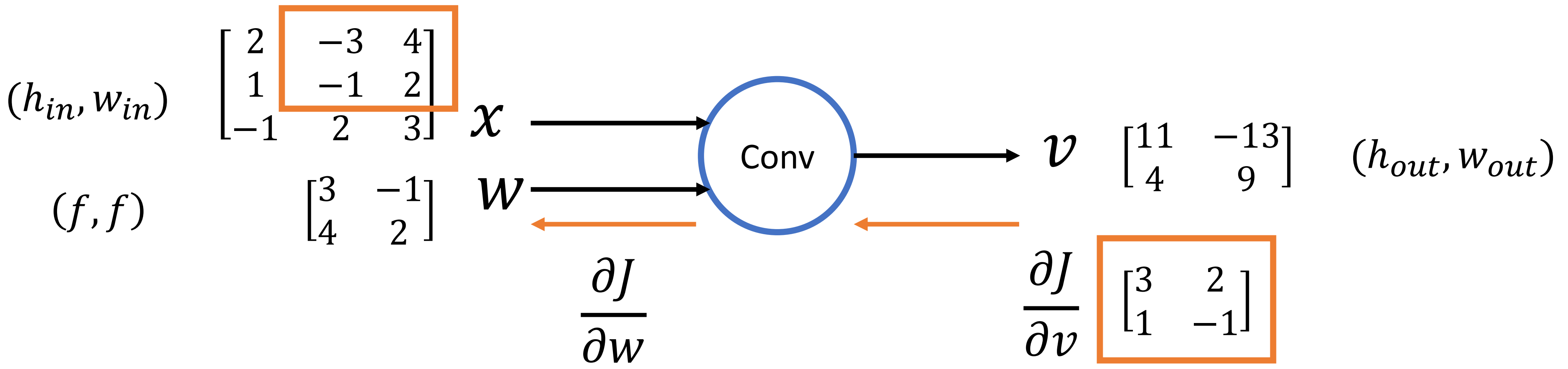
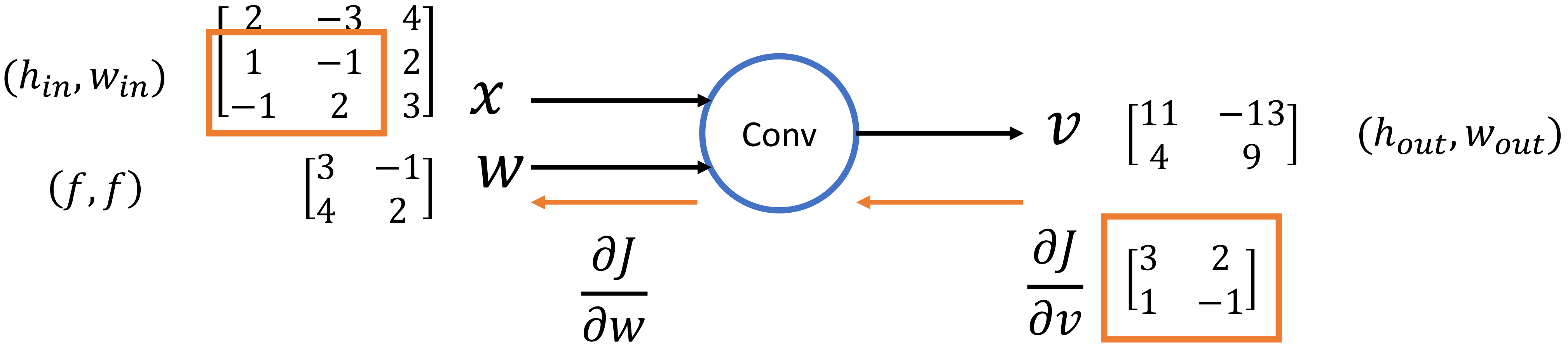


Instead of using Jacobian

$$\frac{\partial J}{\partial w} = x \text{ conv} \frac{\partial J}{\partial v} = \begin{bmatrix} 12 & \boxed{-20} \\ ? & ? \end{bmatrix}$$

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w} \frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}$$



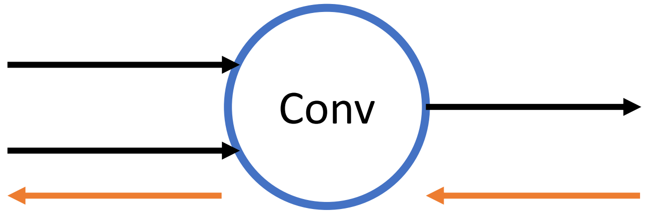
Instead of using Jacobian

$$\frac{\partial J}{\partial w} = x \text{ conv} \frac{\partial J}{\partial v} = \begin{bmatrix} 12 & -20 \\ \boxed{2} & ? \end{bmatrix}$$

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}$$

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

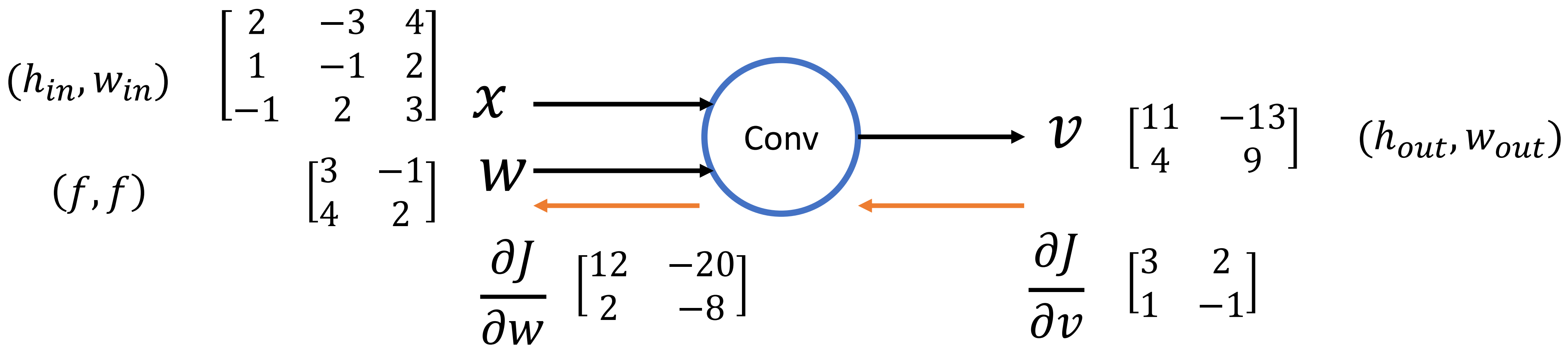
$\frac{\partial J}{\partial w}$

$\frac{\partial J}{\partial v}$ $\boxed{\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}}$

Instead of using Jacobian

$$\frac{\partial J}{\partial w} = x \text{ conv} \frac{\partial J}{\partial v} = \begin{bmatrix} 12 & -20 \\ 2 & \boxed{-8} \end{bmatrix}$$

$$\frac{\partial J}{\partial w} = \frac{\partial v}{\partial w} \frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} & \begin{bmatrix} -3 & 4 \\ -1 & 2 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} & \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 12 & -20 \\ 2 & -8 \end{bmatrix}$$



$$\boxed{\frac{\partial J}{\partial w} = x \text{ conv} \frac{\partial J}{\partial v}}$$

- No Jacobian require at all!
- This convolution yields same result as doing the full 4D-Tensor Jacobian and upstream gradient matrix multiply!
- You will implement this in Assignment 4

Still only considering:

- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)

Matrix of Shape (3,3)

x

$$\frac{\partial J}{\partial x} = \frac{\partial v}{\partial x}\frac{\partial J}{\partial v}$$

$$(3,3,2,2) * (2,2) = (3,3,\ )$$

$\frac{\partial v}{\partial x}$ is shape $(3,3,2,2)$

$\frac{\partial v}{\partial w}$ is shape $(2,2,2,2)$

Matrix of Shape (2,2)

v

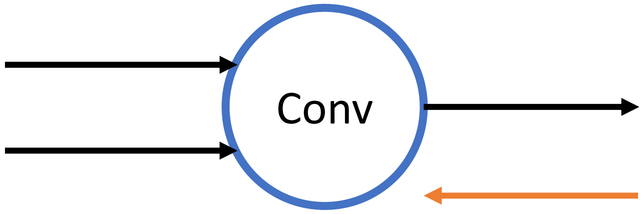
$\frac{\partial J}{\partial v}$ Shape $(2,2)$

Matrix of Shape (2,2)

w

(h_{in}, w_{in}) $\begin{bmatrix}2 & -3 & 4\\ 1 & -1 & 2\\ -1 & 2 & 3\end{bmatrix}$ x

(f, f) $\begin{bmatrix}3 & -1\\ 4 & 2\end{bmatrix}$ w



v $\begin{bmatrix}11 & -13\\ 4 & 9\end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix}$

$$\frac{\partial J}{\partial x} = \boxed{\frac{\partial v}{\partial x}}\frac{\partial J}{\partial v}$$

Jacobian

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

$$x, w \longrightarrow \text{Conv} \longrightarrow v$$

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

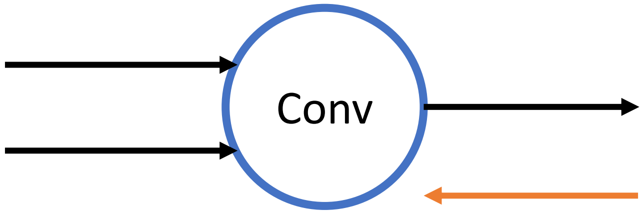
$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial x}$ is shape $((h_{in}, w_{in}) \ , (h_{out}, w_{out}))$

Interpretation: How does each input value affect each feature map element

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial x}$ is shape $((h_{in}, w_{in}) \; , (h_{out}, w_{out}))$

Interpretation: How does each input value affect each feature map element

$$\frac{\partial v}{\partial x} = \begin{bmatrix} \frac{\partial v}{\partial x_{11}} & \frac{\partial v}{\partial x_{12}} & \frac{\partial v}{\partial x_{13}} \\ \frac{\partial v}{\partial x_{21}} & \frac{\partial v}{\partial x_{22}} & \frac{\partial v}{\partial x_{23}} \\ \frac{\partial v}{\partial x_{31}} & \frac{\partial v}{\partial x_{32}} & \frac{\partial v}{\partial x_{33}} \end{bmatrix}$$

(h_{in}, w_{in})

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial x}$ is shape $((h_{in}, w_{in}) \ , (h_{out}, w_{out}))$

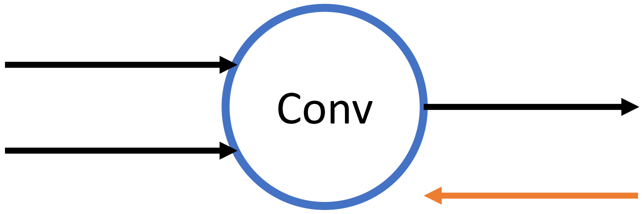
$$\frac{\partial v}{\partial x} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial x_{11}}} & \frac{\partial v}{\partial x_{12}} & \frac{\partial v}{\partial x_{13}} \\ \frac{\partial v}{\partial x_{21}} & \frac{\partial v}{\partial x_{22}} & \frac{\partial v}{\partial x_{23}} \\ \frac{\partial v}{\partial x_{31}} & \frac{\partial v}{\partial x_{32}} & \frac{\partial v}{\partial x_{33}} \end{bmatrix} \qquad \frac{\partial v}{\partial x_{11}} = \begin{bmatrix} \frac{\partial v_{11}}{\partial x_{11}} & \frac{\partial v_{12}}{\partial x_{11}} \\ \frac{\partial v_{21}}{\partial x_{11}} & \frac{\partial v_{22}}{\partial x_{11}} \end{bmatrix}$$

(h_{in}, w_{in}) (h_{out}, w_{out})

Interpretation: How does this one in put value affect each feature map element

(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial x}$ is shape $((h_{in}, w_{in}), (h_{out}, w_{out}))$

$$\frac{\partial v}{\partial x} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial x_{11}}} & \frac{\partial v}{\partial x_{12}} & \frac{\partial v}{\partial x_{13}} \\ \frac{\partial v}{\partial x_{21}} & \frac{\partial v}{\partial x_{22}} & \frac{\partial v}{\partial x_{23}} \\ \frac{\partial v}{\partial x_{31}} & \frac{\partial v}{\partial x_{32}} & \frac{\partial v}{\partial x_{33}} \end{bmatrix} \qquad \frac{\partial v}{\partial x_{11}} = \begin{bmatrix} \frac{\partial v_{11}}{\partial x_{11}} & \frac{\partial v_{12}}{\partial x_{11}} \\ \frac{\partial v_{21}}{\partial x_{11}} & \frac{\partial v_{22}}{\partial x_{11}} \end{bmatrix}$$

(h_{in}, w_{in}) (h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$
$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$
$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$
$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

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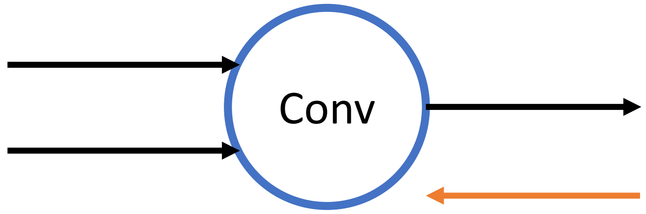
(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial x}$ is shape $((h_{in}, w_{in}), (h_{out}, w_{out}))$

$$\frac{\partial v}{\partial x} = \begin{bmatrix} \boxed{\frac{\partial v}{\partial x_{11}}} & \frac{\partial v}{\partial x_{12}} & \frac{\partial v}{\partial x_{13}} \\ \frac{\partial v}{\partial x_{21}} & \frac{\partial v}{\partial x_{22}} & \frac{\partial v}{\partial x_{23}} \\ \frac{\partial v}{\partial x_{31}} & \frac{\partial v}{\partial x_{32}} & \frac{\partial v}{\partial x_{33}} \end{bmatrix}$$

(h_{in}, w_{in})

$$\frac{\partial v}{\partial x_{11}} = \begin{bmatrix} w_{11} & 0 \\ 0 & 0 \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = \boxed{w_{11}x_{11}} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$
$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$
$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$
$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

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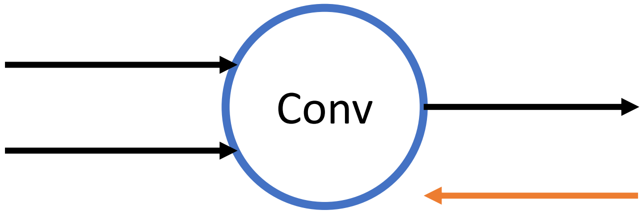
(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\dfrac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\dfrac{\partial v}{\partial x}$ is shape $((h_{in}, w_{in})\ , (h_{out}, w_{out}))$

$$\frac{\partial v}{\partial x} = \begin{bmatrix} \frac{\partial v}{\partial x_{11}} & \boxed{\frac{\partial v}{\partial x_{12}}} & \frac{\partial v}{\partial x_{13}} \\ \frac{\partial v}{\partial x_{21}} & \frac{\partial v}{\partial x_{22}} & \frac{\partial v}{\partial x_{23}} \\ \frac{\partial v}{\partial x_{31}} & \frac{\partial v}{\partial x_{32}} & \frac{\partial v}{\partial x_{33}} \end{bmatrix}$$

(h_{in}, w_{in})

$$\frac{\partial v}{\partial x_{12}} = \begin{bmatrix} w_{12} & w_{11} \\ 0 & 0 \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + \boxed{w_{12}x_{12}} + w_{21}x_{21} + w_{22}x_{22}$$

$$v_{12} = \boxed{w_{11}x_{12}} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$

$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial x}$ is shape $((h_{in}, w_{in}), (h_{out}, w_{out}))$

$$\frac{\partial v}{\partial x} = \begin{bmatrix} \frac{\partial v}{\partial x_{11}} & \frac{\partial v}{\partial x_{12}} & \boxed{\frac{\partial v}{\partial x_{13}}} \\ \frac{\partial v}{\partial x_{21}} & \frac{\partial v}{\partial x_{22}} & \frac{\partial v}{\partial x_{23}} \\ \frac{\partial v}{\partial x_{31}} & \frac{\partial v}{\partial x_{32}} & \frac{\partial v}{\partial x_{33}} \end{bmatrix}$$

(h_{in}, w_{in})

$$\frac{\partial v}{\partial x_{13}} = \begin{bmatrix} 0 & w_{12} \\ 0 & 0 \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + w_{22}x_{22}$$
$$v_{12} = w_{11}x_{12} + \boxed{w_{12}x_{13}} + w_{21}x_{22} + w_{22}x_{23}$$
$$v_{21} = w_{11}x_{21} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$
$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial x}$ is shape $((h_{in}, w_{in}) \ , (h_{out}, w_{out}))$

$$\frac{\partial v}{\partial x} = \begin{bmatrix} \frac{\partial v}{\partial x_{11}} & \frac{\partial v}{\partial x_{12}} & \frac{\partial v}{\partial x_{13}} \\ \boxed{\frac{\partial v}{\partial x_{21}}} & \frac{\partial v}{\partial x_{22}} & \frac{\partial v}{\partial x_{23}} \\ \frac{\partial v}{\partial x_{31}} & \frac{\partial v}{\partial x_{32}} & \frac{\partial v}{\partial x_{33}} \end{bmatrix}$$

(h_{in}, w_{in})

$$\frac{\partial v}{\partial x_{21}} = \begin{bmatrix} w_{21} & 0 \\ w_{11} & 0 \end{bmatrix}$$

(h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + \boxed{w_{21}x_{21}} + w_{22}x_{22}$$

$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + w_{21}x_{22} + w_{22}x_{23}$$

$$v_{21} = \boxed{w_{11}x_{21}} + w_{12}x_{22} + w_{21}x_{31} + w_{22}x_{31}$$

$$v_{22} = w_{11}x_{22} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

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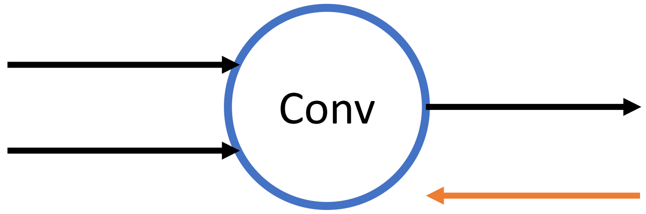
(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial x}$ is shape $((h_{in}, w_{in}) \ , (h_{out}, w_{out}))$

$$\frac{\partial v}{\partial x} = \begin{bmatrix} \frac{\partial v}{\partial x_{11}} & \frac{\partial v}{\partial x_{12}} & \frac{\partial v}{\partial x_{13}} \\ \frac{\partial v}{\partial x_{21}} & \boxed{\frac{\partial v}{\partial x_{22}}} & \frac{\partial v}{\partial x_{23}} \\ \frac{\partial v}{\partial x_{31}} & \frac{\partial v}{\partial x_{32}} & \frac{\partial v}{\partial x_{33}} \end{bmatrix}$$

$$\frac{\partial v}{\partial x_{22}} = \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix}$$

(h_{in}, w_{in}) (h_{out}, w_{out})

$$v_{11} = w_{11}x_{11} + w_{12}x_{12} + w_{21}x_{21} + \boxed{w_{22}x_{22}}$$
$$v_{12} = w_{11}x_{12} + w_{12}x_{13} + \boxed{w_{21}x_{22}} + w_{22}x_{23}$$
$$v_{21} = w_{11}x_{21} + \boxed{w_{12}x_{22}} + w_{21}x_{31} + w_{22}x_{31}$$
$$v_{22} = \boxed{w_{11}x_{22}} + w_{12}x_{23} + w_{21}x_{32} + w_{22}x_{33}$$

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(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w

Conv

v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

$\frac{\partial v}{\partial x}$ is shape $((h_{in}, w_{in})\ , (h_{out}, w_{out}))$

$$\frac{\partial v}{\partial x} = \begin{bmatrix} \frac{\partial v}{\partial x_{11}} & \frac{\partial v}{\partial x_{12}} & \frac{\partial v}{\partial x_{13}} \\ \frac{\partial v}{\partial x_{21}} & \frac{\partial v}{\partial x_{22}} & \frac{\partial v}{\partial x_{23}} \\ \frac{\partial v}{\partial x_{31}} & \frac{\partial v}{\partial x_{32}} & \frac{\partial v}{\partial x_{33}} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} w_{11} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} w_{12} & w_{11} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & w_{12} \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} w_{21} & 0 \\ w_{11} & 0 \end{bmatrix} & \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} & \begin{bmatrix} 0 & w_{22} \\ 0 & w_{12} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ w_{21} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ w_{22} & w_{21} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & w_{22} \end{bmatrix} \end{bmatrix}$$

$((h_{in}, w_{in})\ , (h_{out}, w_{out}))$

This is the full Jacobian

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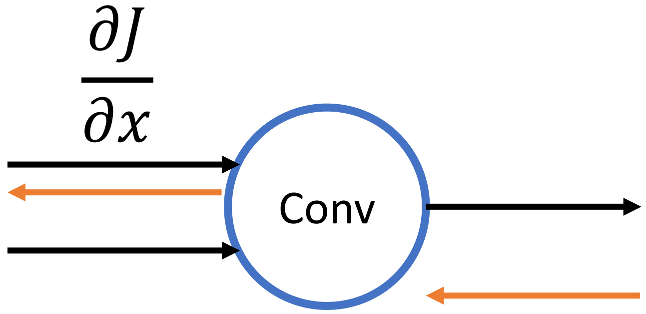
(h_{in}, w_{in}) $\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 2 \\ -1 & 2 & 3 \end{bmatrix}$ x

(f, f) $\begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$ w



v $\begin{bmatrix} 11 & -13 \\ 4 & 9 \end{bmatrix}$ (h_{out}, w_{out})

$\frac{\partial J}{\partial v}$ $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$

Apply chain rule to compute input gradients

$$\frac{\partial J}{\partial x} = \frac{\partial v}{\partial x}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} w_{11} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} w_{12} & w_{11} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & w_{12} \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} w_{21} & 0 \\ w_{11} & 0 \end{bmatrix} & \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} & \begin{bmatrix} 0 & w_{22} \\ 0 & w_{12} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ w_{21} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ w_{22} & w_{21} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & w_{22} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} \\ \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix}$$

$(h_{in}, w_{in}, h_{out}, w_{out})$ $\quad$ (h_{out}, w_{out}) $\quad$ (h_{in}, w_{in})

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Apply chain rule to compute input gradients

$$\frac{\partial J}{\partial x} = \frac{\partial v}{\partial x}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} w_{11} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} w_{12} & w_{11} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & w_{12} \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} w_{21} & 0 \\ w_{11} & 0 \end{bmatrix} & \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} & \begin{bmatrix} 0 & w_{22} \\ 0 & w_{12} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ w_{21} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ w_{22} & w_{21} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & w_{22} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} \\ \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} \end{bmatrix} =$$

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Apply chain rule to compute input gradients

$$\frac{\partial J}{\partial x} = \frac{\partial v}{\partial x}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} w_{11} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} w_{12} & w_{11} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & w_{12} \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} w_{21} & 0 \\ w_{11} & 0 \end{bmatrix} & \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} & \begin{bmatrix} 0 & w_{22} \\ 0 & w_{12} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ w_{21} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ w_{22} & w_{21} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & w_{22} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} \\ \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} \end{bmatrix} =$$

What is the pattern here?

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Apply chain rule to compute input gradients

$$\frac{\partial J}{\partial x} = \frac{\partial v}{\partial x}\frac{\partial J}{\partial v} = \begin{bmatrix} \begin{bmatrix} w_{11} & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} w_{12} & w_{11} \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & w_{12} \\ 0 & 0 \end{bmatrix} \\ \begin{bmatrix} w_{21} & 0 \\ w_{11} & 0 \end{bmatrix} & \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} & \begin{bmatrix} 0 & w_{22} \\ 0 & w_{12} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ w_{21} & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ w_{22} & w_{21} \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & w_{22} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} \\ \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} \end{bmatrix} =$$

What is the pattern here? It's another convolution!

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

Can get the same result with the following convolution

$\frac{\partial J}{\partial v_{11}}$	$\frac{\partial J}{\partial v_{12}}$
$\frac{\partial J}{\partial v_{21}}$	$\frac{\partial J}{\partial v_{22}}$

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Can get the same result with the following convolution

0	0	0	0
0	$\frac{\partial J}{\partial v_{11}}$	$\frac{\partial J}{\partial v_{12}}$	0
0	$\frac{\partial J}{\partial v_{21}}$	$\frac{\partial J}{\partial v_{22}}$	0
0	0	0	0

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Can get the same result with the following convolution

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Can get the same result with the following convolution

$$
\begin{array}{|c|c|c|c|}
\hline
0 & 0 & 0 & 0 \\
\hline
0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\
\hline
0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\
\hline
0 & 0 & 0 & 0 \\
\hline
\end{array}
\quad \text{conv} \quad
\begin{array}{|c|c|}
\hline
w_{21} & w_{11} \\
\hline
w_{22} & w_{12} \\
\hline
\end{array}
$$

90 Degrees

$$
\begin{bmatrix}
w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\
w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\
w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}}
\end{bmatrix}
$$

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Can get the same result with the following convolution

$$
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\
0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\text{ conv }
\begin{bmatrix}
w_{22} & w_{21} \\
w_{12} & w_{11}
\end{bmatrix}
$$

90 Degrees

$$
\begin{bmatrix}
w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\
w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\
w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}}
\end{bmatrix}
$$

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Can get the same result with the following convolution

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} =$$

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Can get the same result with the following convolution

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} =$$

$$\begin{bmatrix} \boxed{w_{11}\frac{\partial J}{\partial v_{11}}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Can get the same result with the following convolution

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} =$$

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & \boxed{w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

Can get the same result with the following convolution

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} =$$

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & \boxed{w_{12}\frac{\partial J}{\partial v_{12}}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Can get the same result with the following convolution

$$
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\
0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\text{ conv }
\begin{bmatrix}
w_{22} & w_{21} \\
w_{12} & w_{11}
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\
\boxed{w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\
w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}}
\end{bmatrix}
$$

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Can get the same result with the following convolution

$$
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\
0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\ \text{conv}\
\begin{bmatrix}
w_{22} & w_{21} \\
w_{12} & w_{11}
\end{bmatrix}
=
$$

$$
\begin{bmatrix}
w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\
w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & \boxed{w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\
w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}}
\end{bmatrix}
$$

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Can get the same result with the following convolution

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} =$$

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & \boxed{w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Can get the same result with the following convolution

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} =$$

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ \boxed{w_{21}\frac{\partial J}{\partial v_{21}}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Can get the same result with the following convolution

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} =$$

$$\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & \boxed{w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}}} & w_{22}\frac{\partial J}{\partial v_{22}} \end{bmatrix}$$

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Can get the same result with the following convolution

$$
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} =
$$

$$
\begin{bmatrix} w_{11}\frac{\partial J}{\partial v_{11}} & w_{12}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{12}} & w_{12}\frac{\partial J}{\partial v_{12}} \\ w_{21}\frac{\partial J}{\partial v_{11}} + w_{11}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{11}} + w_{21}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{21}} + w_{11}\frac{\partial J}{\partial v_{22}} & w_{22}\frac{\partial J}{\partial v_{12}} + w_{12}\frac{\partial J}{\partial v_{22}} \\ w_{21}\frac{\partial J}{\partial v_{21}} & w_{22}\frac{\partial J}{\partial v_{21}} + w_{21}\frac{\partial J}{\partial v_{22}} & \boxed{w_{22}\frac{\partial J}{\partial v_{22}}} \end{bmatrix}
$$

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Can get the same result with the following convolution

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial x_{11}} & \frac{\partial J}{\partial x_{12}} & \frac{\partial J}{\partial x_{13}} \\ \frac{\partial J}{\partial x_{21}} & \frac{\partial J}{\partial x_{22}} & \frac{\partial J}{\partial x_{23}} \\ \frac{\partial J}{\partial x_{31}} & \frac{\partial J}{\partial x_{32}} & \frac{\partial J}{\partial x_{33}} \end{bmatrix}$$

$$\boxed{\frac{\partial J}{\partial x} = pad\left(\frac{\partial J}{\partial v}\right) \text{ conv rot180}(w)}$$

- No Jacobian require at all!
- This convolution yields same result as doing the full 4D-Tensor Jacobian and upstream gradient matrix multiply!
- You will implement this in Assignment 4

Can get the same result with the following convolution

$$
\begin{array}{|c|c|c|c|}
\hline
0 & 0 & 0 & 0 \\
\hline
0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\
\hline
0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\
\hline
0 & 0 & 0 & 0 \\
\hline
\end{array}
\quad \text{conv} \quad
\begin{array}{|c|c|}
\hline
w_{22} & w_{21} \\
\hline
w_{12} & w_{11} \\
\hline
\end{array}
\quad = \quad
\begin{array}{|c|c|c|}
\hline
\frac{\partial J}{\partial x_{11}} & \frac{\partial J}{\partial x_{12}} & \frac{\partial J}{\partial x_{13}} \\
\hline
\frac{\partial J}{\partial x_{21}} & \frac{\partial J}{\partial x_{22}} & \frac{\partial J}{\partial x_{23}} \\
\hline
\frac{\partial J}{\partial x_{31}} & \frac{\partial J}{\partial x_{32}} & \frac{\partial J}{\partial x_{33}} \\
\hline
\end{array}
$$

$$
\boxed{\frac{\partial J}{\partial x} = pad\left(\frac{\partial J}{\partial v}\right) \ \text{conv rot180}(w)}
$$

How much to pad?

- No Jacobian require at all!
- This convolution yields same result as doing the full 4D-Tensor Jacobian and upstream gradient matrix multiply!
- You will implement this in Assignment 4

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From the forward propagation convolution operation:

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \text{ conv } \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}$$

(h_{in}, w_{in}) $\quad$ (f, f) $\quad$ (h_{out}, w_{out})

$$h_{out} = h_{in} - f + 1 \qquad w_{out} = w_{in} - f + 1$$

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From the back propagation convolution operation:

$$\underset{(h'_{out}, w'_{out})}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} \text{ conv } \underset{(f, f)}{\begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix}} = \underset{(h_{in}, w_{in})}{\begin{bmatrix} \frac{\partial J}{\partial x_{11}} & \frac{\partial J}{\partial x_{12}} & \frac{\partial J}{\partial x_{13}} \\ \frac{\partial J}{\partial x_{21}} & \frac{\partial J}{\partial x_{22}} & \frac{\partial J}{\partial x_{23}} \\ \frac{\partial J}{\partial x_{31}} & \frac{\partial J}{\partial x_{32}} & \frac{\partial J}{\partial x_{33}} \end{bmatrix}}$$

$$h_{out} = h_{in} - f + 1 \qquad w_{out} = w_{in} - f + 1$$

$$h'_{out} = h_{out} + 2p_h \qquad w'_{out} = w_{out} + 2p_w$$

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From the back propagation convolution operation:

$$\underset{(h'_{out}, w'_{out})}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} \text{ conv } \underset{(f, f)}{\begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix}} = \underset{(h_{in}, w_{in})}{\begin{bmatrix} \frac{\partial J}{\partial x_{11}} & \frac{\partial J}{\partial x_{12}} & \frac{\partial J}{\partial x_{13}} \\ \frac{\partial J}{\partial x_{21}} & \frac{\partial J}{\partial x_{22}} & \frac{\partial J}{\partial x_{23}} \\ \frac{\partial J}{\partial x_{31}} & \frac{\partial J}{\partial x_{32}} & \frac{\partial J}{\partial x_{33}} \end{bmatrix}}$$

$$h_{out} = h_{in} - f + 1$$
$$h'_{out} = h_{out} + 2p_h$$
$$h_{in} = h'_{out} - f + 1$$

$$w_{out} = w_{in} - f + 1$$
$$w'_{out} = w_{out} + 2p_w$$
$$w_{in} = w'_{out} - f + 1$$

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From the back propagation convolution operation:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial x_{11}} & \frac{\partial J}{\partial x_{12}} & \frac{\partial J}{\partial x_{13}} \\ \frac{\partial J}{\partial x_{21}} & \frac{\partial J}{\partial x_{22}} & \frac{\partial J}{\partial x_{23}} \\ \frac{\partial J}{\partial x_{31}} & \frac{\partial J}{\partial x_{32}} & \frac{\partial J}{\partial x_{33}} \end{bmatrix}$$

(h'_{out}, w'_{out}) (f, f) (h_{in}, w_{in})

$$h_{out} = h_{in} - f + 1$$
$$h'_{out} = h_{out} + 2p_h$$
$$h_{in} = h'_{out} - f + 1$$

$$w_{out} = w_{in} - f + 1$$
$$w'_{out} = w_{out} + 2p_w$$
$$w_{in} = w'_{out} - f + 1$$

$$\boxed{p_h = f - 1}$$

$$\boxed{p_w = f - 1}$$

Summary (m=1, K=1, c_in=1)

$$\frac{\partial J}{\partial w} = x \text{ conv} \frac{\partial J}{\partial v}$$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} \text{ conv } \begin{bmatrix} \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} \\ \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial w_{11}} & \frac{\partial J}{\partial w_{12}} \\ \frac{\partial J}{\partial w_{21}} & \frac{\partial J}{\partial w_{22}} \end{bmatrix}$$

$$\frac{\partial J}{\partial x} = pad\left(\frac{\partial J}{\partial v}\right) \text{ conv rot180}(w)$$

Pad by $p = f - 1$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{22} & w_{21} \\ w_{12} & w_{11} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial x_{11}} & \frac{\partial J}{\partial x_{12}} & \frac{\partial J}{\partial x_{13}} \\ \frac{\partial J}{\partial x_{21}} & \frac{\partial J}{\partial x_{22}} & \frac{\partial J}{\partial x_{23}} \\ \frac{\partial J}{\partial x_{31}} & \frac{\partial J}{\partial x_{32}} & \frac{\partial J}{\partial x_{33}} \end{bmatrix}$$

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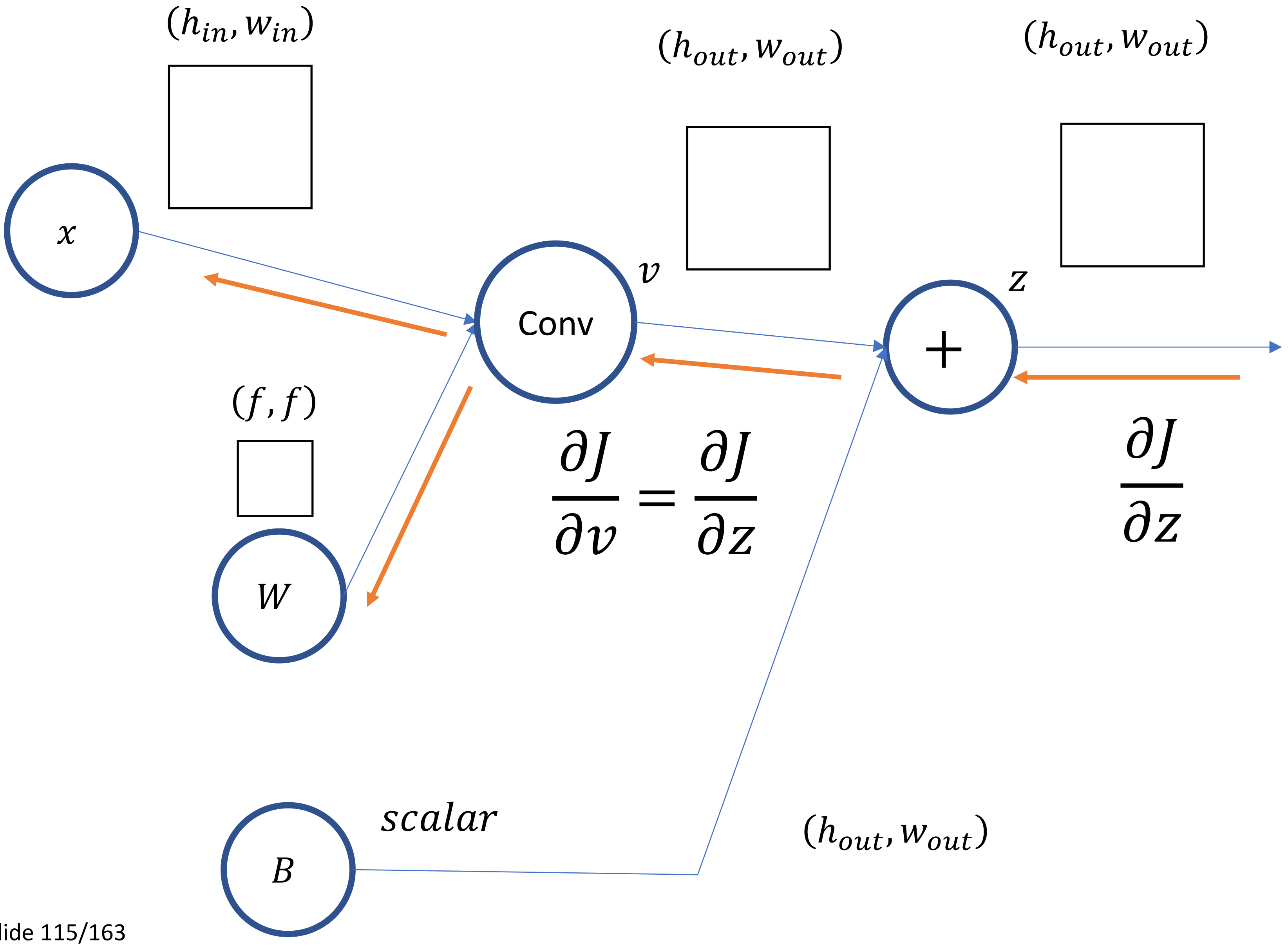
A compute graph for a convolutional layer



Start by considering:

- One sample (m=1)
- One filter (K=1)
- One channel ($c_{in} = 1$)

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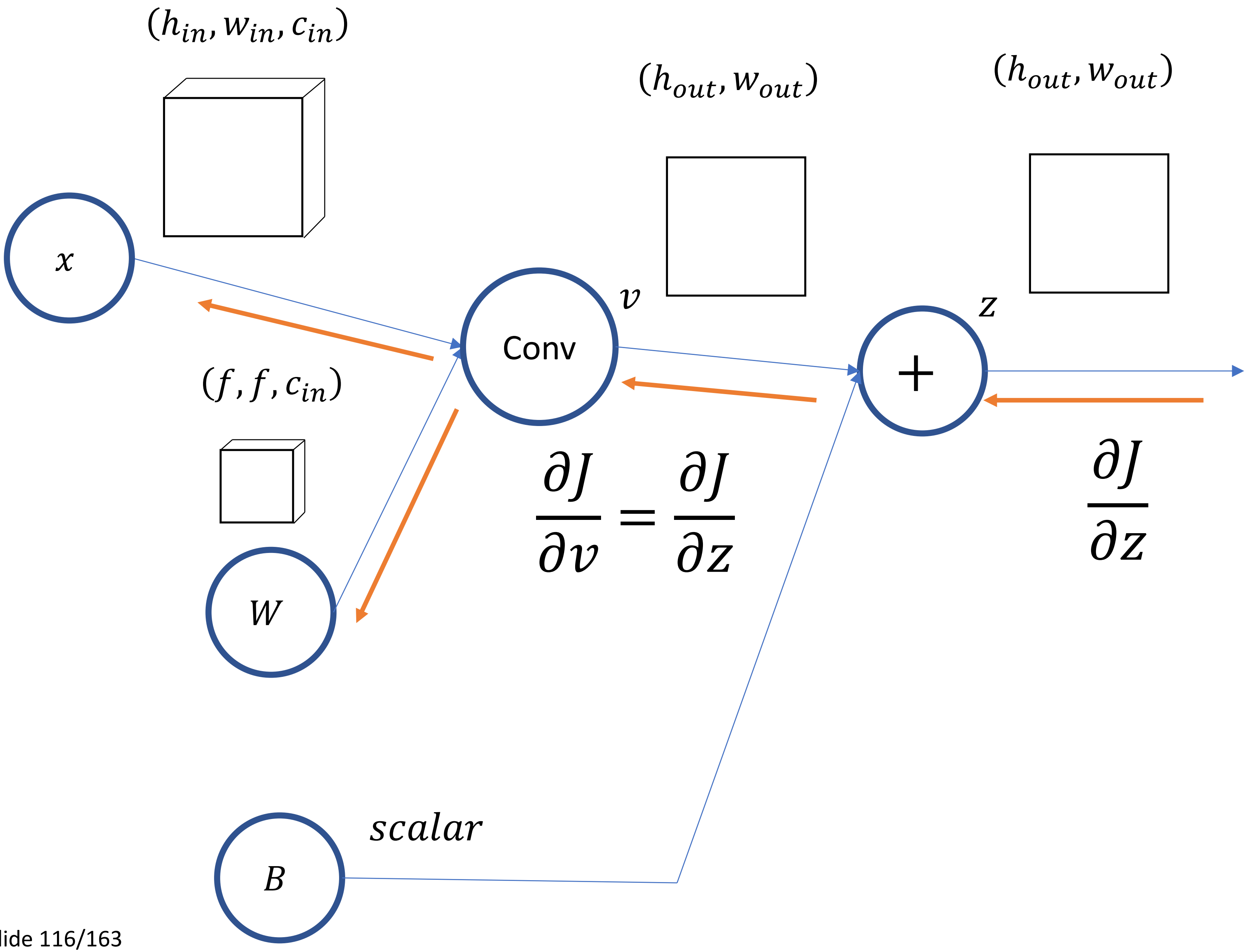
A compute graph for a convolutional layer



Start by considering:

- One sample (m=1)
- One filter (K=1)
- ~~One channel ($c_{in} = 1$)~~

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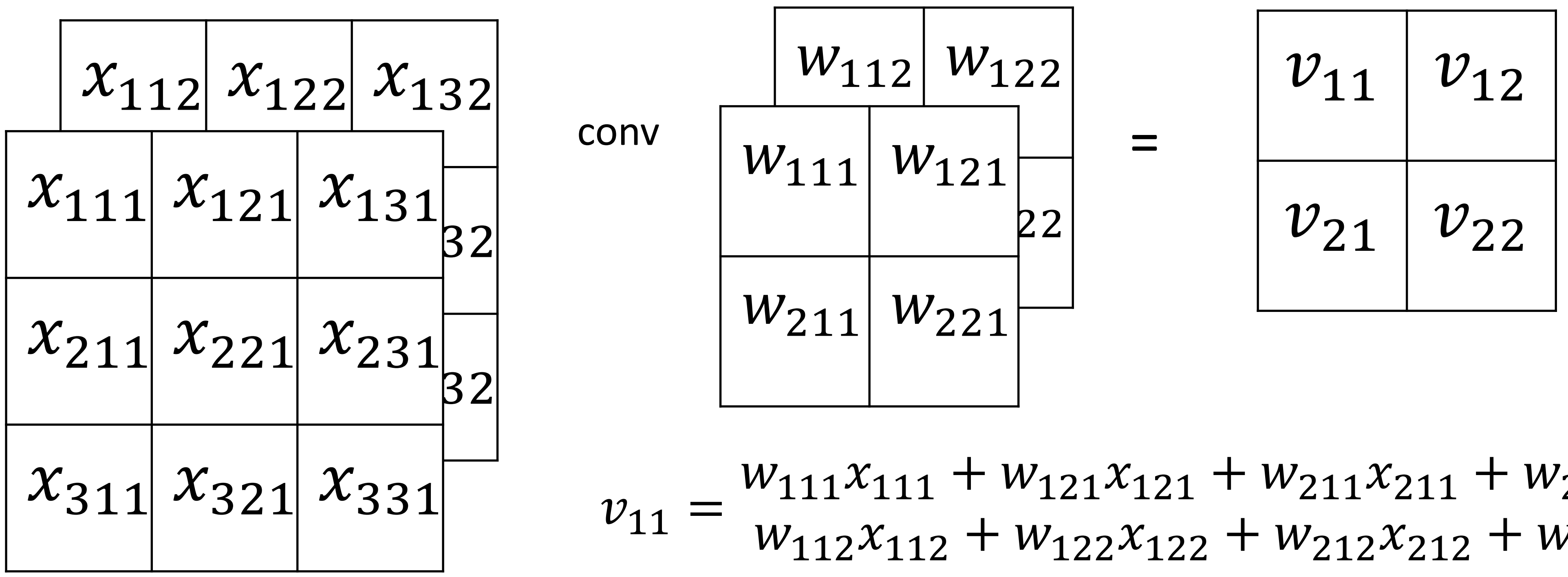
For Weight Gradients $\frac{\partial J}{\partial w}$

- Recall from forward propagation, each channel of weights operate independently on each channel of the input volume.

$(h_{in}, w_{in}) = (3,3,2)$ $\quad (f, f) = (2,2,2)$ $\quad (h_{out}, w_{out}) = (2,2)$



$$v_{11} = \begin{array}{c} w_{111}x_{111} + w_{121}x_{121} + w_{211}x_{211} + w_{221}x_{221} + \\ w_{112}x_{112} + w_{122}x_{122} + w_{212}x_{212} + w_{222}x_{222} \end{array}$$

$$\frac{\partial J}{\partial w} = x \text{ conv} \frac{\partial J}{\partial v}$$

Compute each channel Independently.

For channel 1

$$\begin{bmatrix} x_{111} & x_{121} & x_{131} \\ x_{211} & x_{221} & x_{231} \\ x_{311} & x_{321} & x_{331} \end{bmatrix} \text{ conv } \begin{bmatrix} \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} \\ \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial w_{111}} & \frac{\partial J}{\partial w_{121}} \\ \frac{\partial J}{\partial w_{211}} & \frac{\partial J}{\partial w_{221}} \end{bmatrix}$$

For channel 2

$$\begin{bmatrix} x_{112} & x_{122} & x_{132} \\ x_{212} & x_{222} & x_{232} \\ x_{312} & x_{322} & x_{332} \end{bmatrix} \text{ conv } \begin{bmatrix} \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} \\ \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial w_{112}} & \frac{\partial J}{\partial w_{122}} \\ \frac{\partial J}{\partial w_{212}} & \frac{\partial J}{\partial w_{222}} \end{bmatrix}$$

For Input Volume

$$\frac{\partial J}{\partial x} = pad\left(\frac{\partial J}{\partial v}\right) \text{ conv rot180}(w)$$

For channel 1

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{221} & w_{211} \\ w_{121} & w_{111} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial x_{111}} & \frac{\partial J}{\partial x_{121}} & \frac{\partial J}{\partial x_{131}} \\ \frac{\partial J}{\partial x_{211}} & \frac{\partial J}{\partial x_{221}} & \frac{\partial J}{\partial x_{231}} \\ \frac{\partial J}{\partial x_{311}} & \frac{\partial J}{\partial x_{321}} & \frac{\partial J}{\partial x_{331}} \end{bmatrix}$$

For channel 2

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial J}{\partial v_{11}} & \frac{\partial J}{\partial v_{12}} & 0 \\ 0 & \frac{\partial J}{\partial v_{21}} & \frac{\partial J}{\partial v_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ conv } \begin{bmatrix} w_{222} & w_{212} \\ w_{122} & w_{112} \end{bmatrix} = \begin{bmatrix} \frac{\partial J}{\partial x_{112}} & \frac{\partial J}{\partial x_{122}} & \frac{\partial J}{\partial x_{132}} \\ \frac{\partial J}{\partial x_{212}} & \frac{\partial J}{\partial x_{222}} & \frac{\partial J}{\partial x_{232}} \\ \frac{\partial J}{\partial x_{312}} & \frac{\partial J}{\partial x_{322}} & \frac{\partial J}{\partial x_{332}} \end{bmatrix}$$

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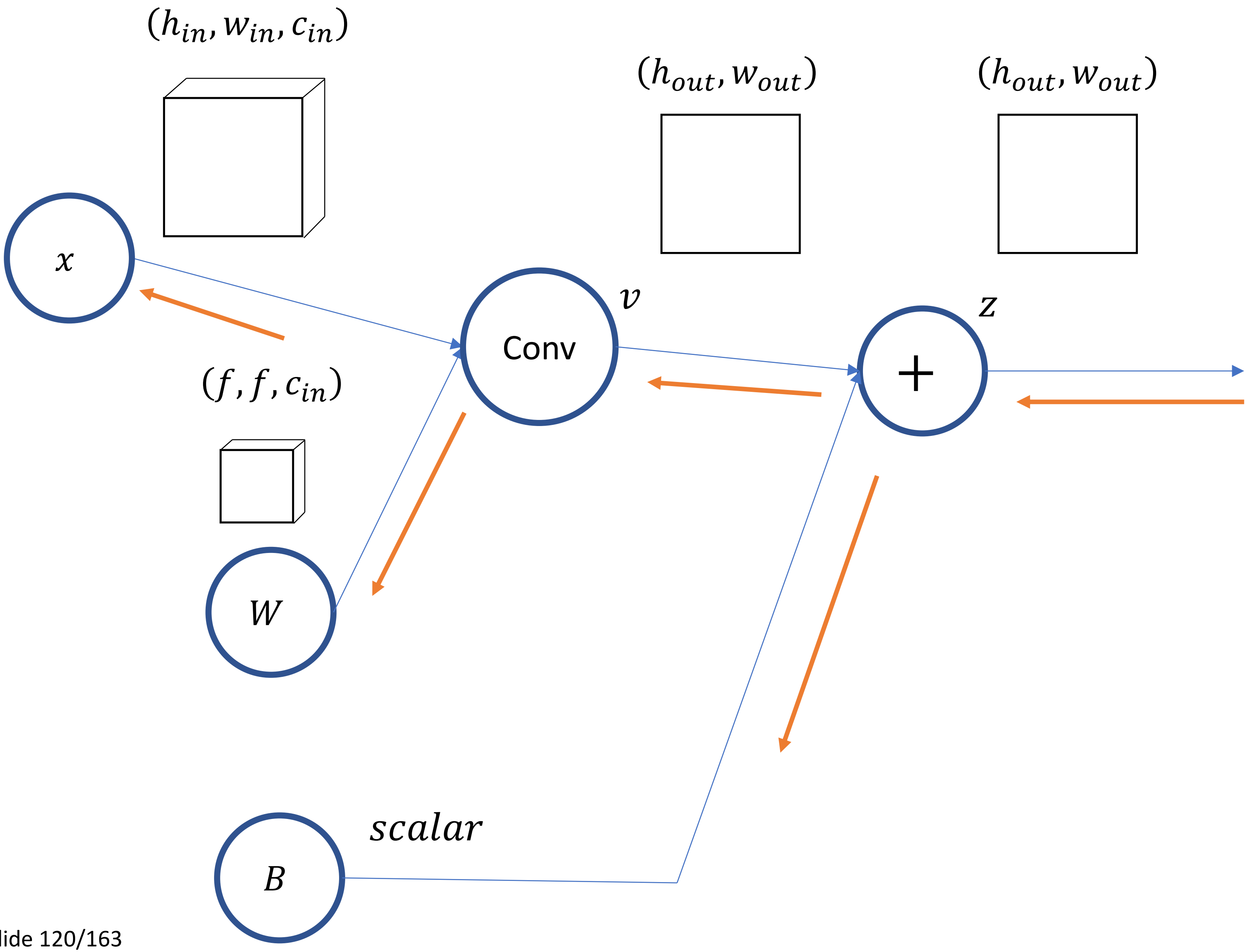
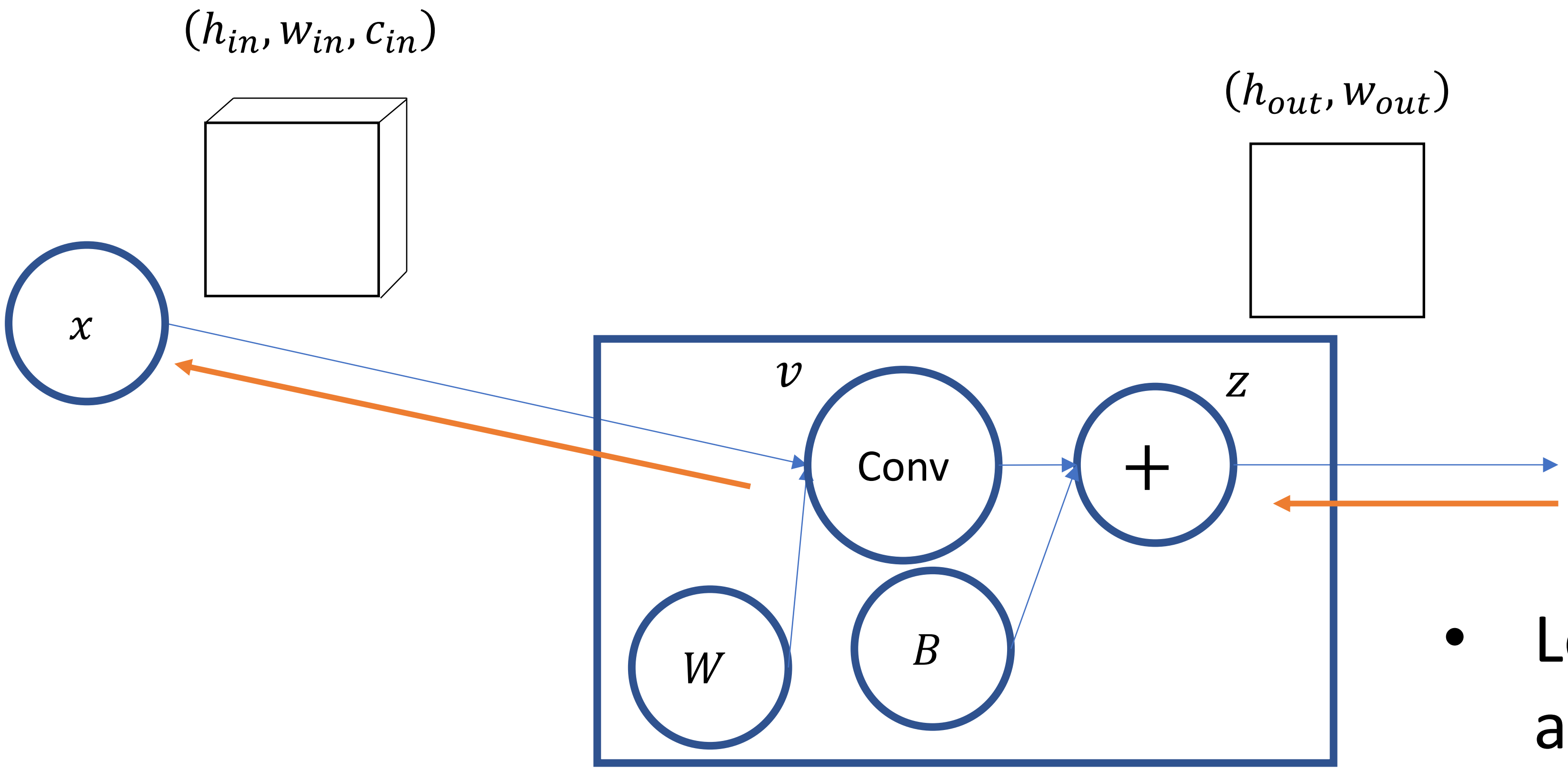
A compute graph for a convolutional layer



We know how to compute all the gradients now for:

- One sample (m=1)
- One filter (K=1)
- ~~One channel~~ $\cancel{(c_{in} = 1)}$

A compute graph for a convolutional layer



- Let's abstract all this part away for now since we know how to do this part

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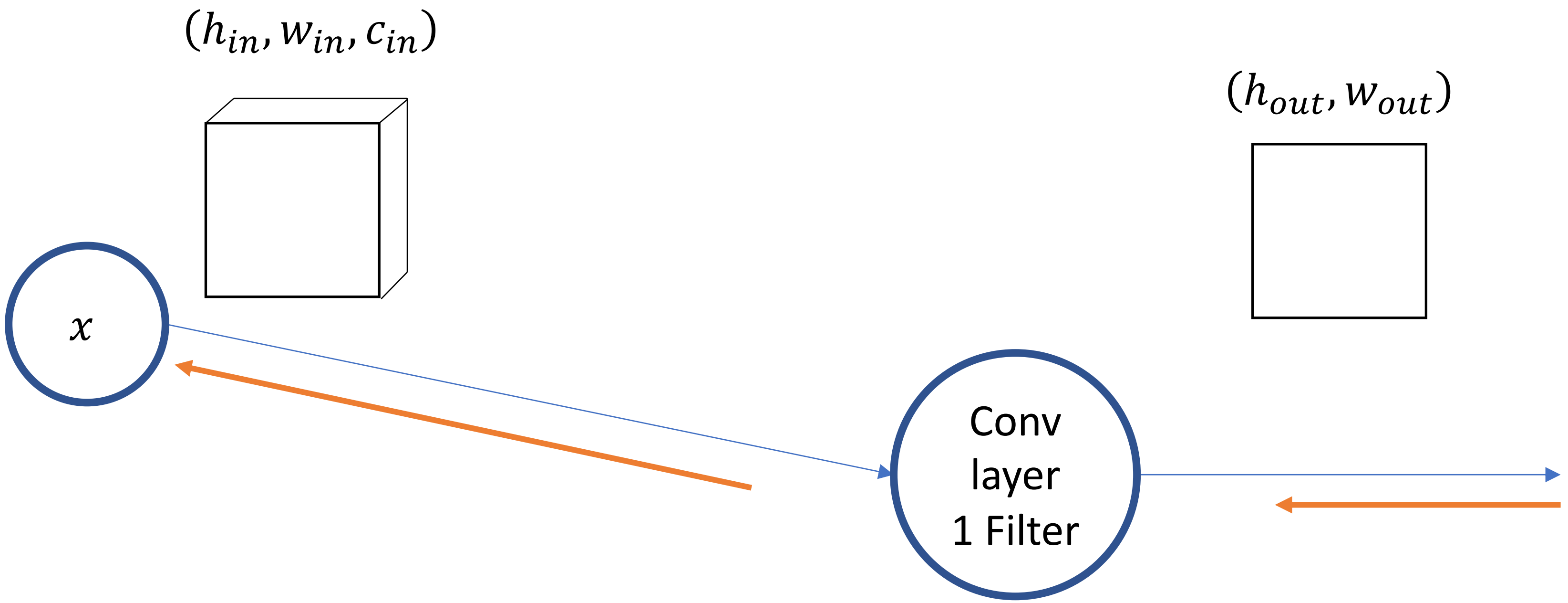
A compute graph for a convolutional layer



- Let’s abstract all this part away for now since we know how to do this part
- Model it as a single compute graph node

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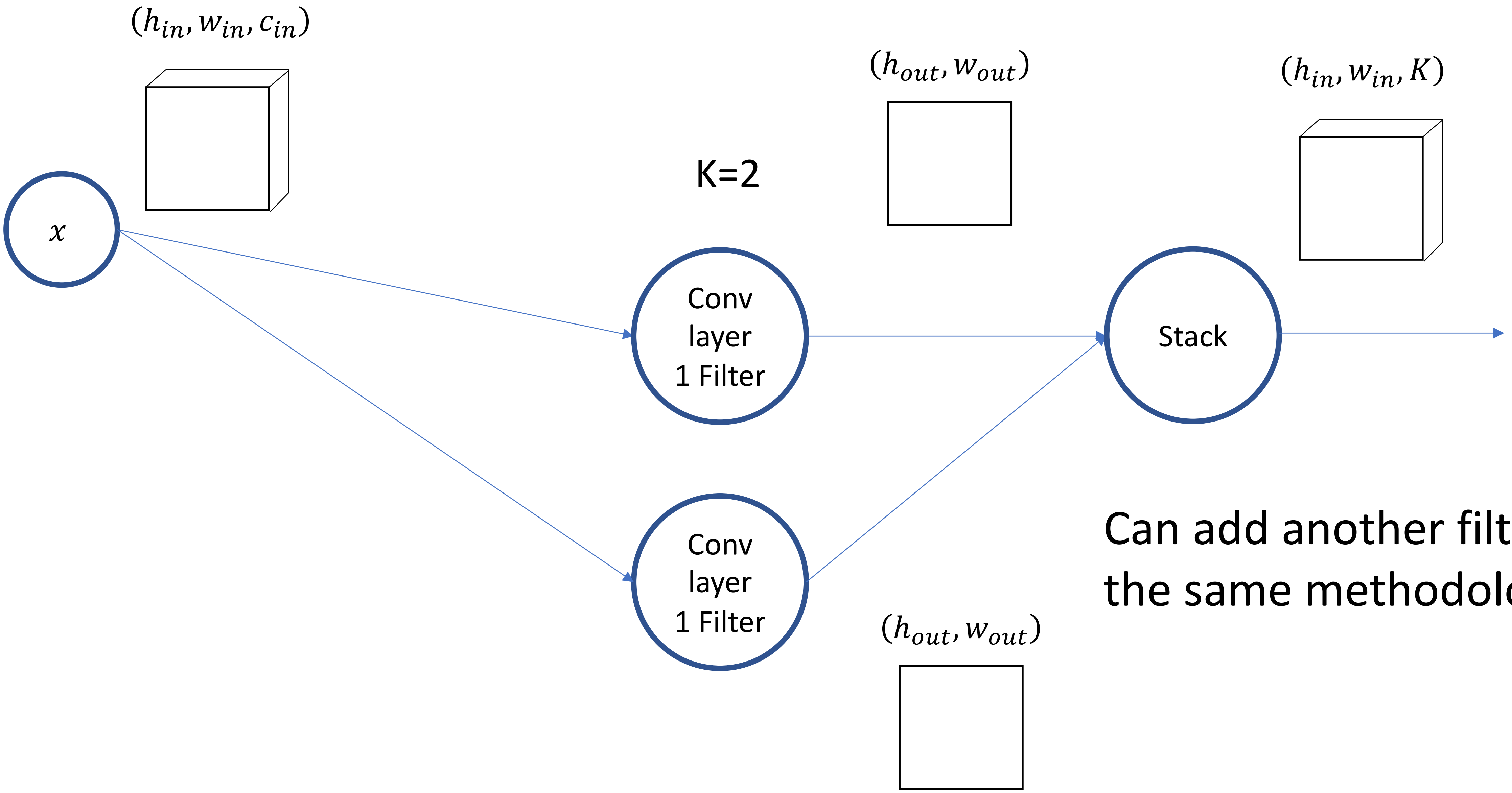
A compute graph for a convolutional layer



Can add another filter using the same methodology

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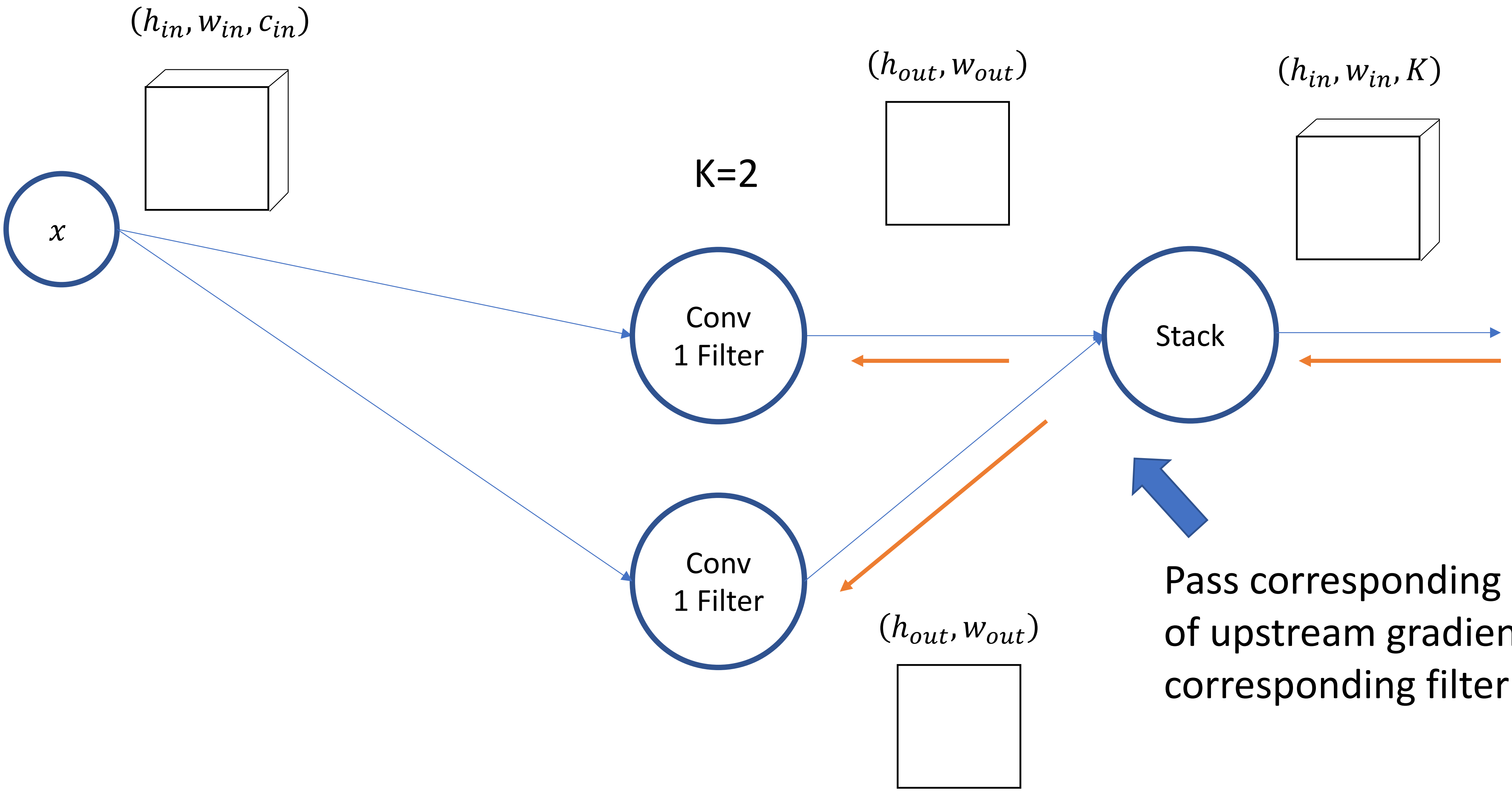
A compute graph for a convolutional layer



Pass corresponding channel of upstream gradients to corresponding filter

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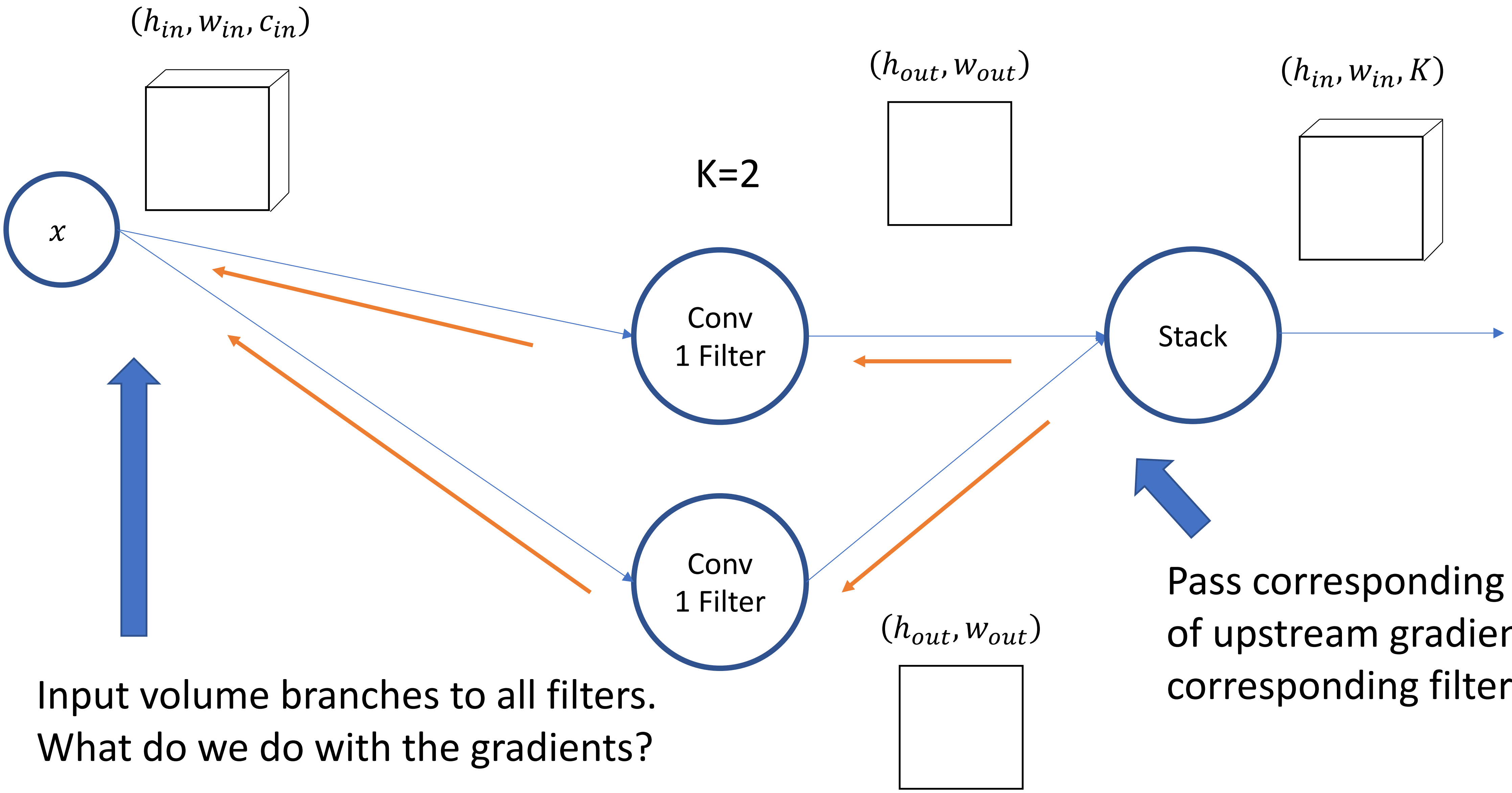
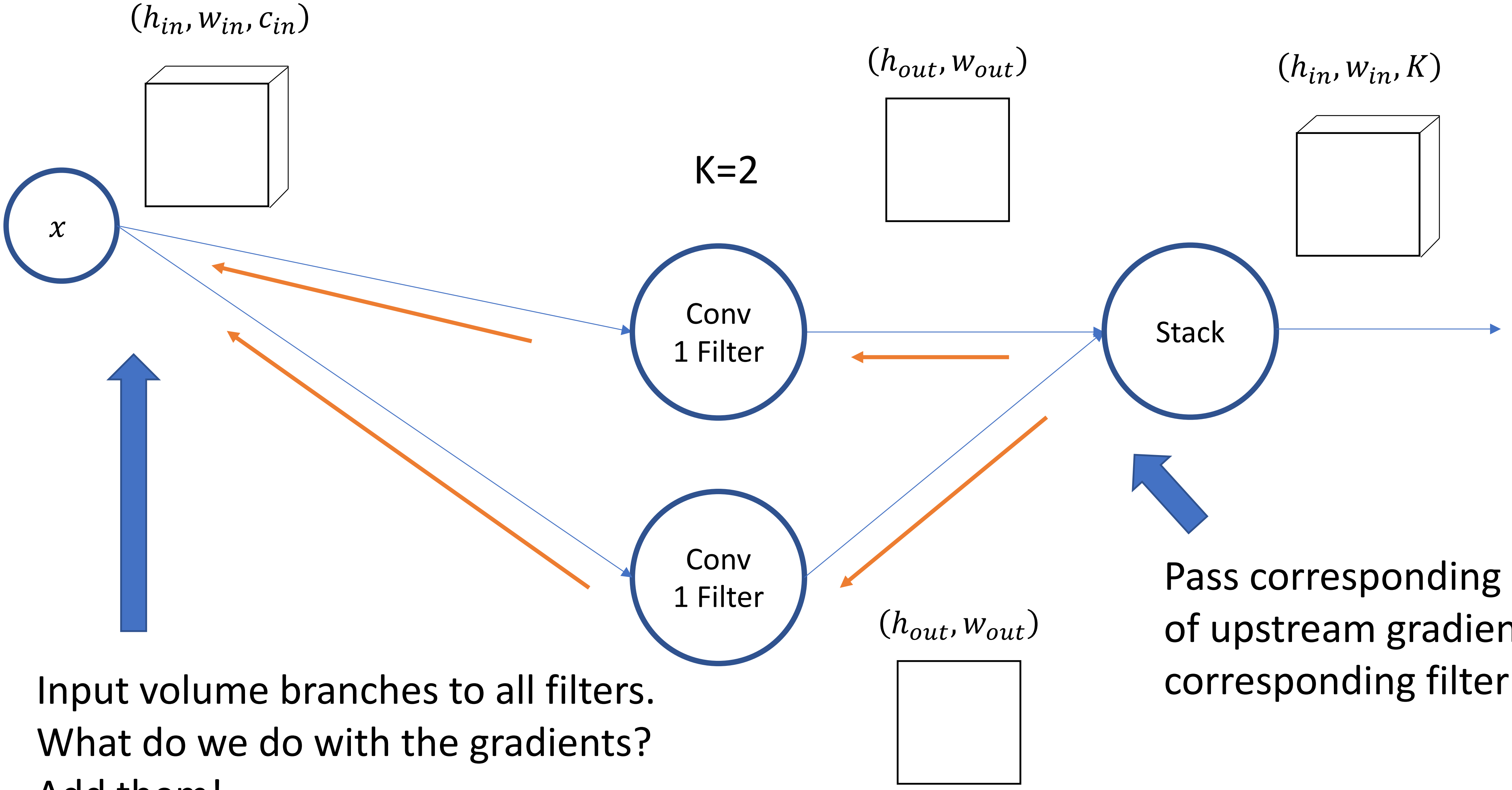
A compute graph for a convolutional layer



Input volume branches to all filters.
What do we do with the gradients?

Pass corresponding channel of upstream gradients to corresponding filter

A compute graph for a convolutional layer



Pass corresponding channel of upstream gradients to corresponding filter

Input volume branches to all filters.
What do we do with the gradients?
Add them!

For Multiple Samples

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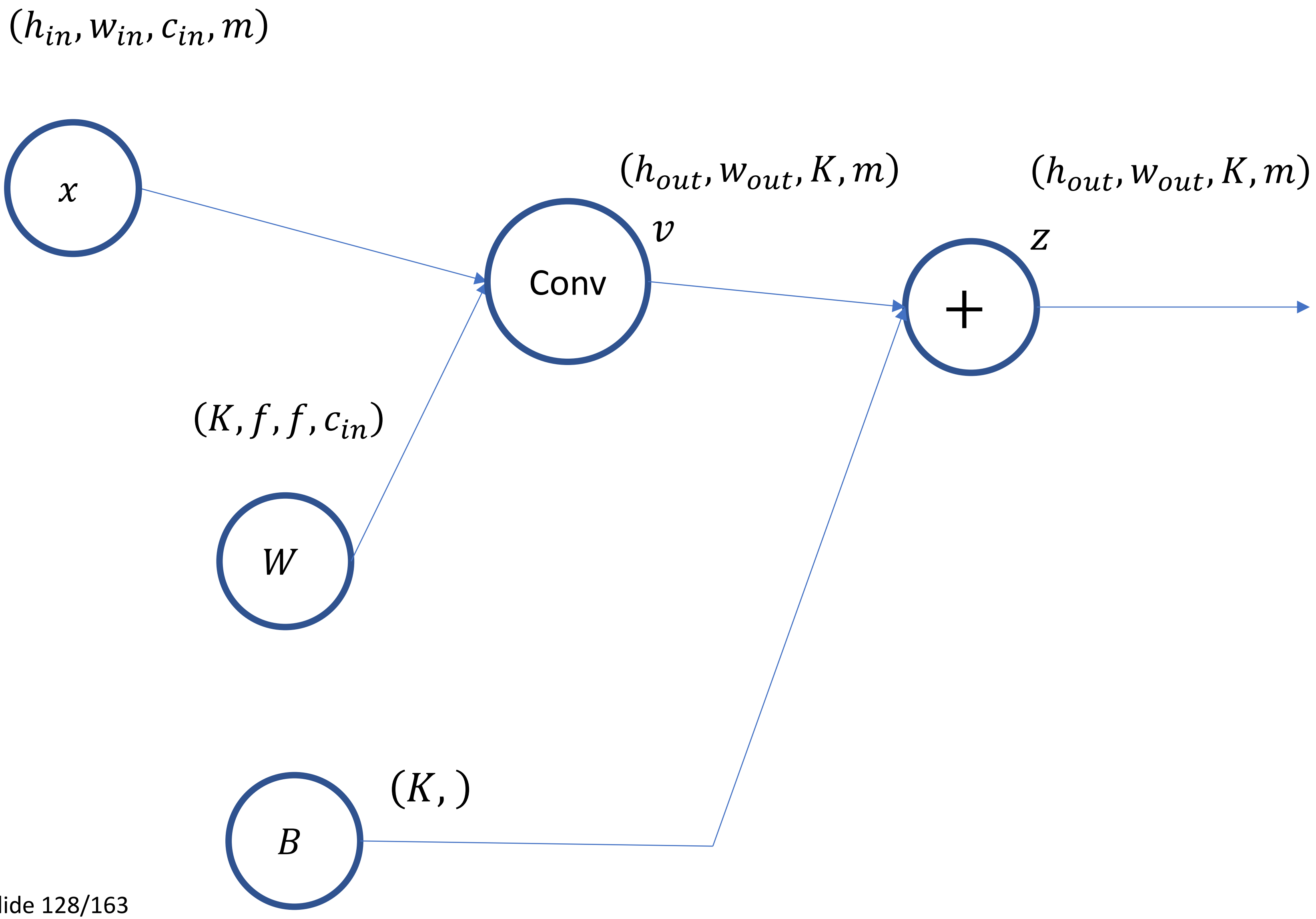
A compute graph for a convolutional layer



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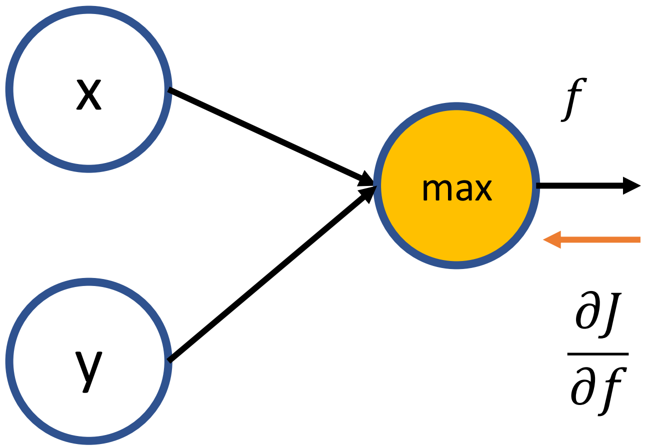
Backpropagation Through MaxPooling

Recall: Backpropagation Through max(x,y)



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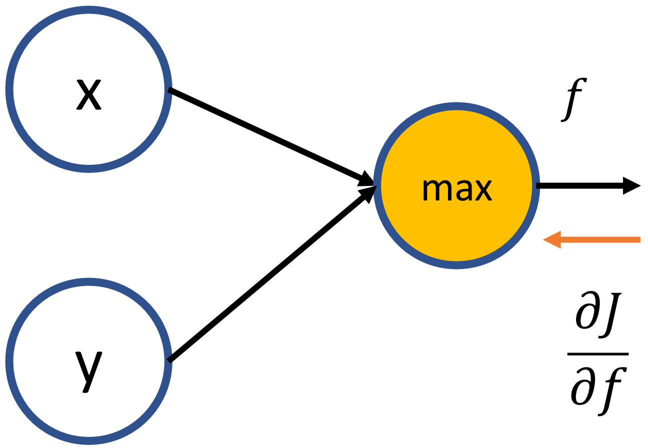
Recall: Backpropagation Through max(x,y)



- Upstream gradient is routed to larger variable
- Intuition: Only one input can affect the output at any time

Backpropgation in MaxPooling Layer

- Recall that MaxPool, you are taking the max in small regions of the input volume (e.g. 2x2 regions)

1	3	10	0
8	4	2	9
7	2	1	3
1	1	0	4

<u>Max Pooling</u>

$(f, f) = (2,2)$

Stride: 2

→

8	10
7	4

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Backpropgation in MaxPooling Layer

- Recall that MaxPool, you are taking the max in small regions of the input volume (e.g. 2x2 regions)

1	3	10	0
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<u>Max Pooling</u>

$(f, f) = (2,2)$

Stride: 2

→

8	10
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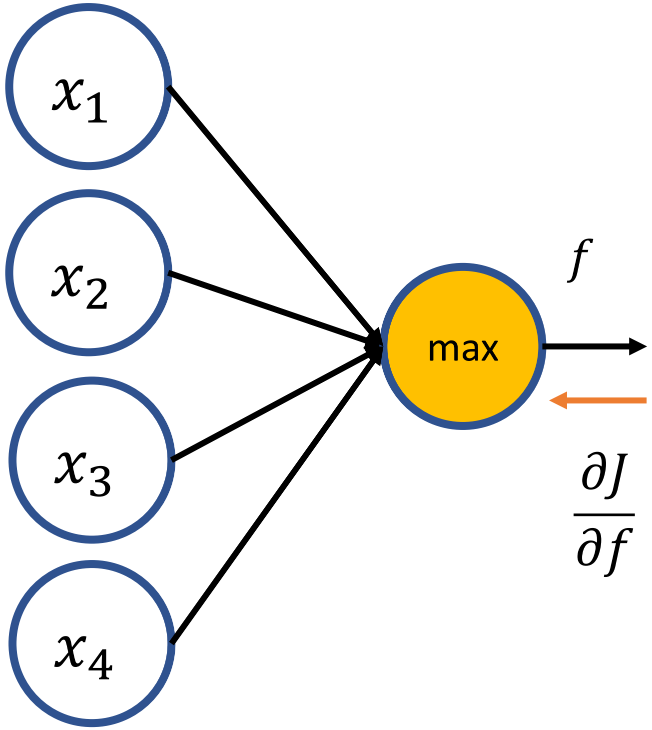

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Aside: Adversarial Inputs to CNNs

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Adversarial Images via Backpropagation

- Pick an input image to modify
- Pick an output class you want to ”trick” the classifier into predicting
- Use a cost function that maximizes that class’ output probability
- Use backpropagation to find changes to the input image (dJ/dx) to maximize cost (i.e.. use gradient ascent)

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Adversarial Images via Backpropagation

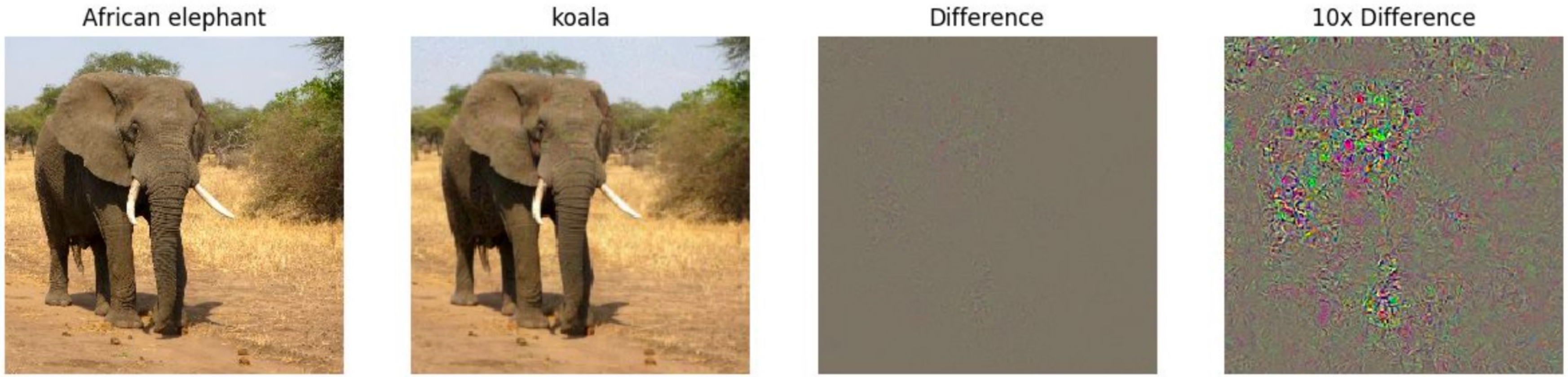


Image Source: CS231n Stanford Course

Adversarial Images via Backpropagation

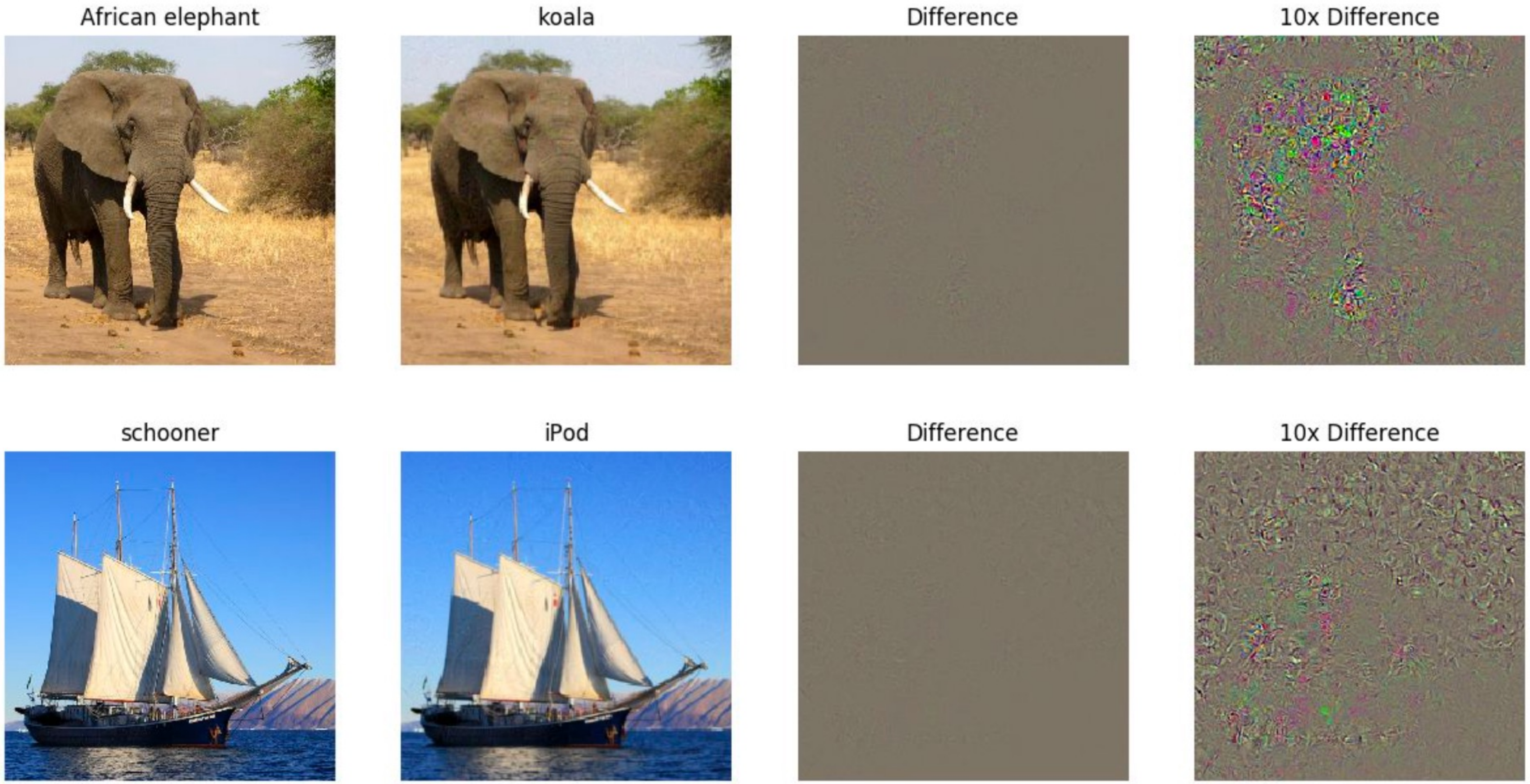
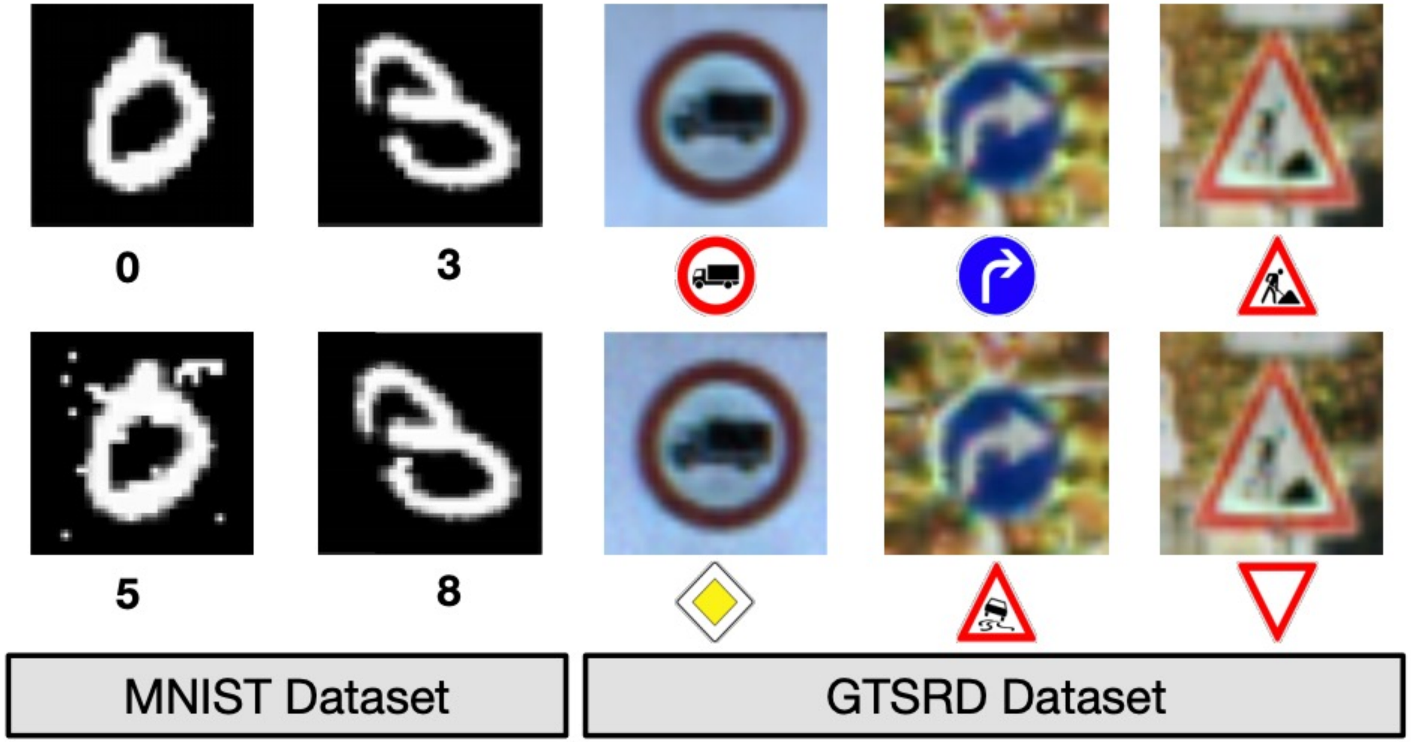


Image Source: CS231n Stanford Course

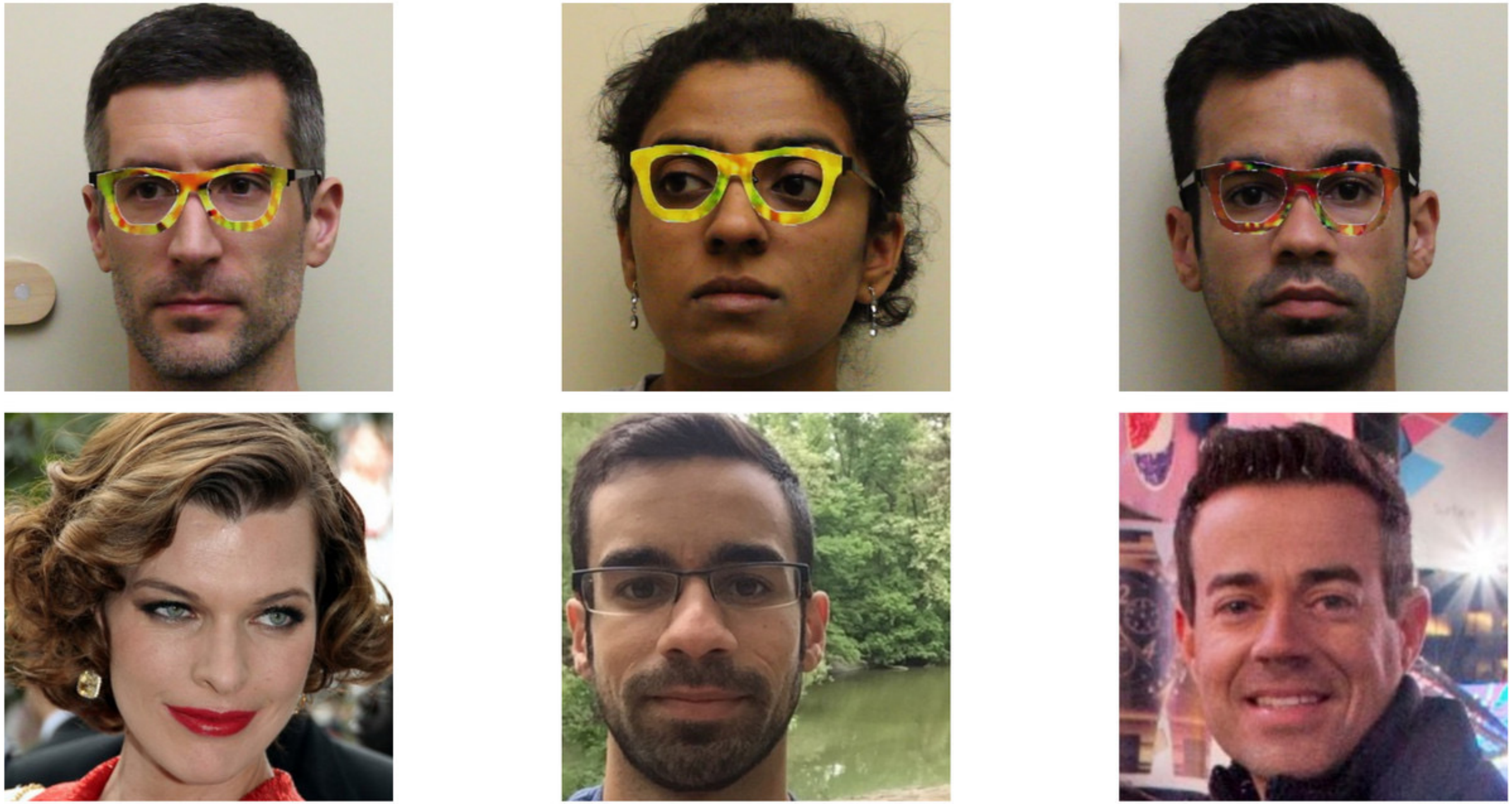
Adversarial Images via Backpropagation



“Practical Black-Box Attacks against Machine Learning”, Papernot et al, 2017, https://arxiv.org/abs/1602.02697

- These were misclassified as a 45mph/hr street sign instead of stop sign

https://arstechnica.com/cars/2017/09/hacking-street-signs-with-stickers-could-confuse-self-driving-cars/



(b) (c) (d)

https://www.theverge.com/2016/11/3/13507542/facial-recognition-glasses-trick-impersonate-fool

Numerical Gradient Checking

- As you saw in assignment 3, easy to make mistakes when implementing backpropagation
- In software development, we want to write tests for our code
- For backprop tests, we need “correct” values
- Computing these is tedious and also error-prone

Test from Assignment 3 for Z=WX+B

```python
# Expected Values
dJ_dx_expected = np.array([[1.32727273, 1.        ],
                           [1.21818182, 1.10909091],
                           [1.10909091, 1.21818182],
                           [1.        , 1.32727273]])
dJ_dw_expected = np.array([[ 1.42857143,  0.51428571, -0.4       , -1.31428571],
                           [ 0.05714286,  0.05714286,  0.05714286,  0.05714286],
                           [-1.31428571, -0.4       ,  0.51428571,  1.42857143]])
dJ_db_expected = np.array([[-1.6],
                           [ 2.22044605e-16],
                           [ 1.6]])

# Test gradient matrix values
assert np.allclose(dJ_dx, dJ_dx_expected), 'Unexpected values for dJ_dx: \n{0}'.format(dJ_dx)
assert np.allclose(dJ_dw, dJ_dw_expected), 'Unexpected values for dJ_dw: \n{0}'.format(dJ_dw)
assert np.allclose(dJ_db, dJ_db_expected), 'Unexpected values for dJ_db: \n{0}'.format(dJ_db)
```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Numerical Gradient Checking

- As you saw in assignment 3, easy to make mistakes when implementing backpropagation
- In software development, we want to write tests for our code
- For backprop tests, we need “correct” values
- Computing these is tedious and also error-prone
- We can instead use Numerical Gradient Checking

Recall from Calculus 101

- Definition of the derivative:

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

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Recall from Calculus 101

- Definition of the derivative:

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

<u>Numerical Gradient Approximation:</u>

- When h is not zero, but small (e.g. 0.00001), we can get a decent approximation to the derivative

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For a multivariable function

- y is a scalar
- x is a vector shaped $(n,)$
- $y = f(x)$
- $\frac{\partial y}{\partial x}$ is shaped $(n,)$

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For a multivariable function

- y is a scalar
- x is a vector shaped $(n,)$
- $y = f(x)$
- $\frac{\partial y}{\partial x}$ is shaped $(n,)$

$$\frac{\partial y}{\partial x_1} \approx \frac{f(x_1 + h, x_2, \dots, x_n) - f(x_1 - h, x_2, \dots, x_n)}{2h}$$

$$\frac{\partial y}{\partial x_2} \approx \frac{f(x_1, x_2 + h, \dots, x_n) - f(x_1, x_2 - h, \dots, x_n)}{2h}$$

$$\vdots$$

$$\frac{\partial y}{\partial x_n} \approx \frac{f(x_1, x_2, \dots, x_n + h) - f(x_1, x_2, \dots, x_n - h)}{2h}$$

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Example

- Let's say: $y = x_1 * x_2 * x_3$ and want gradient at $(x_1, x_2, x_3) = (2,1,3)$

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Example – Using "Analytical" Gradient

- Let's say: $y = x_1 * x_2 * x_3$ and want gradient at $(x_1, x_2, x_3) = (2,1,3)$
- Derive the closed-form symbolic derivatives:

$$\frac{\partial y}{\partial x_1} = x_2 * x_3$$

$$\frac{\partial y}{\partial x_2} = x_1 * x_3$$

$$\frac{\partial y}{\partial x_3} = x_1 * x_2$$

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Example – Using "Analytical" Gradient

- Let's say: $y = x_1 * x_2 * x_3$ and want gradient at $(x_1, x_2, x_3) = (2,1,3)$
- Derive the closed-form symbolic derivatives:

$$\frac{\partial y}{\partial x_1} = x_2 * x_3 = 1 * 3 = 3$$

$$\frac{\partial y}{\partial x_2} = x_1 * x_3 = 2 * 3 = 6$$

$$\frac{\partial y}{\partial x_3} = x_1 * x_2 = 2 * 1 = 2$$

Example – Using Numerical Gradient Approx.

- Let's say: $y = x_1 * x_2 * x_3$ and want gradient at $(x_1, x_2, x_3) = (2,1,3)$
- Using numerical gradient approximation (h=0.00001)

$$\frac{\partial y}{\partial x_1} \approx \frac{(x_1+h) * x_2 * x_3 - (x_1-h) * x_2 * x_3}{2h}$$

$$\frac{\partial y}{\partial x_2} \approx \frac{x_1 * (x_2 + h) * x_3 - x_1 * (x_2 - h) * x_3}{2h}$$

$$\frac{\partial y}{\partial x_3} \approx \frac{x_1 * x_2 * (x_3 + h) - x_1 * x_2 * (x_3 - h)}{2h}$$

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Example – Using Numerical Gradient Approx.

- Let's say: $y = x_1 * x_2 * x_3$ and want gradient at $(x_1, x_2, x_3) = (2,1,3)$
- Using numerical gradient approximation (h=0.00001)

$$\frac{\partial y}{\partial x_1} \approx \frac{(x_1+h) * x_2 * x_3 - (x_1-h) * x_2 * x_3}{2h} = 3$$

$$\frac{\partial y}{\partial x_2} \approx \frac{x_1 * (x_2 + h) * x_3 - x_1 * (x_2 - h) * x_3}{2h} = 6$$

$$\frac{\partial y}{\partial x_3} \approx \frac{x_1 * x_2 * (x_3 + h) - x_1 * x_2 * (x_3 - h)}{2h} = 2$$

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Example – Compare

- Let's say: $y = x_1 * x_2 * x_3$ and want gradient at $(x_1, x_2, x_3) = (2,1,3)$

$$\frac{\partial y}{\partial x_1} \approx \frac{(x_1+h) * x_2 * x_3 - (x_1-h) * x_2 * x_3}{2h} = 3 \qquad \frac{\partial y}{\partial x_1} = x_2 * x_3 = 1 * 3 = 3$$

$$\frac{\partial y}{\partial x_2} \approx \frac{x_1 * (x_2 + h) * x_3 - x_1 * (x_2 - h) * x_3}{2h} = 6 \qquad \frac{\partial y}{\partial x_2} = x_1 * x_3 = 2 * 3 = 6$$

$$\frac{\partial y}{\partial x_3} \approx \frac{x_1 * x_2 * (x_3 + h) - x_1 * x_2 * (x_3 - h)}{2h} = 2 \qquad \frac{\partial y}{\partial x_3} = x_1 * x_2 = 2 * 1 = 2$$

Gradient Check For Backpropagation

1. Need to implement these from Assignment 3 (Z=WX+B)
   z = linear_forward_propagate(x, w, b)
   dJ_dw, dJ_b, dJ_dx = linear_backward_propagate(dJ_dz)
2. You typically have the forward prop symbolic equations, so computing expected values for tests is straight-forward
3. So assuming you have correctly implemented the forward prop function, you can use it for numerical gradient checking for your backprop function

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Example – Backprop (Scalar)

Forward Propagation:

$$z = f(w_2, x_2, w_1, x_1, b) = \; w_2 x_2 + w_1 x_1 + b$$

```python
def f(x2, x1, w2, w1, b):
    z = w2*x2 + w1*x1 + b
    return z
```

Backward Propagation:

$$\frac{\partial z}{\partial w_2} = \; x_2 \qquad \frac{\partial z}{\partial x_2} = \; w_2$$

$$\frac{\partial z}{\partial w_1} = \; x_1 \qquad \frac{\partial z}{\partial x_1} = w_1$$

$$\frac{\partial z}{\partial b} = 1$$

```python
def df(x2, x1, w2, w1, b):
    dz_dx2 = w2
    dz_dx1 = w1
    dz_dw2 = x2
    dz_dw1 = x1
    dz_db  = 1
    return dz_dx2, dz_dx1, dz_dw2, dz_dw1, dz_db
```


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Example – Backprop (Scalar)

Forward Propagation:

$$z = f(w_2, x_2, w_1, x_1, b) = w_2x_2 + w_1x_1 + b$$

```python
def f(x2, x1, w2, w1, b):
    z = w2*x2 + w1*x1 + b
    return z
```

Test: Is this correct?

Backward Propagation:

$$\frac{\partial z}{\partial w_2} = x_2 \qquad \frac{\partial z}{\partial x_2} = w_2$$

$$\frac{\partial z}{\partial w_1} = x_1 \qquad \frac{\partial z}{\partial x_1} = w_1$$

$$\frac{\partial z}{\partial b} = 1$$

```python
def df(x2, x1, w2, w1, b):
    dz_dx2 = w2
    dz_dx1 = w1
    dz_dw2 = x2
    dz_dw1 = x1
    dz_db  = 1
    return dz_dx2, dz_dx1, dz_dw2, dz_dw1, dz_db
```


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```python
# For some set of inputs
x2, x1, w2, w1, b = np.random.randn(5)

# Compute analytical gradient
dz_dx2, dz_dx1, dz_dw2, dz_dw1, dz_db = df(x2, x1, w2, w1, b)
```

Are these 5 numbers correct?

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```python
# For some set of inputs
x2, x1, w2, w1, b = np.random.randn(5)

# Compute analytical gradient
dz_dx2, dz_dx1, dz_dw2, dz_dw1, dz_db = df(x2, x1, w2, w1, b)
```

Are these 5 numbers correct?

Example: Checking one of the results dz_dx2

```python
# Compute numerical gradient
h = 1e-5
dz_dx2_num = (f(x2+h, x1, w2, w1, b) - f(x2-h, x1, w2, w1, b))/(2*h)
print('dz_dx2 numerical: {0:0.14f} analytic: {1:0.14f} diff: {2}'.format(
    dz_dx2_num, dz_dx2, dz_dx2_num-dz_dx2))
```

```
dz_dx2 numerical: 0.97873798412529 analytic: 0.97873798410574

diff: 1.9549251106809606e-11
```

```python
# For some set of inputs
x2, x1, w2, w1, b = np.random.randn(5)

# Compute analytical gradient
dz_dx2, dz_dx1, dz_dw2, dz_dw1, dz_db = df(x2, x1, w2, w1, b)
```

Are these 5 numbers correct?

Example: Checking one of the results dz_dx2

```python
# Compute numerical gradient
h = 1e-5
dz_dx2_num = (f(x2+h, x1, w2, w1, b) - f(x2-h, x1, w2, w1, b))/(2*h)
print('dz_dx2 numerical: {0:0.14f} analytic: {1:0.14f} diff: {2}'.format(
    dz_dx2_num, dz_dx2, dz_dx2_num-dz_dx2))
```

```
dz_dx2 numerical: 0.97873798412529 analytic: 0.97873798410574
diff: 1.954925106809606e-11
```

Check that difference is within some tolerance

```
dz_dx2_num = (f(x2+h, x1, w2, w1, b) - f(x2-h, x1, w2, w1, b))/(2*h)
dz_dx1_num = (f(x2, x1+h, w2, w1, b) - f(x2, x1-h, w2, w1, b))/(2*h)
dz_dw2_num = (f(x2, x1, w2+h, w1, b) - f(x2, x1, w2-h, w1, b))/(2*h)
dz_dw1_num = (f(x2, x1, w2, w1+h, b) - f(x2, x1, w2, w1-h, b))/(2*h)
dz_db_num  = (f(x2, x1, w2, w1, b+h) - f(x2, x1, w2, w1, b-h))/(2*h)
```

```
dz_dx2 numerical: 0.97873798412529 analytic: 0.97873798410574 diff: 1.9549251106809606e-11
dz_dx1 numerical: 2.24089319917908 analytic: 2.24089319920146 diff: -2.2374102570665855e-11
dz_dw2 numerical: 1.76405234597610 analytic: 1.76405234596766 diff: 8.432810005842839e-12
dz_dw1 numerical: 0.40015720834674 analytic: 0.40015720836722 diff: -2.0482504581309513e-11
dz_db  numerical: 1.00000000000655 analytic: 1.00000000000000 diff: 6.551204023708124e-12
```

```python
def df(x2, x1, w2, w1, b):
    dz_dx2 = w2*2
    dz_dx1 = w1
    dz_dw2 = x2
    dz_dw1 = x1
    dz_db  = 1
    return dz_dx2, dz_dx1, dz_dw2, dz_dw1, dz_db
```

Say I made a mistake in my analytic gradient implementation

Correct:

`dz_dx2 numerical: 0.97873798412529 analytic: 0.97873798410574`

`diff: 1.9549251106809606e-11`

With Mistake:

`dz_dx2 numerical: 0.97873798412529 analytic: 1.95747596821148`

`diff: -0.97873798408619`

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Why not just always use this for backprop?

- Too slow
- You need to make two calls to your forward prop function for each gradient value.

Learning Objectives

- More practice with backpropagation
- Understand backprop being applied to convolutional layer
- See some of the “shortcuts” in convolutional layer backprop
- Understand use of numerical gradient