Convolutional Neural Networks

Deep Learning

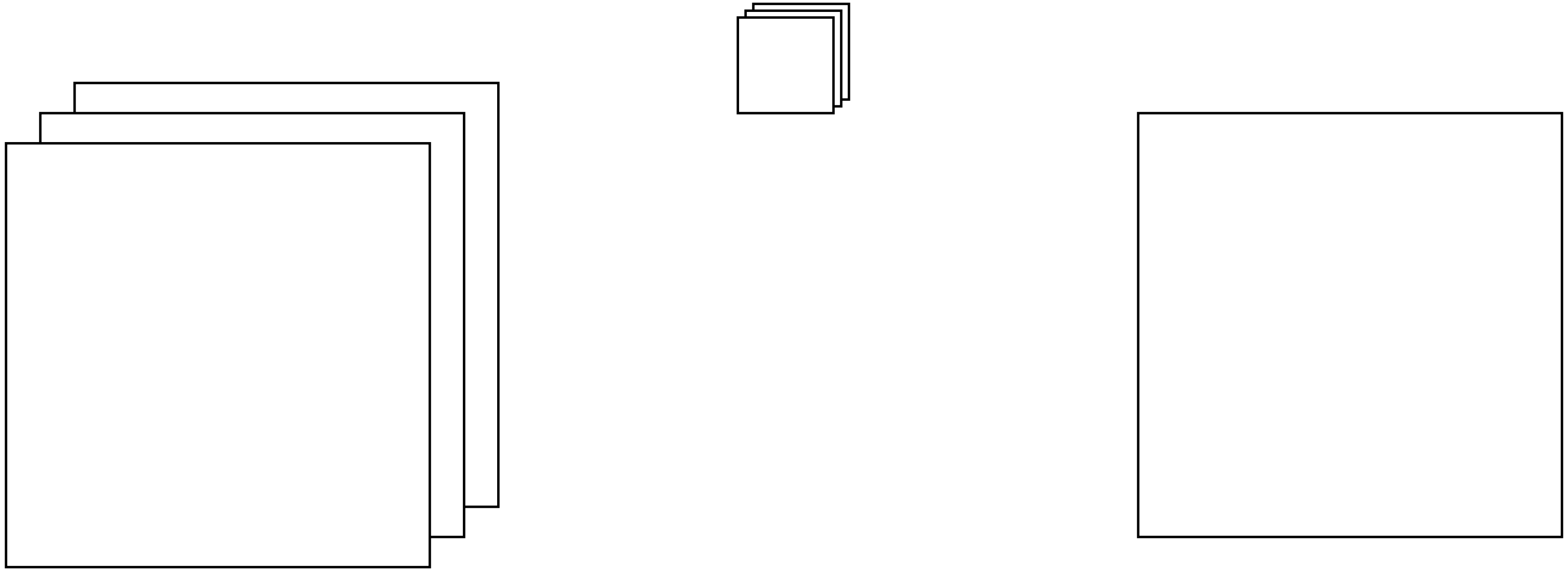
[Brad Quinton](), [Scott Chin]()

Learning Objectives

- Understand how a Convolutional Layer works
- Understand how we can build a Convolutional Neural Network using Convolutional and Fully-Connected Layers
- Understand how we can ”transform” a Convolutional Layer to a Fully-Connected Layer and vice-versa
- Understand what stride and padding are
- Understand how Pooling Layers work

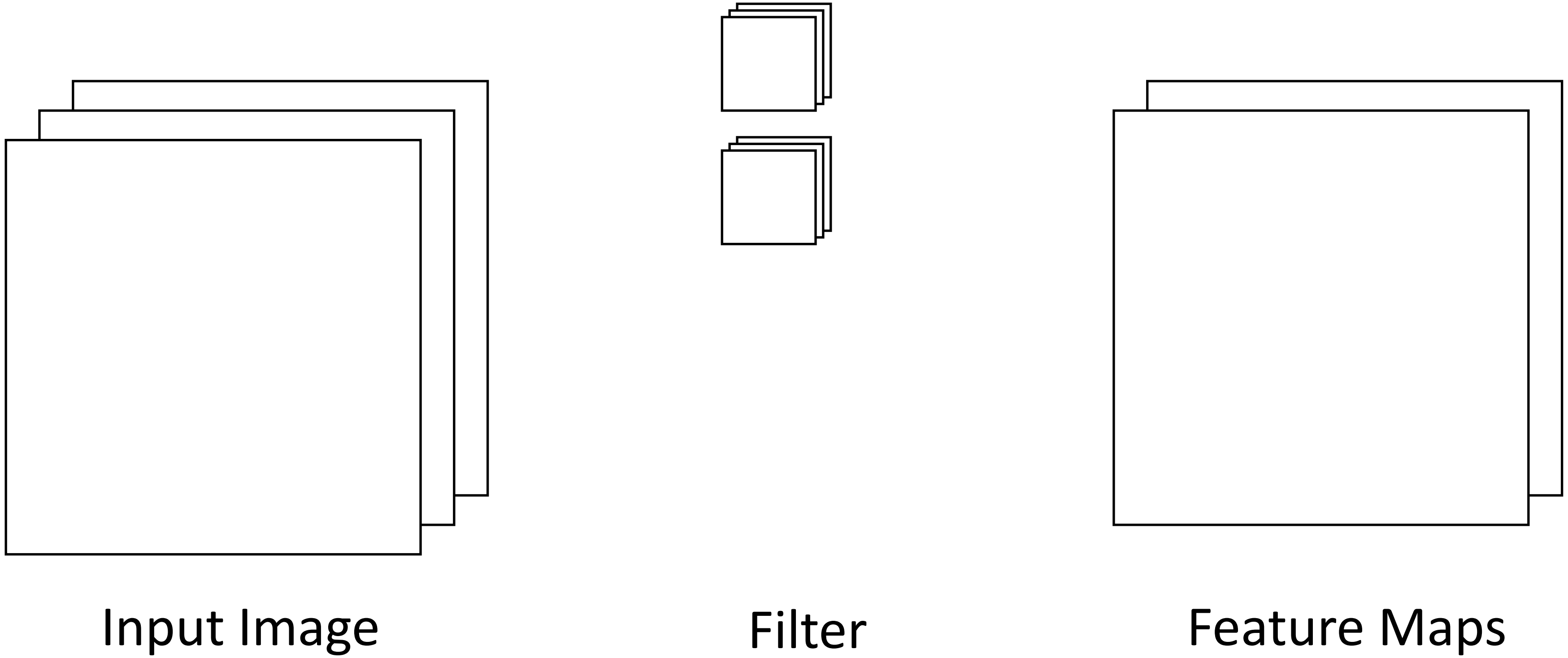
Convolutional Layer

So far, talking about one filter



Input Image Filter Feature Map

Want to look for more than one feature



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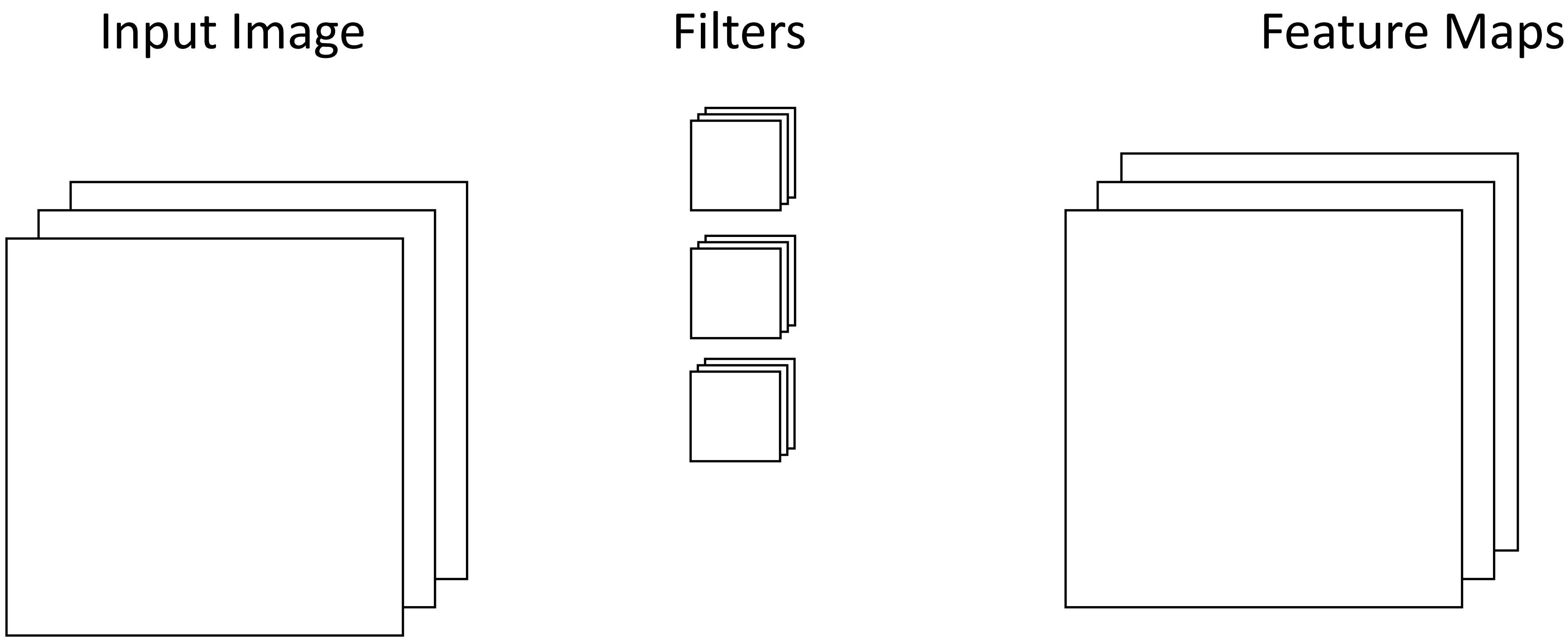
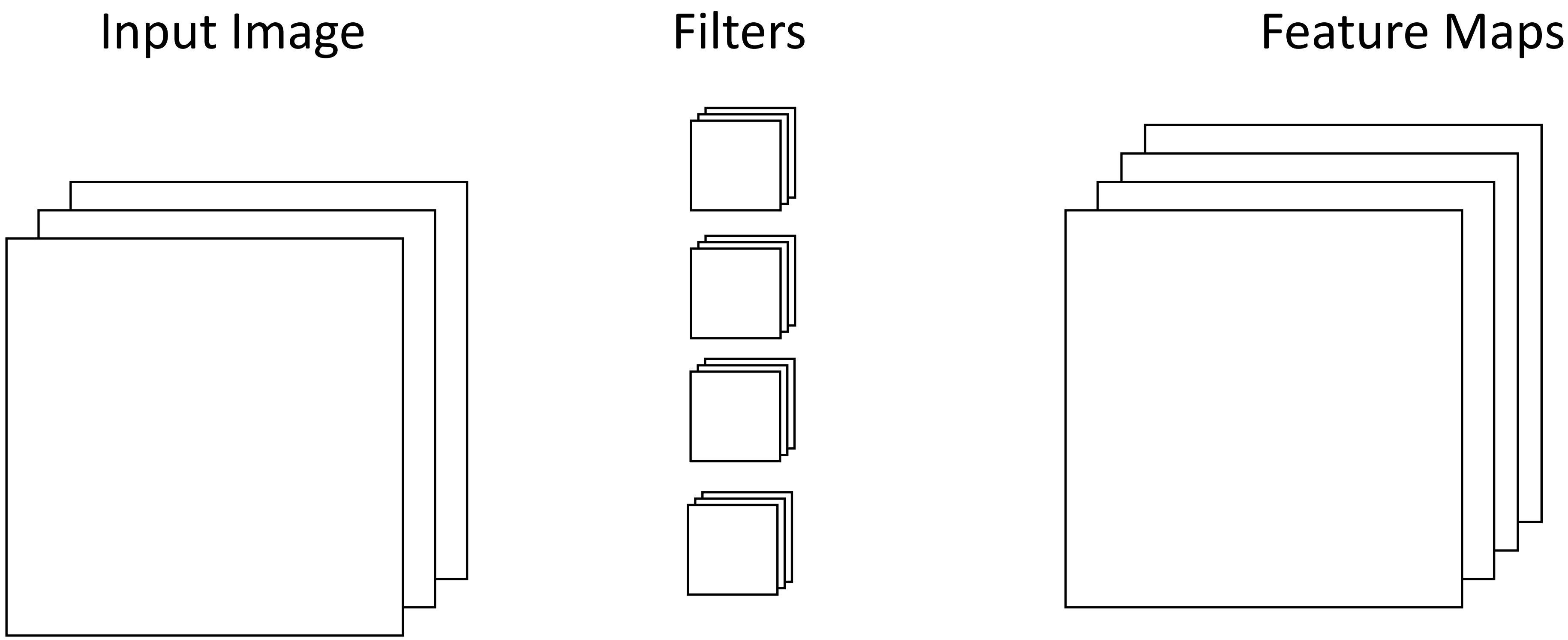
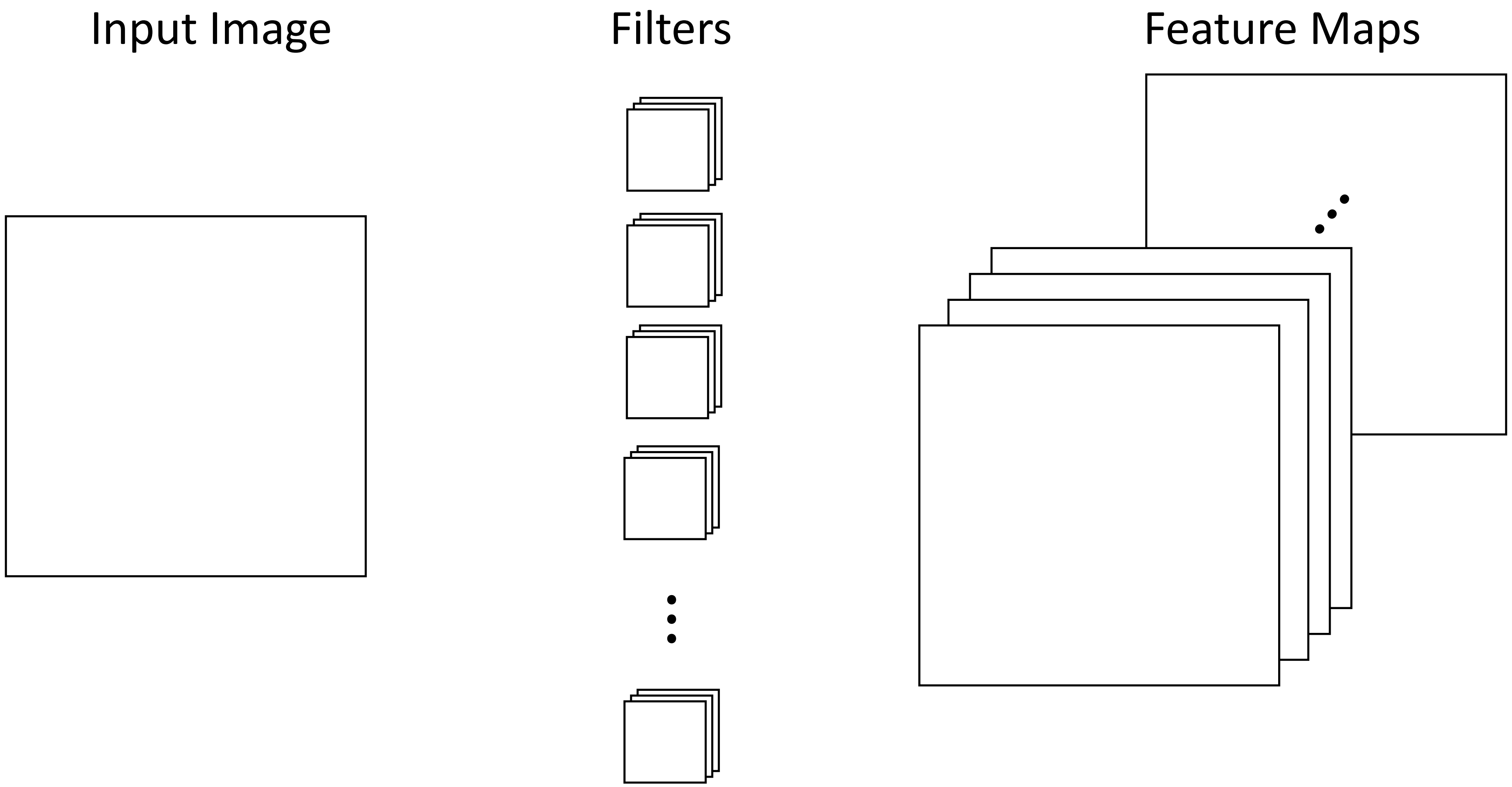
Working towards defining a Convolutional Layer



Working towards defining a Convolutional Layer



Working towards defining a Convolutional Layer



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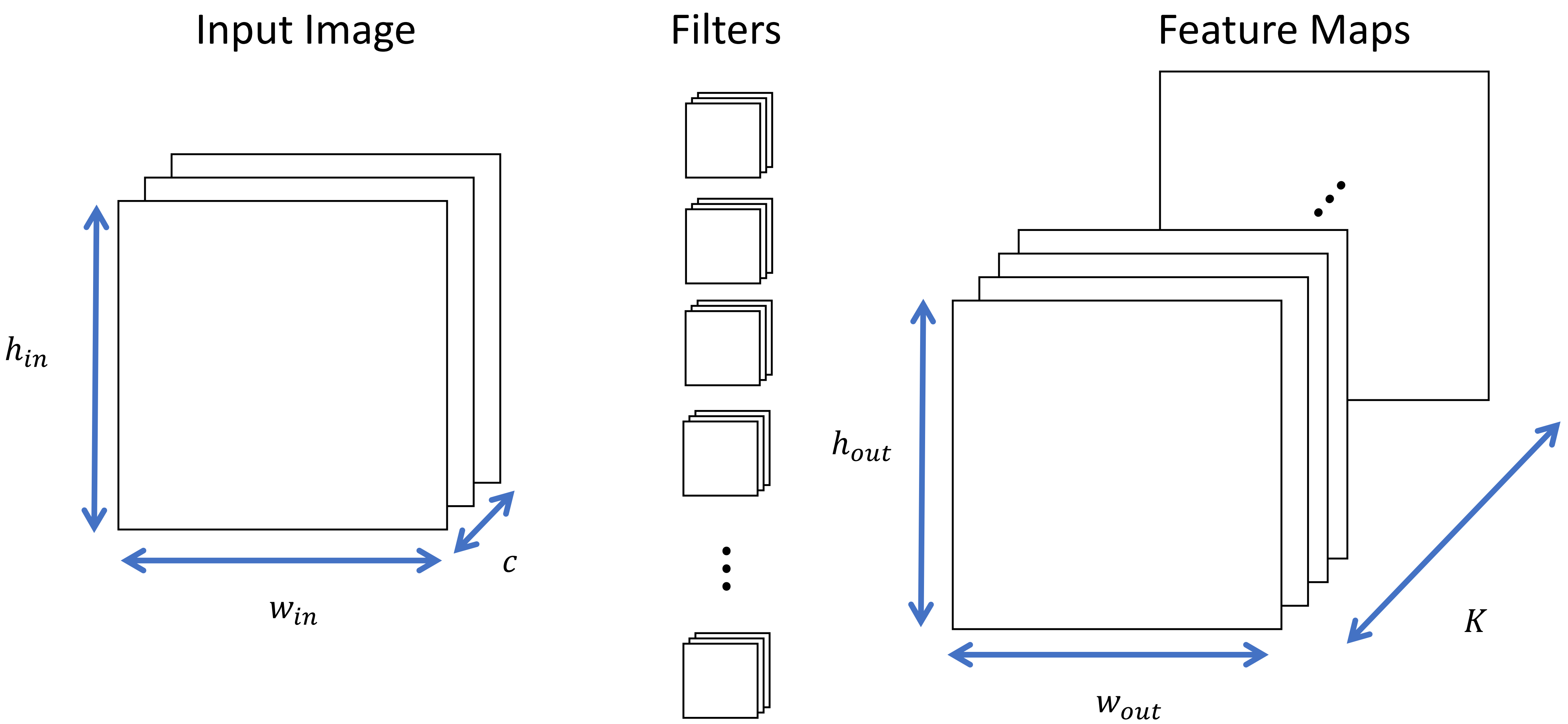
Working towards defining a Convolutional Layer



- K number of (f, f, c)
- Can think of each filter as a "neuron"

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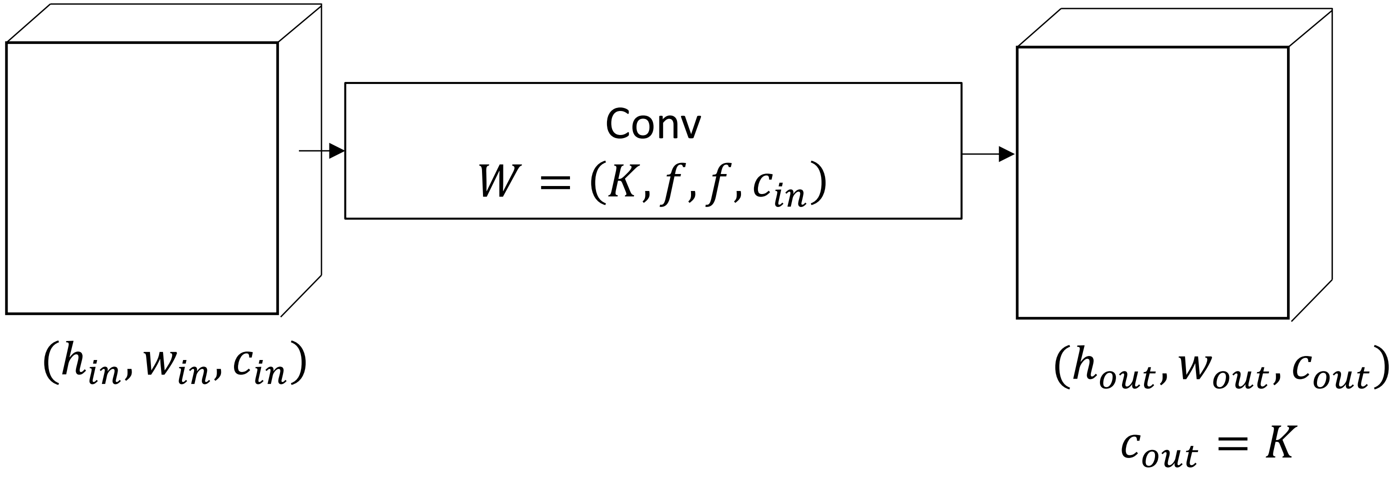
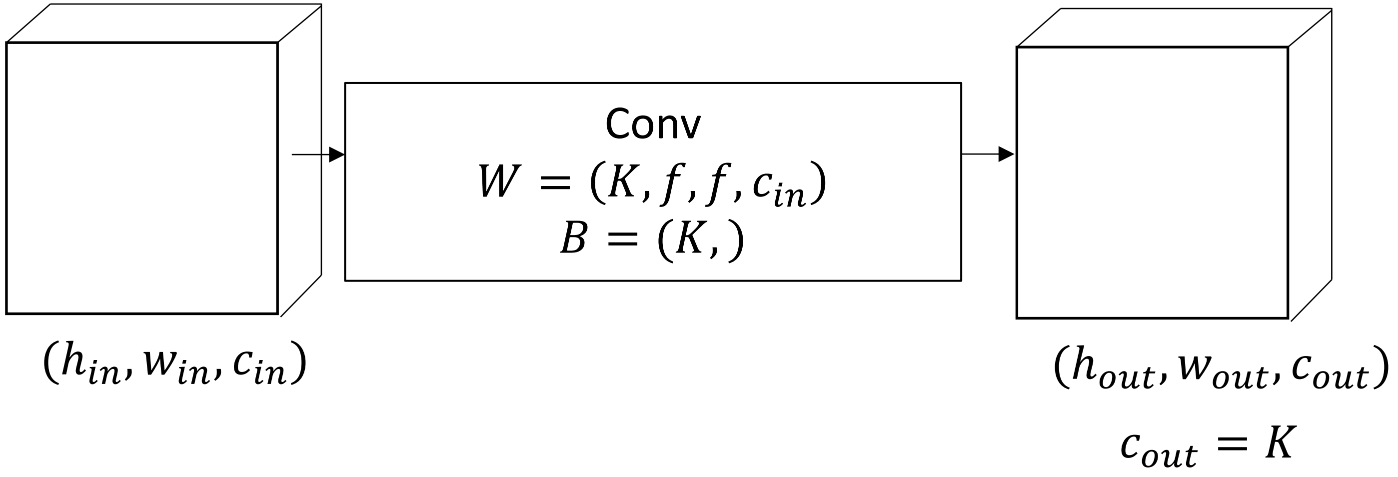
Working towards defining a Convolutional Layer



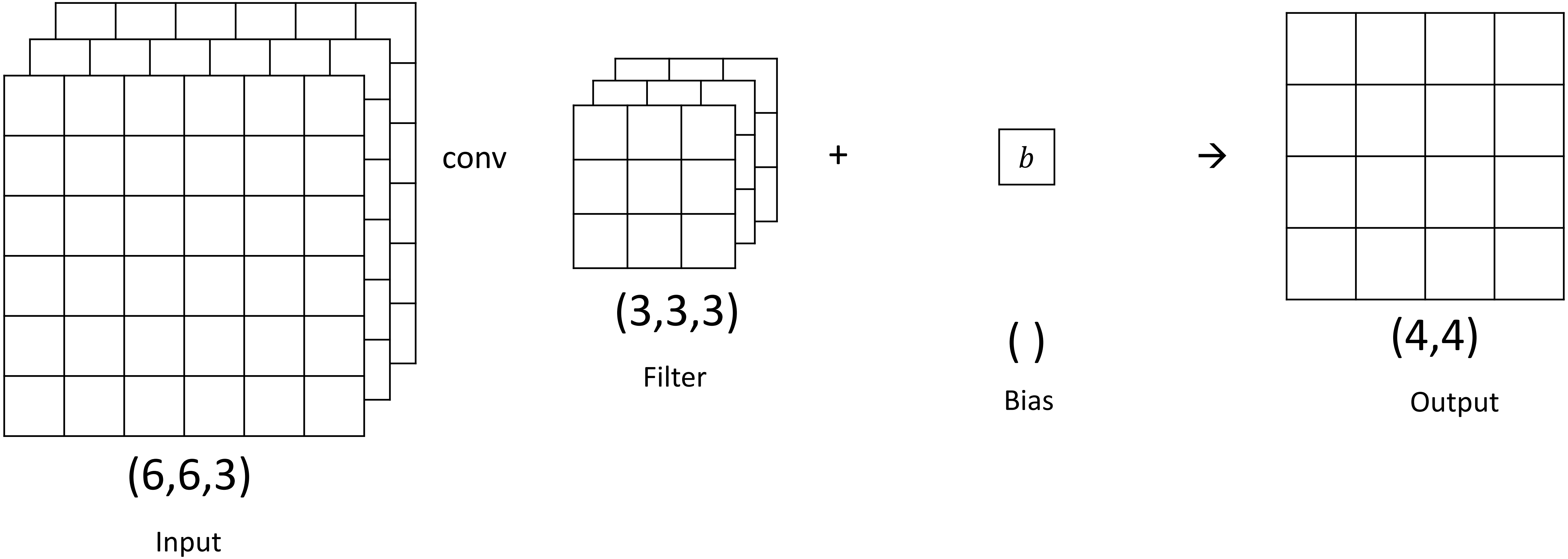
- Collection of filters can be represented as a single (K, f, f, c) weight tensor
- We don't quite have a convolutional layer yet.
- Still need bias parameters and activations

Working towards defining a Convolutional Layer



- If we think of each filter as a neuron, then we need one bias per filter

Computation for ONE filter



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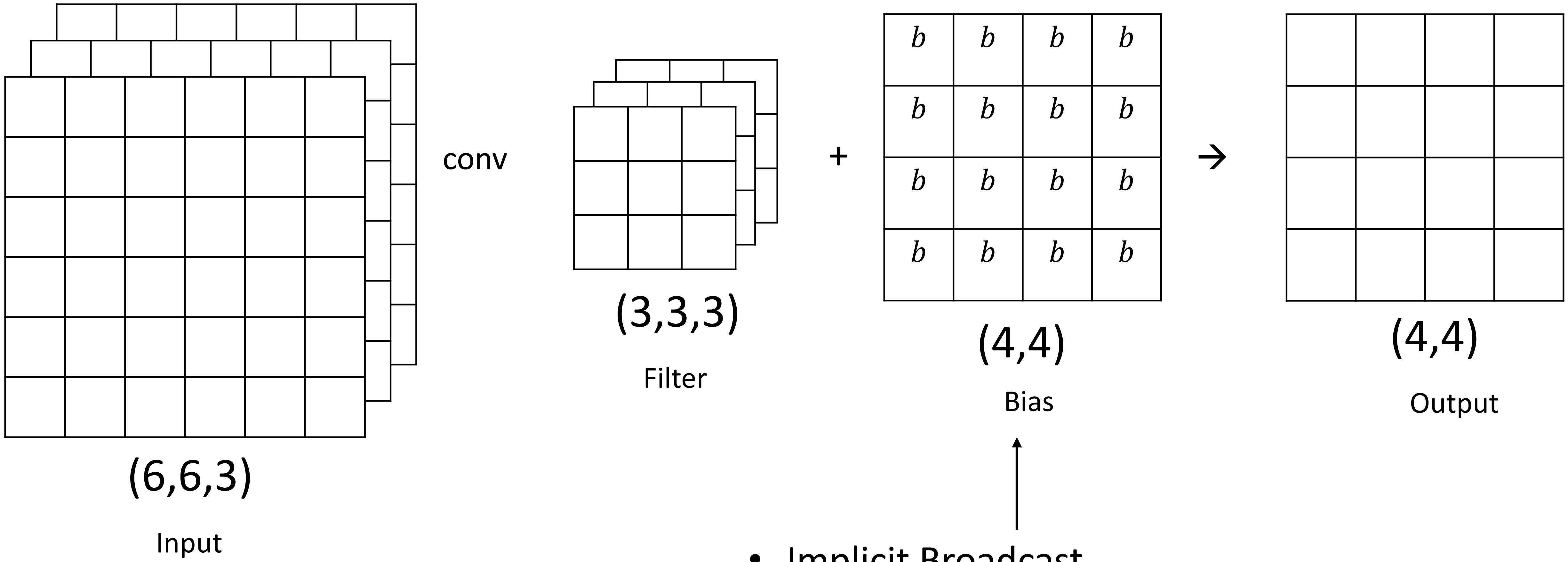
Computation for ONE filter



- Implicit Broadcast.
- There is still only **ONE** bias parameter

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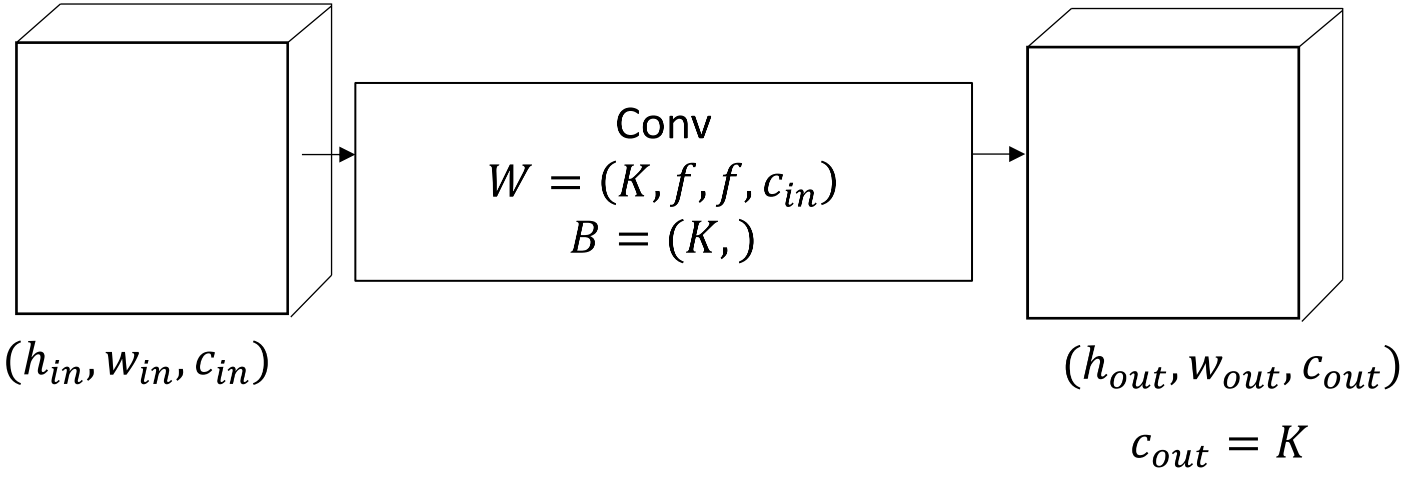
Working towards defining a Convolutional Layer



- So far we've defined the "linear" part of the layer. Similar to the z=WX+B part of the fully-connected layer
- Convolution is a linear operation
- Still need nonlinear activation

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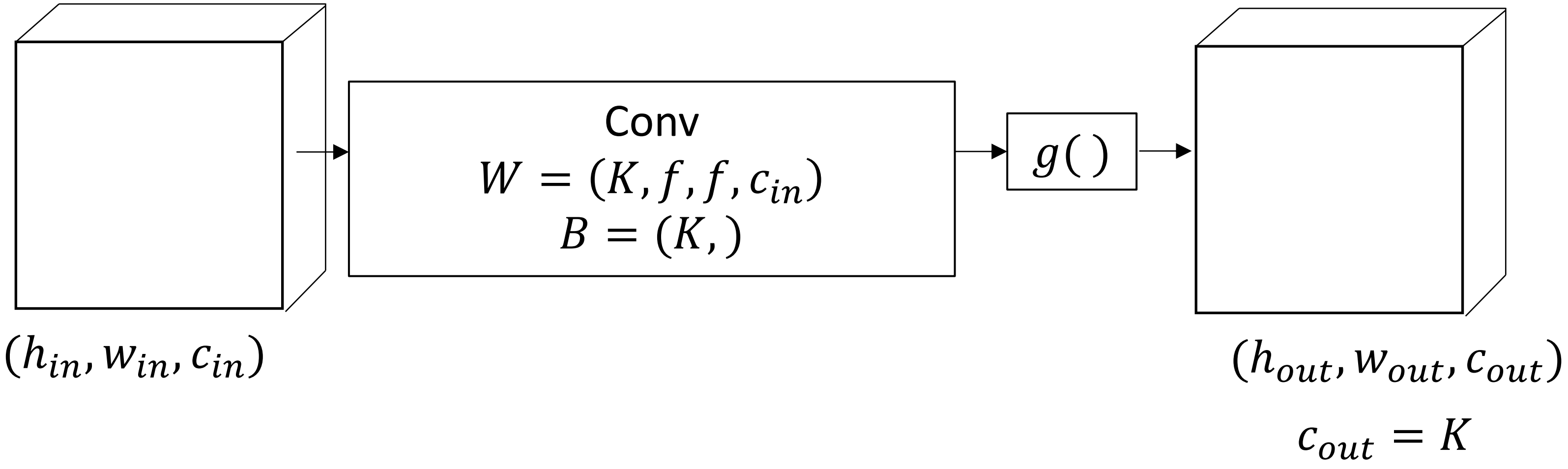
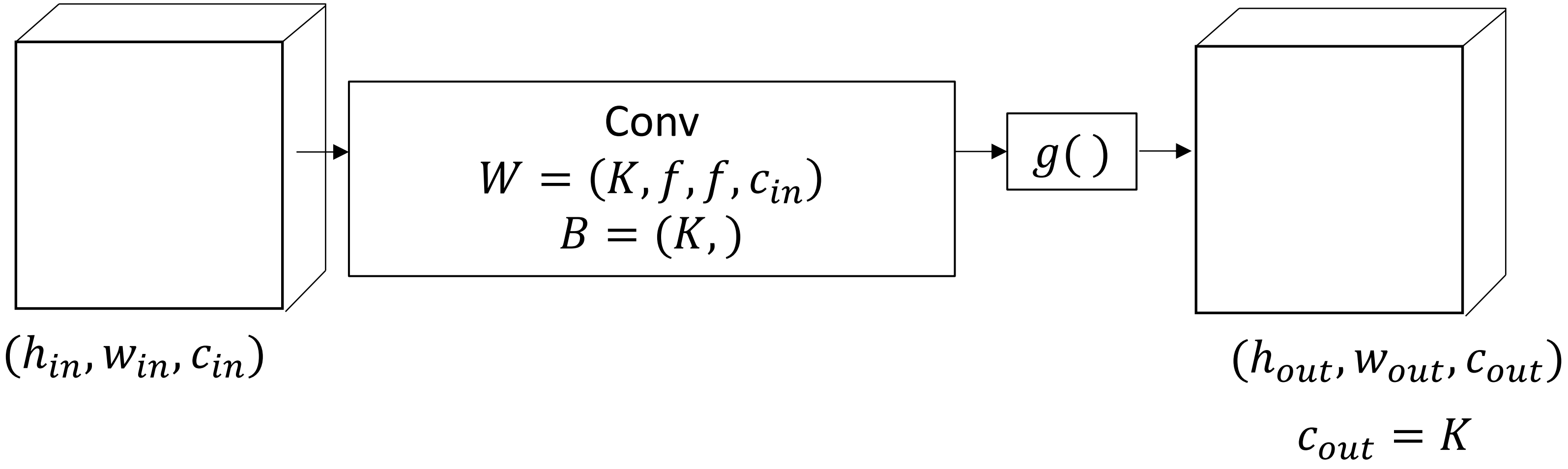
This is a Convolutional Layer

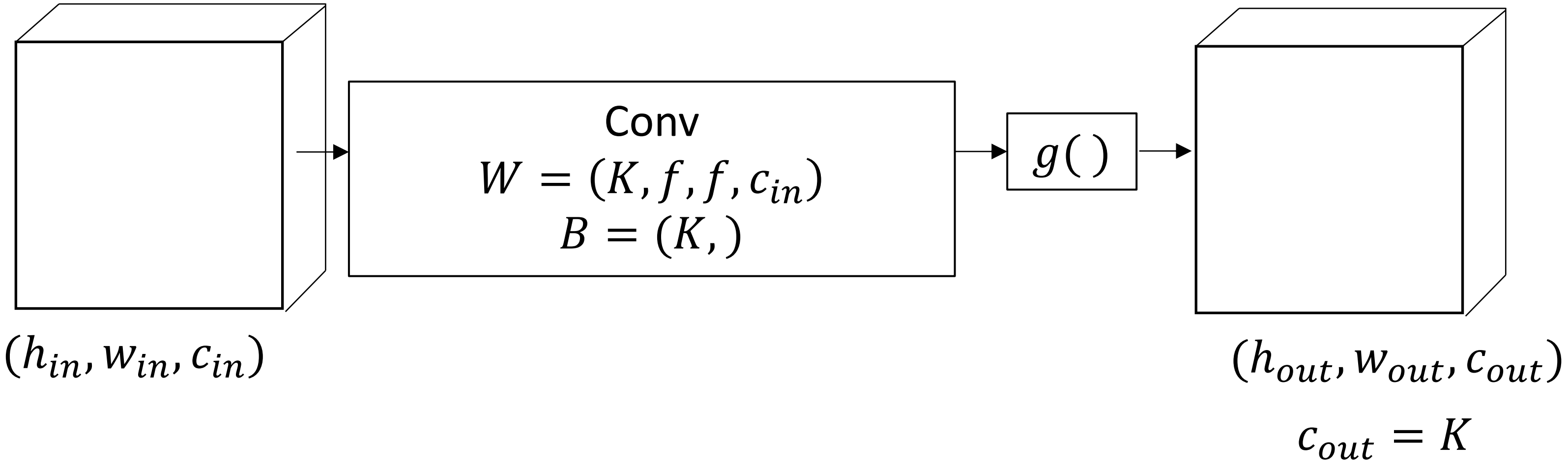


- Remember that activation is applied to each element separately

Number of parameters in a Conv Layer?



Number of parameters in a Conv Layer?



- Weight parameters: $K * f * f * c_{in}$
- Bias parameters: K
- Total: $K(f * f * c_{in} + 1)$

Number of parameters doesn't change if input size changes

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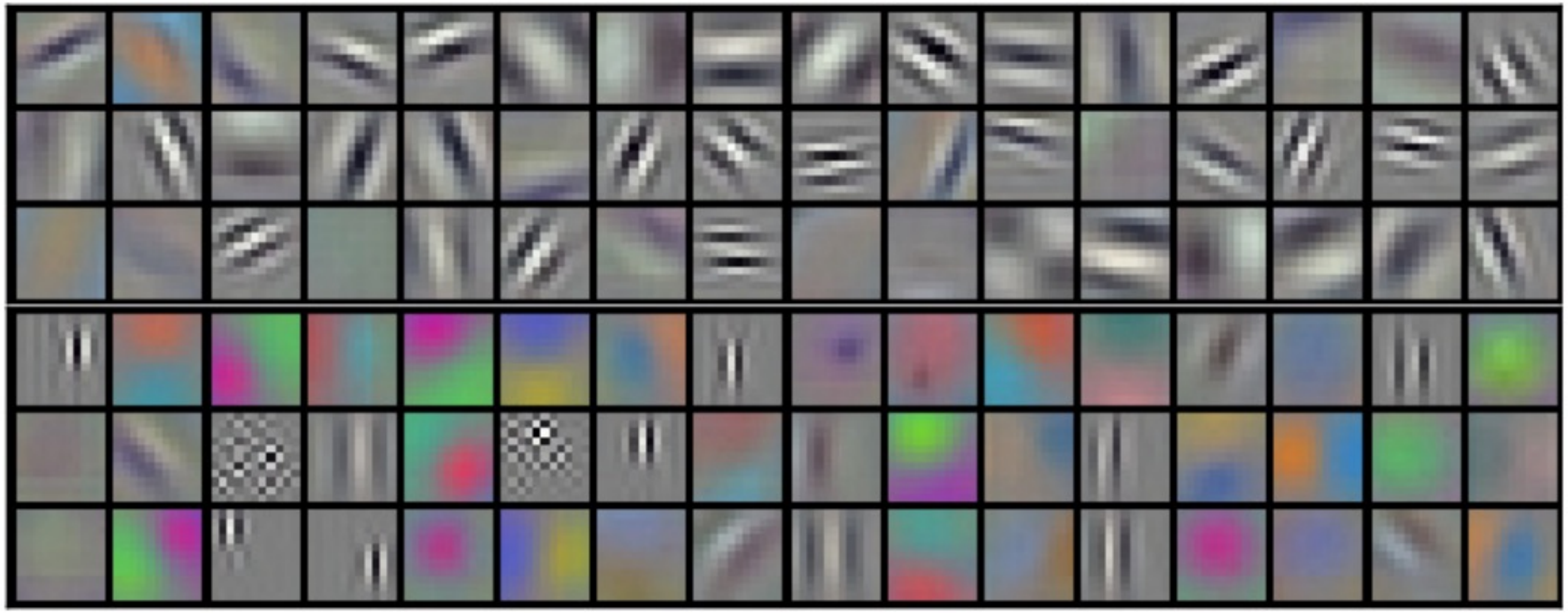
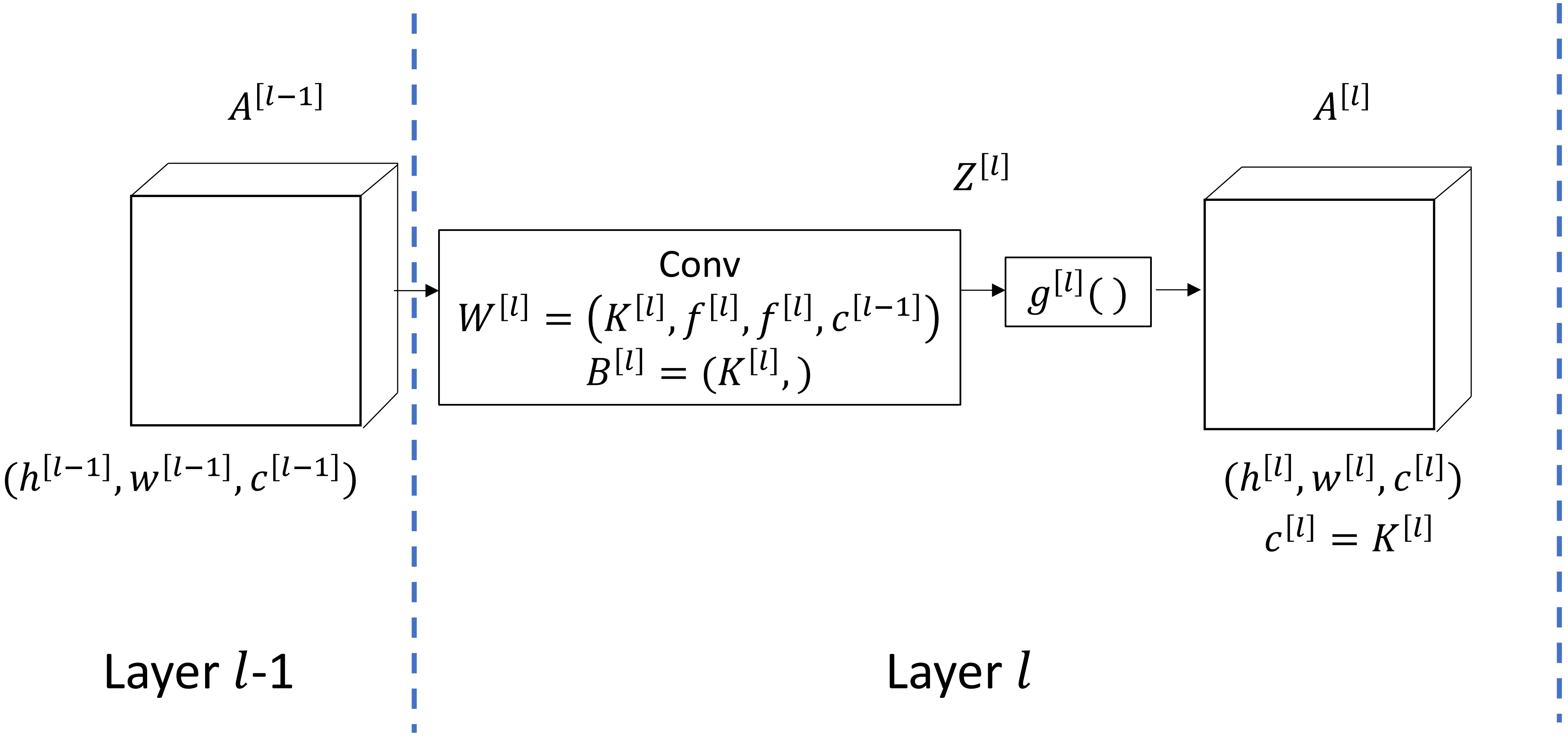
First Layer Filters of AlexNet (11x11)



“ImageNet Classification with Deep Convolutional Neural Networks”, Krizhevsky, Sutskever, Hinton, 2012,
https://papers.nips.cc/paper/4824-imagenet-classification-with-deep-convolutional-neural-networks.pdf

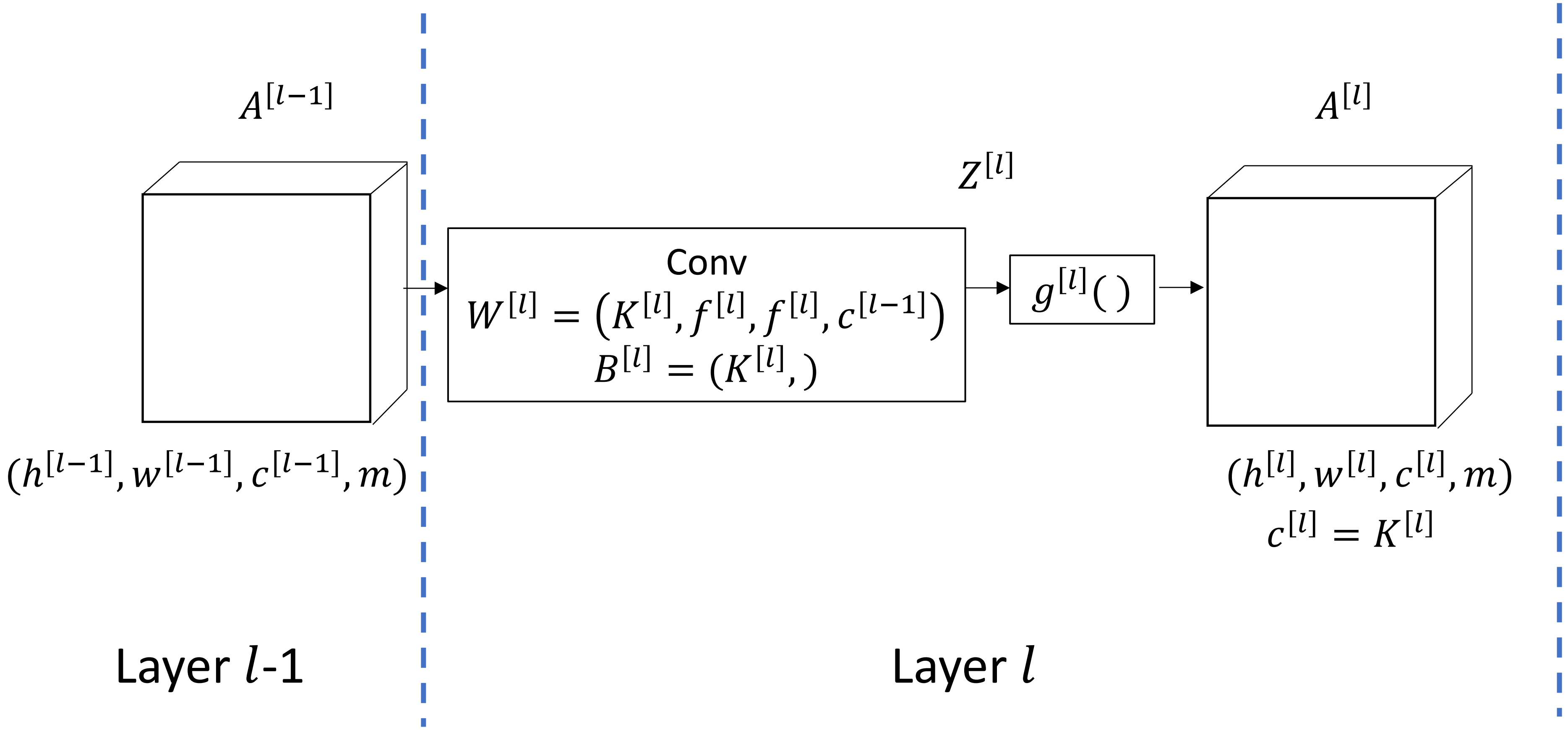
Stacking Convolution Layers

Generalize



- $[l]$ denotes layer l

Vectorizing for multiple samples (Next Lecture)



- $[l]$ denotes layer l
- m denotes batch-size

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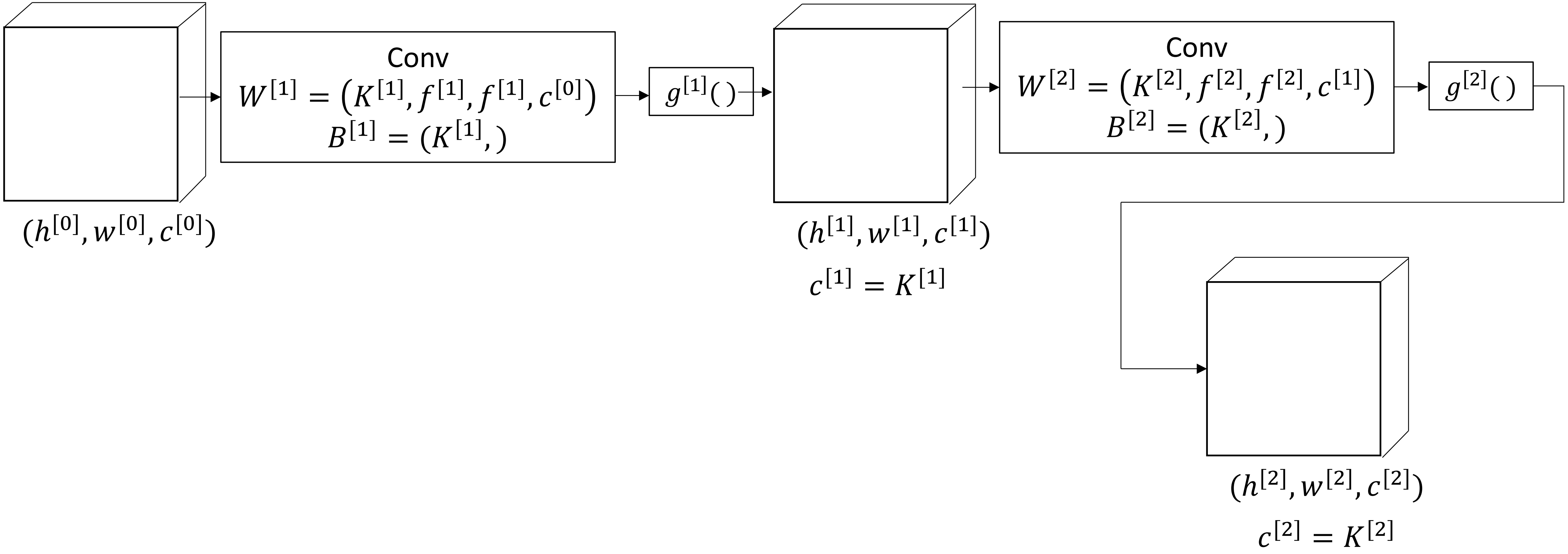
Two Convolutional Layers Stacked



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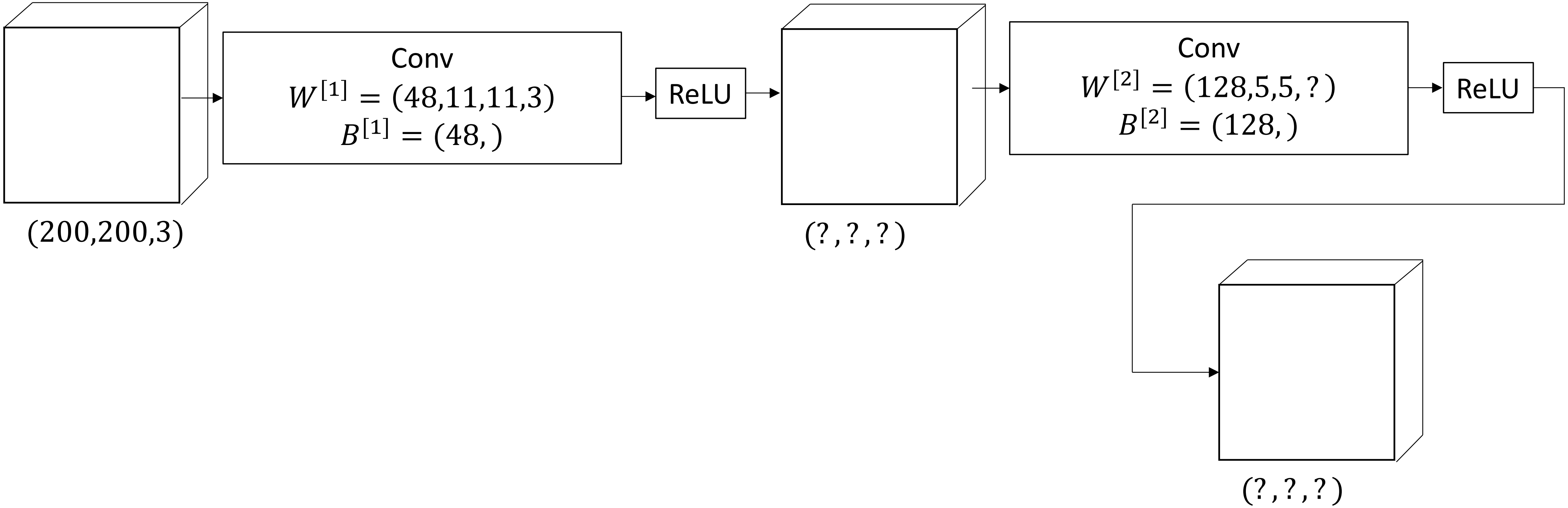
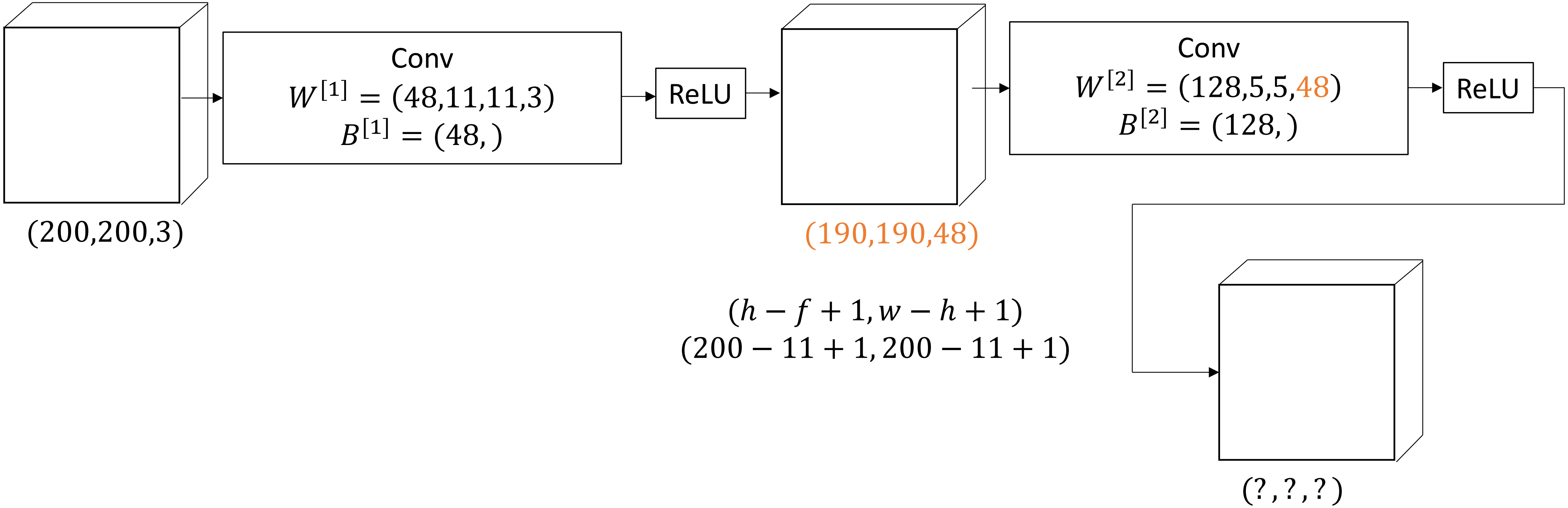
Now with some real numbers



Simple CNN Example with numbers



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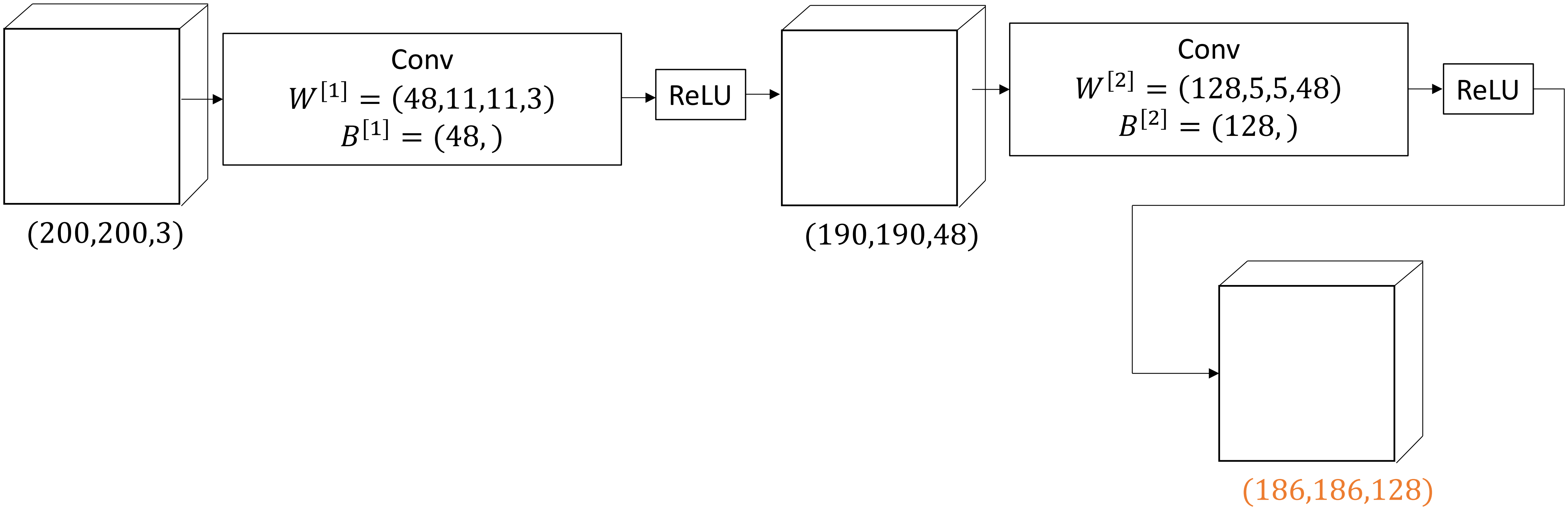
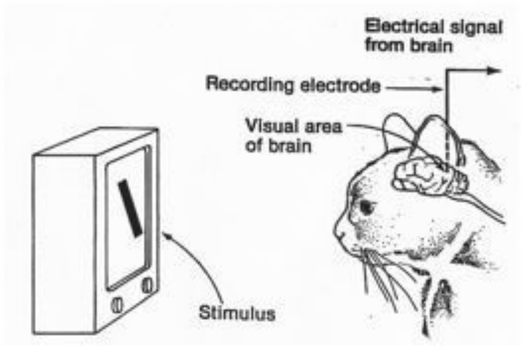
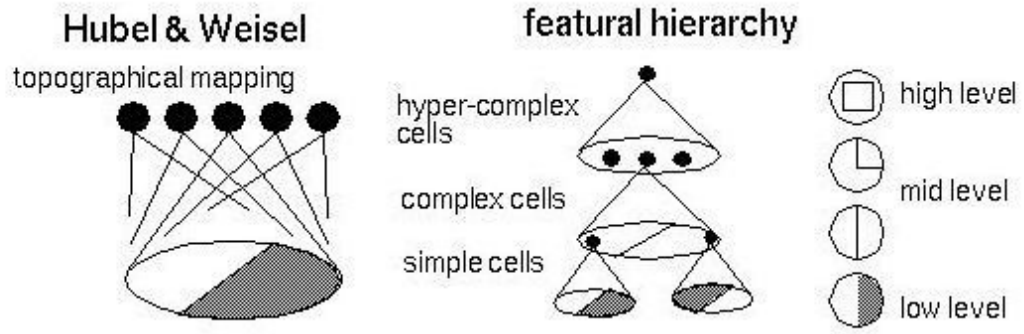
Simple CNN Example with numbers



$$(h - f + 1, w - h + 1)$$
$$(190 - 5 + 1, 190 - 5 + 1)$$

Interpretation of Stacking Convolutions





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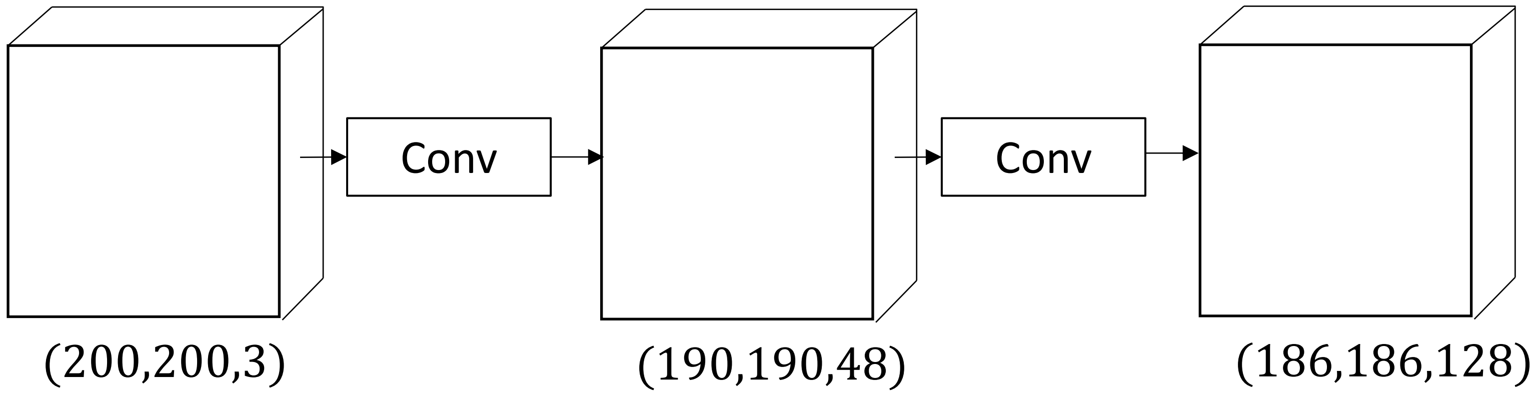
Why do filters look across all channels?

- Beyond the first layer, each channel of a volume is the activation map of a lower level feature
- In order to build filters that look for **compositions** of lower level features, must look at multiple activation maps.



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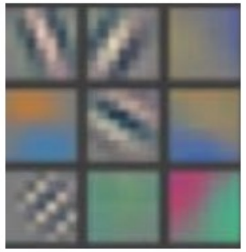
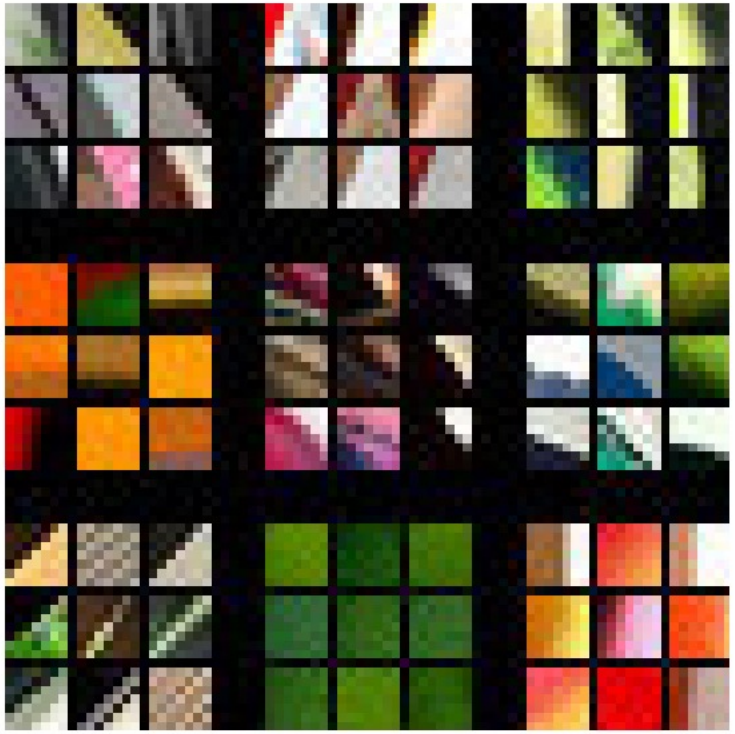
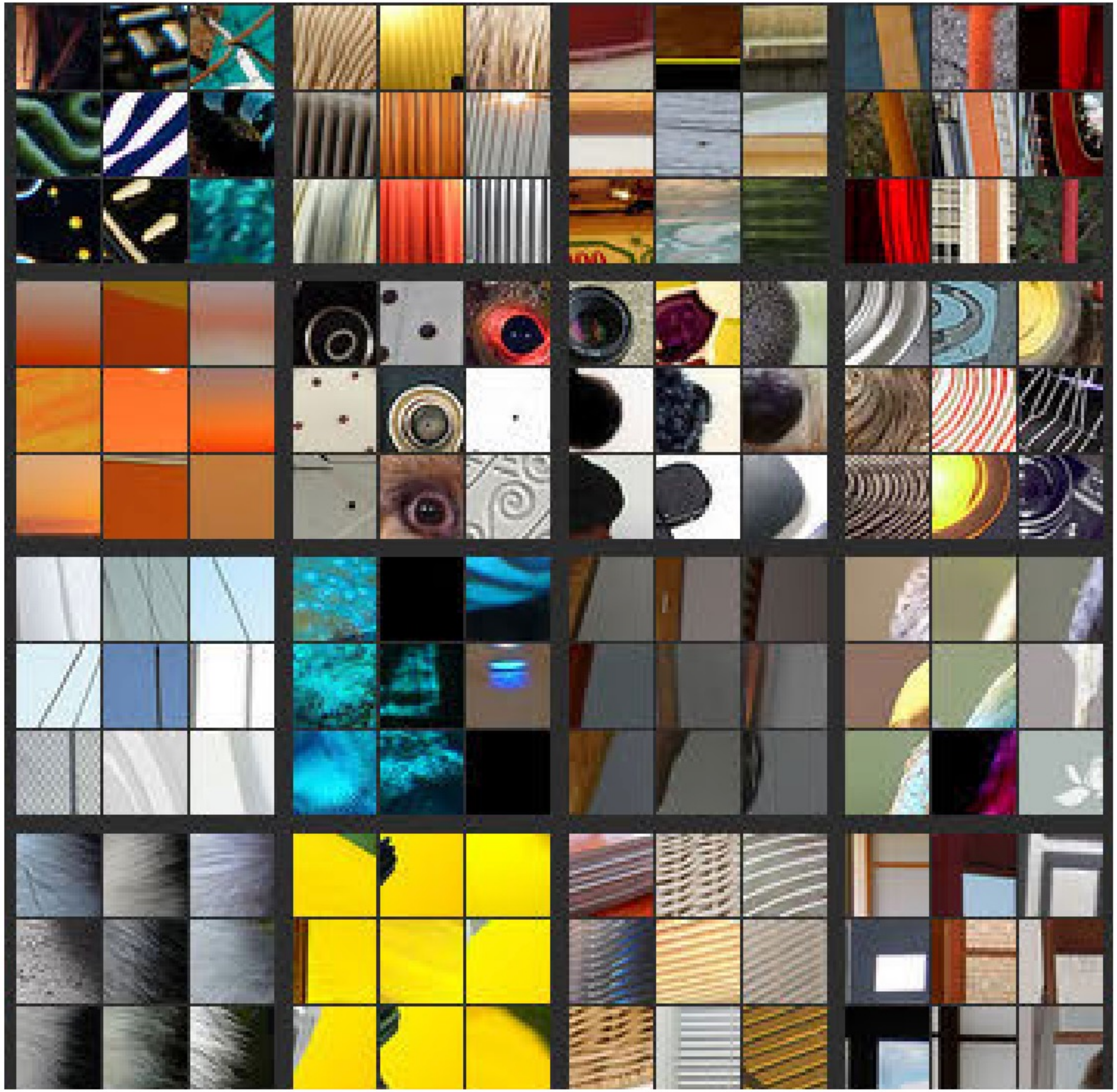
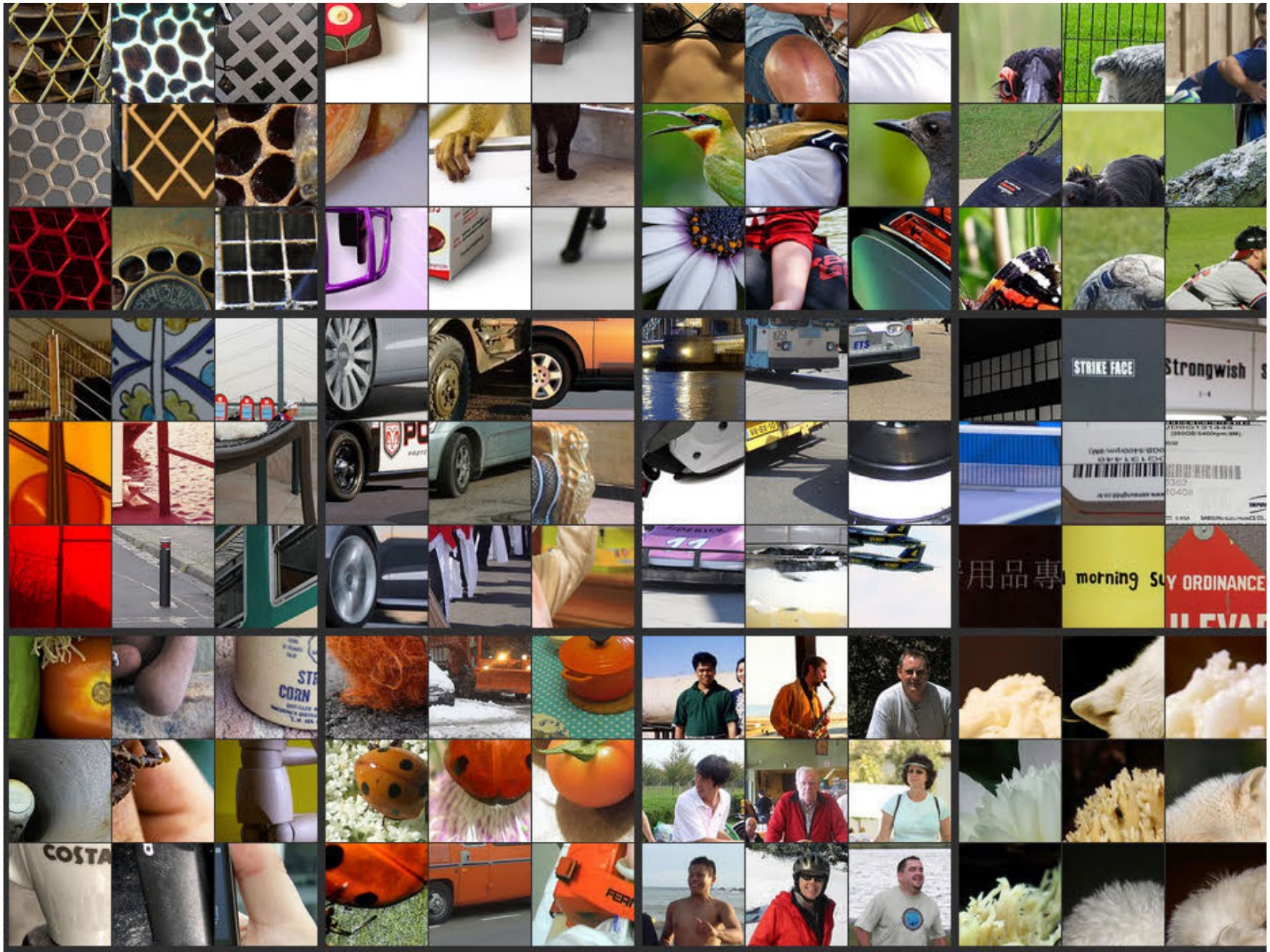


Layer 1



Layer 2





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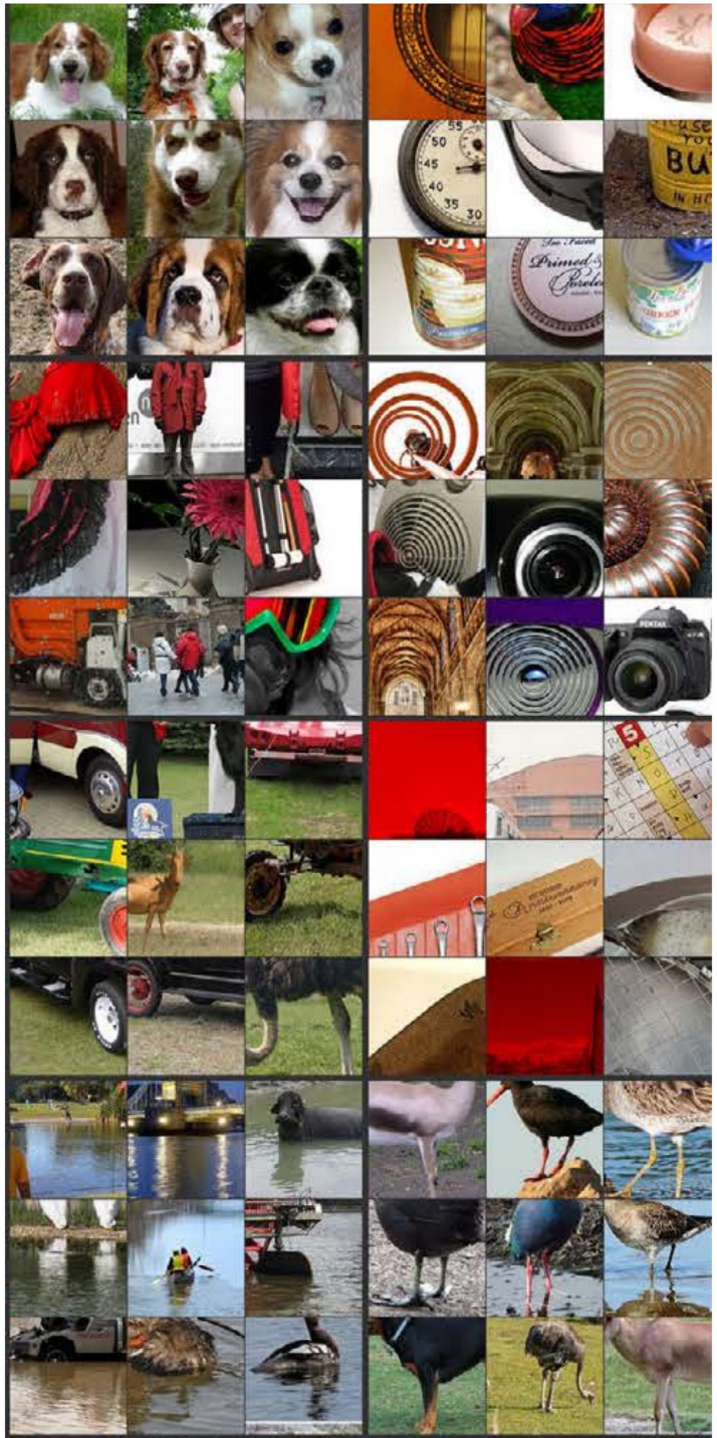


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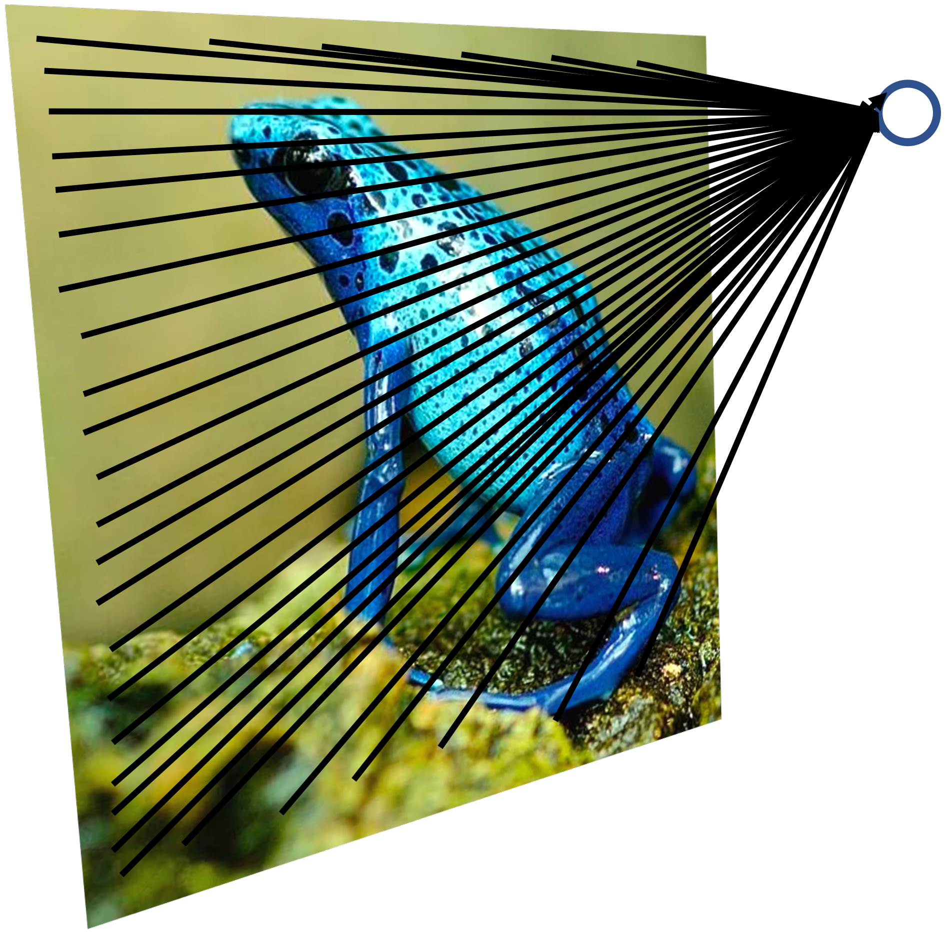
Relationship between Convolutional and Fully-Connected Layer

Recall Motivation for Convolutional Layer



One fully connected neuron shown

- We said that Fully-Connected neural networks leads to a lot of parameters
- And doesn't make use of locality of visual features spatial correlation
- Not making use of locality of visual features spatial correlation

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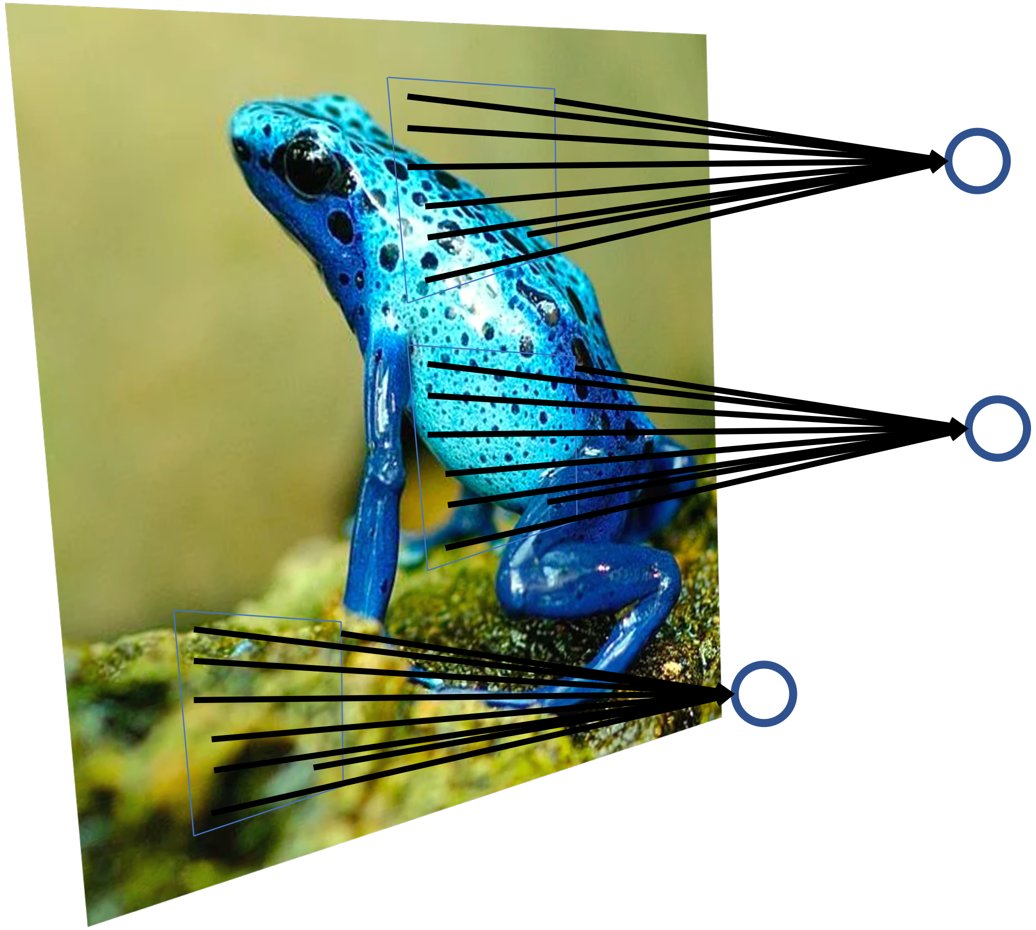
Recall Motivation for Convolutional Layer



- We said that Fully-Connected neural networks leads to a lot of parameters
- And doesn’t make use of locality of visual features spatial correlation
- We then said that maybe we could solve these problems using some sort of well thought-out sparsely-connected organization

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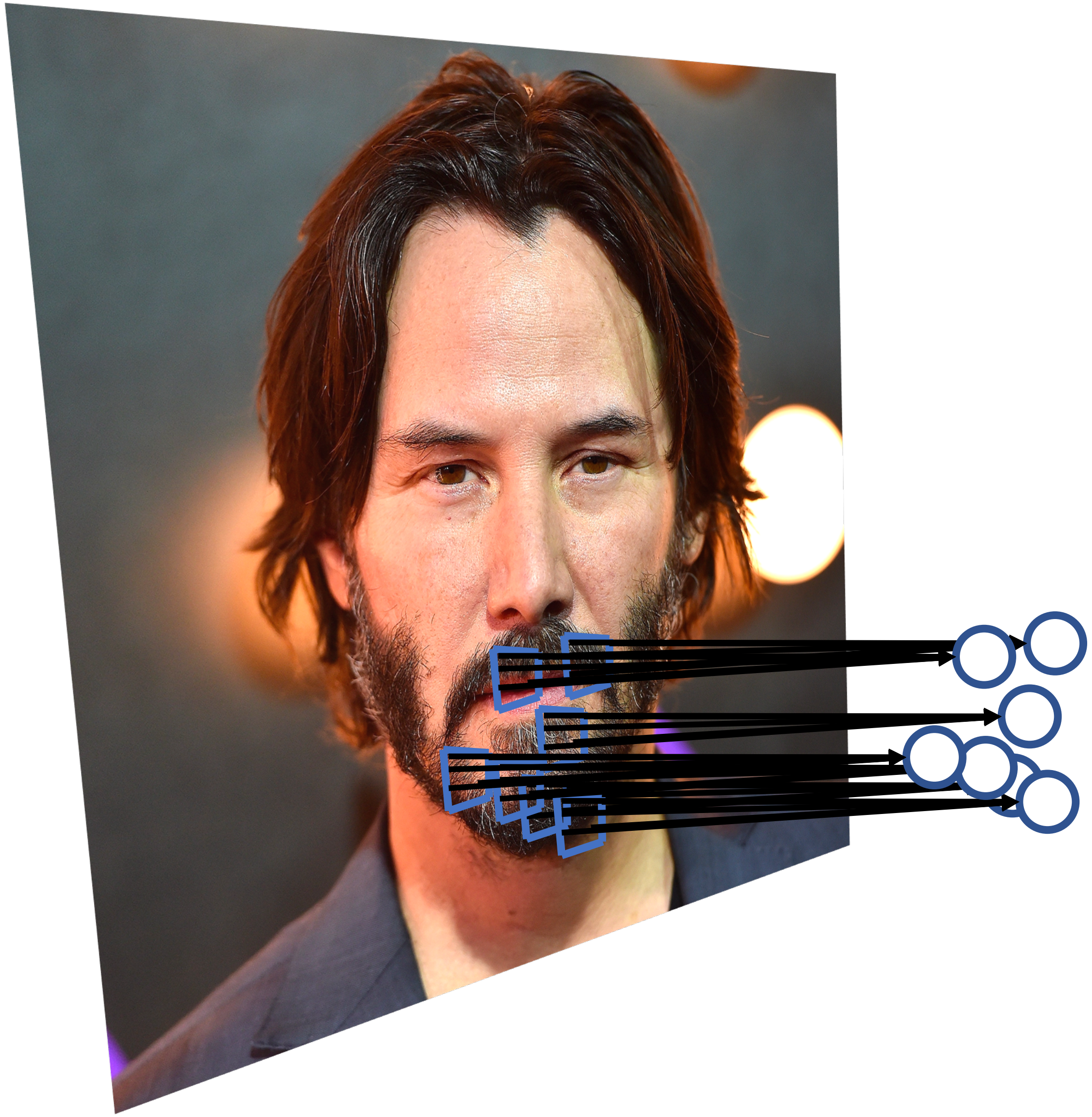
Recall Motivation for Conv Layer



- We said that even for only localized connections, we would have redundant neurons
- And that maybe we could solve this with parameter sharing

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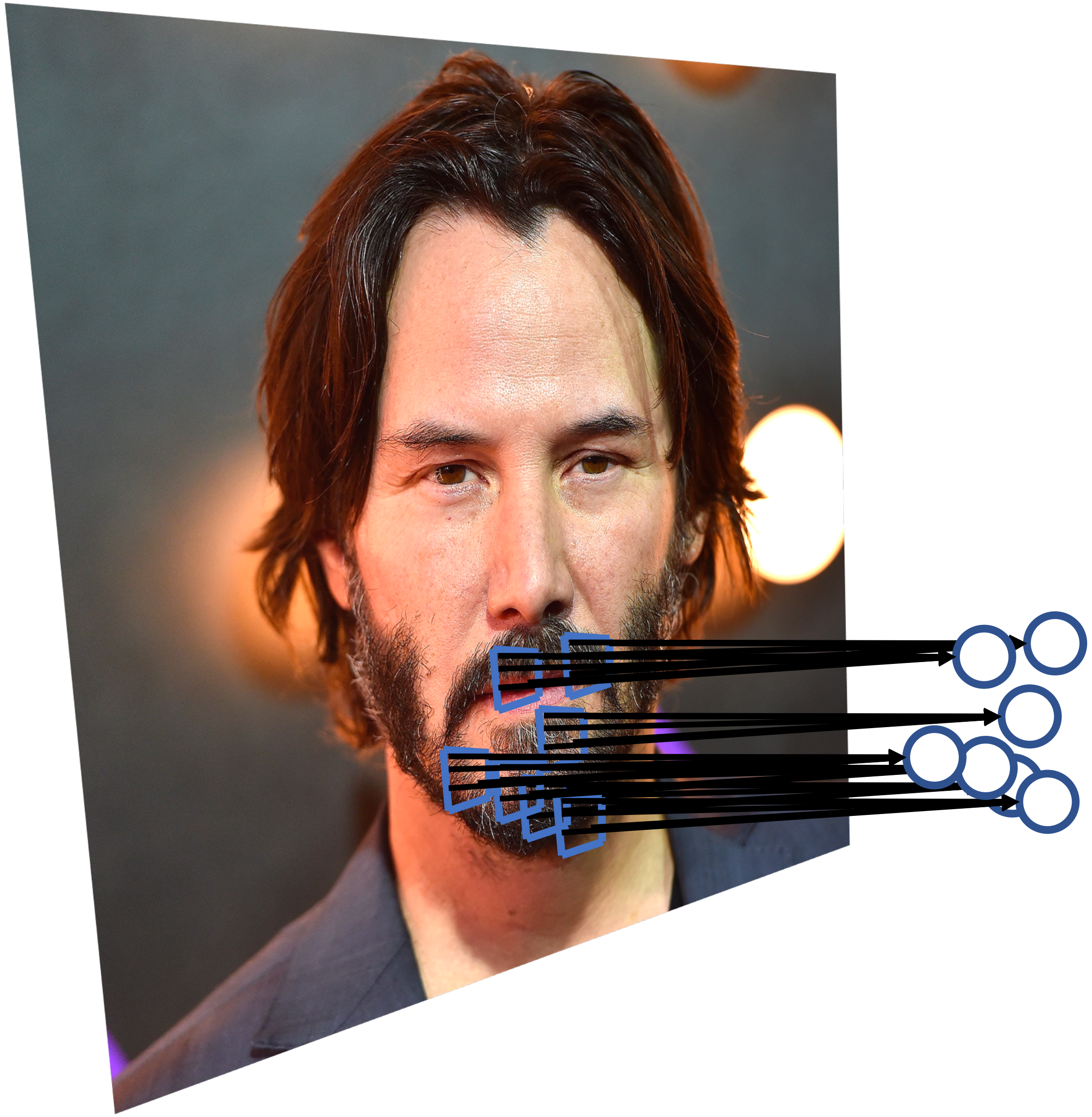
Recall Motivation for Conv Layer



- We said that even for only localized connections, we would have redundant neurons
- And that maybe we could solve this with parameter sharing

Note: For all these problems, given a fully-connected network with enough capacity, and a large enough training data set, we could get around these problems. But you'd need a massive network and a, likely, infeasibly large data set. CNNs are much much more efficient

Nothing magical about Convolutional Layers

- Convolution layer basically let us achieve sparse connectivity between layers while also taking advantage of spatial structure of image data to allow parameter sharing!
- Although it employs a lot of inspiration and intuition from our understanding of human vision, it is mathematically just a sparsely connected version of the fully connected layer with parameter sharing.
- Let's illustrate with an example how we transform a fully-connected layer to a convolutional layer

Example for one filter:

$$\underset{3\text{x}3}{\begin{array}{|c|c|c|}\hline x_1 & x_2 & x_3 \\ \hline x_4 & x_5 & x_6 \\ \hline x_7 & x_8 & x_9 \\ \hline \end{array}} \quad \text{conv} \quad \underset{2\text{x}2}{\begin{array}{|c|c|}\hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array}} \quad + \quad \begin{array}{|c|}\hline b_1 \\ \hline \end{array} \quad = \quad \underset{2\text{x}2}{\begin{array}{|c|c|}\hline z_1 & z_2 \\ \hline z_3 & z_4 \\ \hline \end{array}}$$

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$$\begin{array}{|c|c|c|}\hline x_1 & x_2 & x_3 \\ \hline x_4 & x_5 & x_6 \\ \hline x_7 & x_8 & x_9 \\ \hline \end{array} \quad \text{conv} \quad \begin{array}{|c|c|}\hline w_1 & w_2 \\ \hline w_3 & w_4 \\ \hline \end{array} \quad + \quad \begin{array}{|c|}\hline b_1 \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|}\hline z_1 & z_2 \\ \hline z_3 & z_4 \\ \hline \end{array}$$

3x3 2x2 2x2

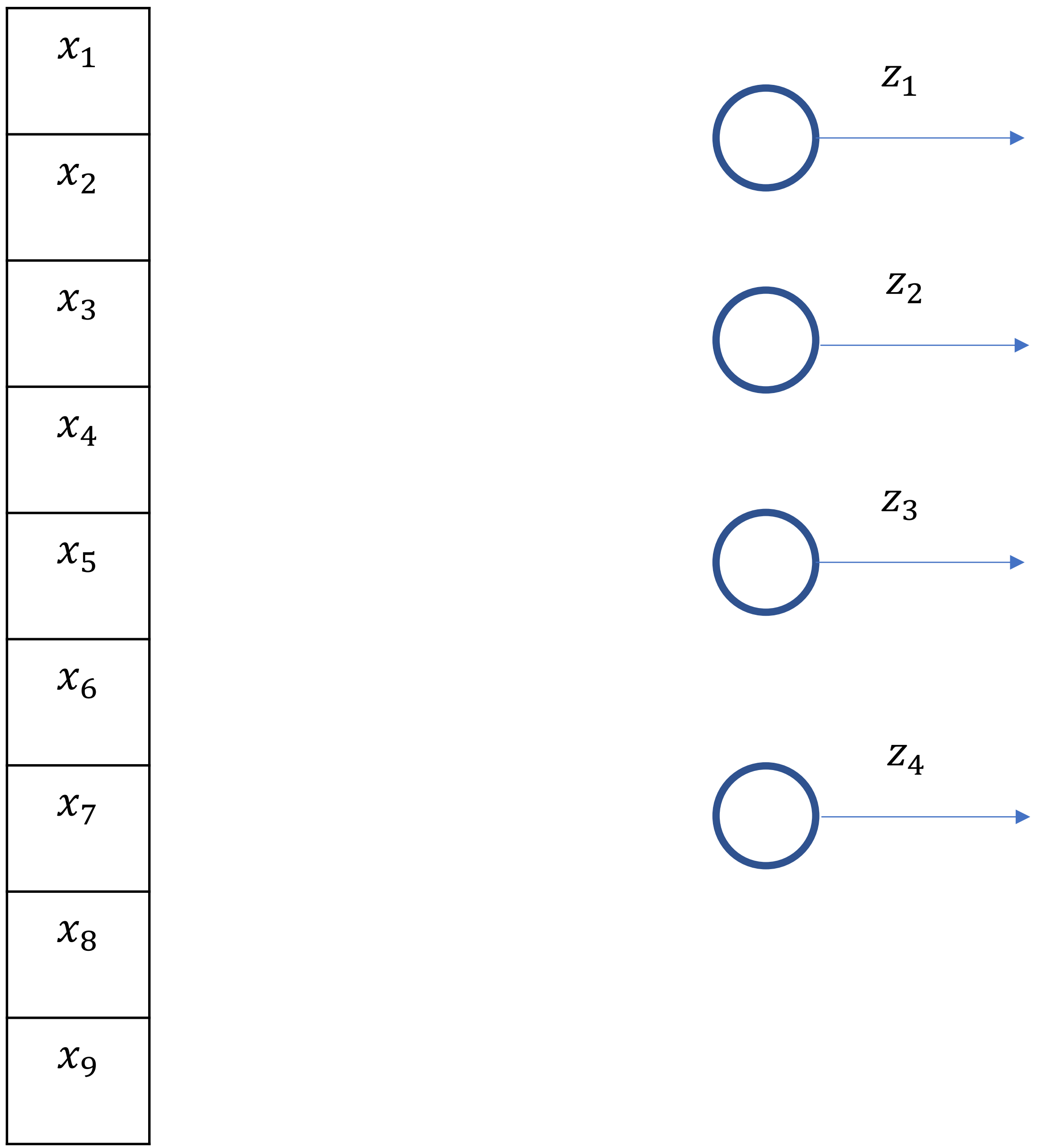
$$z_1 = w_1 x_1 + w_2 x_2 + w_3 x_4 + w_4 x_5 + b_1$$

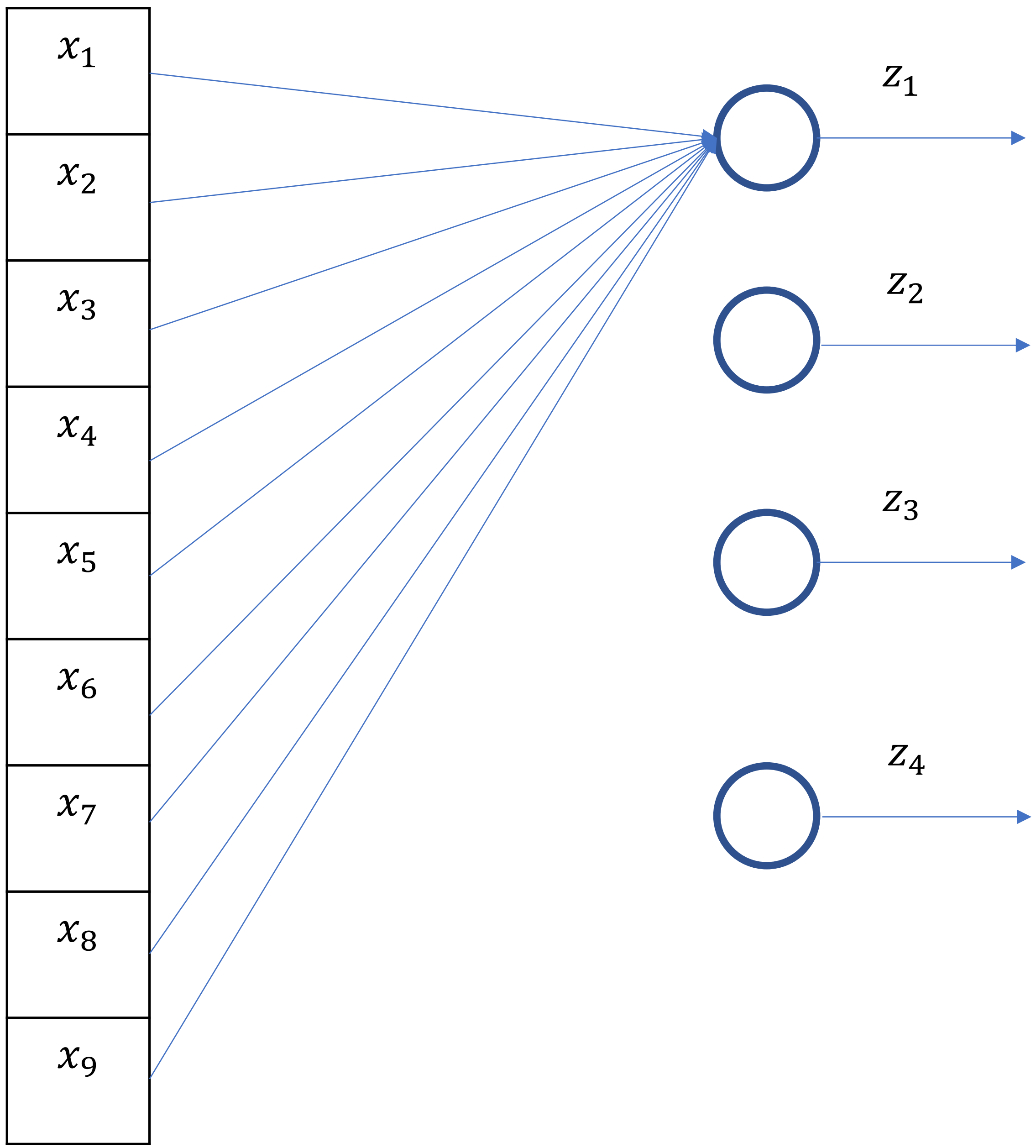
$$z_2 = w_1 x_2 + w_2 x_3 + w_3 x_5 + w_4 x_6 + b_1$$

$$z_3 = w_1 x_4 + w_2 x_5 + w_3 x_7 + w_4 x_8 + b_1$$

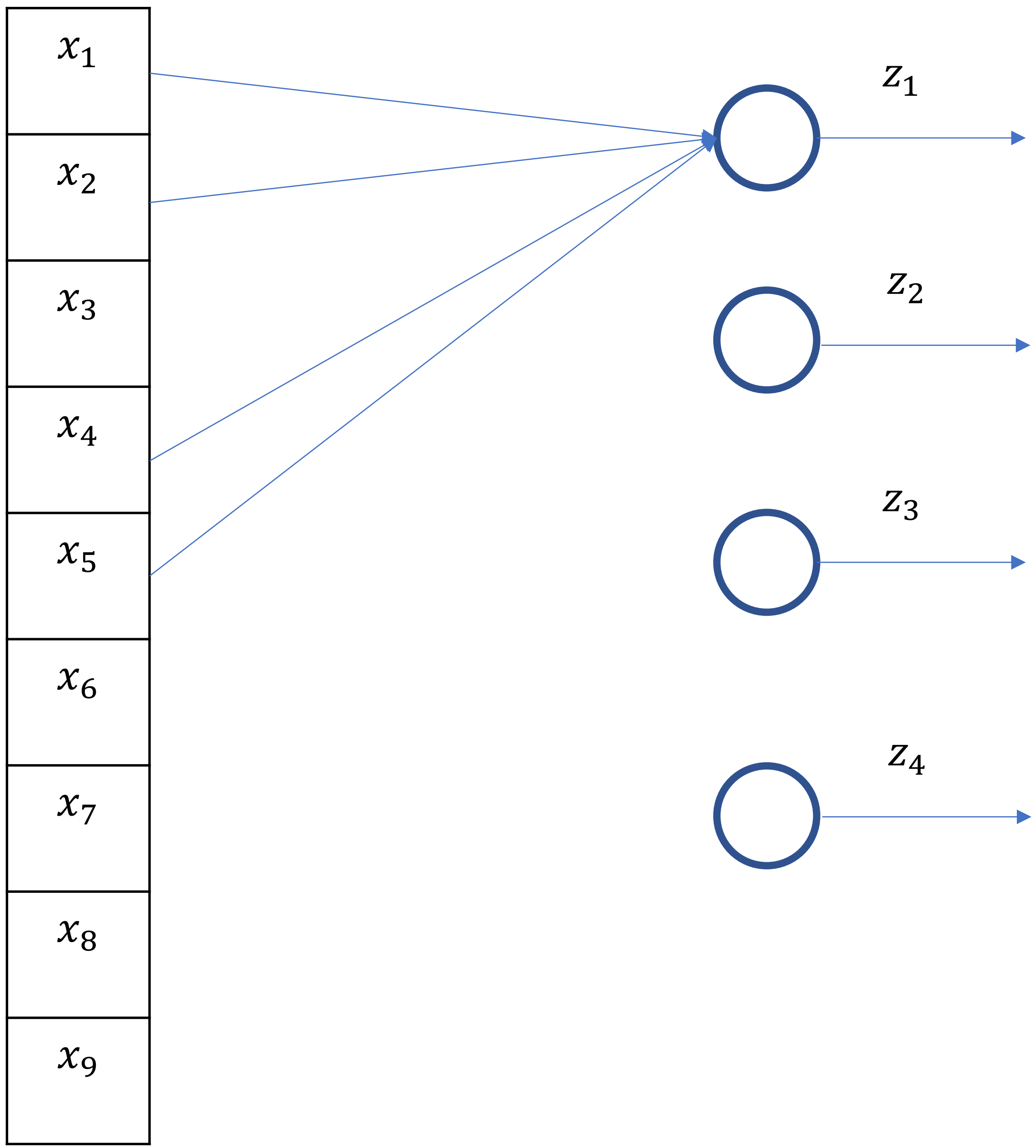
$$z_4 = w_1 x_5 + w_2 x_6 + w_3 x_8 + w_4 x_9 + b_1$$

- Consider a Fully Connected Layer with 4 units





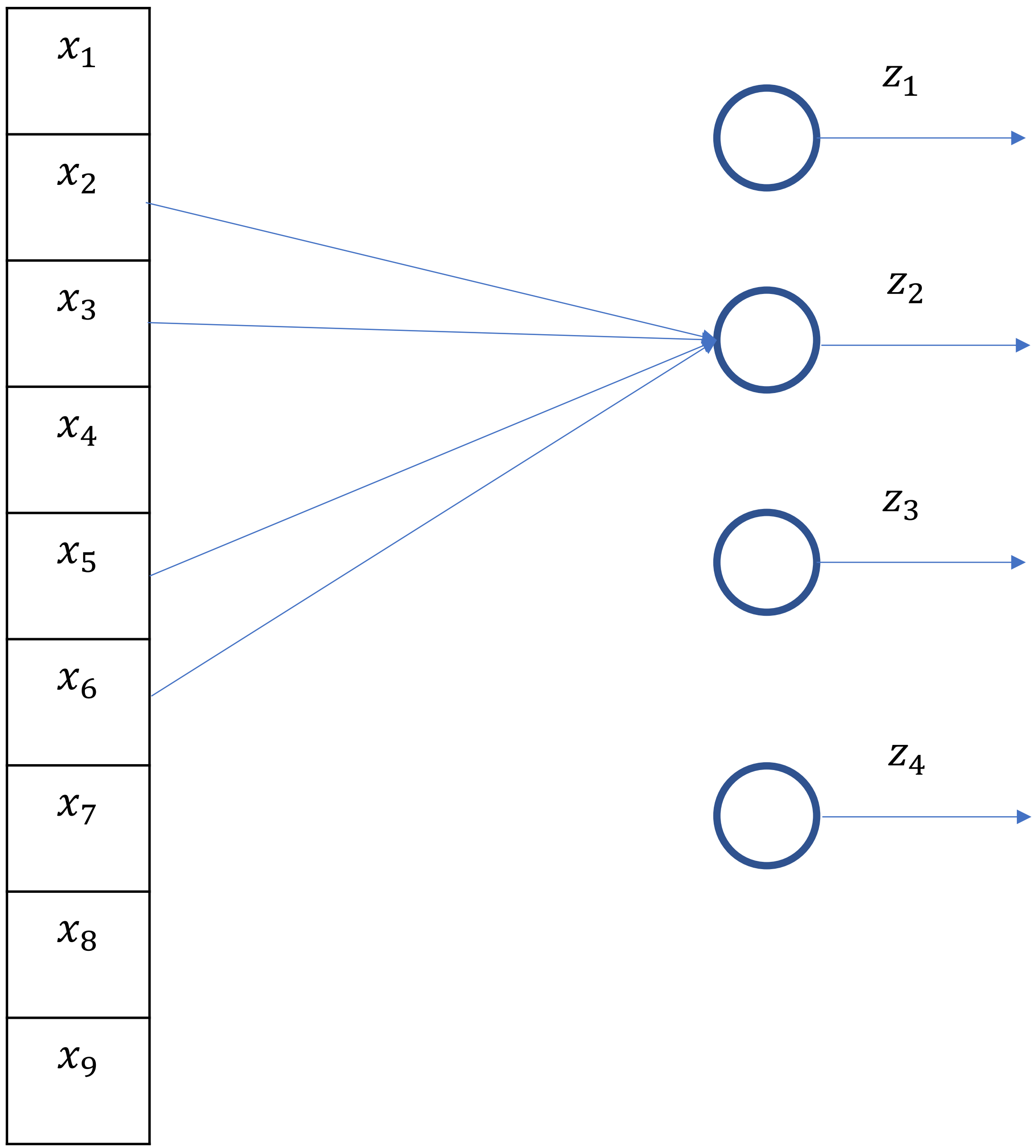
- Consider a Fully Connected Layer with 4 units
- Each unit would have a "connection" and weight for each input to the layer



- Consider a Fully Connected Layer with 4 units
- Each unit would have a “connection” and weight for each input to the layer

Sparsely Connected

- Now let’s get rid of some of these connections (in this case, instead of 9 connections, only 4)



- Consider a Fully Connected Layer with 4 units
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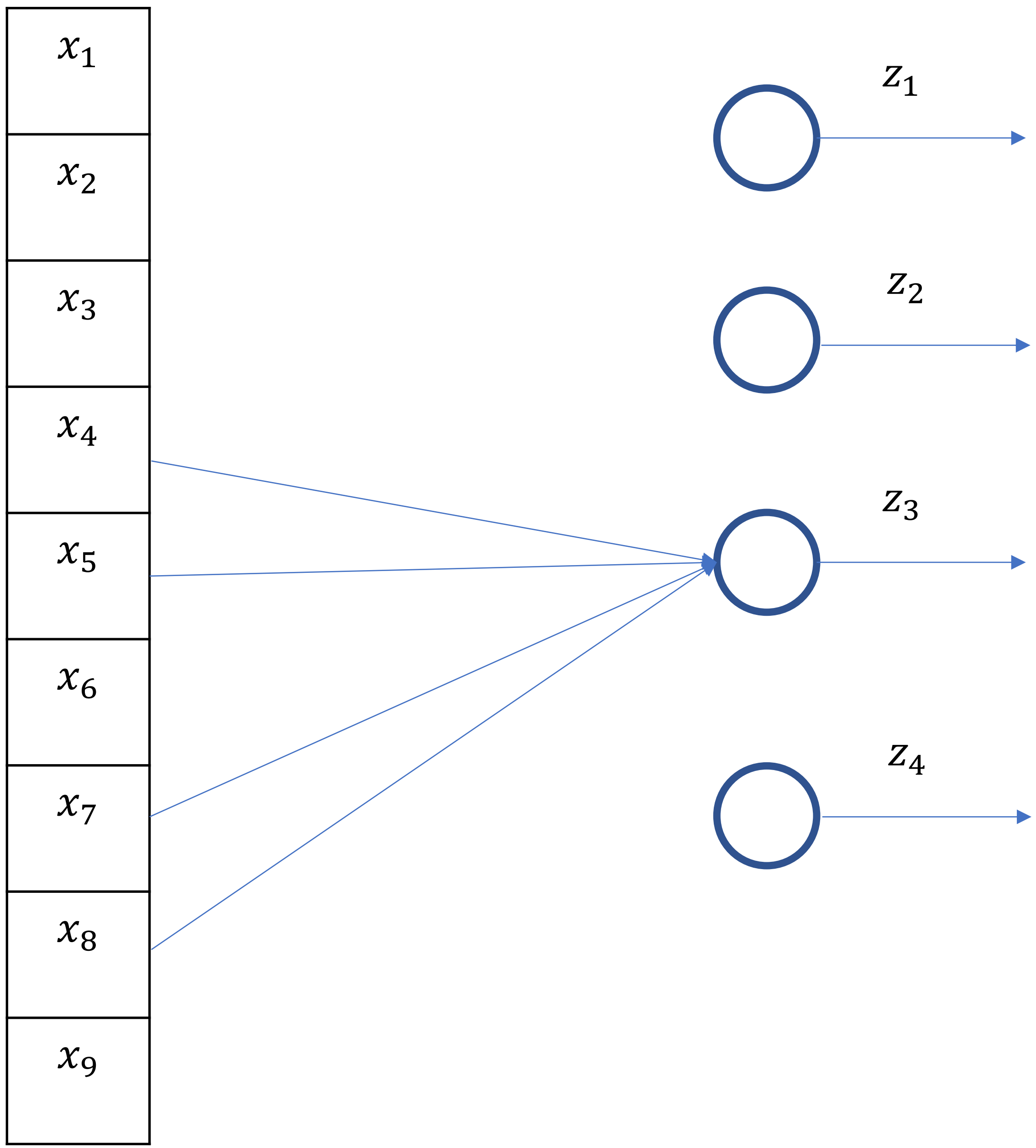
Sparsely Connected

- Now let's get rid of some of these connections (in this case, instead of 9 connections, only 4)
- Each neuron is connected to a different subset of the inputs



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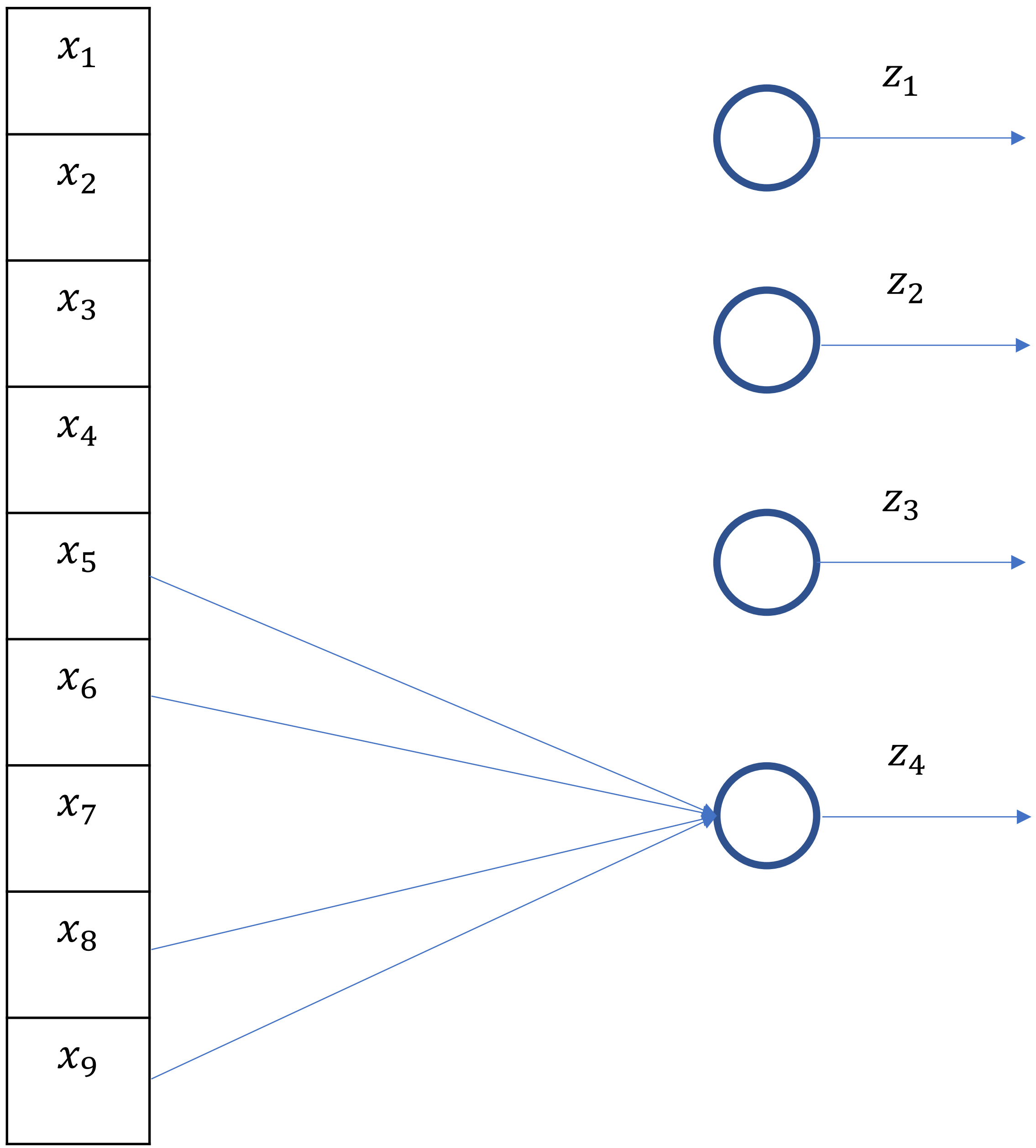

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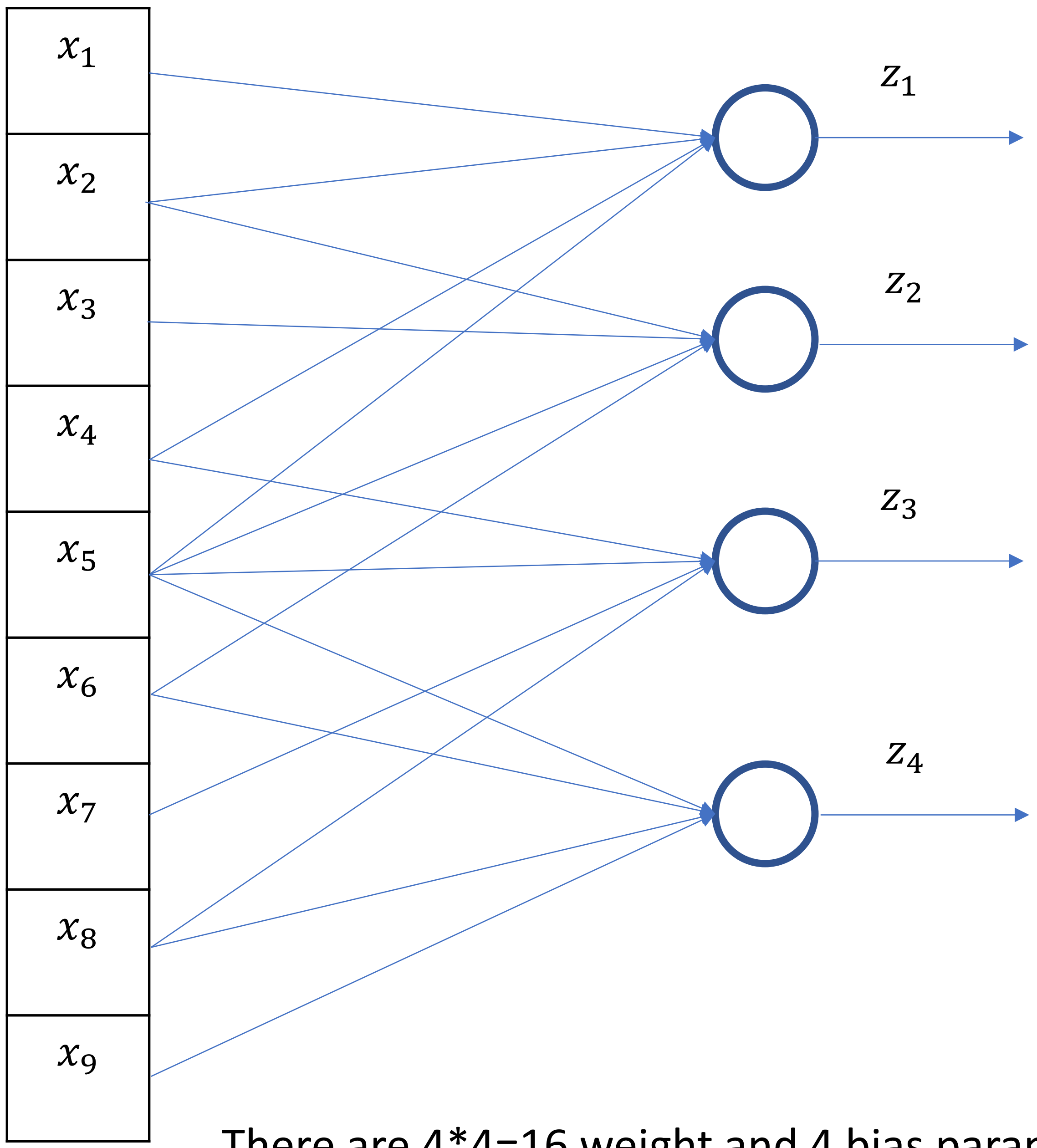

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Sparsely Connected

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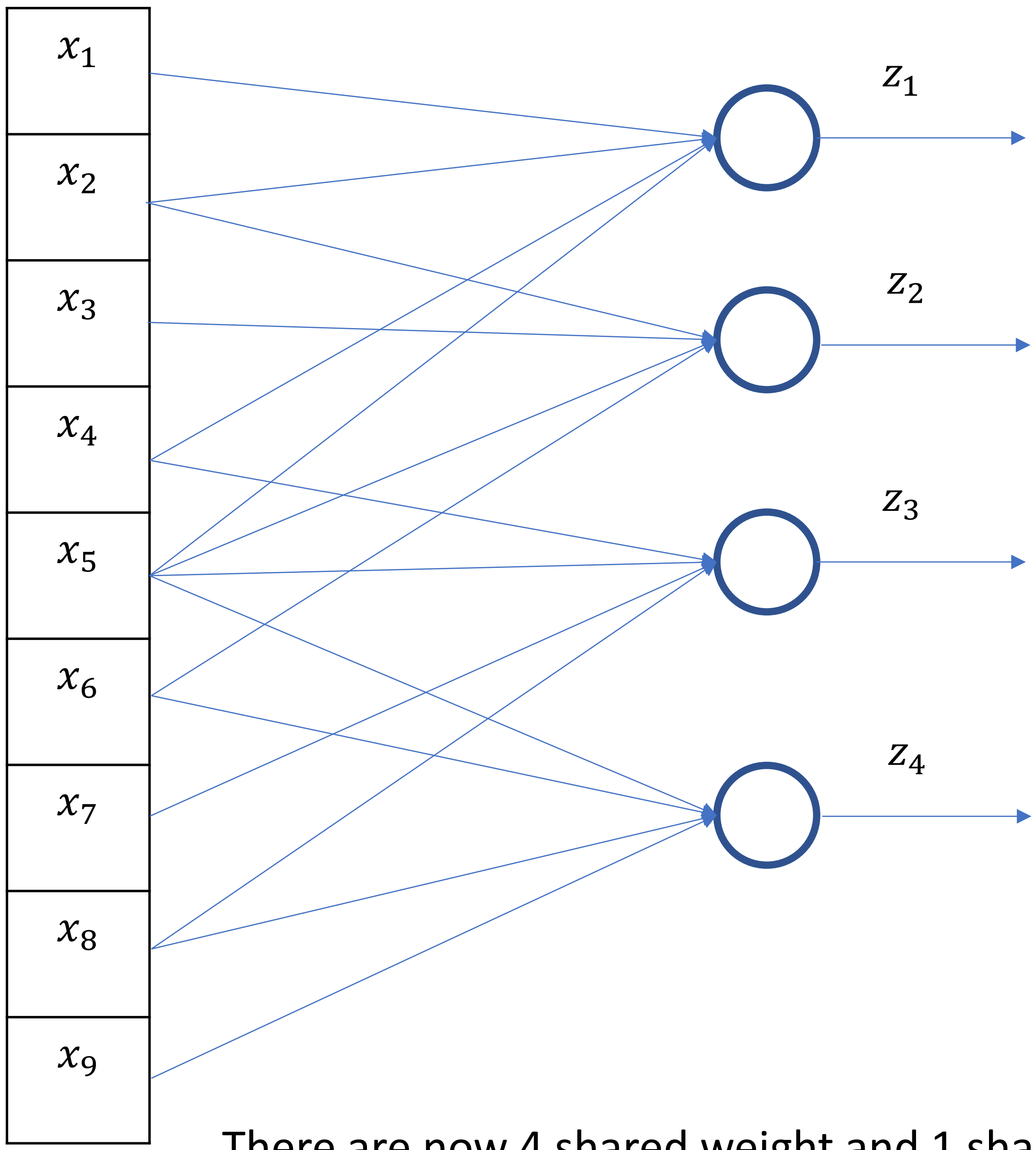
- Each neuron is connected to a different subset of the inputs

There are 4*4=16 weight and 4 bias parameters



- Consider a Fully Connected Layer with 4 units
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Sparsely Connected

- Now let's get rid of some of these connections (in this case, instead of 9 connections, only 4)
- Each neuron is connected to a different subset of the inputs

Parameter Sharing

- Instead of each neuron having it's own 4 weight and 1 bias parameters, they all share the same 4 weights and 1 bias parameter

There are now 4 shared weight and 1 shared bias parameters

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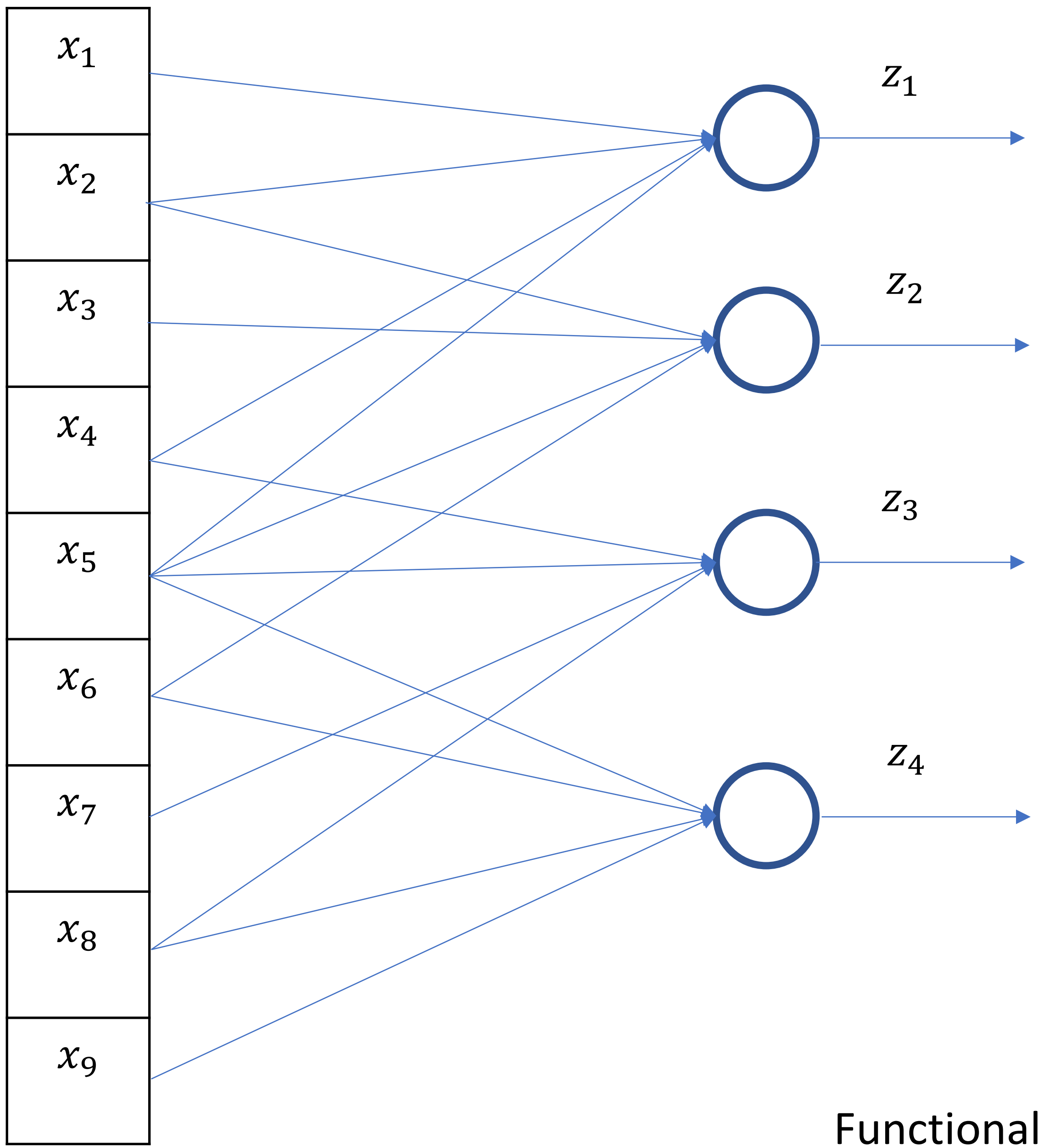

- Consider a Fully Connected Layer with 4 units
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Sparsely Connected

- Now let's get rid of some of these connections (in this case, instead of 9 connections, only 4)
- Each neuron is connected to a different subset of the inputs

Parameter Sharing

- Instead of each neuron having it's own 4 weight and 1 bias parameters, they all share the same 4 weights and 1 bias parameter

$$z_1 = w_1 x_1 + w_2 x_2 + w_3 x_4 + w_4 x_5 + b_1$$
$$z_2 = w_1 x_2 + w_2 x_3 + w_3 x_5 + w_4 x_6 + b_1$$
$$z_3 = w_1 x_4 + w_2 x_5 + w_3 x_7 + w_4 x_8 + b_1$$
$$z_4 = w_1 x_5 + w_2 x_6 + w_3 x_8 + w_4 x_9 + b_1$$

Functionally/Mathematically equivalent to our convolutional layer

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Example for more than one filter filter:

$$\underset{\text{3x3}}{\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix}} \ \text{conv} \ \overset{\text{2x2}}{\begin{bmatrix} w_1 & w_2 \\ w_3 & w_4 \end{bmatrix}} + \boxed{b_1} = \overset{\text{2x2}}{\begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix}}$$

Example for more than one filter filter:

$$\begin{array}{c}\begin{bmatrix}x_1 & x_2 & x_3\\ x_4 & x_5 & x_6\\ x_7 & x_8 & x_9\end{bmatrix}\\ \text{3x3}\end{array} \ \text{conv}\ \begin{array}{c}\text{2x2}\\ \begin{bmatrix}w_1 & w_2\\ w_3 & w_4\end{bmatrix}\end{array} + \begin{bmatrix}b_1\end{bmatrix} = \begin{array}{c}\text{2x2}\\ \begin{bmatrix}z_1 & z_2\\ z_3 & z_4\end{bmatrix}\end{array}$$

$$\begin{bmatrix}w_5 & w_6\\ w_7 & w_8\end{bmatrix} + \begin{bmatrix}b_2\end{bmatrix} = \begin{bmatrix}z_5 & z_6\\ z_7 & z_8\end{bmatrix}$$

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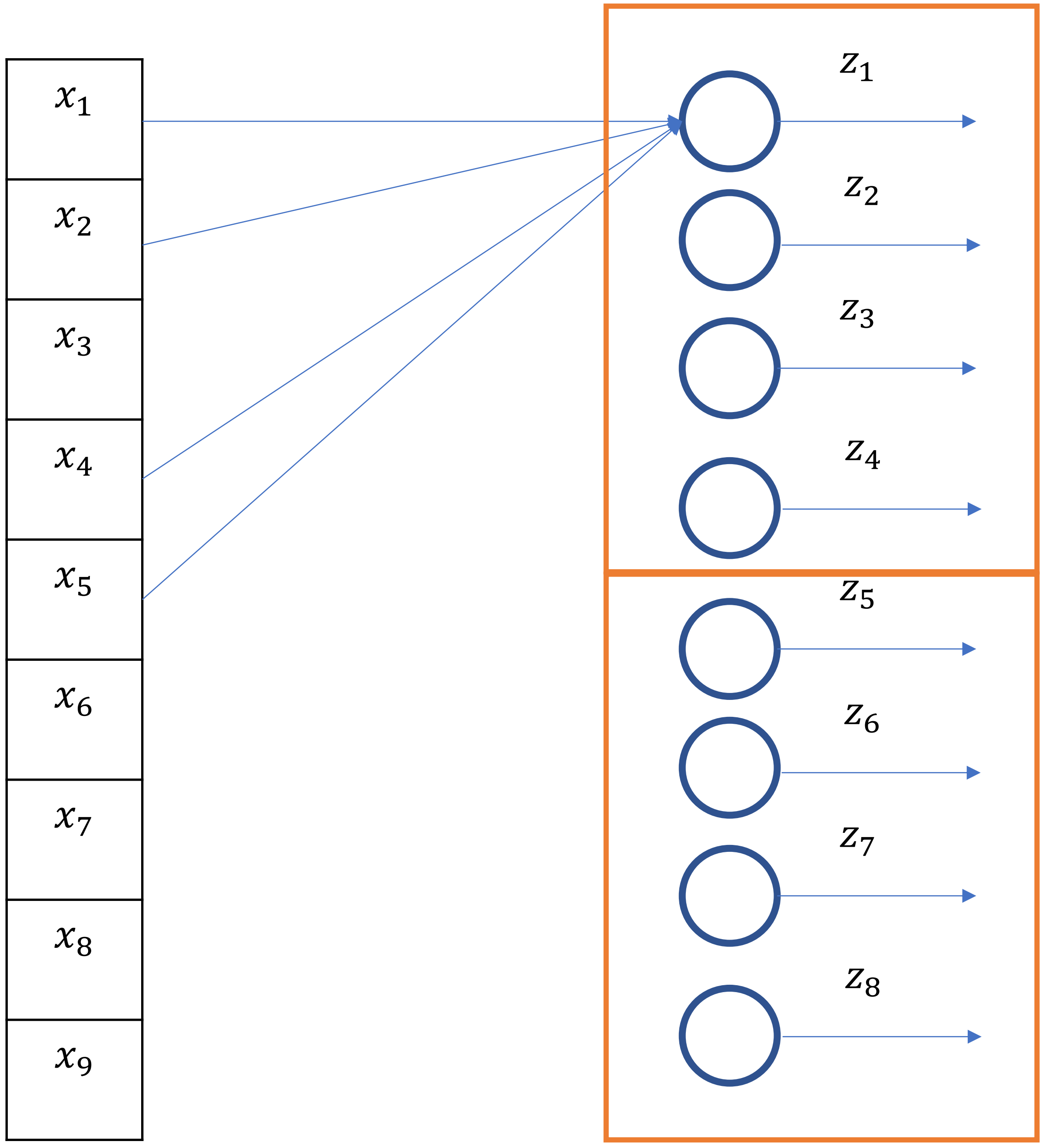


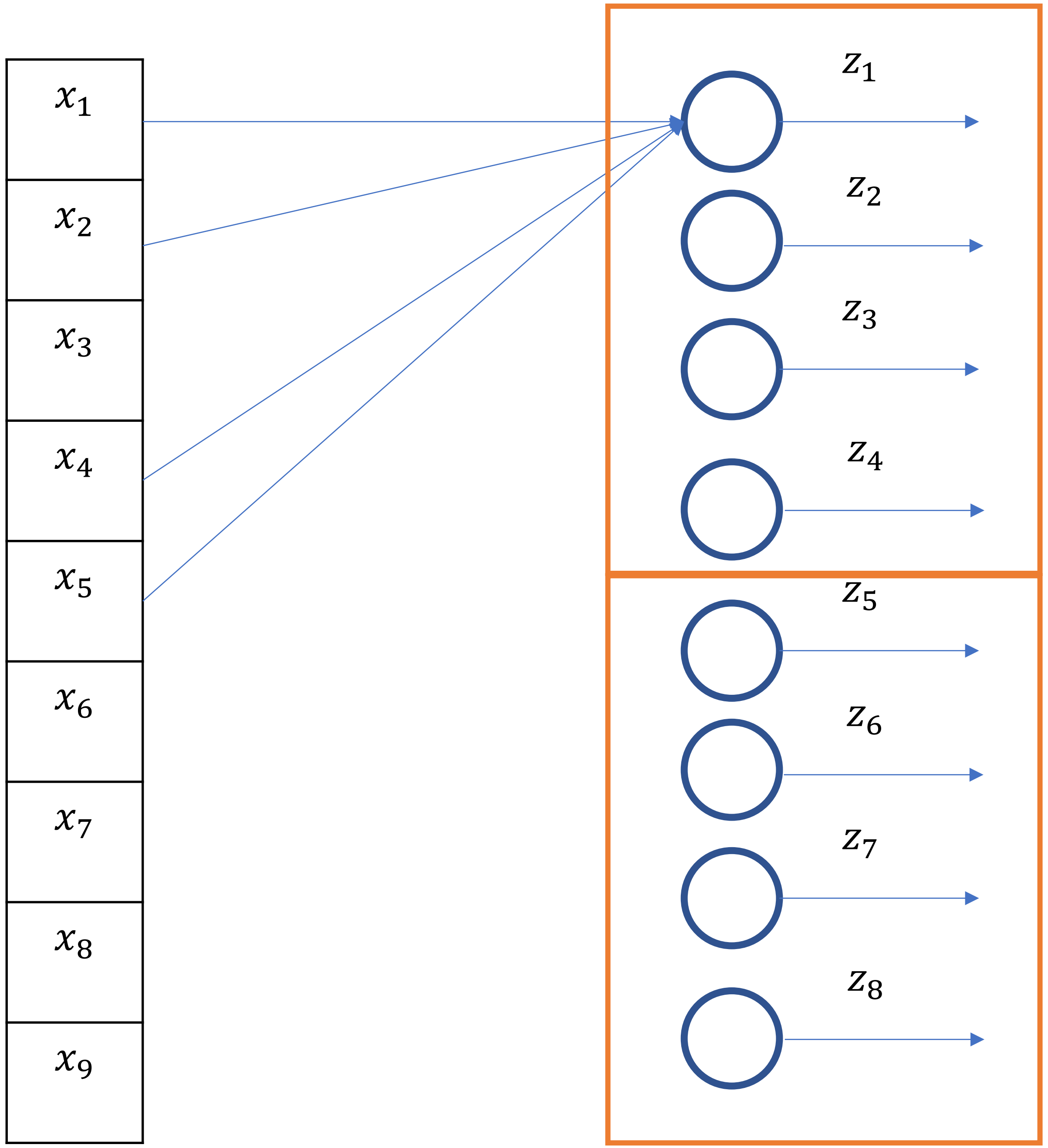
(Only connections for one neuron shown)

Filter 1

$$z_1 = w_1x_1 + w_2x_2 + w_3x_4 + w_4x_5 + b_1$$
$$z_2 = w_1x_2 + w_2x_3 + w_3x_5 + w_4x_6 + b_1$$
$$z_3 = w_1x_4 + w_2x_5 + w_3x_7 + w_4x_8 + b_1$$
$$z_4 = w_1x_5 + w_2x_6 + w_3x_8 + w_4x_9 + b_1$$

Filter 2

$$z_5 = w_5x_1 + w_6x_2 + w_7x_4 + w_8x_5 + b_2$$
$$z_6 = w_5x_2 + w_6x_3 + w_7x_5 + w_8x_6 + b_2$$
$$z_7 = w_5x_4 + w_6x_5 + w_7x_7 + w_8x_8 + b_2$$
$$z_8 = w_5x_5 + w_6x_6 + w_7x_8 + w_8x_9 + b_2$$



(Only connections for one neuron shown)

Filter 1

$$z_1 = w_1x_1 + w_2x_2 + w_3x_4 + w_4x_5 + b_1$$
$$z_2 = w_1x_2 + w_2x_3 + w_3x_5 + w_4x_6 + b_1$$
$$z_3 = w_1x_4 + w_2x_5 + w_3x_7 + w_4x_8 + b_1$$
$$z_4 = w_1x_5 + w_2x_6 + w_3x_8 + w_4x_9 + b_1$$

Filter 2

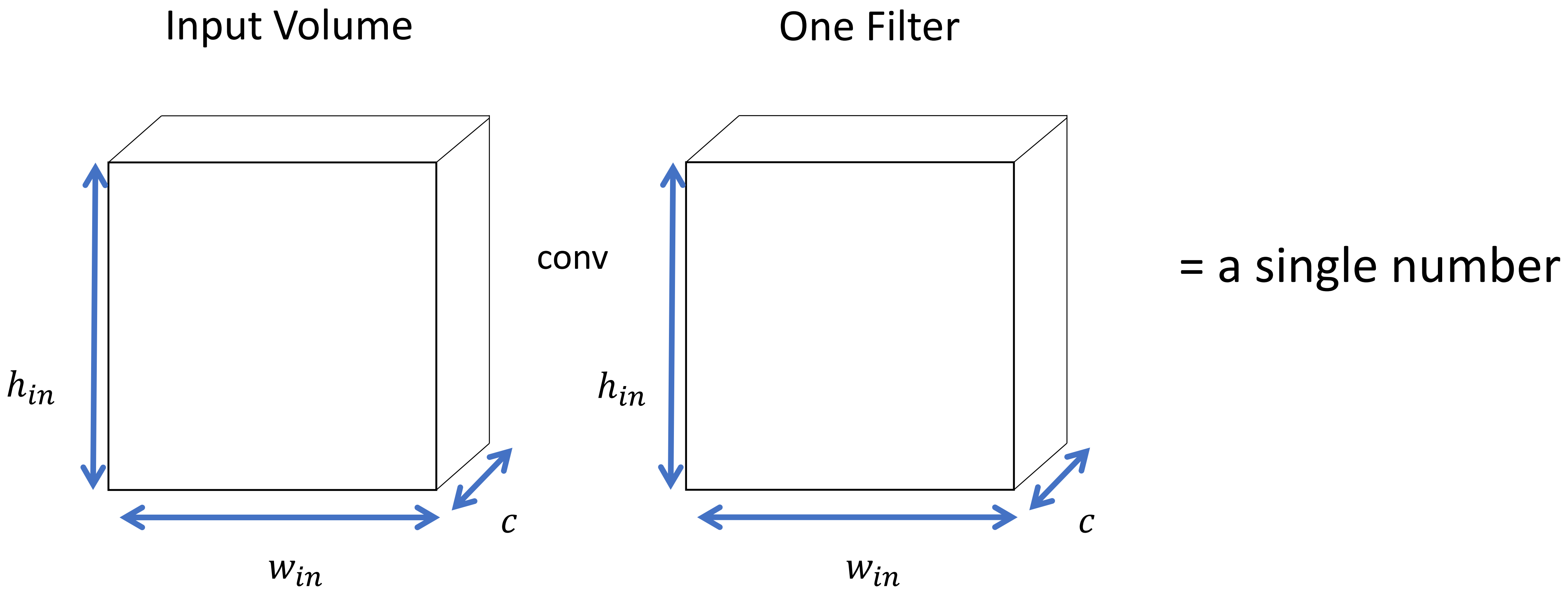
$$z_5 = w_5x_1 + w_6x_2 + w_7x_4 + w_8x_5 + b_2$$
$$z_6 = w_5x_2 + w_6x_3 + w_7x_5 + w_8x_6 + b_2$$
$$z_7 = w_5x_4 + w_6x_5 + w_7x_7 + w_8x_8 + b_2$$
$$z_8 = w_5x_5 + w_6x_6 + w_7x_8 + w_8x_9 + b_2$$

Key Takeaway: Convolution Layer is just a Fully-Connected layer with sparse connectivity and parameter sharing

Comparison to Fully Connected

- Fully-Connected Neural Network Layer:
  - $a^{[l]} = g\left(W^{[l]} \cdot a^{[l-1]} + b^{[l]}\right)$
- Convolutional Neural Network Layer
  - $a^{[l]} = g\left(conv\left(W^{[l]}, a^{[l-1]}\right) + b^{[l]}\right)$
- You can still use backward propagation and optimization techniques as before to train a CNN

Consider a filter the same shape as the input



Example for one filter (K=1):

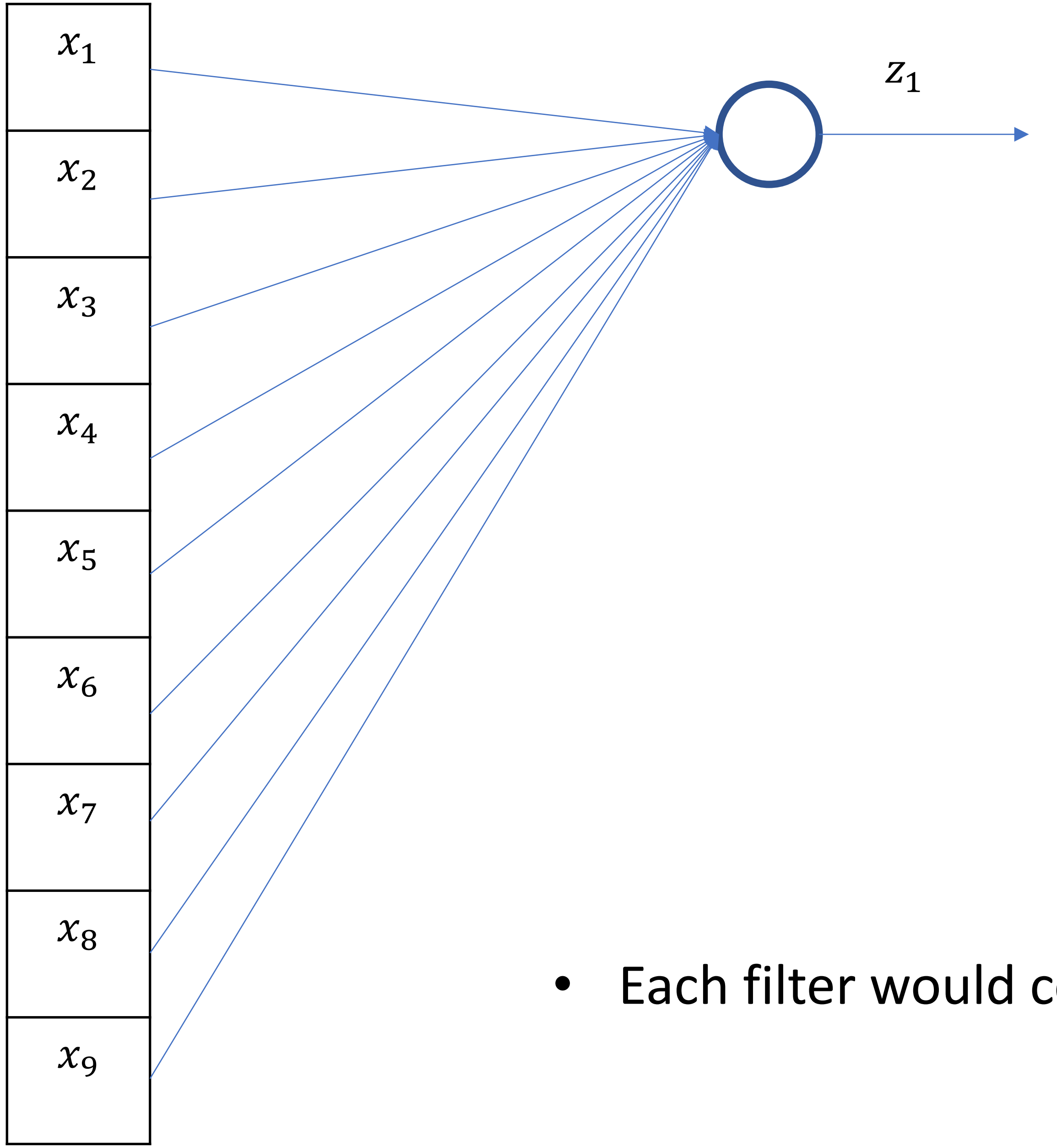
$$\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix} \text{ conv } \begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{bmatrix} + \boxed{b_1} = \boxed{z_1}$$

3x3 3x3 2x2

$$z_1 = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5x_5 + w_6x_6 + w_7x_7 + w_8x_8 + w_9x_9 + b_1$$

If we make the filter the same shape as the input, we have the equivalent of a single fully-connected neuron

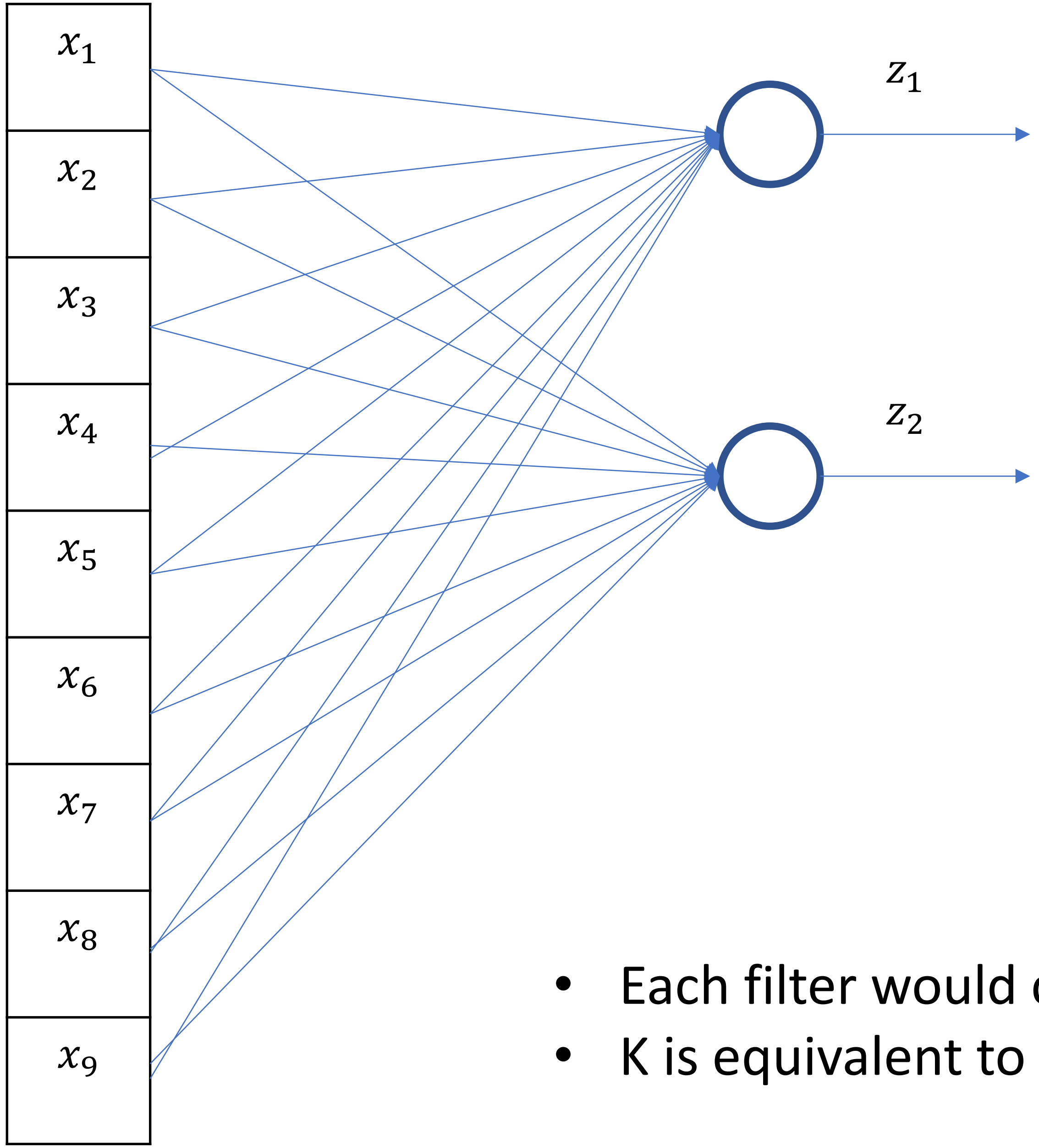
Number of filters: K=1



$$z_1 = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5x_5$$
$$+w_6x_6 + w_7x_7 + w_8x_8 + w_9x_9 + b_1$$

- Each filter would correspond to a single fully-connected neuron

Number of filters: K=2



$$z_1 = w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4 + w_5x_5 + w_6x_6 + w_7x_7 + w_8x_8 + w_9x_9 + b_1$$

$$z_2 = w_{10}x_1 + w_{11}x_2 + w_{12}x_3 + w_{13}x_4 + w_{14}x_5 + w_{15}x_6 + w_{16}x_7 + w_{17}x_8 + w_{18}x_9 + b_2$$

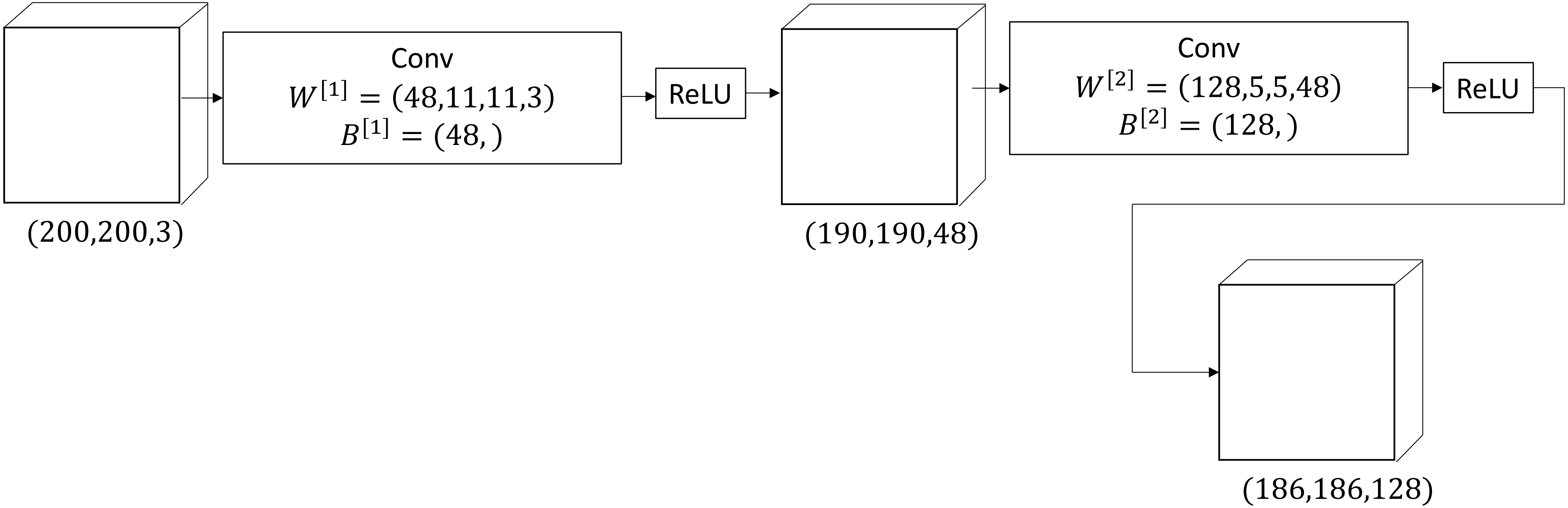
- Each filter would correspond to a single fully-connected neuron
- K is equivalent to n_h

Summary of Fully-Connected to Conv and Back

- You can transform a Fully-Connected layer to a Convolutional layer by employing sparse connectivity and parameter sharing
- You can transform a Convolutional layer to a Fully-Connected layer by making the filters the same shape as the input, and having one filter for each fully-connected neuron.
- We will see the latter case come up again when we talk about Object Detection

Convolutional Layers: Padding

Simple CNN Example with numbers



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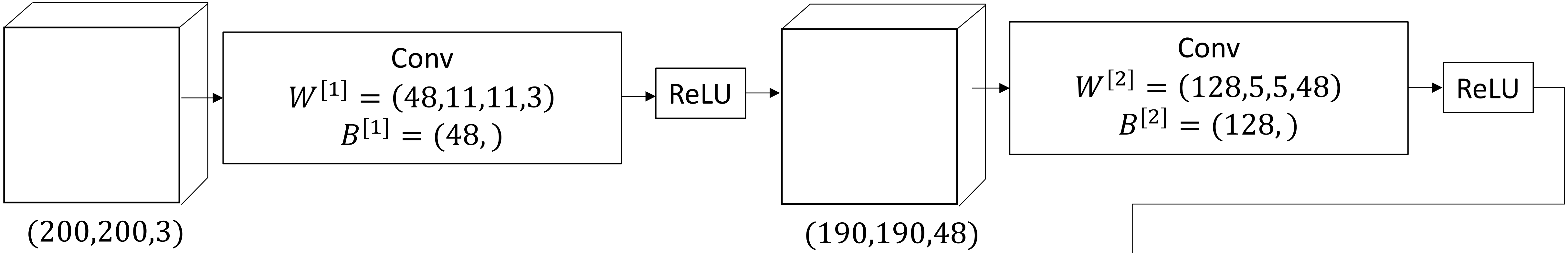
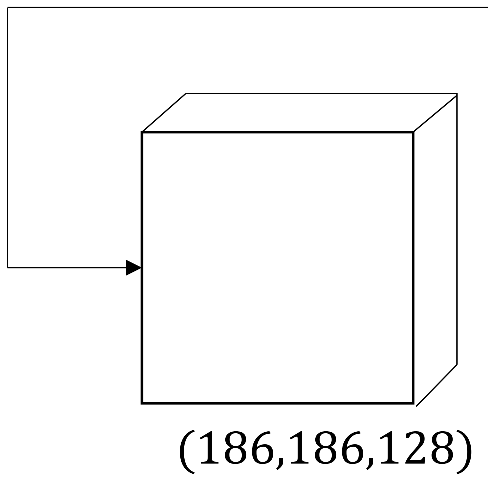
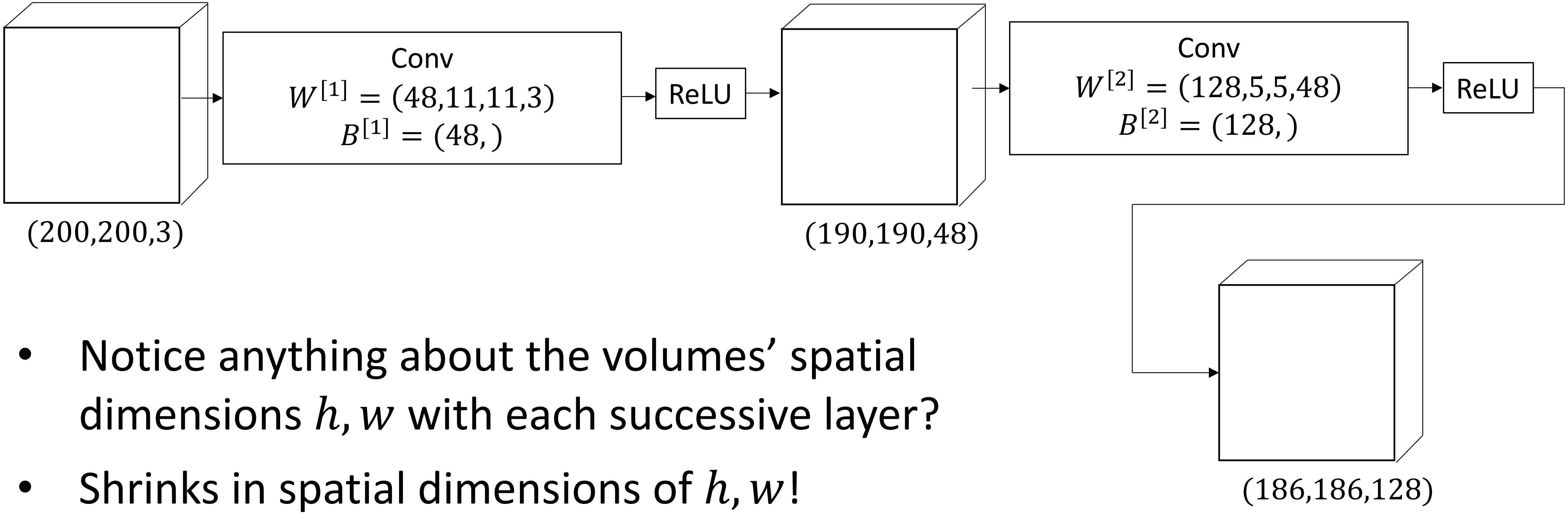
Simple CNN Example with numbers





- Notice anything about the volumes' spatial dimensions h, w with each successive layer?

Simple CNN Example with numbers



- Notice anything about the volumes' spatial dimensions h, w with each successive layer?
- Shrinks in spatial dimensions of h, w!
- Number of output activations depend on how many times you can fit the filter in the input

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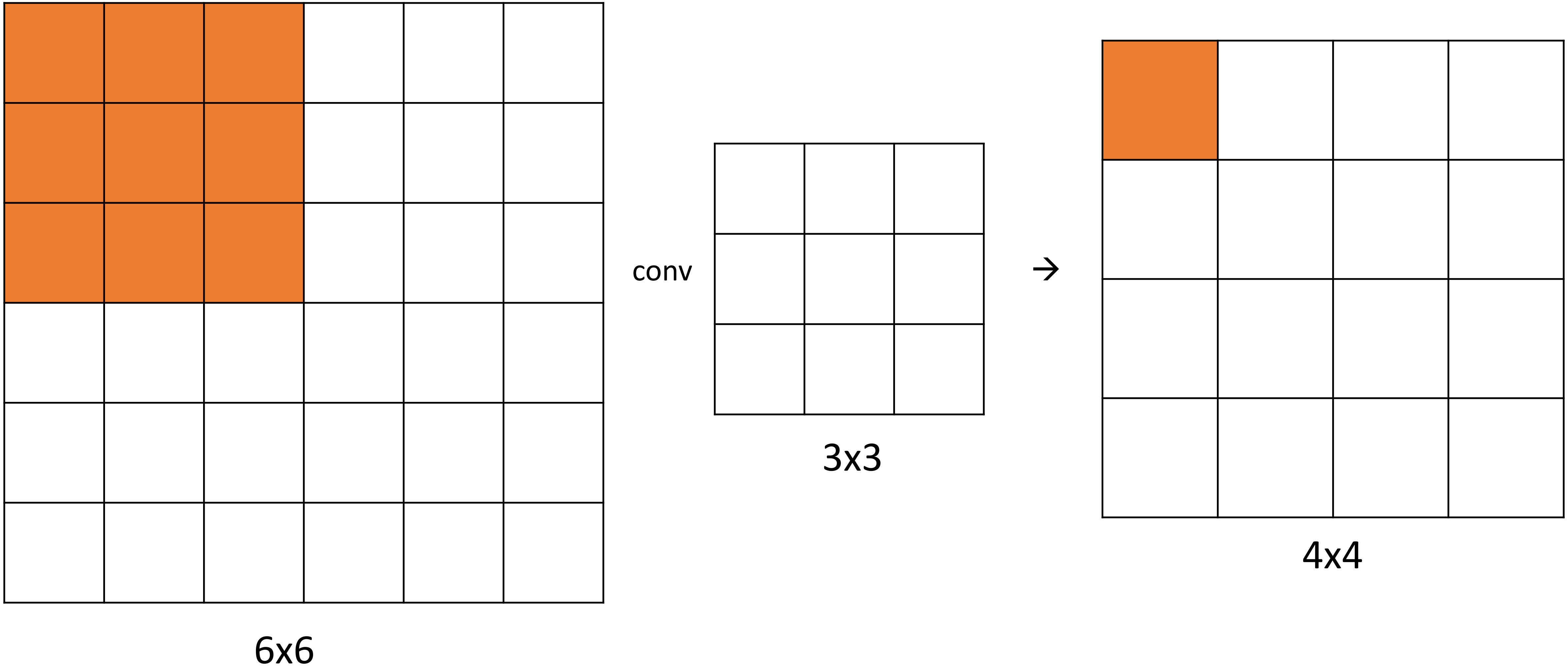
Problem 1: Data Shrinks

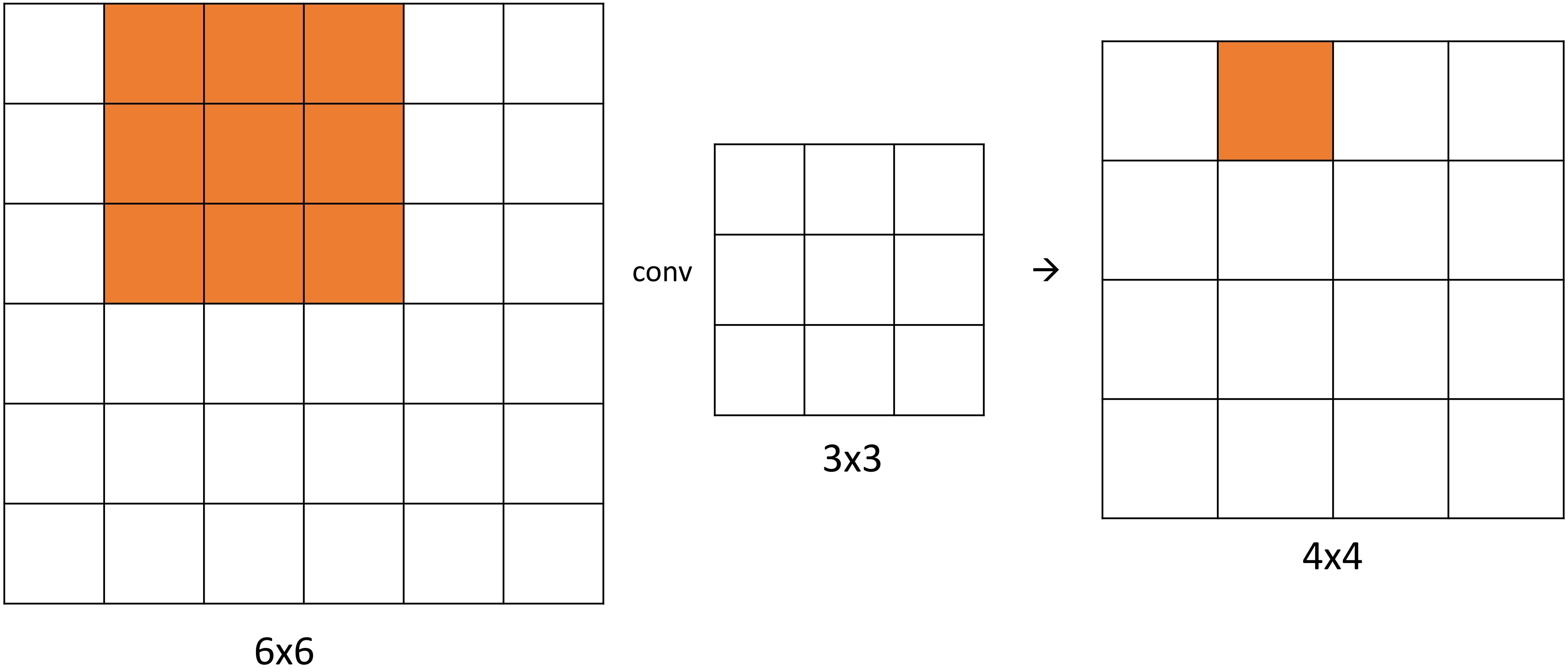
- Want to use the output of a Conv layer as the input to the next. But output volume's spatial dimensions shrink.
- This may limit how deep you can make your CNN

(32,32,3) conv (3,3) → (30,30) conv (3,3) → (28,28) conv (5,5) → (24,24) → …

Shrinking Volumes



Shrinking Volumes



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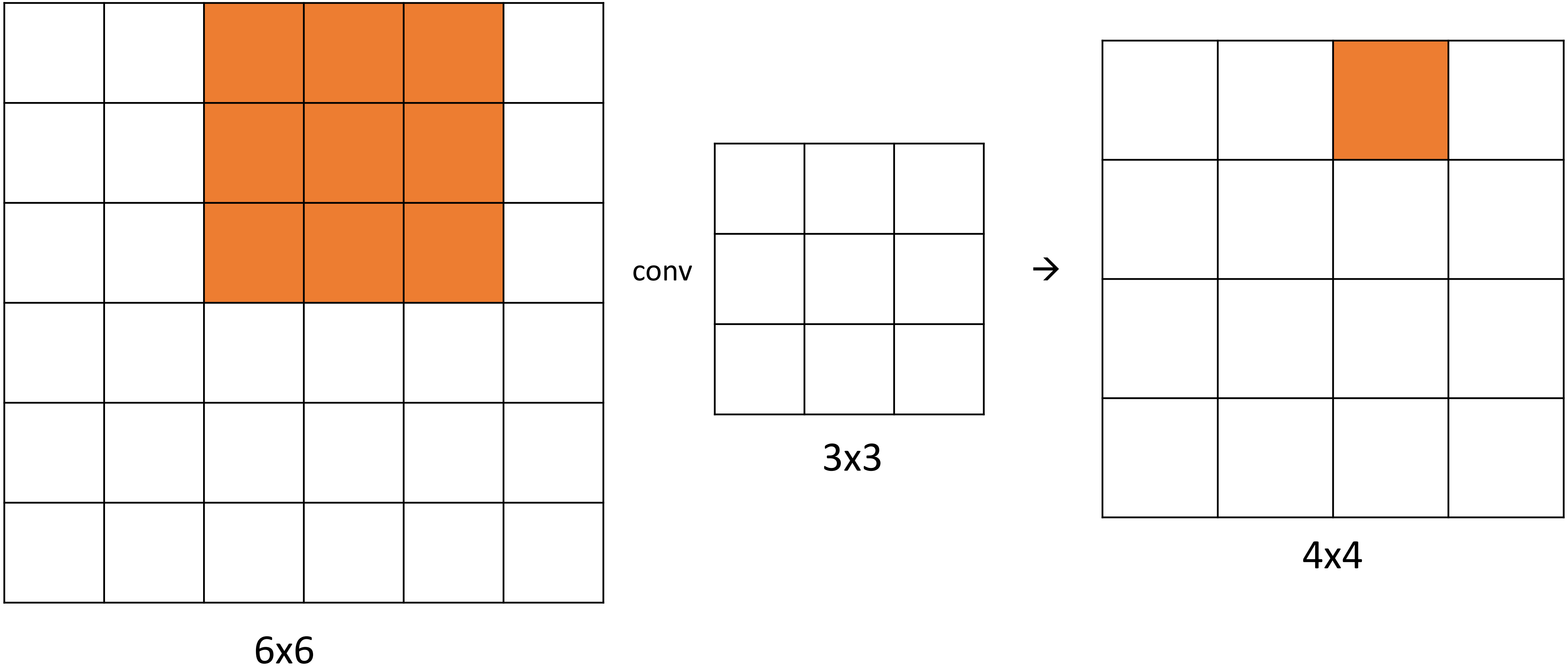
Shrinking Volumes



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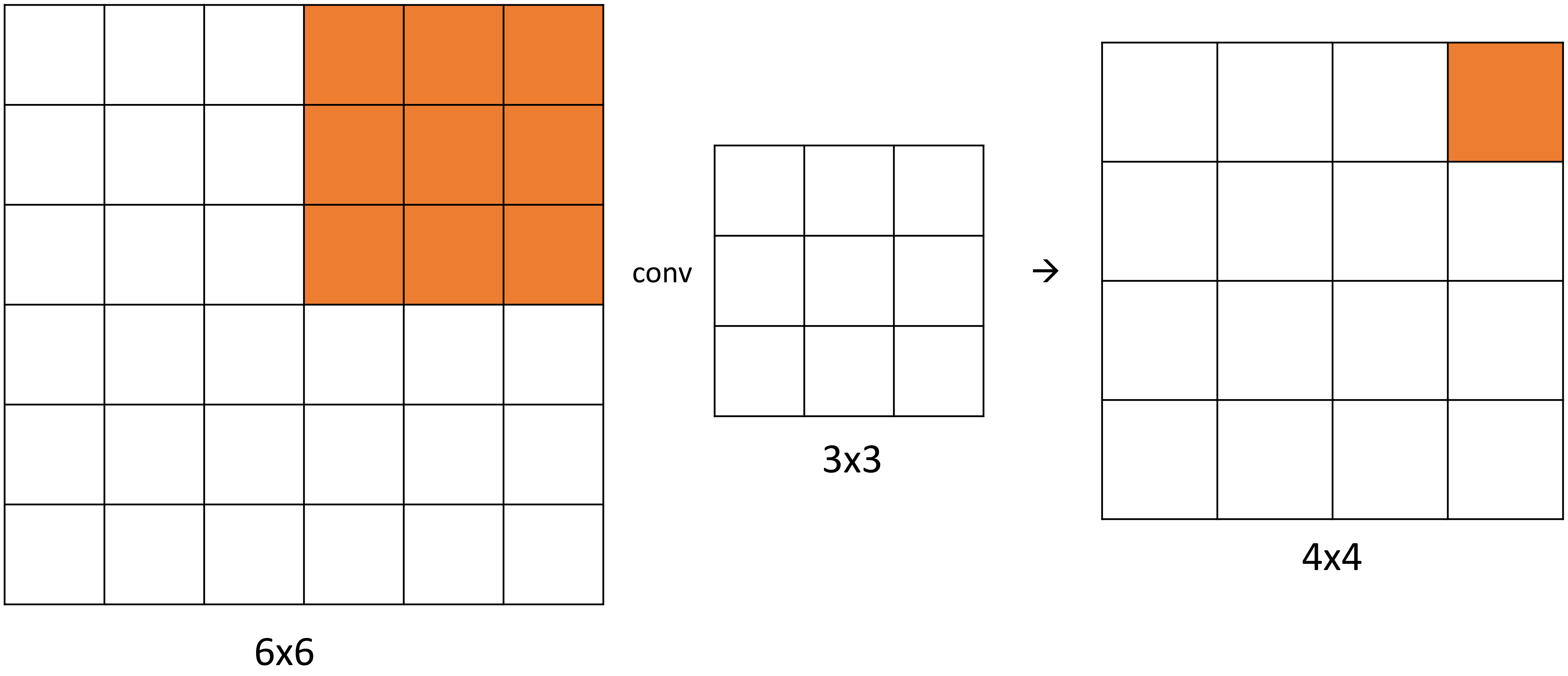
Shrinking Volumes



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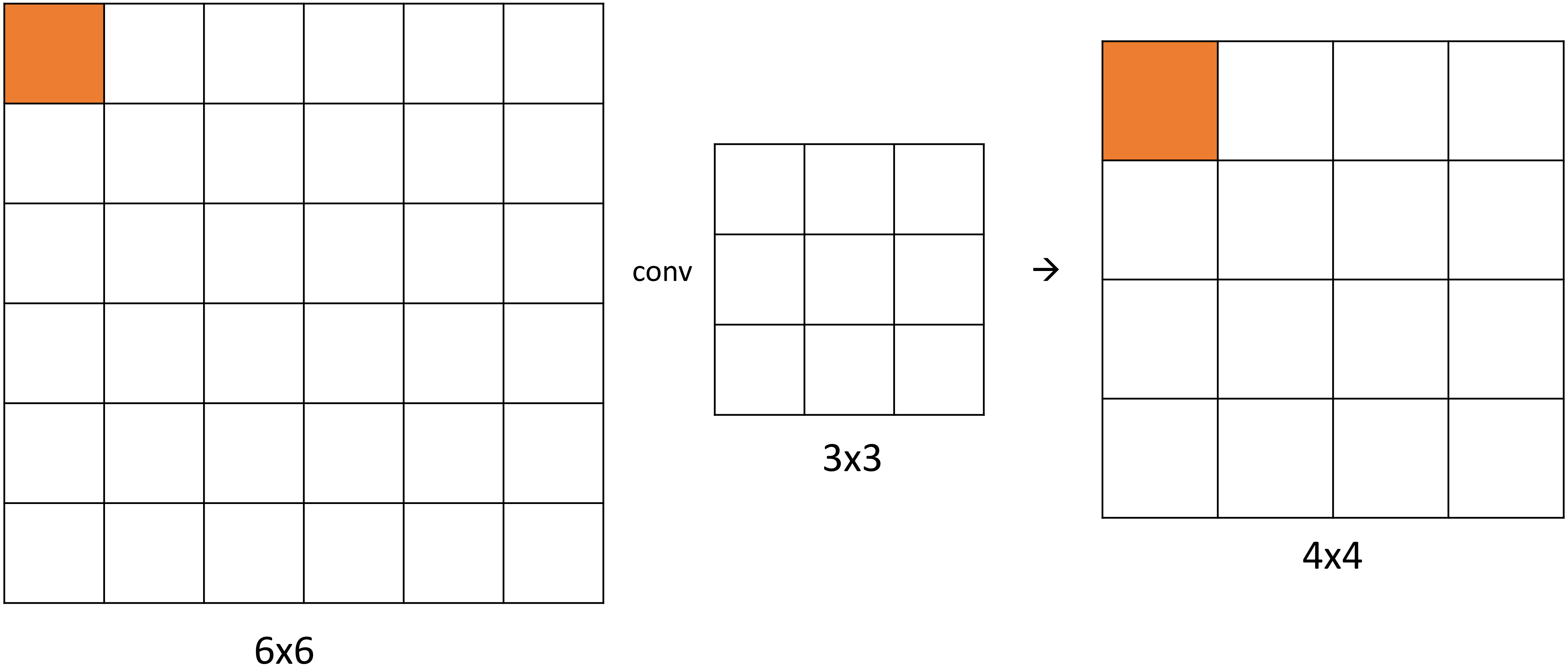
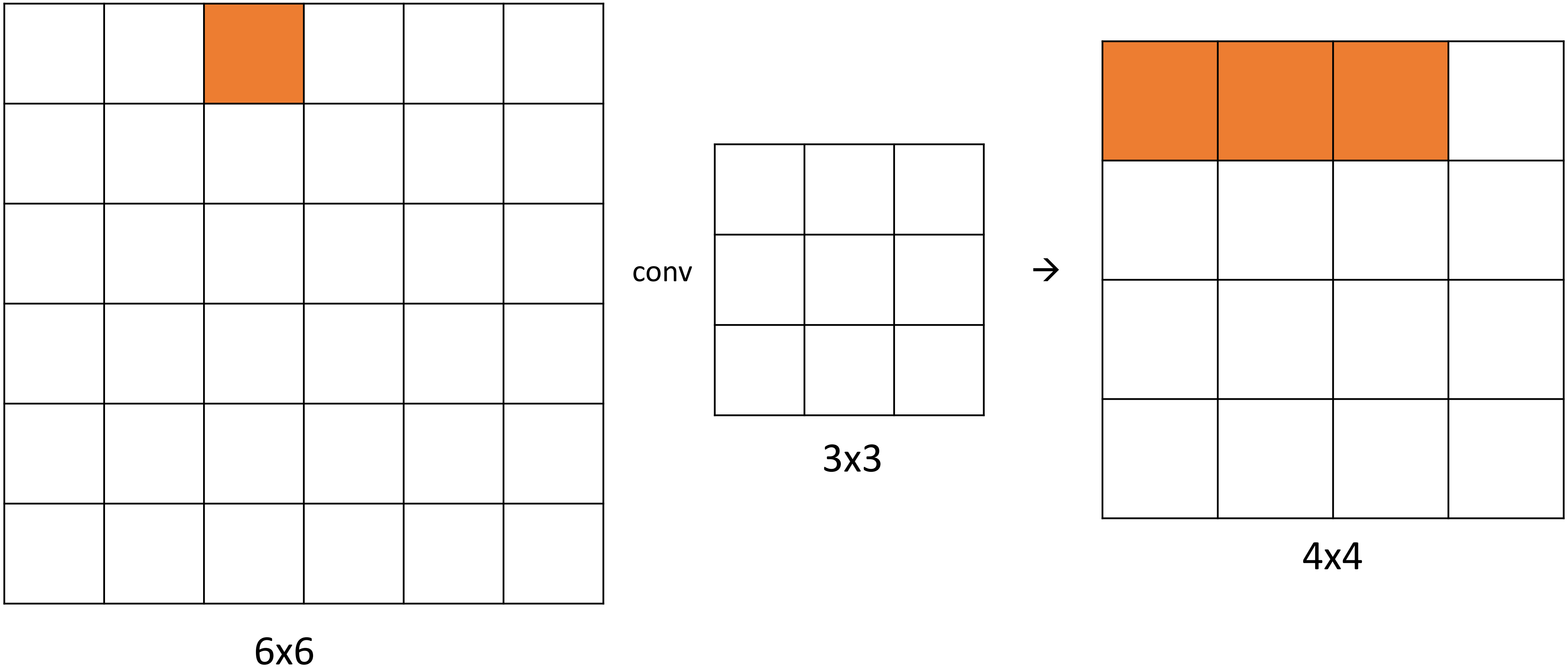
Problem 2: Data at the edge

- Input data at the edges influence fewer output values than input data in the middle.

Data at Corner



Data at Edge (non-corner)



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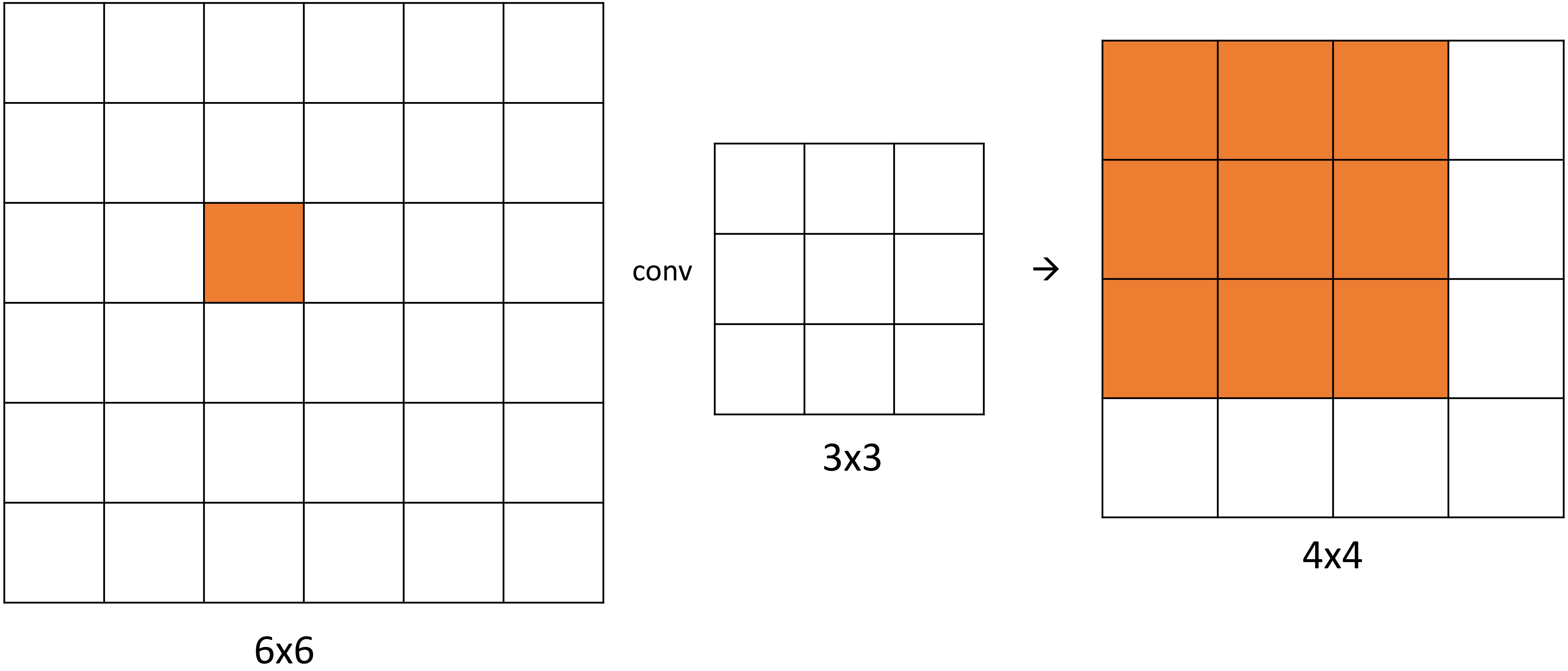
Data in the middle



Solution: Padding

- Pad the perimeter of the input volume before convolution operation

Example:

- Pad a 6x6 image with 1 element **all around** perimeter becomes 8x8
- Output is now 6x6 → This preserves original spatial dimensions
- Original perimeter data now affects more output values
- Both problems solved …

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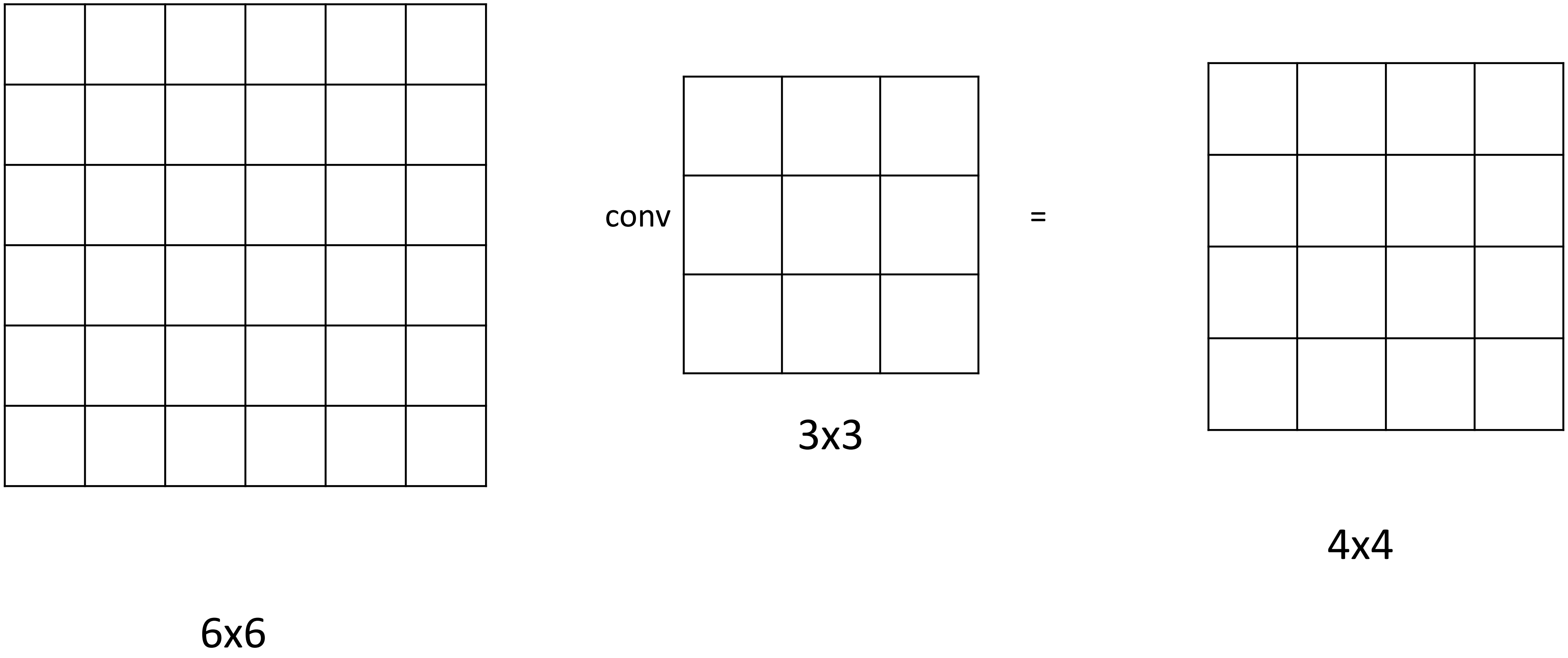
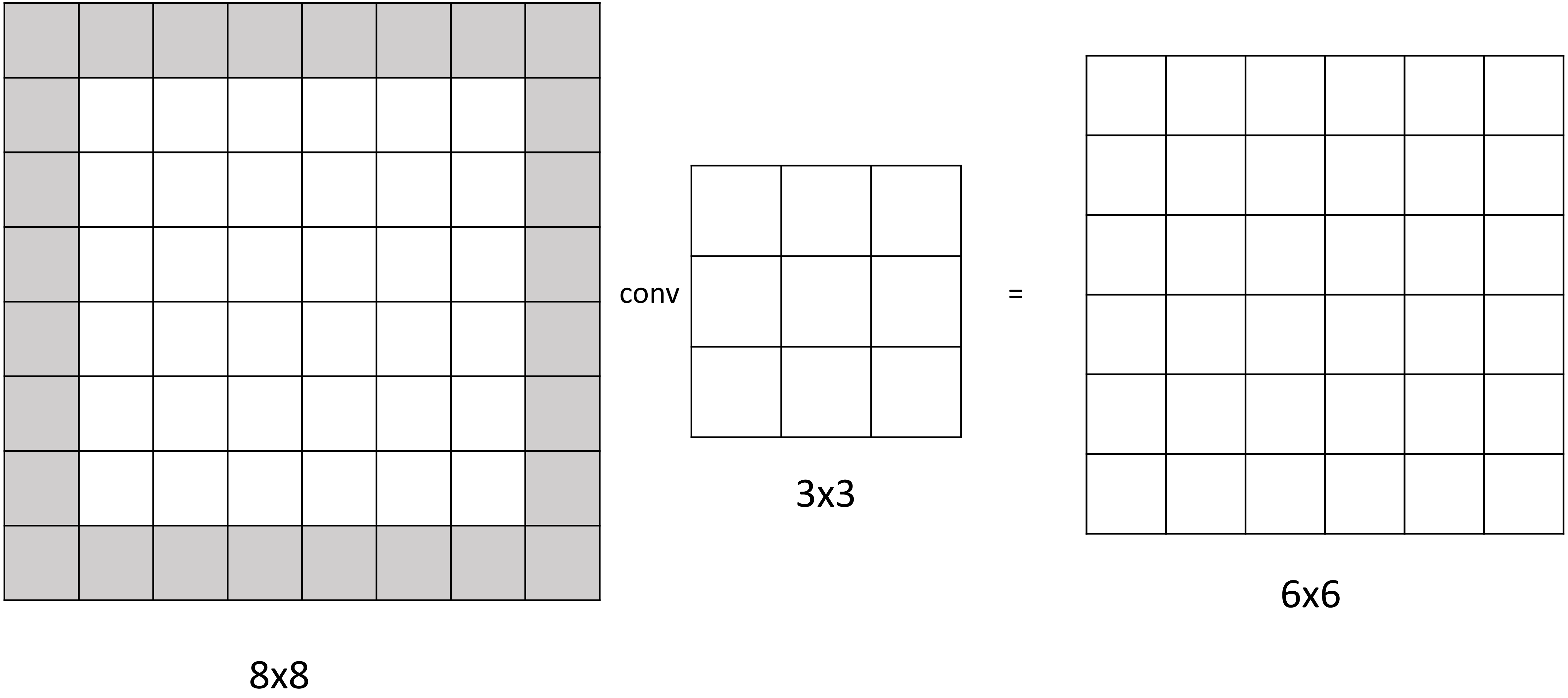
Example: Padding



Example: Padding



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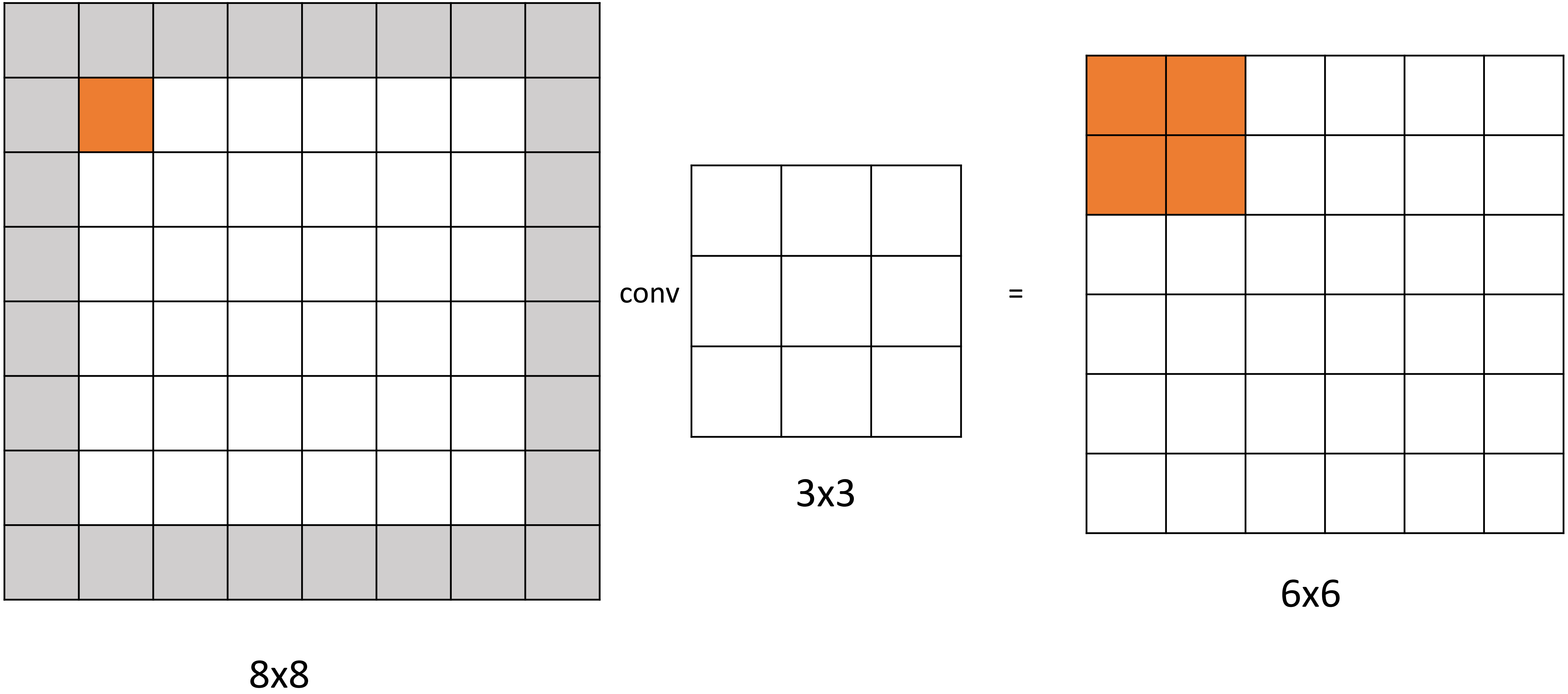
Example: Corner



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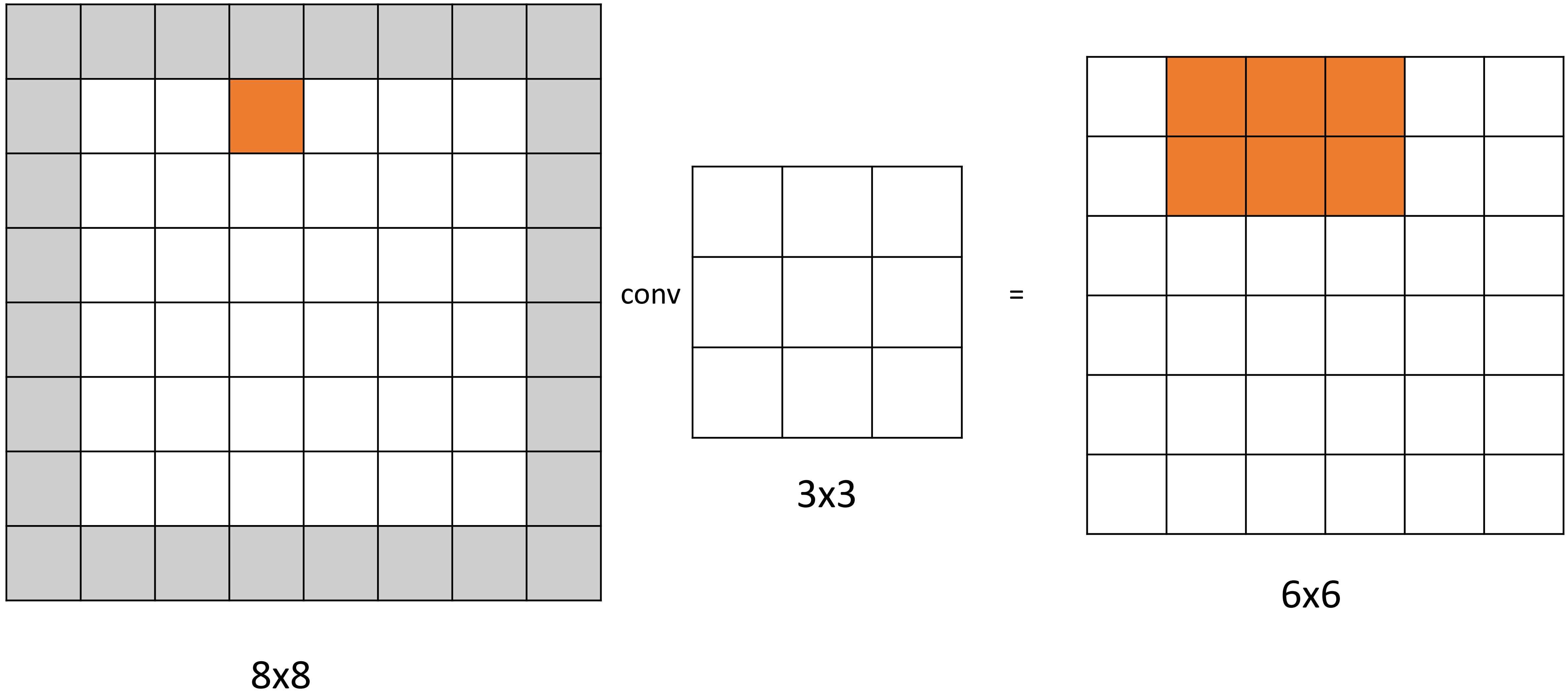
Example: Edge



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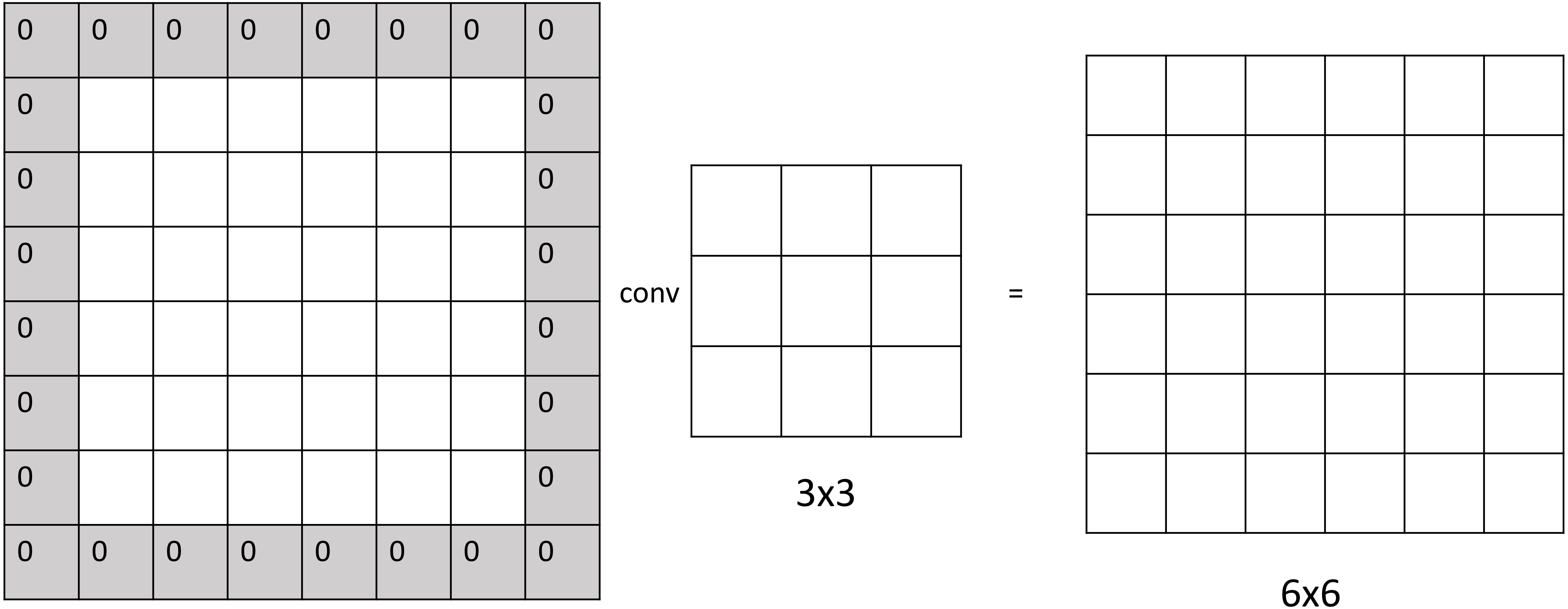
Typically pad with a value of 0



8x8

3x3

6x6

Output Volume Dimensions with Padding

- Input volume: (h, w, c)
- Filter size: (f, f, c)
- Output volume without padding: $(h - f + 1, w - f + 1)$
- With padding $(h + 2p - f + 1, W + 2p - f + 1)$

Padding In Practice

- Typically pad in one of two ways:
  1. Don't pad
  2. Pad so that output volume spatial dimensions is the same as the input volume.
- For the latter, you usually don't manually specify p

Padding In Practice

- Typically pad in one of two ways:
    1. Don't pad
    2. Pad so that output volume spatial dimensions is the same as the input volume.
- For the latter, you usually don't manually specify p

Some terminology:

- No padding ← VALID
- Pad so that output volume is the same as the input volume ← SAME

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“Same” Padding

- Pad so that output volume is the same as the input volume

<u>Example</u>:

- Input is (h, w), output is $(h + 2p - f + 1, w + 2p - f + 1)$
- What to set p so that input is equal to output?

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"Same" Padding

- Pad so that output volume is the same as the input volume

<u>Example</u>:

- Input is (h, w), output is $(h + 2p - f + 1, w + 2p - f + 1)$
- What to set p so that input is equal to output?
- $h = h + 2p - f + 1 \rightarrow p = \frac{f-1}{2}$
- $w = w + 2p - f + 1 \rightarrow p = \frac{f-1}{2}$

- Doesn't depend on input volume. Just depends on filter size.

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“Same” Padding

- Works well for odd sized filters
- What about even? You can do asymmetric padding (i.e. one side has more padding than the other)
- But due to long traditional reasons in computer vision, filters are almost always odd and square
  - Nice to have a central pixel and notion of top/bottom/left/right

- Just use odd sized filters …

Same Padding

- What if p doesn't work out to an integer?

$$p = \frac{f - 1}{2}$$

Same Padding

- What if p doesn't work out to an integer?

$$p = \frac{f-1}{2}$$

- You would typically design your CNN architecture so that p works out to be an integer

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Stride

Stride

- So far, when we perform the convolutions, the filter “slides” 1 value at a time across the data
- But you could slide by larger steps.
- This is another hyperparameter of your CNN architecture
- The amount by which you step is referred to as the “stride”

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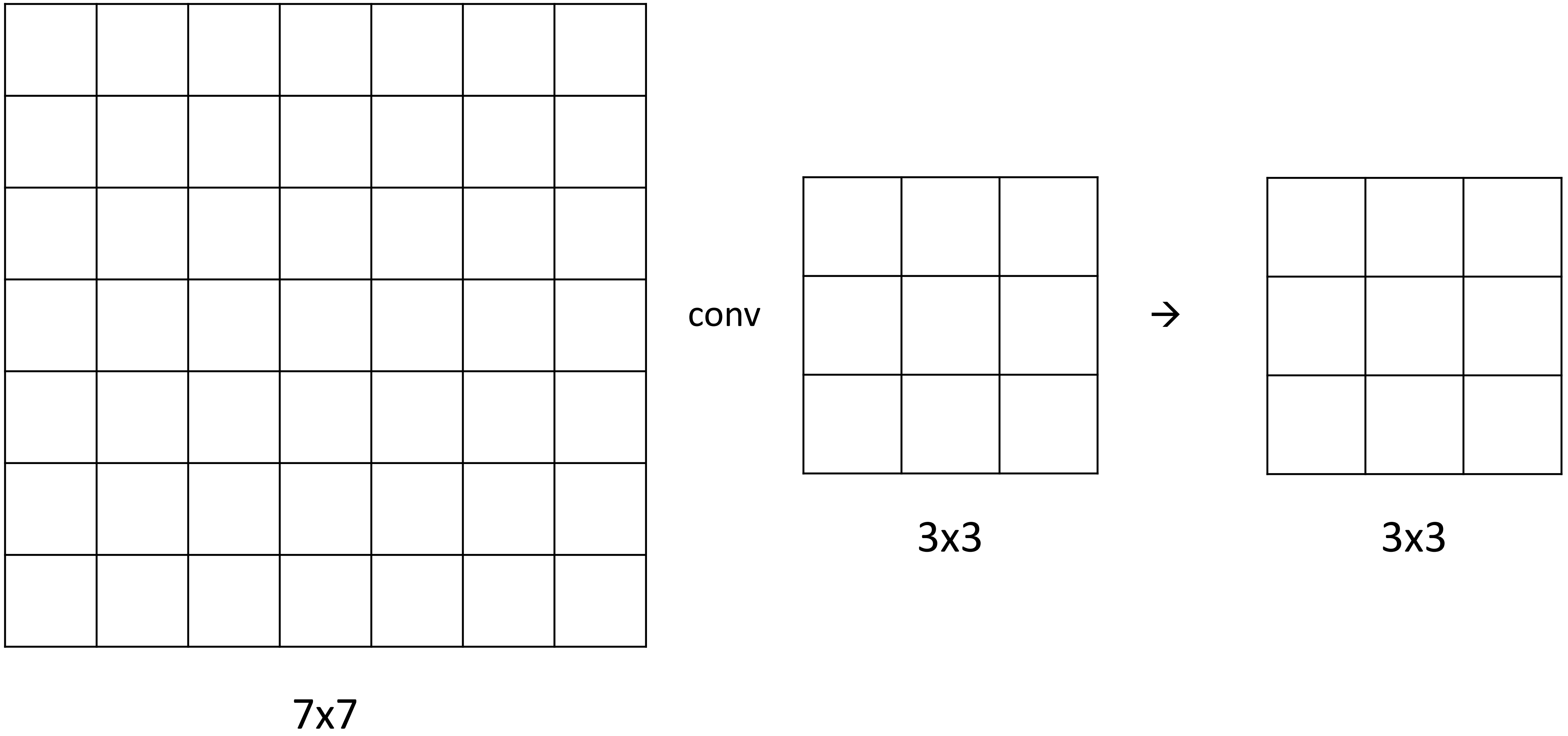
Example: Stride = 2



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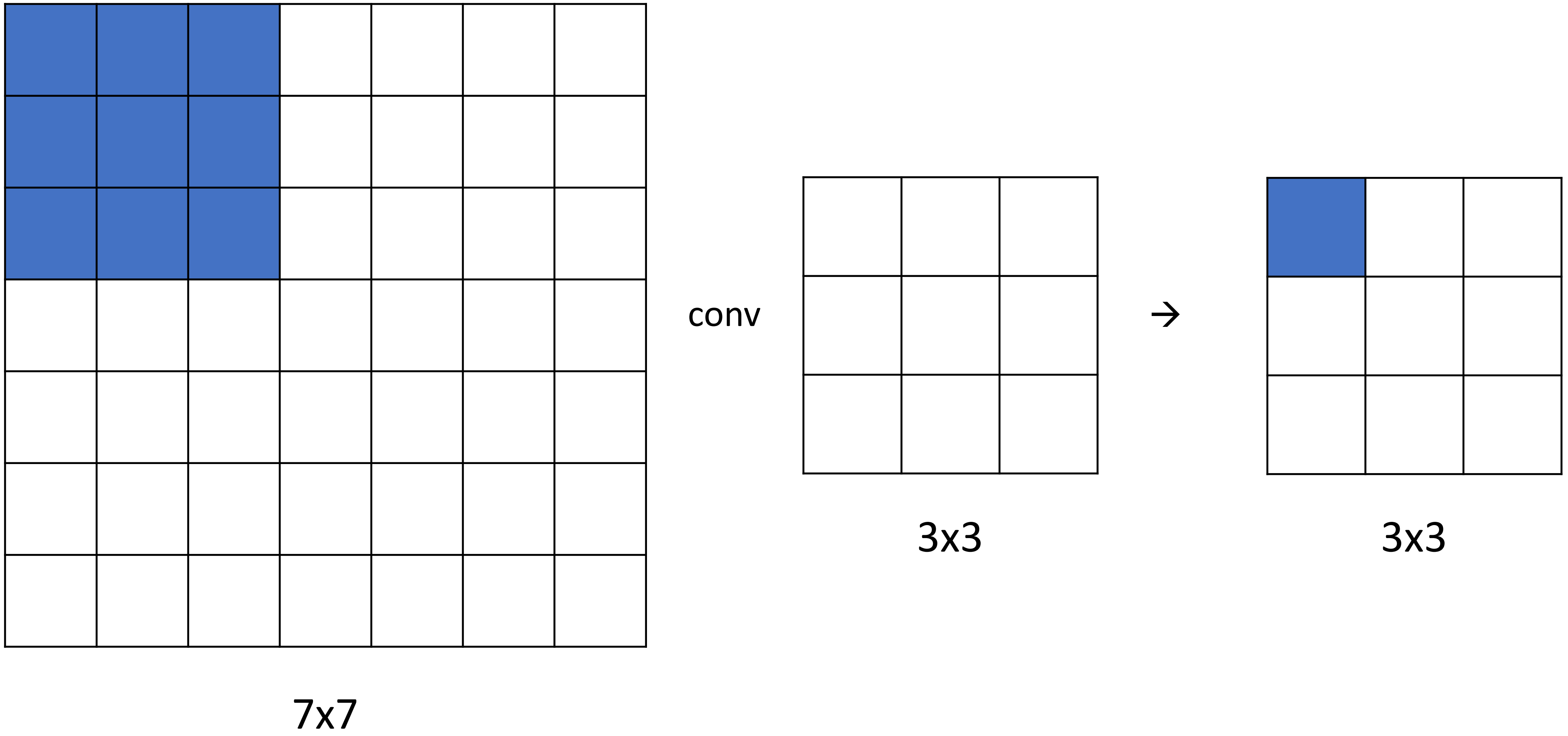
Example: Stride = 2



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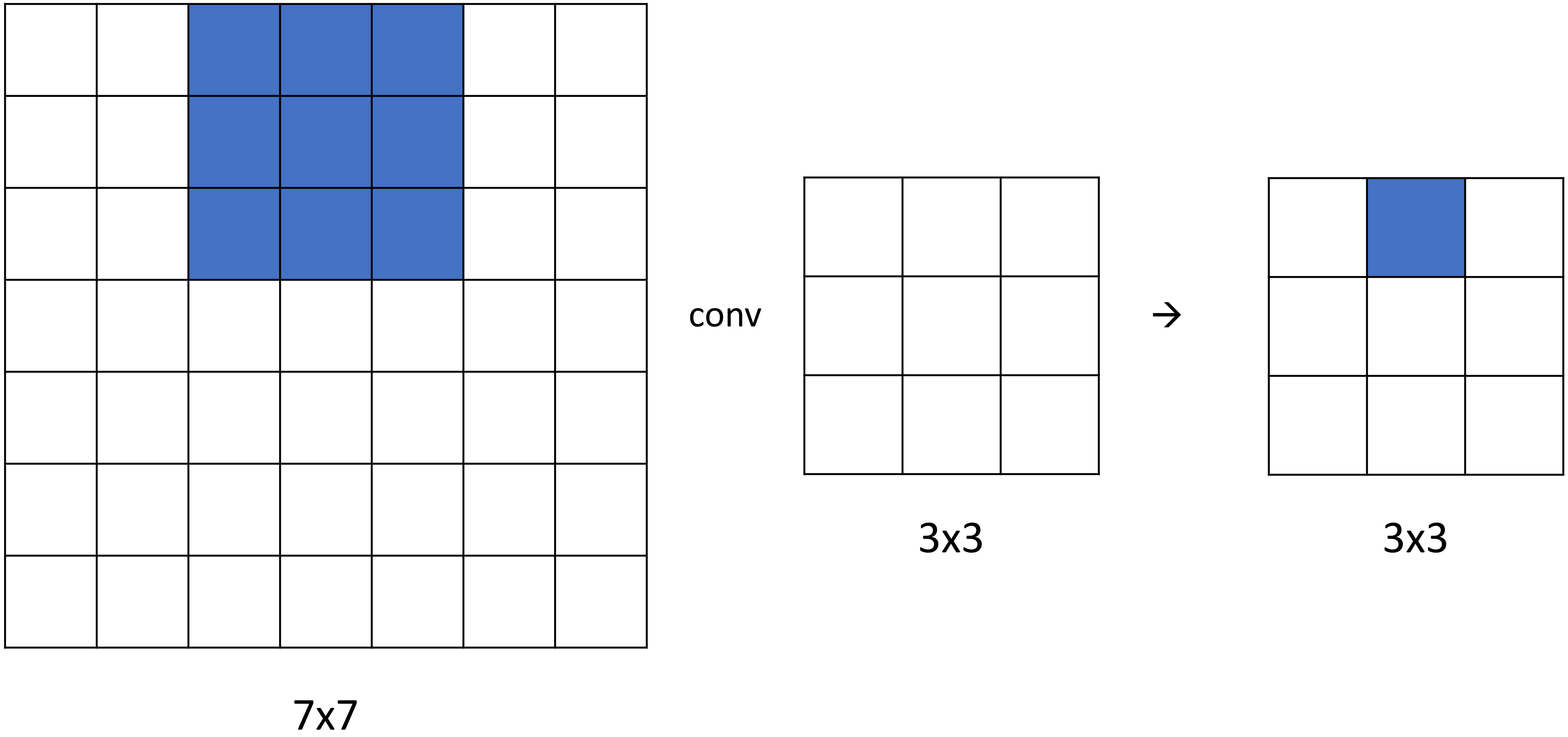

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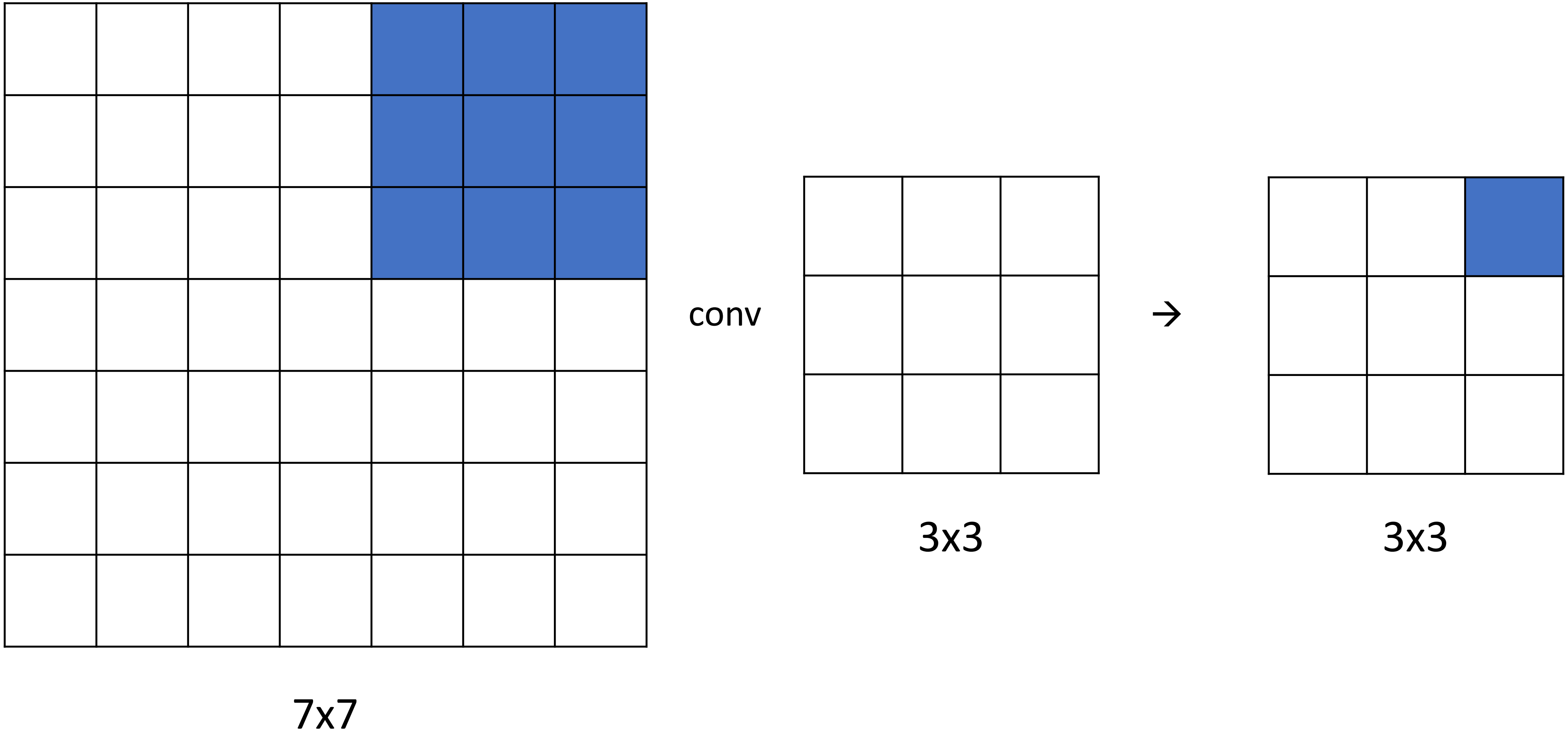

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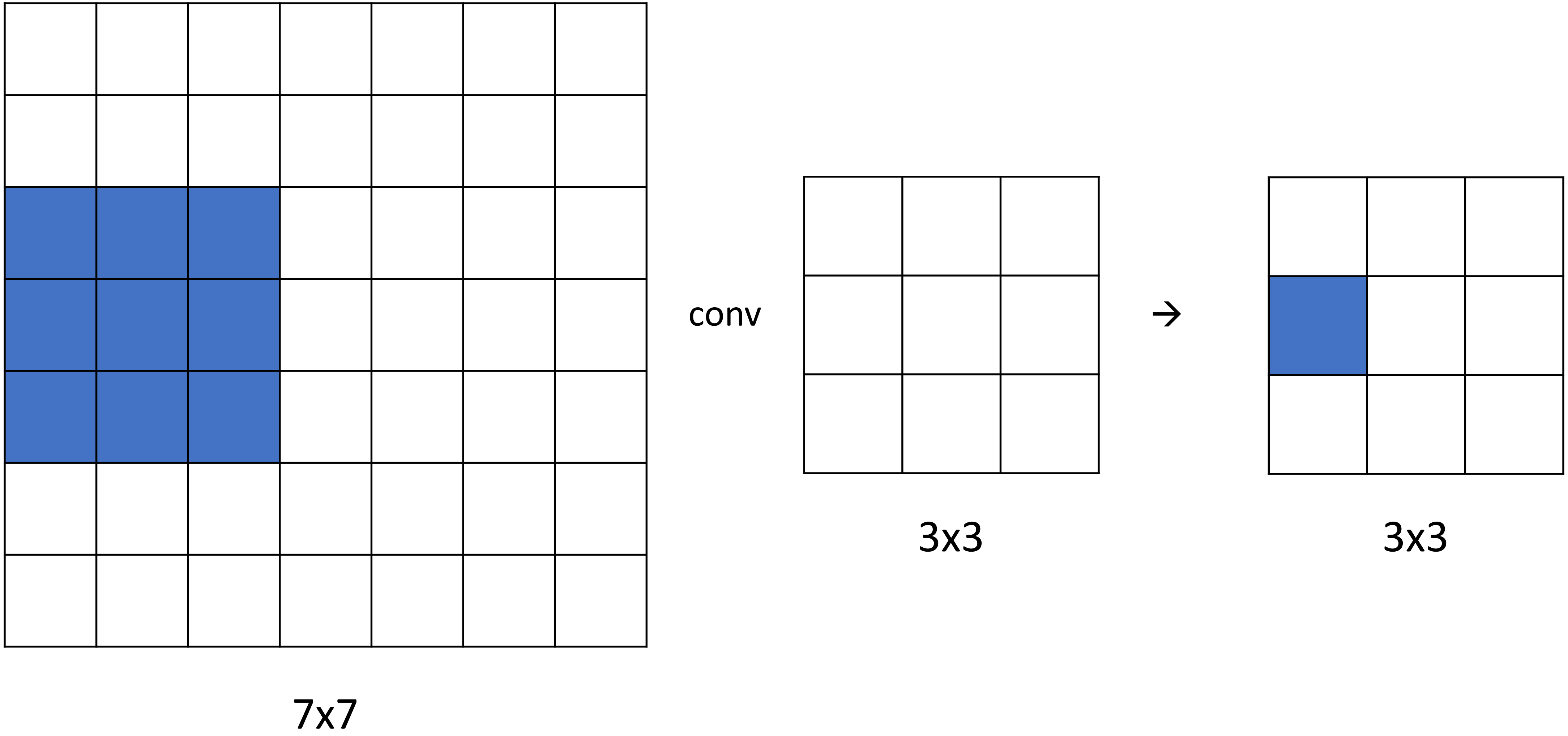

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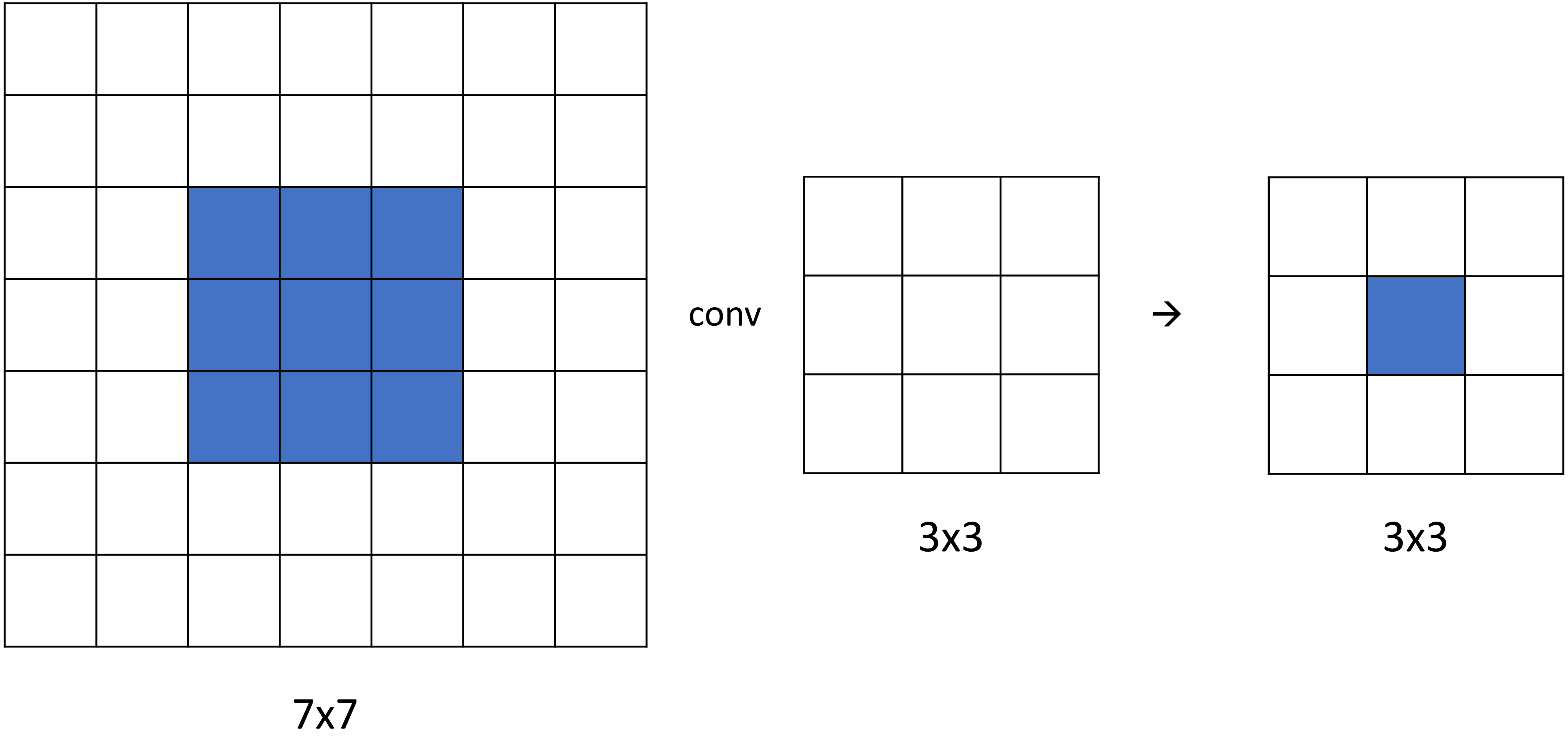

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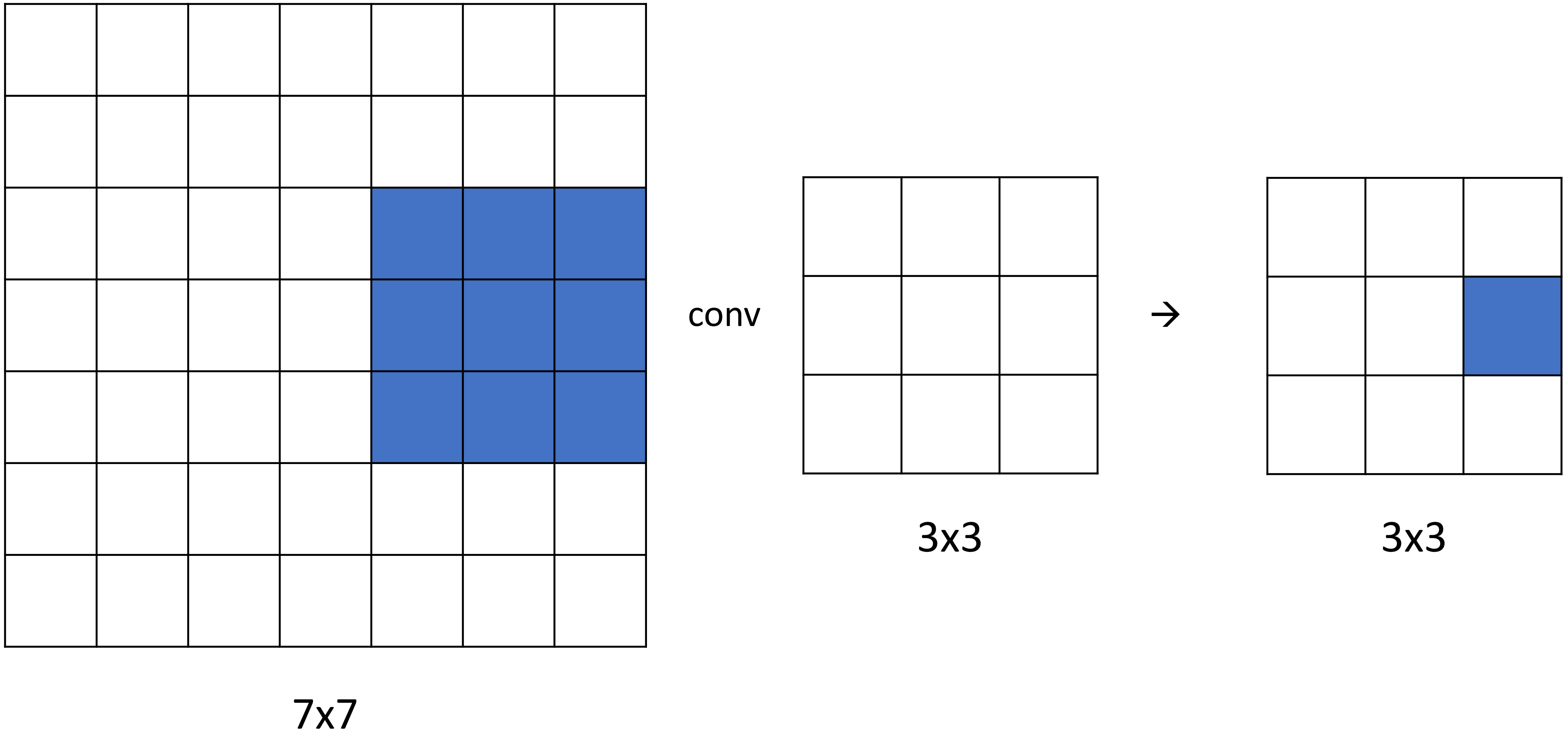

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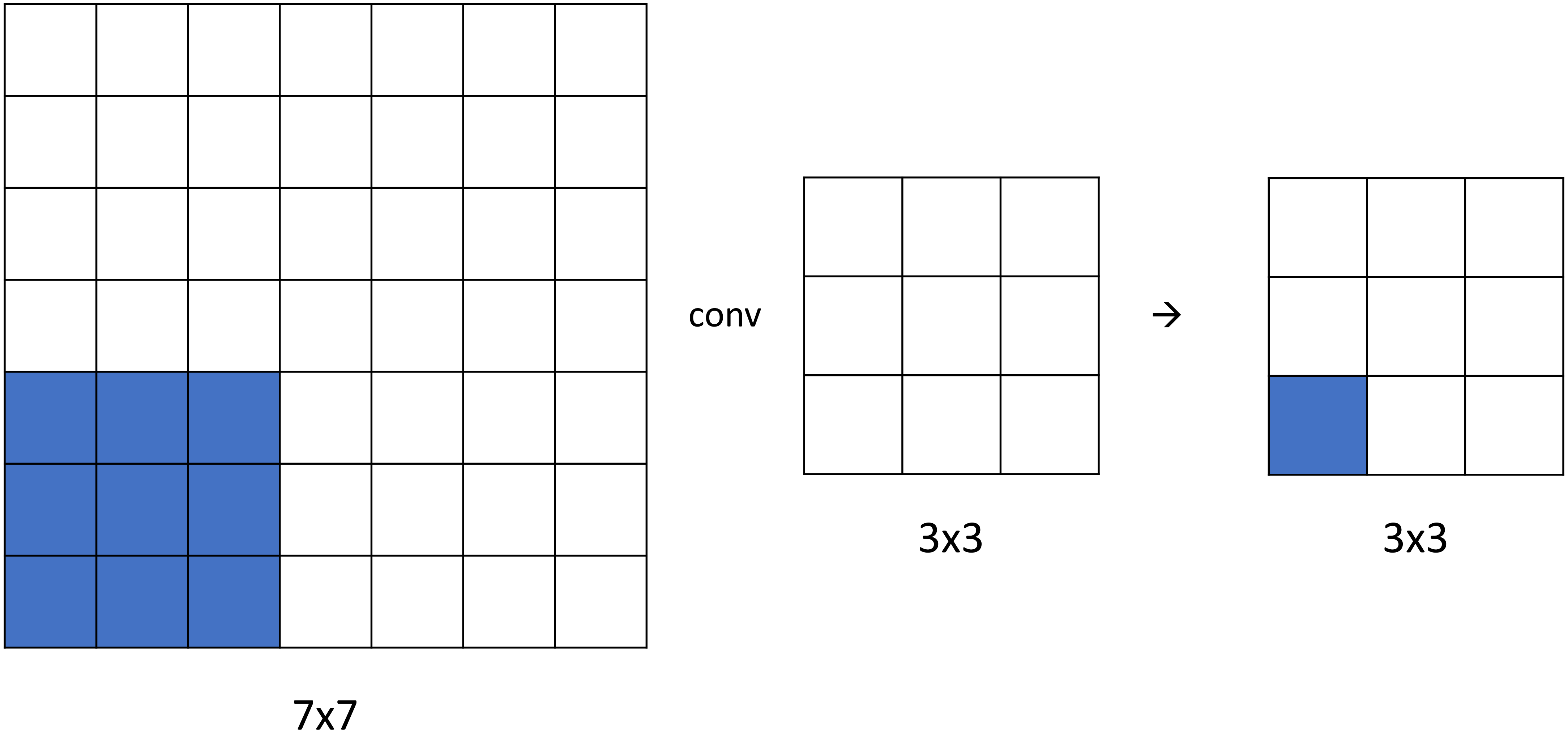

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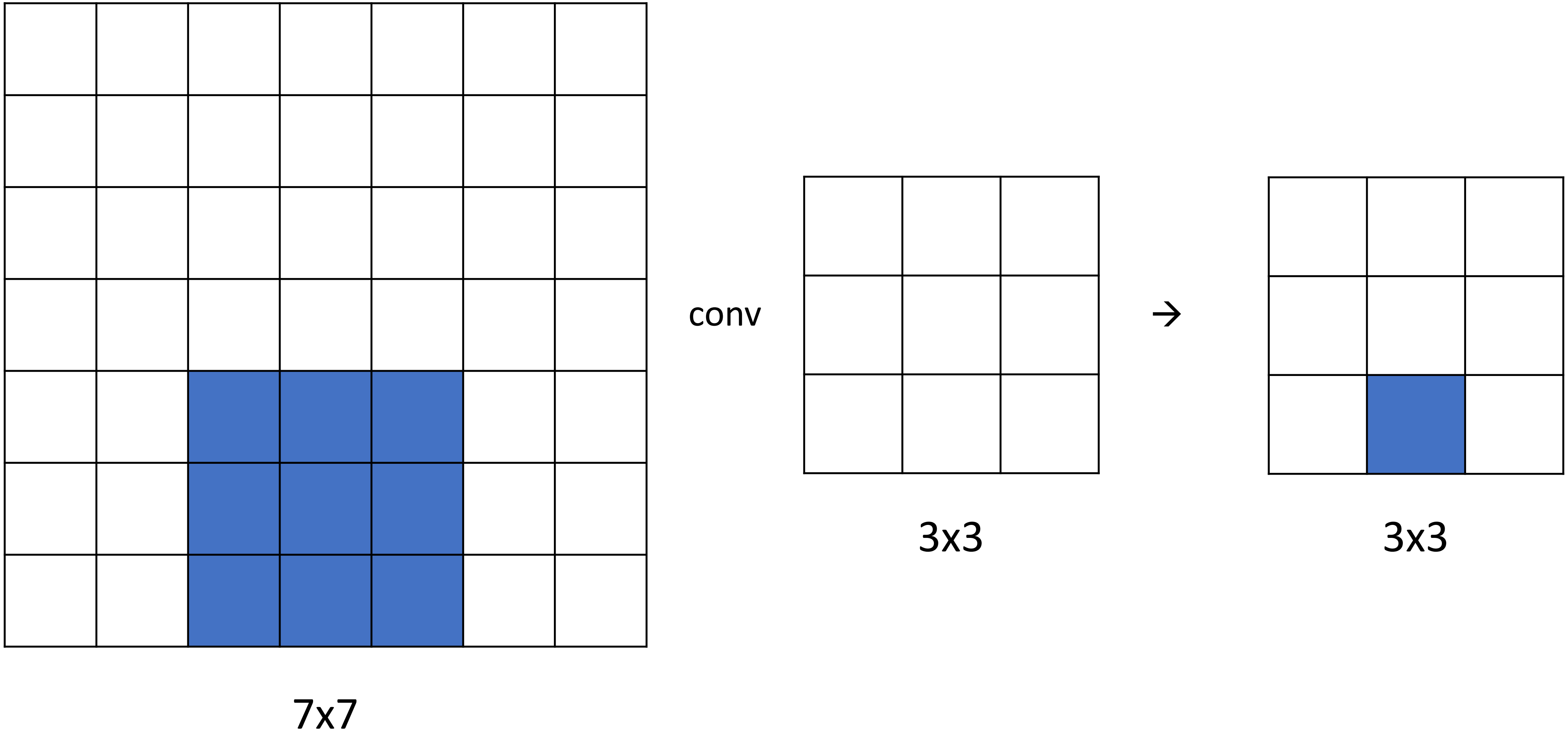

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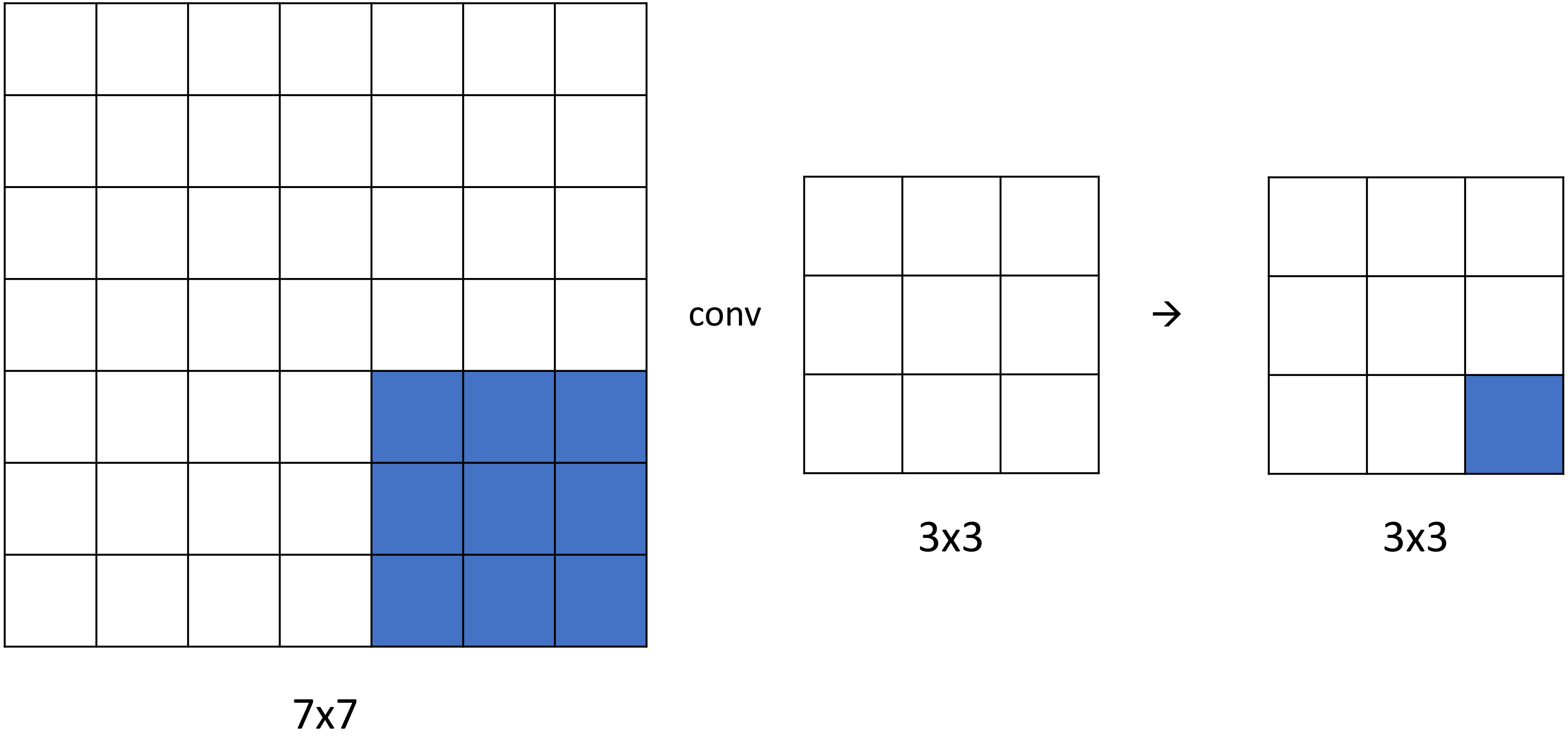

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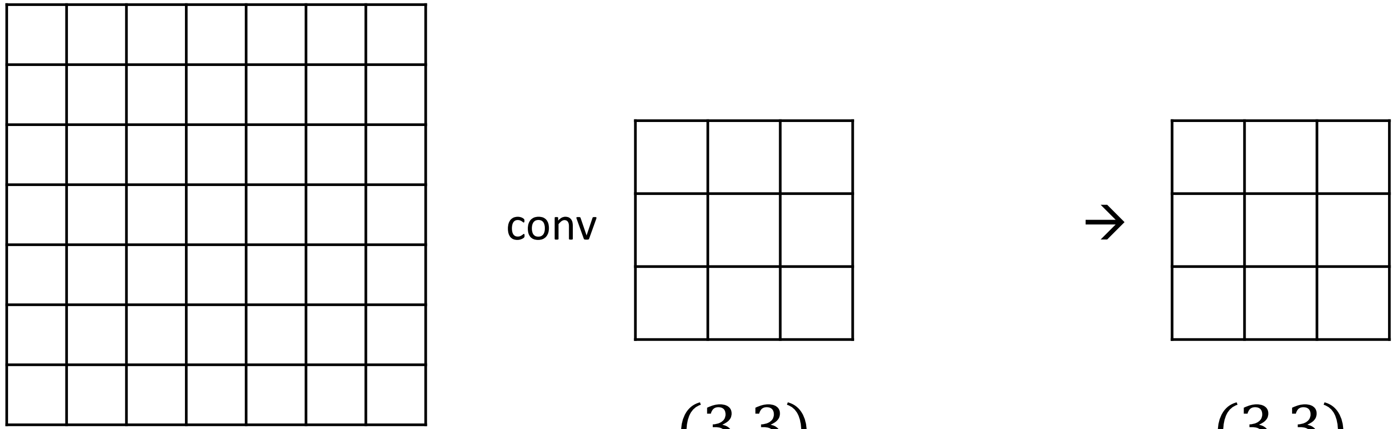
Output Volume Size With Striding

- Input Volume: (h, w, c)
- Output Volume = $\left(\frac{h+2p-f}{s} + 1, \frac{w+2p-f}{s} + 1, K\right)$



$(3,3)$ $(3,3)$

$$(h, w) = (7,7)$$
$$p = 0$$
$$s = 2$$

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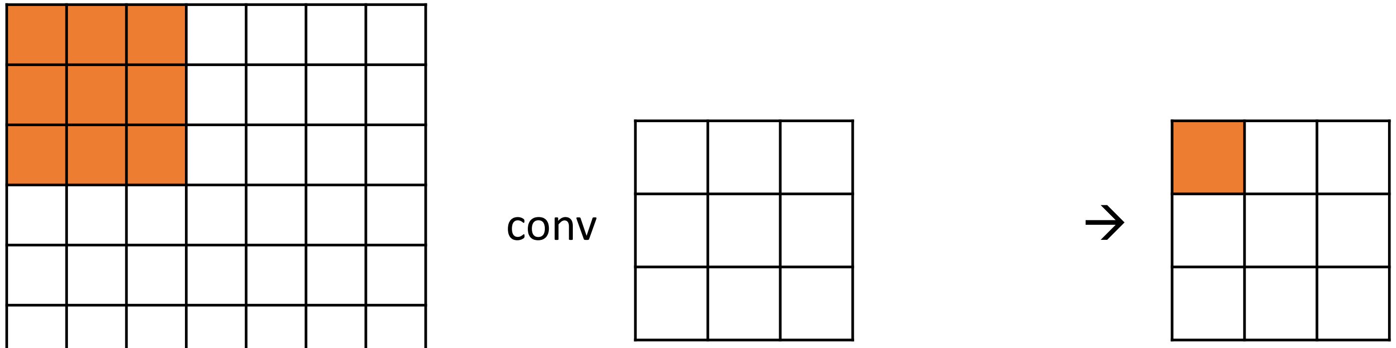

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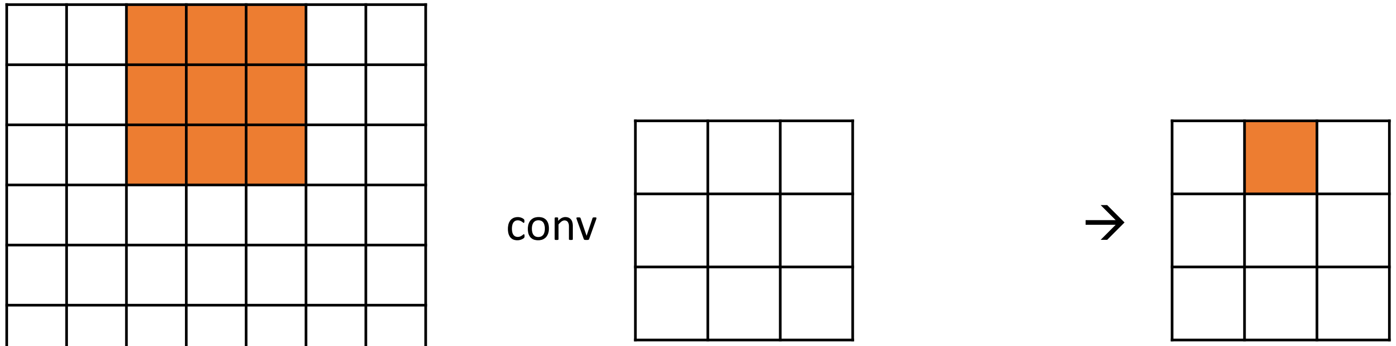

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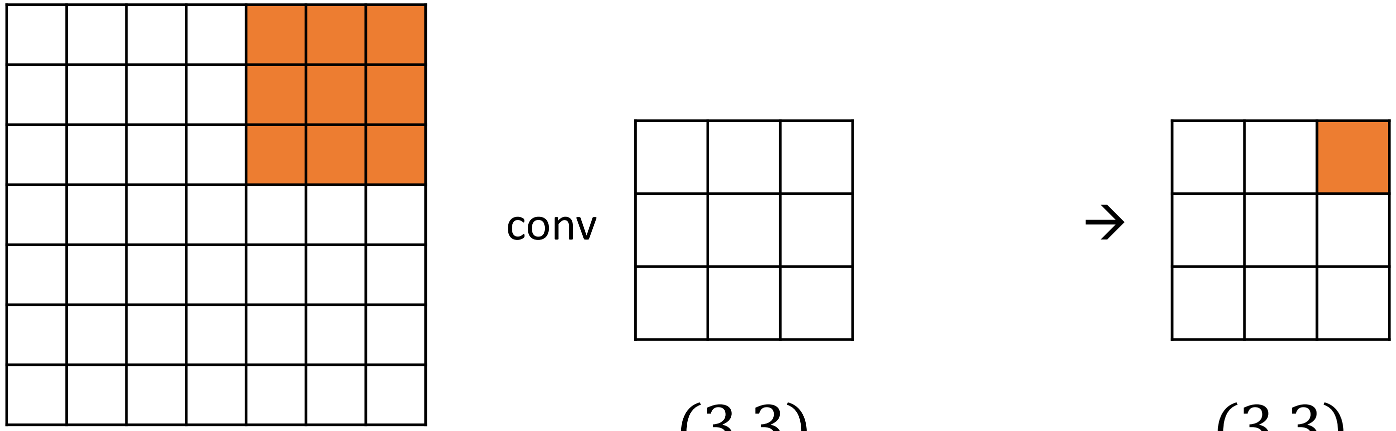

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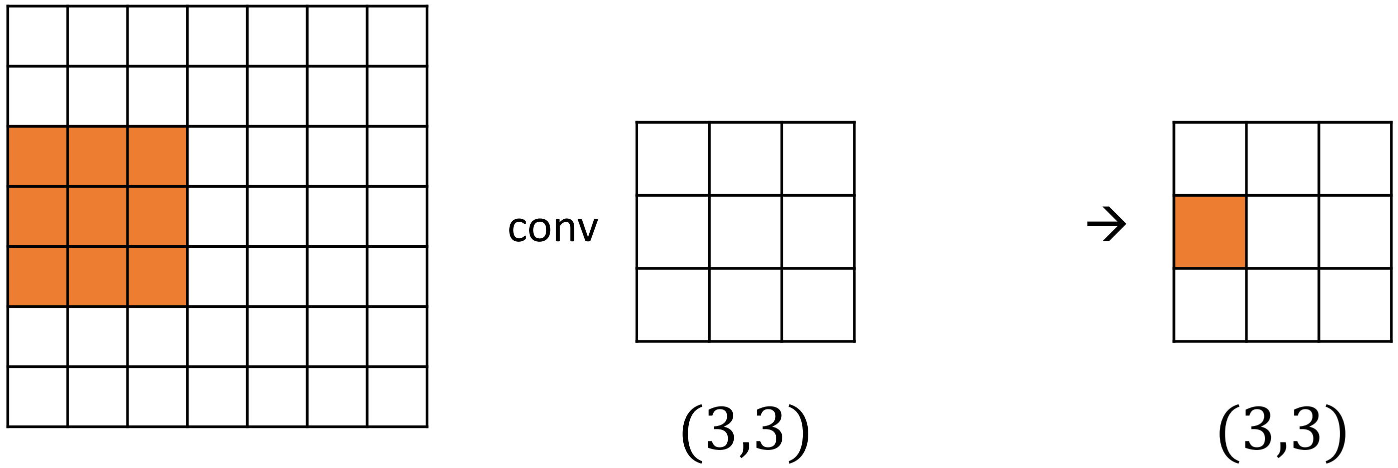

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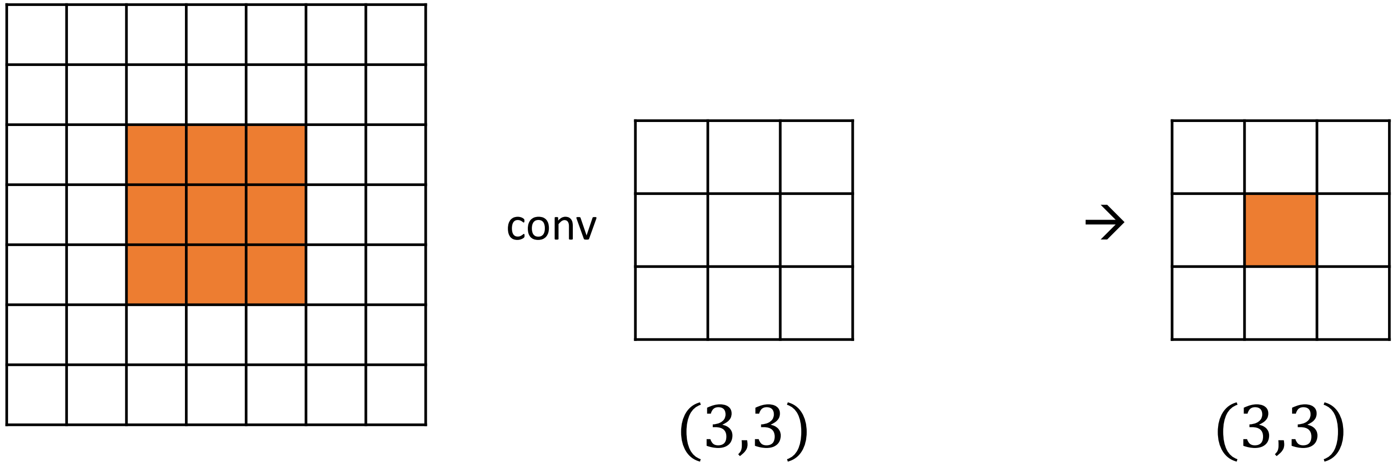

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Output Volume Size With Striding

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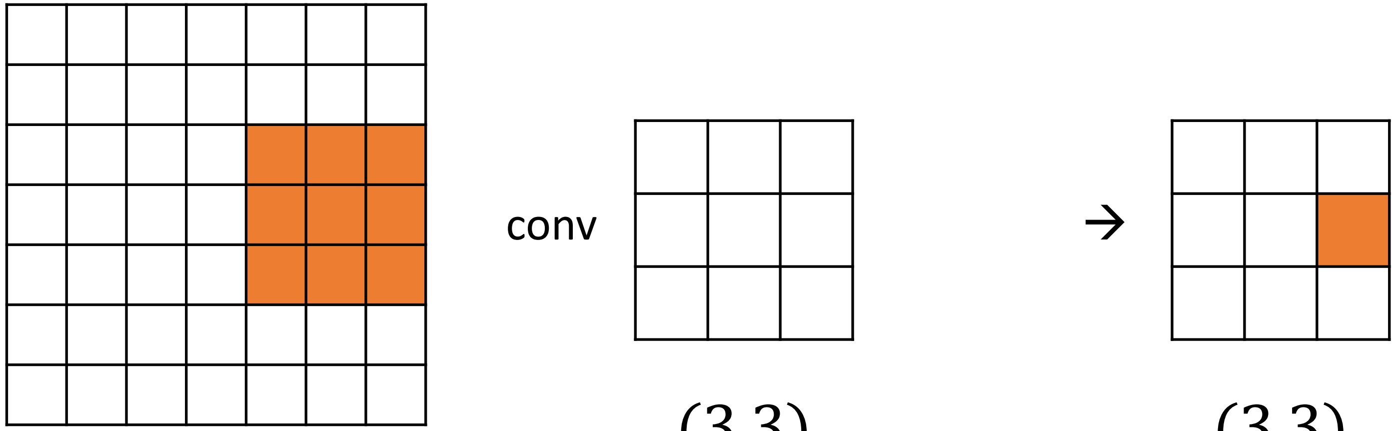

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$(3,3)$

$(3,3)$

Output Volume Size With Striding

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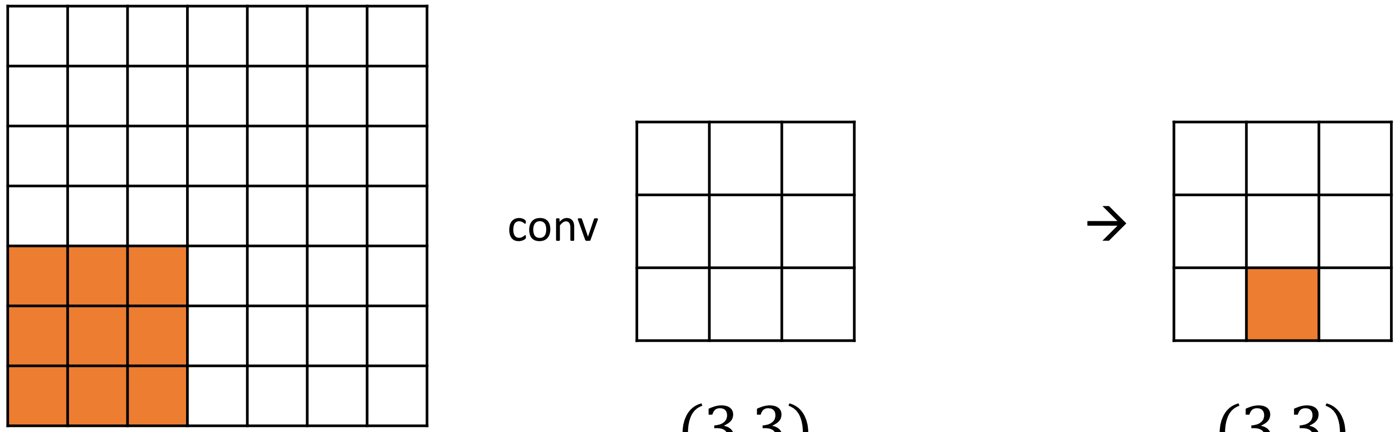

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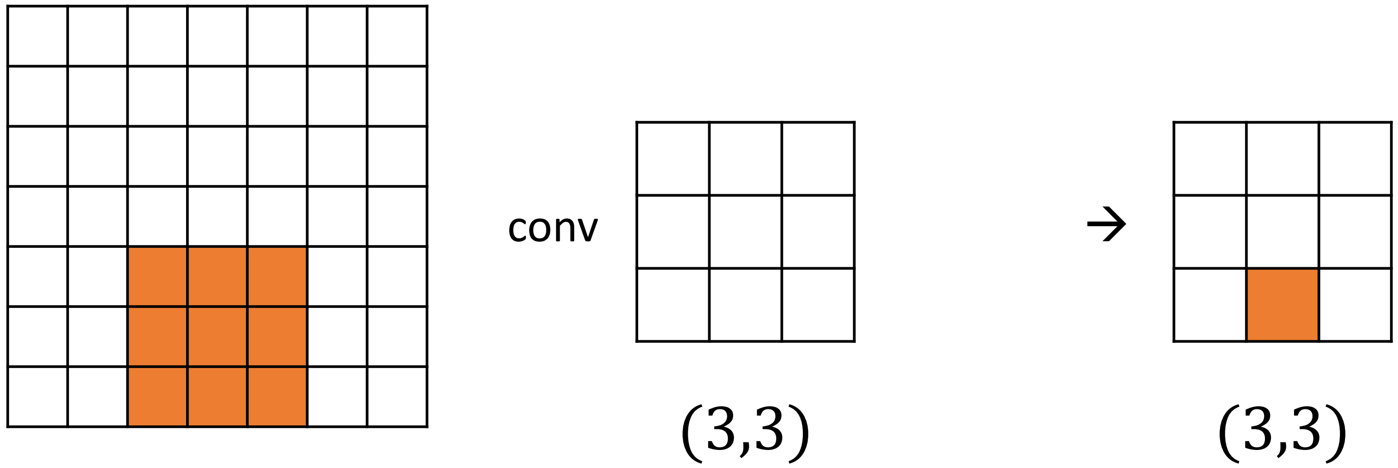

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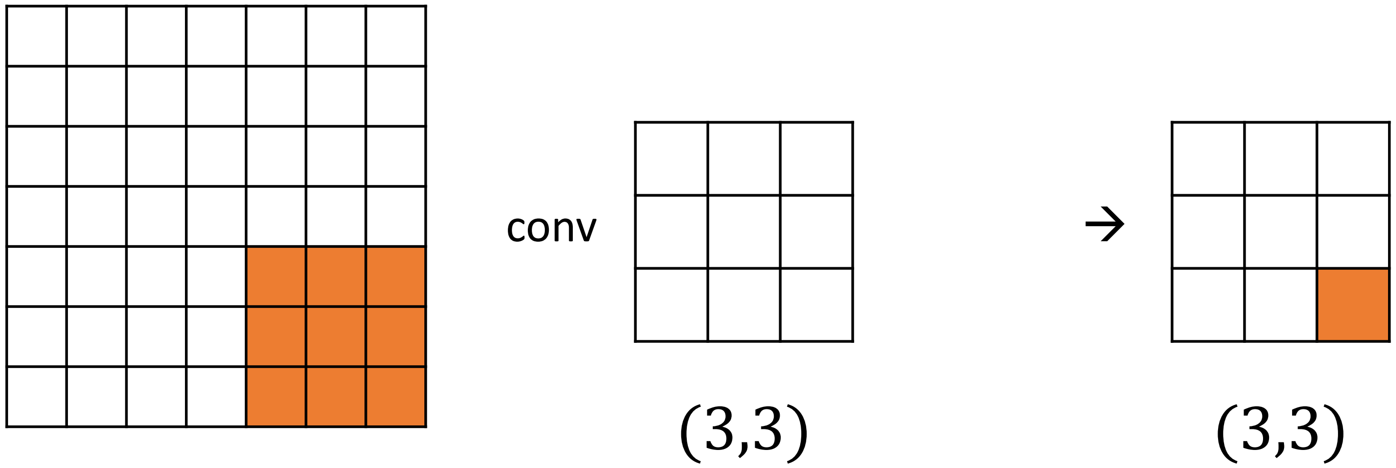

$(h, w) = (7,7)$
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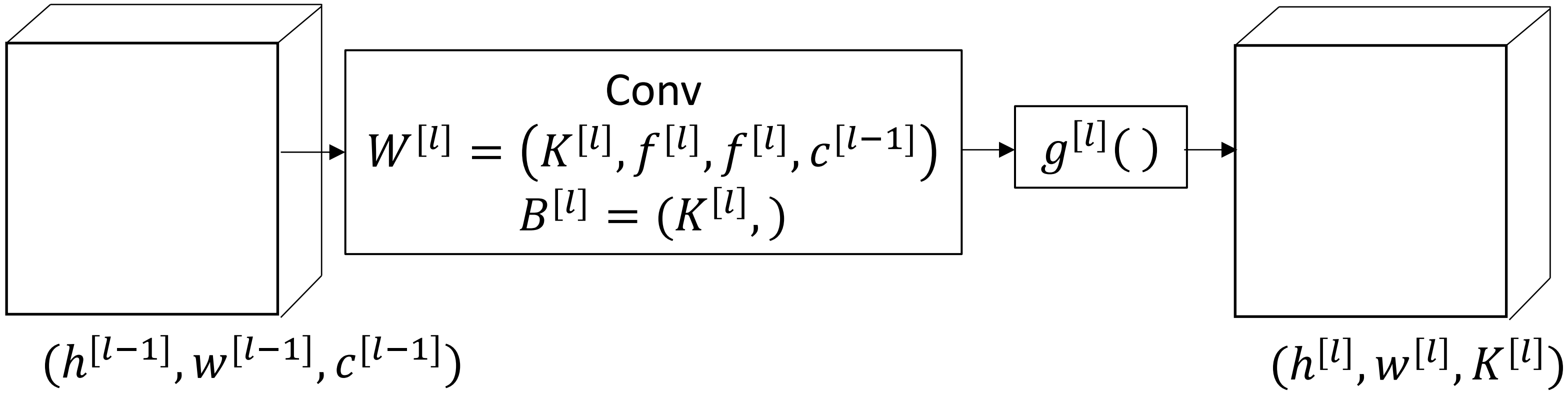
Why Stride > 1?

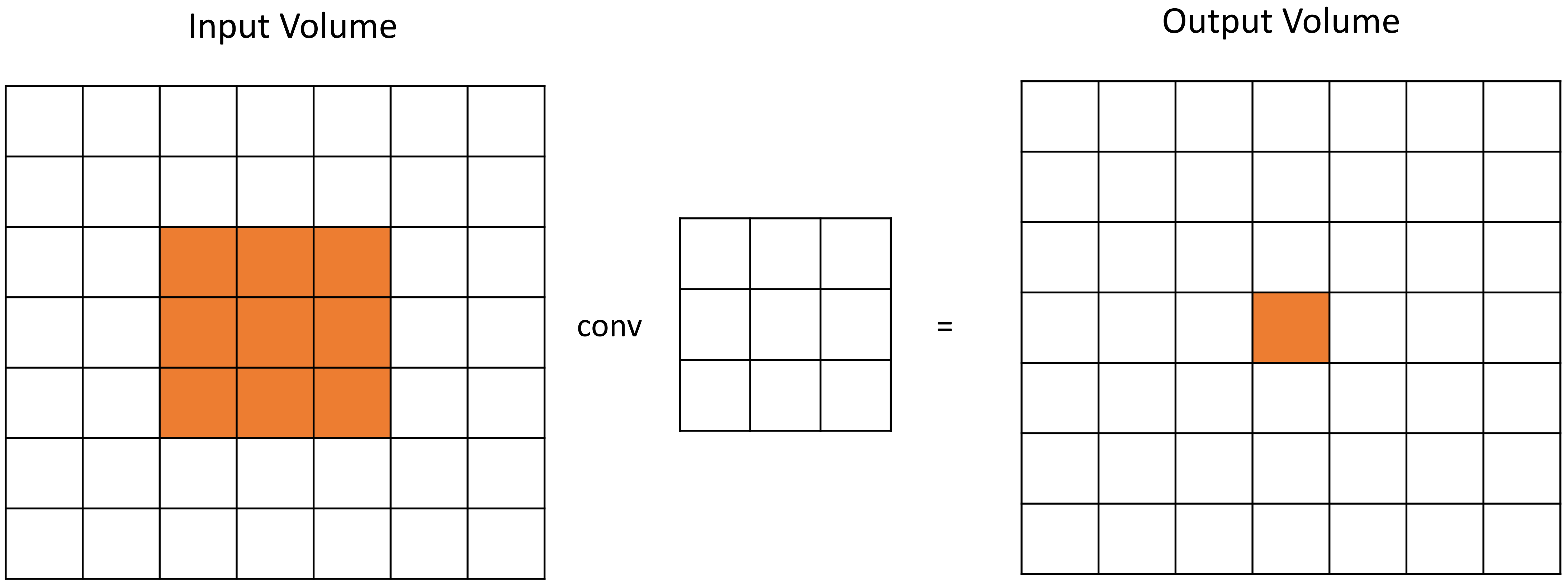
- A form of compression/downsampling of the feature map
- A way to shrink the volumes in a **controlled** fashion
- Shrinking volume is often necessary to control size before the final layer ← more on this soon
- Controls how quickly the receptive fields grows between layers

Final Summary of Convolutional Layer

- Hyperparameters
  - Number of filters K
  - Filter size (f, f)
  - Stride s
  - Padding p
- Input Volume
  - $(h^{[l-1]}, w^{[l-1]}, c^{[l-1]})$
- Output Volume
  - $\left(\frac{h^{[l-1]}+2p-f}{s} + 1, \frac{w^{[l-1]}+2p-f}{s} + 1, K\right)$
- # Learned parameters
  - $K * \left(f * f * c^{[l-1]} + 1\right)$



Receptive Fields



- Assuming a 3x3 filter in this layer
- Each output element "sees" a 3x3 region of the input

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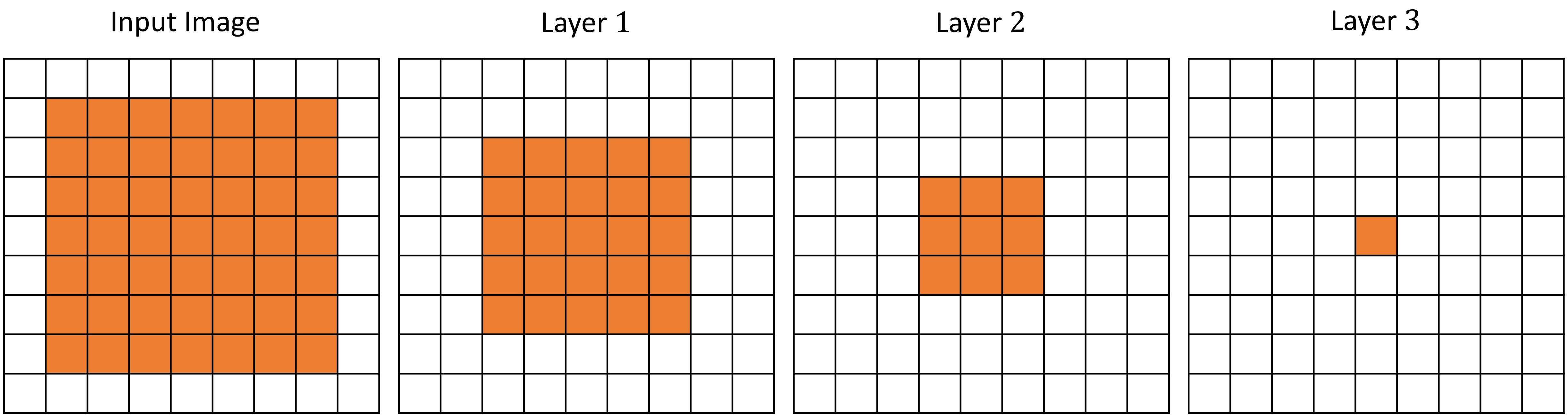
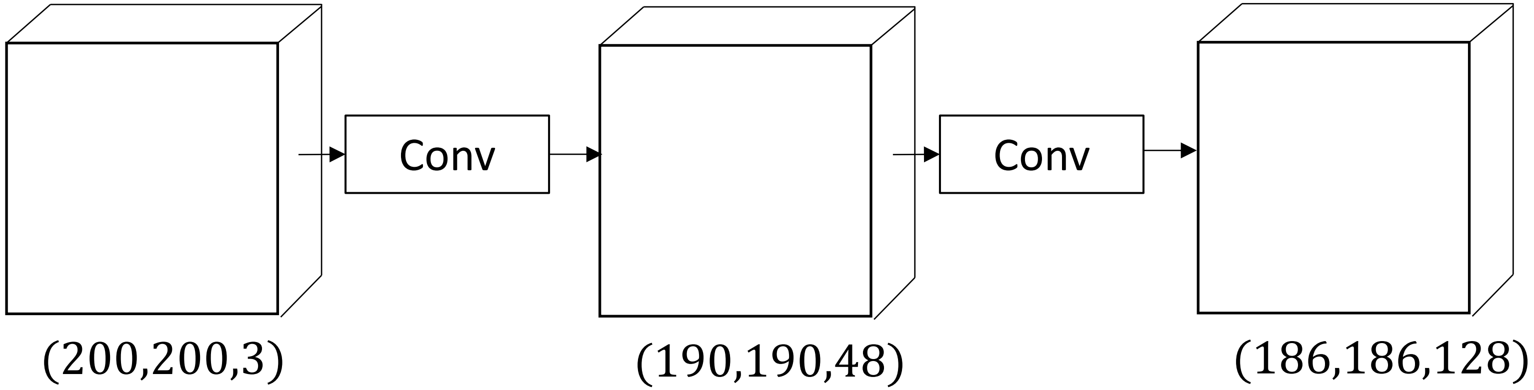
Receptive Fields



- Assuming 3x3 filters in all layers
- Each output element of each layer “sees” a 3x3 region of its input
- You can work backwards to see how much an output element ”sees” w.r.t outputs of earlier layer

Final Layers of a CNN

- Ok if we stack these, we still end up with a volume at the end
- Need a way to make a prediction using the final volume (e.g. final collection of feature activation maps)



Final Layers of a CNN Image Classifier

- Flatten the final volume
- Use one or more fully connected layer
- Final volume must therefore be of a manageable size
- Can think of conv layers as feature extractors

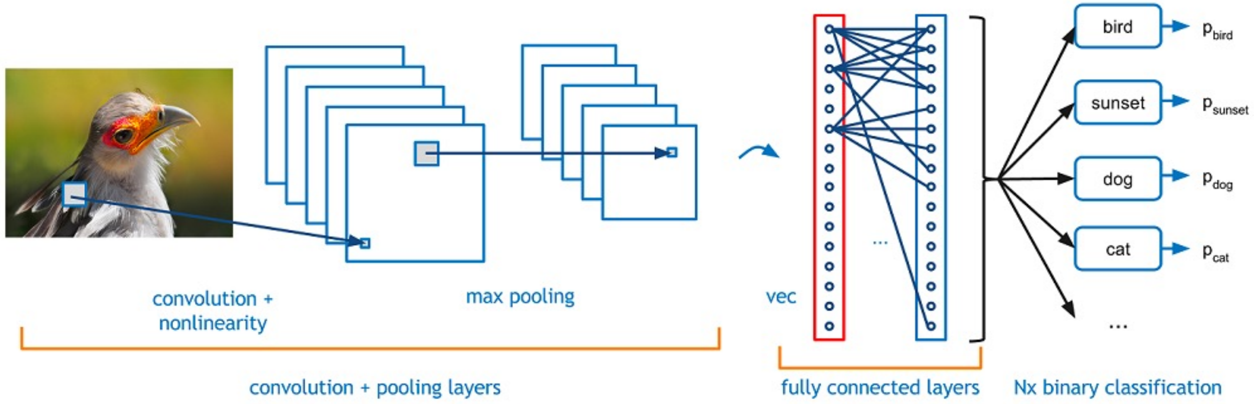
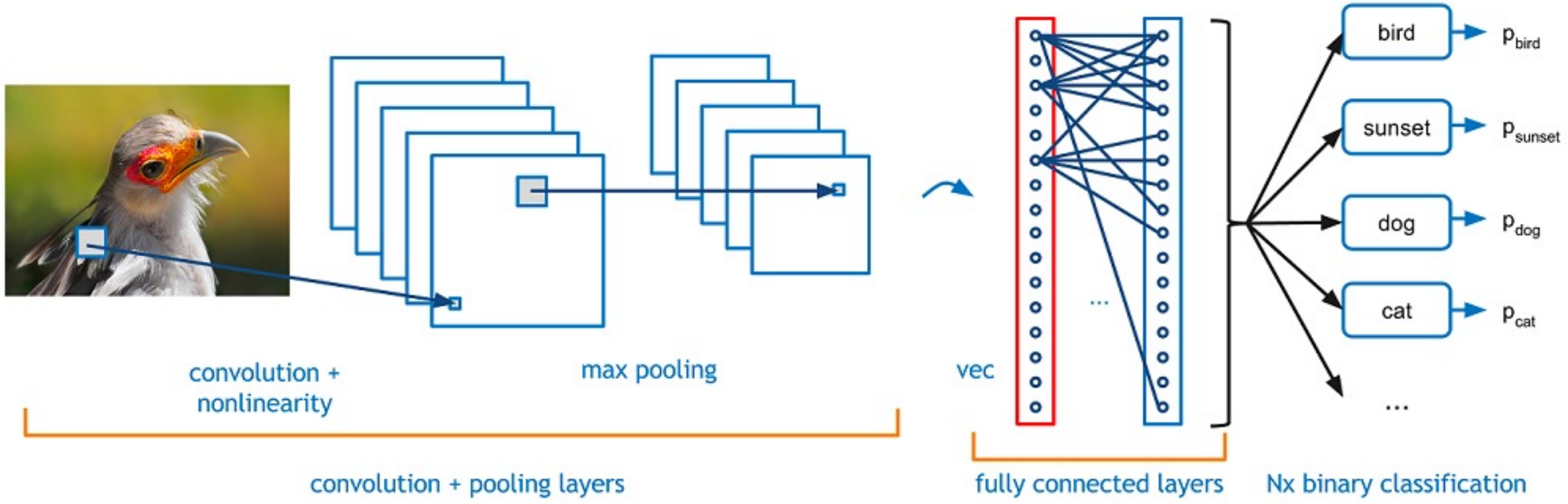


Image source: https://adeshpande3.github.io/A-Beginner%27s-Guide-To-Understanding-Convolutional-Neural-Networks/

Aside – One interpretation: Conv Layers are Feature Extractors



- Compress the image into a “signature”. Use the signature for classification
- Learn structure from unstructured data

Image source: https://adeshpande3.github.io/A-Beginner%27s-Guide-To-Understanding-Convolutional-Neural-Networks/

How do we control the final volume size?

- When using only Convolutional layers, your final volume depends on
- Stride, padding of your conv layers,

Pooling Layer

Example: Max Pooling

- Output is max value within each region

1	3	10	0
8	4	2	9
7	2	1	3
1	1	0	4

Max Pooling
$(f, f) = (2,2)$
Stride: 2

→

Example: Max Pooling

- Output is max value within each region

1	3	10	0
8	4	2	9
7	2	1	3
1	1	0	4

<u>Max Pooling</u>
$(f, f) = (2,2)$
Stride: 2

→

8	

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Example: Max Pooling

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8	4	2	9
7	2	1	3
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<u>Max Pooling</u>

$(f, f) = (2,2)$

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→

8	10

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7	

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$(f, f) = (2,2)$

Stride: 2

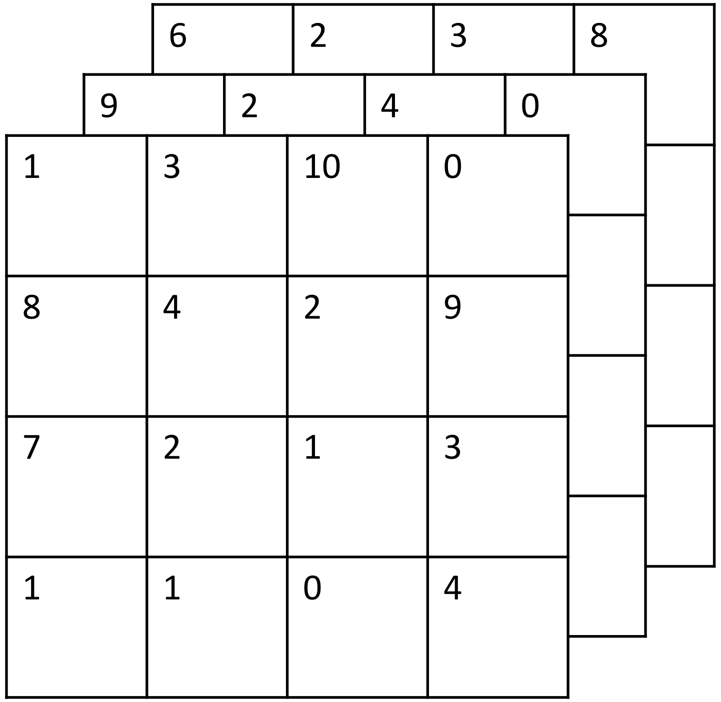
→

8	10
7	4

After pooling volume size has been reduced!

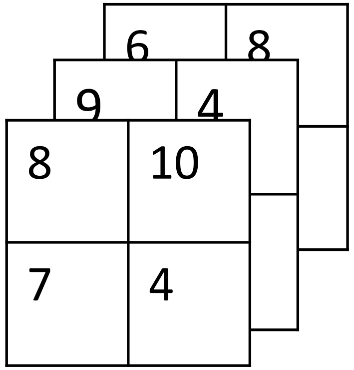
Pool each channel independently

- Apply this process to each activation map independently
- Does not change your channels size, only spatial dimensions of (h, w)



Max Pooling
$(f, f) = (2,2)$
Stride: 2

→



Max Pooling Intuition

- Compressing the data. Discarding all but the "strongest" signal
- Max Pooling adds a bit of flexibility to feature detection in the form of tolerance to translation (jitter)
- Once we know that a specific feature is in the original input volume, it's exact location is not as important as its relative location to other features.

1	3	10	0
8	4	2	9
7	2	1	3
1	1	0	4

<u>Max Pooling</u>

$(f, f) = (2{,}2)$

Stride: 2

→

8	10
7	4

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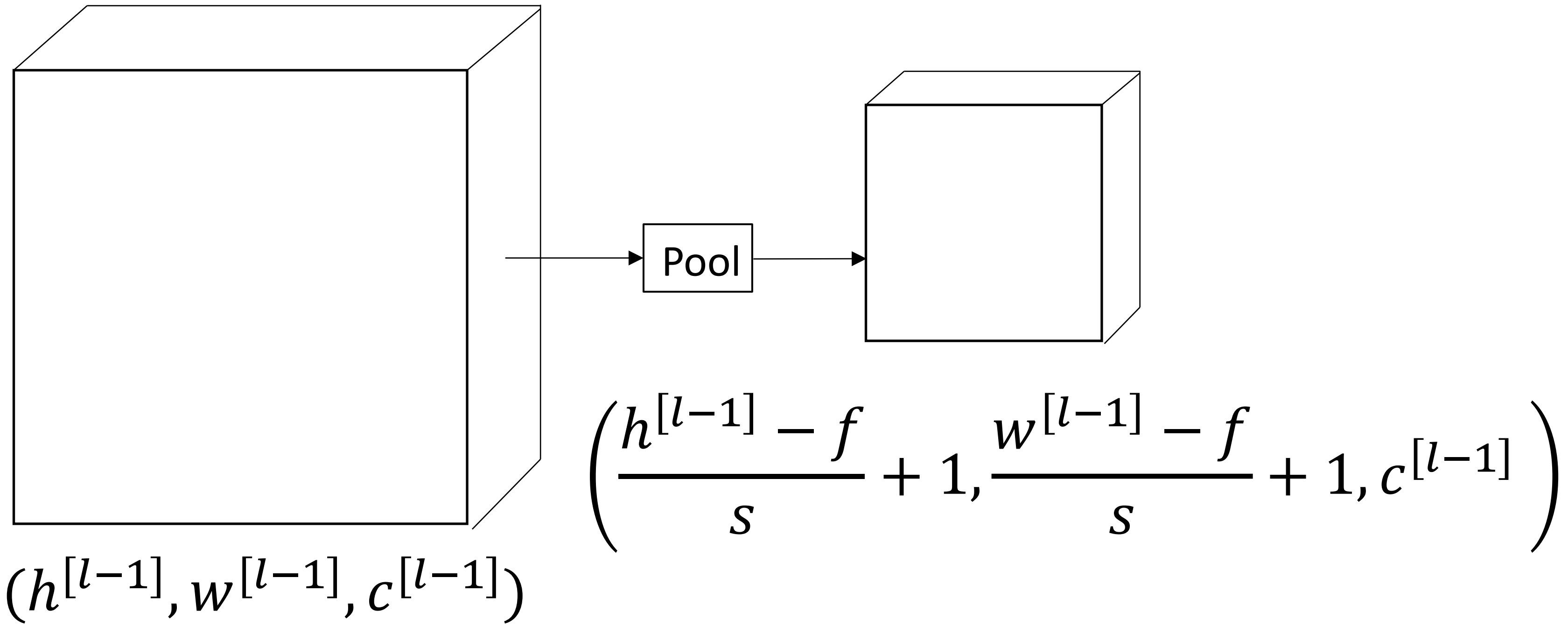
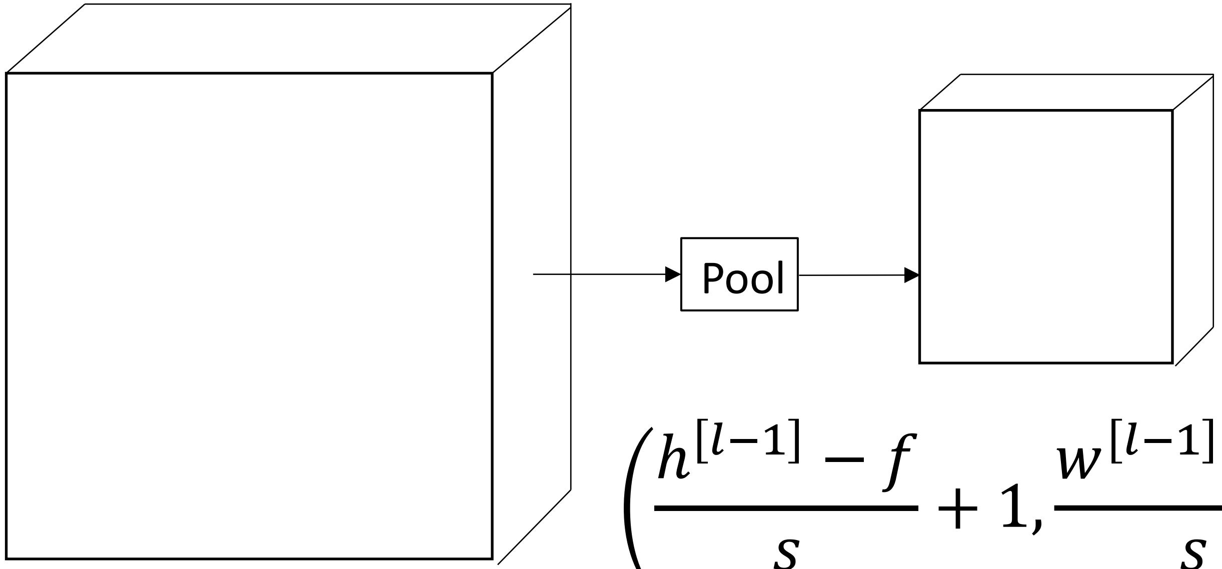
Pooling

- Hyperparameters
  - Pooling function
  - Pool size (f, f)
  - Stride s



- No learned parameters!
- Reduces spatial dimensions but NOT channel dimension

Pooling



$$\left(h^{[l-1]}, w^{[l-1]}, c^{[l-1]}\right)$$

$$\left(\frac{h^{[l-1]} - f}{s} + 1, \frac{w^{[l-1]} - f}{s} + 1, c^{[l-1]}\right)$$

Usually, $f = s$

$$\left(\frac{h^{[l-1]}}{s}, \frac{w^{[l-1]}}{s}, c^{[l-1]}\right)$$

- Hyperparameters
  - Pooling function
  - Pool size (f, f)
  - Stride s
- No learned parameters!
- Reduces spatial dimensions but NOT channel dimension

Example: Average Pooling

Example: Average Pooling

- Output is **average** value within each region

1	3	10	1
8	4	4	9
7	2	4	3
1	1	0	4

<u>Avg Pooling</u>
$(f, f) = (2,2)$
Stride: 2

→

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Example: Average Pooling

- Output is **average** value within each region

1	3	10	1
8	4	4	9
7	3	4	3
1	1	0	5

Avg Pooling
$(f, f) = (2,2)$
Stride: 2

→

4	

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Example: Average Pooling

- Output is **average** value within each region

1	3	10	1
8	4	4	9
7	3	4	3
1	1	0	5

<u>Avg Pooling</u>
$(f, f) = (2,2)$
Stride: 2

→

4	6

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Example: Average Pooling

- Output is **average** value within each region

1	3	10	1
8	4	4	9
7	3	4	3
1	1	0	5

<u>Avg Pooling</u>
$(f, f) = (2,2)$
Stride: 2

→

4	6
3	

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Example: Average Pooling

- Output is **average** value within each region

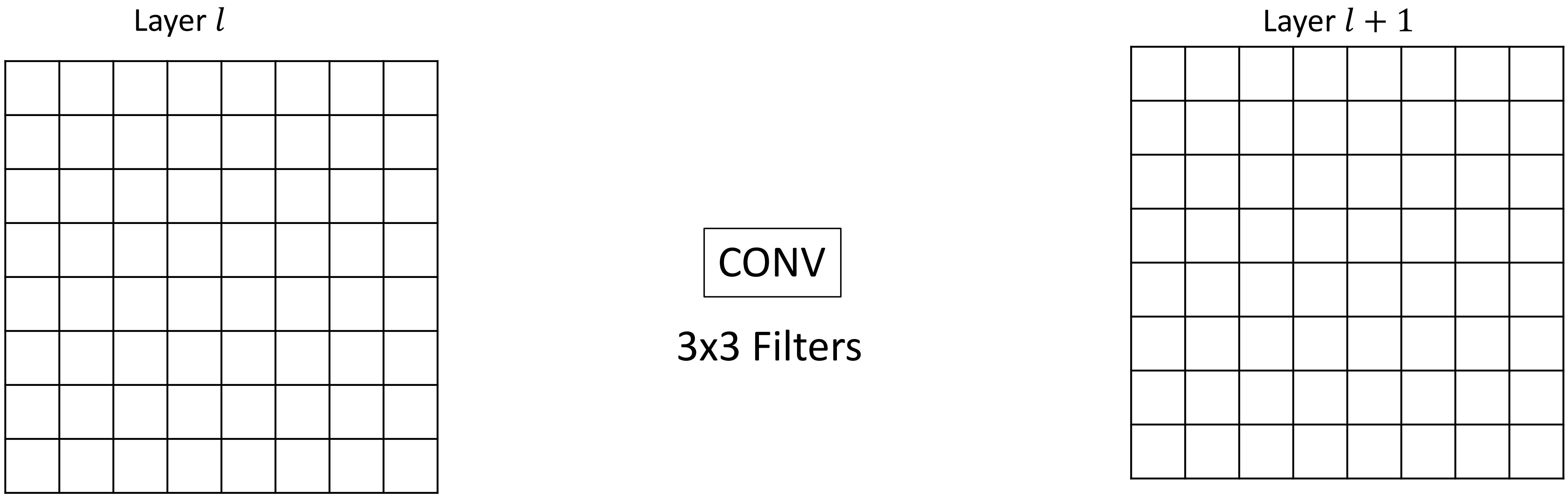
1	3	10	1
8	4	4	9
7	3	4	3
1	1	0	5

Avg Pooling
$(f, f) = (2,2)$
Stride: 2

→

4	6
3	3

Pooling and Receptive Fields



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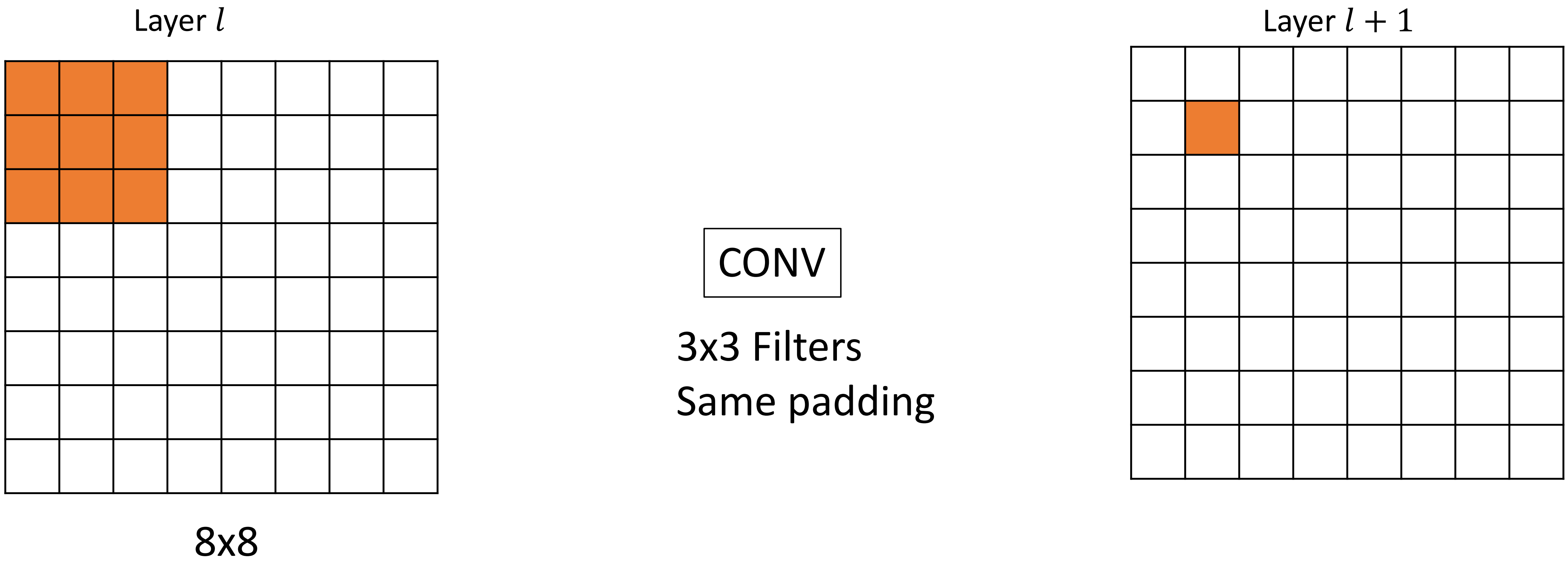
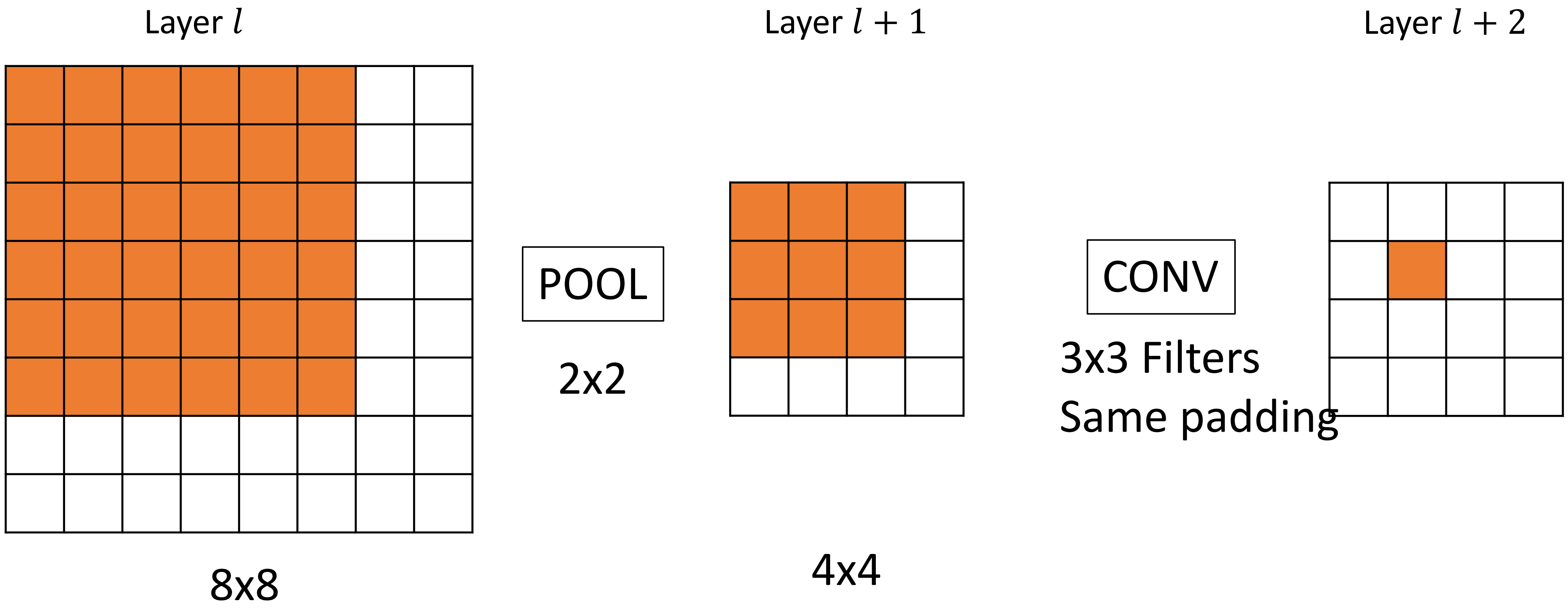
Pooling and Receptive Fields



Pooling and Receptive Fields



Summary of Pooling Layer

- Reduces the size of the feature maps (i.e. volume size)
  - Controlled shrinkage of volume is required for final fully connected layer
- Pooling operation is specified → not learned (no parameters)
- Most often, stride is equal to filter size. $s = f$
- Max Pooling adds some translation tolerance. But you need to think about whether this is what you want for your application
- Operates over each activation map **independently**. Does not reduce channel size, only spatial dimensions
- People generally use Max Pooling over Average Pooling

Learning Objectives

- Understand how a Convolutional Layer works
- Understand how we can build a Convolutional Neural Network using Convolutional and Fully-Connected Layers
- Understand how we can ”transform” a Convolutional Layer to a Fully-Connected Layer and vice-versa
- Understand what stride and padding are
- Understand how Pooling Layers work

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