Centralized and Game Theoretical Solutions of Joint Source and Relay Power Allocation for AF Relay based Network

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***Abstract*—**[1] **Relaying is an emerging technique for 3G/4G high bandwidth networks in order to improve the capacity of edge nodes. As the deployment cost is high, there might be a few number of relay nodes in the cell which can help the edge nodes to transmit their data. From this perspective, one of the key problems in a relay equipped node is to make decision which edge nodes to be helped and how much power need to be disseminated among them in order to maximize the system capacity. This problem is formulated as an optimization problem given individual node and total available power constraints. The objective function of the formulated problem is non-convex and we solve this using geometric program (GP). Since the solution of this problem is computationally expensive, we propose a low complexity suboptimal solution for this problem. Having noticed the selfless nature of the sources in the centralized solution, we also provide a game theoretical solution. Two separate Stackelberg games are required to solve this power allocation problem. Moreover, given the total power constraint, a centralized entity is necessary to connect these two games. For assigning power among the sources, the centralized entity plays the buyer level game, whereas the sources act as power sellers. On the other hand, to disseminate relay power among the sources, roles of the players are just interchanged. Besides, before staring the game, the centralized entity determines, of total power, how much is for the transmit operation of the sources and how much is for their relay operation. We show that there is a unique Stackelberg Equilibrium (SE) for both games under certain convergence condition. Finally, the proposed game theoretical solution can achieve comparable performance in terms of resource allocation with the centralized optimal one.**

***Index Terms*—Power Allocation, Convex Optimization, Amplify and Forward Relay, Geometric Programming, Stackelberg Game.**

I. Introduction

Emerging relay technique in 3G/4G networks, such as Long Term Evolution (LTE) and LTE-Advanced has brought numerous problems to optimize its deployment. In recent years, cooperative relay networks have got tremendous attention. Two most popular cooperation protocols are considered - decode and forward (DF); amplify and forward (AF) [1]. DF relays retransmit just the replica of source's transmitted signal. Whereas AF means, the relay nodes amplify source's signal first, and then retransmit towards the destination. We adopt the AF protocol in our work because of its simple signal processing mechanism. Our problem presented in this paper focus on a network having one relay node, a set of sources, and single destination. Given fixed amount of power to be distributed among the sources and relay is an optimization problem, and exhibits the trade off between fair resource allocation and overall network performance. If resource is limited in the network, serving all sources may lead to the degradation of network capacity. In this situation, best possible few sources should be selected for the transmission while ignoring others.

To put our work into context, we outline the chronological order of evolved research on the power optimization of AF relayed networks. At the beginning of such exploration, people focused on a simple network with one source and one relay [2]. And, then, relay power allocation for a single-source multi-relay network has extensively been studied varying different optimization criterion, i.e., minimizing outage probability, or minimizing sum relay power under the source's SNR (Signal to Noise Ratio) or outage probability constraint [3], [4], [5]. Performance optimization of multi-source single-relay network has also been given attention from different angles, such as interference cancellation schemes are proposed in [6] and [7]. In [8] and [9], network decoding is applied to combat interference among the users. In [10], the resource allocation problem including both subcarrier and relay power is studied to maximize the sum rate. At the same time, joint multi-source multi-relay power optimization has appeared in a few papers [11], [12] under different optimization criterion, i.e., sum rate maximization, sum power minimization etc. Although in [11], each source is essentially served by only one relay, [12] made performance improvement by assisting single source with the help of multiple relays. Drawback of their approach is, they have totally ignored SNR due to the direct link transmission in their optimization formulation.

Due to the inefficient bandwidth utilization of relays while transmitting on the orthogonal channels, at one point, many people gave emphasis on the best relay selection schemes. Multi-relay selection and their power allocation recently appeared in some works [13], [14], [15] because of their ability to attain better performance. Joint relay and opportunistic source selection for a bidirectional network has been considered in [16] to optimize the outage probability and BER (Bit Error Rate).

In the first portion of this paper, we have formulated the problem considering individual source, relay, and total system power constraints. Total power constraint is imposed to reduce the interference in the neighboring network. Our formulated problem is very similar to [11], [12] although [11] has not taken individual source and total power constraints, and in [12], each source is served by multiple relays[2]. The main drawback of their work is, they have totally skipped SNR due to the direct link in their formulation. Putting the direct link SNR in the original formulation, we have solved the problem using GP. Because of the direct link SNR, the original problem is not amenable to GP. Hence, we have used single condensation method in order to solve this problem using GP. Since the original problem has many variables, and single condensation method may need many iterations to attain convergence, we have transformed our problem to the problem with two variables (one source and one relay) which is much computationally cheap. Once we obtain the source transmit power, we subdivide it among all original sources using greedy and fair algorithms. For dividing the relay power among them, we apply water filling method given that the sources are assigned with transmit power using the proposed heuristics.

In the aforementioned solution of our defined problem, the nodes in the network are assumed to be altruistic, and willing to cooperate in optimizing the overall network performance. In many practical solutions, however, nodes are selfish and aim to optimize their own benefits or quality of service, which may result in conflict of sharing resource among them. Game theory is a flexible and natural tool to model, and analyze these behaviors of the nodes. This tool has extensively been applied for several problems in cooperative relay networks. In [17], uplink of a network with multiple users can form a coalition in order to maximize their transmission rate or utilities. In [18], [19], a bargaining game has been designed in order to show the interaction between two users, where one user acts as a relay for the other one with the purpose of attaining bandwidth [18] and power allocation [19]. In [20], the users define the payment rates for the relays, and they share the payments among themselves who are willing to help the users. [21] shows cooperation for two nodes in two different mesh networks. An analytical framework is proposed to determine when the cooperation is beneficial, and the expected performance gain is estimated.

Game theoretical power allocation for a multi-user multi-relay network has also extensively been studied. For example, non-cooperative game theory has been applied to show the competition among the relays in order to assign optimal power. The authors in [22] have presented a solution for relay power allocation of such network, where two kinds of users are considered: variable, constant rate users. They have used dual decomposition method in order to solve their formulated problem, and based on that, they have given a distributed solution. For a single-user multi-relay network, two level Stackelberg game has been proposed in [23] for selecting the best relays and determine their power. For a multi-user single-relay network, source power allocation problem has also been modeled using two level Stackelberg game [24]. The relay sets price for the users, and they play a non-cooperative game in order to maximize their individual utility. One more recent work for a multi-source single-relay network is [25] for the purpose of relay power allocation following some fair resource allocation rule. The relay acts as a leader and sets the price for power, whereas the users work as customers or followers for the purpose of maximizing their own utility. In this work, since the relay needs to know the complete information of the network, same authors proposed fully distributed relay power allocation scheme among the users in [26].

Having observed the non-convexity nature of our problem, and to model the selfish nature of the sources, we have proposed a game theoretical solution of it in two steps[3]. Two Stackelberg games operating in two consecutive steps have been designed in order to solve this problem. For connecting these two games, we have introduced one centralized entity, which is aware of the complete channel state information (CSI) in the network. In the first step, this entity plays as a buyer level game for buying power from the source nodes. The sources are non-cooperative among themselves in terms of selling their power to the centralized entity. On the other hand, the centralized entity is willing to maximize its own utility by deciding optimal amount of power when the prices are announced by the sources. In the second step, on behalf of the relay node, the centralized entity plays as a seller level game for selling/disseminating relay power to the sources. In turn, the sources themselves compete for the relay power given the price set by the centralized entity. As there is total power constraint in the system, at the beginning of the game, the centralized entity applies some intelligent measure in order to determine how much power is dedicated for the transmit operation of the sources and how much is for their relay operation[4]. Although there are some works on source power control [24], and on relay power allocation [25] for a multi-source single-relay network, there is no complete solution for joint sources and relay power allocation for such network in the literature, which we believe to fill up. Moreover, the work in [24] considers that the source nodes transmit simultaneously in the same frequency/time domain which results in interference among them, and not a practical notion of a relayed system. Through the extensive simulation while showing the results from different stand points, we have justified that game theoretical solution achieves comparable performance with the centralized optimal one.

The rest of the paper is organized as follows. System model with its detailed mechanism is explained in Section II. The centralized and game theoretical solutions of defined problem are given in Section III and Section IV, respectively. We evaluate the performance of both solutions in Sections III-C and IV-D, respectively. The last one also compares the perfor-

[2]For the sake of simplicity without losing the generality, we consider, the sources are served by a single relay node.

[3]Since the joint sources and relay power allocation of this network is a non-convex problem, it is impossible to capture this problem with a single game.

[4]Except the centralized entity, no other nodes need to know the CSI of other nodes. At the beginning of these games, the centralized entity uses the complete CSI to decide the amount of power for the transmit and relay operations in aggregate level. However, two games decide the amount of power for the individual source. In each game, the interaction between the sources and the centralized entity is completely distributed.

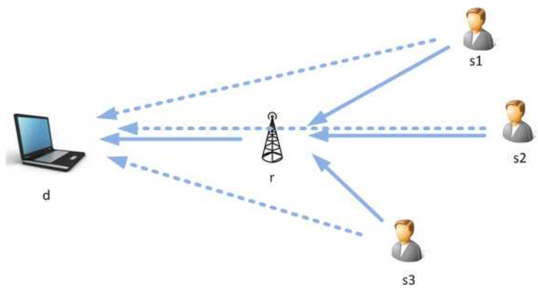


Fig. 1. System model.

mance of both solutions, and finally Section V concludes the paper.

II. System Model and Problem Formulation

In this section, we consider a system, where there exists an access point, one relay node, and N number of source nodes which need help from the relay to get their packets transmitted to the access point. The edge nodes essentially act as servers which have some special applications, and the destination node needs to get the content of that application on time. For example, application could be some video which needs to be displayed on the destination. the relay node amplifies the received signal from the source nodes, and then forwards towards the destination, as depicted in Figure 1. Although the system has one destination, proposed solution can easily be adapted to a multi-destination network.[5] Transmit channels of the sources and relay are orthogonal, either in time or frequency domain. We assume a block-fading channel (or quasi-static) model: the channels remain invariant over a time slot whose duration is less than the coherence time of the channels. Denote the channel gain between source s_i and destination d is $G_{s_i,d}$; the channel gain between source s_i and relay r as $G_{s_i,r}$, and the channel gain between relay r and destination d is $G_{r,d}$.

The entire transmission operation using the AF relay consists of two phases (i.e., time slots). At each phase, the sources or relay use orthogonal frequency channel for multiple transmissions. At the first phase, source s_i broadcasts its information to both destination d and relay node r. The received signals $y_{s_i,d}$ and $y_{s_i,r}$ at destination d and relay r can be expressed as

$$y_{s_i,d} = \sqrt{E_{s_i} G_{s_i,d}} x_{s_i} + \eta_{s_i,d} \text{ and } y_{s_i,r} = \sqrt{E_{s_i} G_{s_i,r}} x_{s_i} + \eta_{s_i,r}, \quad (1)$$

where E_{s_i} represents the transmit power at node s_i, x_{s_i} is the broadcast information symbol with unit energy from source s_i to nodes d and r. $\eta_{s_i,d}$ and $\eta_{s_i,r}$ are the additive noises received at destination d and relay r, respectively. In the second step, the relay amplifies its received signal and forwards it to destination d. Denote the power the relay uses to help source s_i is E_{r_i}. The signal received at destination d for source s_i can be shown

$$y_{r_i,d} = \frac{\sqrt{E_{r_i} G_{r,d}} \left(\sqrt{E_{s_i} G_{s_i,r}} x_{s_i} + \eta_{s_i,r} \right)}{\sqrt{E_{s_i} G_{s_i,r} + \sigma^2}} + \eta_{r_i,d}. \quad (2)$$

$\eta_{r_i,d}$ is the received noise from relay r to destination d (for source s_i). Without loss of generality, we assume that the noise power is the same additive white gaussian noise for all links, denoted by σ^2. After maximum ratio combining of both the direct and relay path, the effective received SNR for source s_i's transmission can be given by

$$\Gamma_{s_i,r,d} = \frac{E_{s_i} G_{s_i,d}}{\sigma^2} + \frac{E_{s_i} G_{s_i,r} E_{r_i} G_{r,d}}{\sigma^2 \left(E_{s_i} G_{s_i,r} + E_{r_i} G_{r,d} + \sigma^2 \right)}. \quad (3)$$

If the set consisting of the source nodes is $L_s = \{s_1, s_2, \cdots, s_N\}$, total capacity achieved by the system can be given by

$$R_{s,r,d} = \gamma_L W \sum_{s_i \in L_s} log_2 \left(1 + \Gamma_{s_i,r,d} \right). \quad (4)$$

Because of the orthogonal transmissions, $\gamma_L = 1/(2N)$ and W is the aggregate bandwidth in the system. Since W and γ_L are constants, we skip these terms in the subsequent discussion.

Our goal is to allocate power optimally among the sources and relay so that the system capacity is maximized. Likewise traditional network resource optimization problems, there are constraints on the sources and relay power. Moreover, in order to mitigate the interference imposed on another network due to the transmission operations in this network, there is a total power constraint, meaning total power allocated to the sources and relay node cannot exceed E^{max}.[6] For the sake of simplicity, we have converted the maximization problem into the minimization one by introducing minus sign in front of the objective function, i.e, $R_{s,r,d}$.

$$\begin{aligned} \underset{\{E_{s_i}\}_{s_i \in L_s}, \{E_{r_i}\}_{s_i \in L_s}}{\arg\min} \quad & \Pi \left(\{E_{s_i}\}_{s_i \in L_s}, \{E_{r_i}\}_{s_i \in L_s}, \sigma^2 \right) \quad (5) \\ \text{where} \quad & E_{s_i} \le E_s^{max}, \quad s_i \in L_s, \\ & \sum_{s_i \in L_s} E_{r_i} \le E_r^{max}, \\ & \sum_{s_i \in L_s} E_{s_i} + \sum_{s_i \in L_s} E_{r_i} \le E^{max}, \\ & \{E_{s_i}\}_{s_i \in L_s} \ge 0, \ \{E_{r_i}\}_{s_i \in L_s} \ge 0. \end{aligned}$$

The aforementioned optimization problem is valid if and only if $\sum_{s_i \in L_s} E_{s_i} + E_r^{max} > E^{max}$, and $\sum_{s_i \in L_s} E_{s_i}^{max} > E^{max}$.

[5]Centralized solution is general enough to incorporate multiple relays even when one relay serves multiple sources and one source is served by multiple relays. If one relay serves only one source and one source is served by only one relay, by trivially modifying the formulation, we should be able to reach to an elegant game theoretical solution. Similar solution method for the relay power allocation game has been given in [25]. On the other hand, if each source is served by multiple relays, and each relay serves multiple sources, we would need to formulate the games in a different manner and it will be definitely complex.

[6]Our network can be the forbidden direction of some MIMO based [27] or cognitive radio [28] networks. To regulate the interference of those networks, they can impose this total transmit power constraint on our network.

$$\Pi\left(\{E_{s_i}\}_{s_i\in L_s},\{E_{r_i}\}_{s_i\in L_s},\sigma^2\right)=\prod_{s_i\in L_s}\frac{\sigma^2(\sigma^2+E_{s_i}G_{s_i,r}+E_{r_i}G_{r,d})}{(\sigma^2+E_{s_i}G_{s_i,d})(\sigma^2+E_{s_i}G_{s_i,r}+E_{r_i}G_{r,d})+E_{s_i}G_{s_i,r}E_{r_i}G_{r,d}}$$

III. Centralized Solution

The problem in Equation 5 is not convex due to the non-convexity property of the objective function. This statement can be proved very easily by the help of special type of convex optimization formulation, i.e., GP [29], [30]. A GP is a type of mathematical optimization problem characterized by the objective and constraint functions that has a special form. It focuses on monomial and posynomial functions. A monomial is a function, $h : R^n \to R$, where the domain contains all real vectors with non-negative components, $h(x) = c{x_1}^{a_1}{x_2}^{a_2}\cdots{x_n}^{a_n}$. A posynomial is a sum of monomials, $f(x) = \sum_k c_k {x_1}^{a_{1k}}{x_2}^{a_{2k}}\cdots{x_n}^{a_{nk}}$. GP is an optimization problem with the form

$$minimize\ \ f_0(x)\ \ s.t.\ \ f_i(x)\le 1,\ h_j(x)=1,$$

where f_0 and f_i are posynomials, and h_j are monomials. This problem in the above form is not convex. However, with a change of variables: $y_i = log x_i$ and $b_{ik} = log c_{ik}$, we can transform it into convex form given the assumption that the logarithm of a sum of exponentials is a convex function.

As mentioned, the objective function is the ratio of two posynomials which cannot be solved by GP. There are ways to transform such type of problem to GP form, i.e., single condensation method, double condensation method [30]. We have used single condensation method which requires to approximate the denominator of the objective function by some monomial term. We denote the denominator by $F\left(\{E_{s_i}\}_{s_i\in L_s},\{E_{r_i}\}_{s_i\in L_s}\right)$, and the monomial is given by

$$\begin{aligned}&\prod_{s_i\in L_s}(\sigma^2+E_{s_i}G_{s_i,d})(\sigma^2+E_{s_i}G_{s_i,r}+E_{r_i}G_{r,d})\\&+E_{s_i}G_{s_i,r}E_{r_i}G_{r,d}\\&\approx\lambda\prod_{s_i\in L_s}E_{s_i}^{a_i}E_{r_i}^{b_i},\end{aligned}$$

where $a_i = \frac{E_{s_i}}{F\left(\{E_{s_i}\}_{s_i\in L_s},\{E_{r_i}\}_{s_i\in L_s}\right)}\frac{\partial F\left(\{E_{s_i}\}_{s_i\in L_s},\{E_{r_i}\}_{s_i\in L_s}\right)}{\partial E_{s_i}}$, $b_i = \frac{E_{r_i}}{F\left(\{E_{s_i}\}_{s_i\in L_s},\{E_{r_i}\}_{s_i\in L_s}\right)}\frac{\partial F\left(\{E_{s_i}\}_{s_i\in L_s},\{E_{r_i}\}_{s_i\in L_s}\right)}{\partial E_{r_i}}$, and $\lambda = \frac{F\left(\{E_{s_i}\}_{s_i\in L_s},\{E_{r_i}\}_{s_i\in L_s}\right)}{\prod_{s_i\in L_s}E_{s_i}^{a_i}E_{r_i}^{b_i}}$.

The derivations are below

$$\begin{aligned}&\frac{\partial F\left(\{E_{s_i}\}_{s_i\in L_s},\{E_{r_i}\}_{s_i\in L_s}\right)}{\partial E_{s_i}}=\\&[G_{s_i,d}(\sigma^2+E_{s_i}G_{s_i,r}+E_{r_i}G_{r,d})+(\sigma^2+E_{s_i}G_{s_i,d})G_{s_i,r}\\&+G_{s_i,r}E_{r_i}G_{r,d}]\\&\prod_{s_j\in L_s,s_j\neq s_i}[(\sigma^2+E_{s_j}G_{s_j,d})(\sigma^2+E_{s_j}G_{s_j,r}\\&+E_{r_j}G_{r,d})+E_{s_j}G_{s_j,r}E_{r_j}G_{r,d}],\\&\frac{\partial F\left(\{E_{s_i}\}_{s_i\in L_s},\{E_{r_i}\}_{s_i\in L_s}\right)}{\partial E_{r_i}}=\\&\left[G_{r,d}(\sigma^2+E_{s_i}G_{s_i,d})+E_{s_i}G_{s_i,r}G_{r,d}\right]\\&\prod_{s_j\in L_s,s_j\neq s_i}[(\sigma^2+E_{s_j}G_{s_j,d})(\sigma^2+E_{s_j}G_{s_j,r}\\&+E_{r_j}G_{r,d})+E_{s_j}G_{s_j,r}E_{r_j}G_{r,d}].\end{aligned}$$

Finally, the overall procedure for the joint sources and relay power allocation is given as follows.

1) Set the initial value of power $\mathbf{E}^{(0)} := \left[E_{s_1}^{(0)},\cdots,E_{s_N}^{(0)},E_{r_1}^{(0)},\cdots,E_{r_N}^{(0)}\right]$, $n := 1$.
2) Determine $\left[a_1^{(n)},\cdots,a_N^{(n)}\right]$, $\left[b_1^{(n)},\cdots,b_N^{(n)}\right]$ and $\lambda^{(n)}$.
3) Solve the optimization problem with the help of GP.
4) Denote the optimal power allocation in the nth round as $\mathbf{E}^{(n)}$.
5) If $||\mathbf{E}^{(n)}-\mathbf{E}^{(n-1)}||\le\epsilon$, where ϵ is a pre-defined threshold, the enumerations stop; otherwise, $n := n+1$ and reiterate from step 2 to 5.

Notice that the above procedure updates $2N$ principle variables in every iteration. And, each iteration needs to update $2N+1$ number of intermediate variable to assist updating the principle variables. In order to simplify this procedure, we can consider, the system has only one source denoted by s^*. Node s^* is the representative of all sources. The gain between node s^* and relay r is weighted average of the gains between the sources and relay. In the similar manner, the gain between node s^* and destination d is determined. After this transformation, the objective function is still the ratio of two posynomials. In order to cast it to GP, we can approximate the denominator (denoted by $H(E_{s^*},E_r)$) of it by monomial

$$\begin{aligned}&(\sigma^2+E_{s^*}G_{s^*,d})(\sigma^2+E_{s^*}G_{s^*,r}+E_rG_{r,d})\\&+E_{s^*}G_{s^*,r}E_rG_{r,d}\\&\approx\mu E_{s^*}^cE_r^d,\end{aligned}$$

where $c = \frac{E_{s^*}}{H(E_{s^*},E_r)}\frac{\partial H(E_{s^*},E_r)}{\partial E_{s^*}}$, $d = \frac{E_r}{H(E_{s^*},E_r)}\frac{\partial H(E_{s^*},E_r)}{\partial E_r}$, and $\mu = \frac{H(E_{s^*},E_r)}{E_{s^*}^cE_r^d}$.

Furthermore,

$$\begin{aligned}\frac{\partial H}{\partial E_{s^*}}&=G_{s^*,d}(\sigma^2+E_{s^*}G_{s^*,r}+E_rG_{r,d})\\&+(\sigma^2+E_{s^*}G_{s^*,d})G_{s^*,r}+G_{s^*,r}E_rG_{r,d},\\\frac{\partial H}{\partial E_{s^*}}&=(\sigma^2+E_{s^*}G_{s^*,d})G_{r,d}+E_{s^*}G_{s^*,r}G_{r,d}.\end{aligned}$$

The iterative procedure for obtaining optimal E_{s^*} and E_r follows the same procedure mentioned above. However, it requires to update 2 principle variables, and 3 auxiliary variables in each iteration in order to achieve convergence. We have two algorithms, greedy and fair [31] in the next subsection (neater version is in our technical report [32]) for distributing power $E_{s^*}^*$ among the original sources, and the following subsection is for disseminating relay power E_r^* among them.

A. Suboptimal Source Power Allocation

From the optimal solution, we have observed that the sources with better channel condition obtain more power compared to others. Since each source has individual power constraint and this power is moderately lower than the total allowable power for all sources, we can propose a greedy power allocation for the source nodes given the total allowable for them is $E_{s^*}^*$. If the direct link SNR of a source is better than its relayed link SNR, it is likely, that source obtains 0 relay power. Therefore, it is rational to distribute $E_{s^*}^*$ among the sources taking the direct link SNR into account. We sort $G_{s_i,d}, s_i \in L_s$ in decreasing order and allocate maximum individual power to each sorted source until there is no left over power. The drawback of this approach is, the sources with worse channel may starve and may never get chance to transmit as they are assigned zero power. This reminds us one important issue which is called fairness. In order to tackle fairness, we have proposed a fair algorithm which disseminates $E_{s^*}^*$ among the original sources in a fair manner [32].

B. Suboptimal Relay Power Allocation

In order to subdivide relay power E_r^* among the sources, we have adopted water filling approach, and the resultant formulated problem is given by

$$\underset{\sum_{s_i \in L_s} E_{r_i} = E_r^*}{\arg\max} \sum_{s_i \in L_s} log_2 \left(1 + \frac{E_{s_i} G_{s_i,r} E_{r_i} G_{r,d}}{\sigma^2 (E_{s_i} G_{s_i,r} + E_{r_i} G_{r,d} + \sigma^2)}\right). \tag{6}$$

By invoking the lagrange multiplier μ for the total relay power constraint of the problem in Equation 6, we obtain the lagrangian $\sum_{s_i \in L_s} log_2 \left(1 + \frac{E_{s_i} G_{s_i,r} E_{r_i} G_{r,d}}{\sigma^2 (E_{s_i} G_{s_i,r} + E_{r_i} G_{r,d} + \sigma^2)}\right) + \mu(E_r^* - \sum_{s_i \in L_s} E_{r_i})$. Following the K.K.T condition, we take the differentiation of the lagrangian with respect to E_{r_i}, and we obtain the following equation

$$\begin{aligned} &\mu\sigma^2(\sigma^2 + E_{s_i} G_{s_i,r} + E_{r_i} G_{r,d})^2 + \mu E_{s_i} G_{s_i,r} E_{r_i}^2 G_{r,d}^2 + \\ &\quad \mu(\sigma^2 + E_{s_i} G_{s_i,r}) E_{s_i} G_{s_i,r} E_{r_i} G_{r,d} - \\ &\quad (\sigma^2 + E_{s_i} G_{s_i,r}) E_{s_i} G_{s_i,r} G_{r,d} = 0. \end{aligned} \tag{7}$$

After simplifying, resultant E_{r_i} is $-\frac{E_{s_i} G_{s_i,r} + 2\sigma^2}{2G_{r,d}} + \frac{1}{2G_{r,d}} \sqrt{E_{s_i}^2 G_{s_i,r}^2 + \frac{4E_{s_i} G_{s_i,r} G_{r,d}}{\mu}}$. Substituting $E_{r_i}, s_i \in L_s$ back into the equation $\sum_{s_i \in L_s} E_{r_i} - E_r^* = 0$, we obtain the upper bound of μ which is $\frac{\sum_{s_i \in L_s} \sqrt{4E_{s_i} G_{s_i,r} G_{r,d}}}{2G_{r,d} E_r^* + \sum_{s_i \in L_s} (2\sigma^2 + E_{s_i} G_{s_i,r})}$. As the lower bound of μ is 0, we apply a bisection search between these two bounds in order to obtain optimal μ. Replacing optimal μ in E_{r_i}, finally we obtain optimal E_{r_i}, i.e., $E_{r_i}^*, s_i \in L_s$.

C. Performance Evaluation

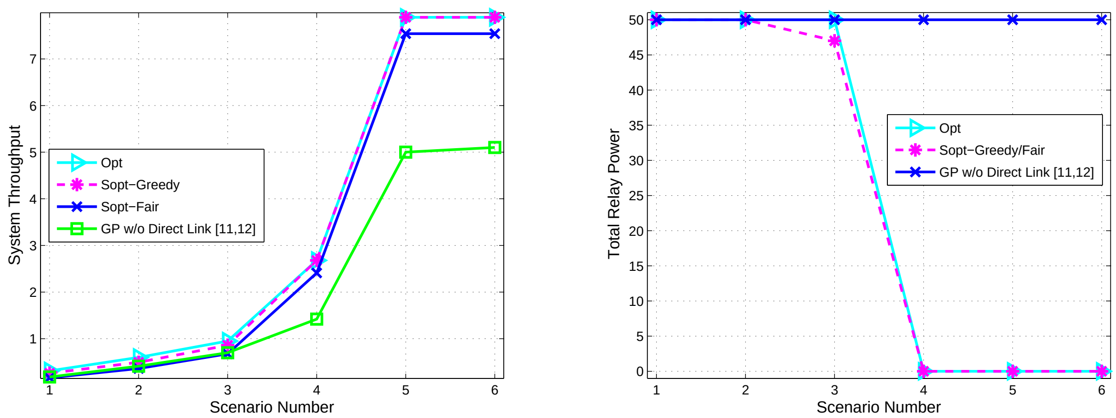
In this subsection, we will evaluate the performance of our proposed solutions. Section III-C1 is for the methodology we have adopted to evaluate the performance, and the following one presents the results while comparing with the approach proposed in [11] and [12].

1) Simulation Methodology: We presume a simple network where there are 5 source nodes, one relay, and one destination node. Maximum power of individual source is $E_s^{max} = 30$ mW, and that of relay node is $E_r^{max} = 50$ mW, total available power in the system is $E^{max} = 120$ mW. Noise variance σ^2 has been set as 1. Channel between two nodes suffers from the shadowing, and Rayleigh fading effects. We take the same channel model, and the similar values of its parameters as mentioned in [33]. Moreover, we assume, each channel has a unit capacity. One of the major assumptions of the works [11], [12] is channel condition between the source and destination is always worse than that between the source and relay. However, that not necessarily happens in practice, and for the counter scenario, their model fails to provide optimal solution. In order to fix the model up, we have considered the SNR due to the direct link in our formulation, and the resultant solution is optimal which is able to give better performance even when the direct link's channel is better than that of relayed link. In order to evaluate the performance of our solution, we have selected 6 different scenarios, each of which has 5 distinct source nodes. In the evaluation part, we have denoted each scenario by Scenario Number. The positions of all nodes are in the following coordinates.

- Destination: (0,12).
- Relay: (0,6).
- Sources: X-coordinates are fixed at $\{-1, -2, -3, -4, -5\}$ for all scenarios. If Scenario Number is denoted by n_s, their Y-coordinates is $2n_s, n_s \in \{1, 2, 3, 4, 5\}$, and $2n_s, n_s = 6$.

All the results we have presented here is the average of 100 simulation runs.

2) Simulation Results: Figure 2(a) presents the total average system throughput with respect to 6 different positions of the source nodes. Notice that, the positions of the relay and destination are fixed, we are varying the positions of 5 sources towards the destination. As the sources move to the destination, resultant channel gain becomes better for them, hence gradually their throughput get improved. For the 5th and 6th scenarios, absolute distance between the sources and destination is very close, however they are on the different sides of the destination. Since their absolute distances are very close, resultant throughput are very close for these two scenarios. Now if we intuitively compare all approaches, for scenarios 1 and 2, direct link's channel condition is worse than the relayed link, resultant outcome proposed by [11], [12] does not deviate much from our optimal solution. For scenario 3, direct link's channel quality is very close to the relayed link and from this scenario, procedure without considering the direct link SNR starts to differ from our optimal approach. And, for the scenarios 4, 5 and 6, relayed link's channel is worse than that of direct link. For these scenarios, Figure 2(b) shows that allocated power for the relay is 0, and total allowable power is distributed among the sources considering their channel gain towards the destination. However, the technique without considering the direct link SNR always assigns full power to the relay no matter the relayed link is worse or better than the direct link. Since our suboptimal approach for allocating power to the source and relay is based on somewhat weighted averaging of all gains, for scenario 3, relayed power by this approach is little less than the optimal one. For rest of the other scenarios, the suboptimal approach confers to the optimal one. We have noticed that, once we obtain total allowable power for the sources, we can distribute this power among the sources by 2 techniques, i.e., greedy and fair algorithms. From Figure 2(a), the greedy one has very



(a) System throughput.

(b) Total relay power.

Fig. 2. Comparison between our centralized solutions and others.

close performance comparing with the optimal one. Greedy algorithm gives privilege to the sources with better channel condition, and makes starvation for others. For being fair to the sources with worse channel condition, the resultant system throughput by the fair algorithm deteriorates compared to the other one. Even for scenarios 1, 2, and 3, achieved throughput by this algorithm is worse compared to the GP solution without considering the direct link SNR, however for the other scenarios, it outperforms.

IV. Game Theoretical Solution

Since joint sources and their relay power allocation is a non-convex problem, by employing a single cooperative or non-cooperative game, it is not possible to assign transmit and relay power of a source jointly.[7] Hence, we plan to consider sources and their relay power allocation as two distinct problems. First, putting some assumption on relay power distribution, the sources will decide their optimal power independently. In the next step, in order to improve the performance of the network further, optimal relay power distribution for the sources is decided. However, in order to connect these two problems, we need an entity in the network whom we call ”Network”. Primarily, Network is aware of the complete CSI of the sources. It is also aware of the individual source, relay, and total system power constraints.

In the centralized solution of this problem, all nodes in the network work selflessly to maximize the network capacity. However, in the real world, selfish nodes may not have a common goal or belong to a common authority. Therefore, a reimbursement mechanism is required such that the sources can earn some benefits while contributing power towards maximizing the capacity of the network. Since there is a restriction on the total power taken from the sources, the authority node ”Network” is likely to choose the most beneficiary sources whose contribution are better compared to others. Following the characteristics of the sources and Network, a Stackelberg game is appropriate to model this problem while considering their joint benefits. Network plays the buyer level game since it aims to achieve the best performance from power given by the sources while giving the least possible reimbursement to them. On the other hand, each source plays the seller level game, in which it aims to earn the payment that not only covers its power cost but also gains as much extra profit as possible.

In order to improve the performance of the network further, Network wants the optimal distribution of relay power among the sources. However, due to the selfish nature, the sources may want to have as much power as possible to maximize their own SNR. Furthermore, relay power is limited, if one source takes the whole power, it results in starvation for others. Given the total relay power budget, the objective of the sources is non-cooperative. To discourage the sources having as much power as they want, pricing is an effective mechanism. If price is imposed on the relay power, they will ask for the amount of power which maximizes their individual utility in order to keep the balance between power and price. In order to model such behaviors of the sources for their relay power, we have introduced second Stackelberg game. In this game, Network is the entity which will decide the price of unit relay power. Given the unit power price, the sources demand some power which maximizes their own utility or benefit.

In order to ensure the correct and unique convergence of the first game, it is required that Network knows how much total power is dedicated for the transmit operation of the sources. Rest of the power subtracted from total network power is for the relay operation. Following the formula given below, total relay power of the sources is determined. For the first game, if the relay power is not 0, Network assumes that available relay power is subdivided equally among the sources which need the help of relay.

$$s_i \in \begin{cases} L_s^1 & \text{if } G_{s_i,d} < \frac{G_{s_i,r}G_{r,d}}{(G_{s_i,r}+G_{r,d})} \\ L_s^2 & \text{Otherwise} \end{cases}, \quad (8)$$

[7]If we would formulate the problem with only one game, we would not be able to design any cost function which is convex with respect to both transmit and relay power of each source. However, this is the fundamental requirement of formulating a game: the cost function has to be convex with respect to its variable (transmit power, relay power or price).

$$E'_r = \begin{cases} 0 & \text{if } L_s^1 \text{ is empty} \\ E_r^{max} & \text{Otherwise} \end{cases}. \tag{9}$$

If L_s^1 is non-empty, $E'_{r_i} = E'_r/|L_s^1|, s_i \in L_s^1$. $|L_s^1|$ is the length of set L_s^1.

A. Game Theoretical Source Power Allocation

This game is to identify the number of sources served by Network, and how much power to be disseminated among them given that total allowable power is E'_s and the maximum allowable power for source s_i is $E_{s_i}^{max}$. We formulate the game below.

1) Network/Buyer: Network can be configured as a buyer, and it aims to attain most benefits at the least possible payment. So, the utility function of Network can be defined as

$$U_n^s = aR'_{s,r,d} - P. \tag{10}$$

$R'_{s,r,d}$ is the aggregate SNR for all sources $\sum_{s_i \in L_s} \left[\frac{E_{s_i} G_{s_i,d}}{\sigma^2} + \frac{E_{s_i} G_{s_i,r} E'_{r_i} G_{r,d}}{\sigma^2 \left(\sigma^2 + E_{s_i} G_{s_i,r} + E'_{r_i} G_{r,d} \right)} \right]$[8]. a denotes the gain per unit of SNR achievement. P refers to the total payment paid by Network to the source nodes, $P = \sum_{s_i \in L_s} p_i E_{s_i}$. p_i is the price per unit of power selling from source s_i to Network, and E_{s_i} represents the amount of power Network would like to buy from source s_i when the prices are advertised by them. Suppose, the sources preferred by Network consists a set, represented by $L_s = \{s_1, s_2, \cdots, s_N\}$, then the optimization problem for Network can be given by

$$\max_{\{E_{s_i}\}_{s_i \in L_s}} U_n^s = aR'_{s,r,d} - P, \ s.t. \ E_{s_i} \geq 0, s_i \in L_s. \tag{11}$$

2) Sellers/Sources: Each source s_i can be assumed as a seller, and wants to earn the payment which not only covers its cost due to the contribution towards the overall system performance but also achieve as much extra profit as possible. Introducing one parameter $c_i, s_i \in L_s$, the cost of unit power which is a reflection of the sources' perception about whether they can actually get profit, the utility function of source s_i can be defined as

$$U_{s_i} = p_i E_{s_i} - c_i E_{s_i} = (p_i - c_i) E_{s_i}. \tag{12}$$

E_{s_i} is source s_i's outcome by optimizing U_n^s illustrated in Equation 11. It is important to note that optimal p_i depends not only on source s_i's channel condition towards the destination and relay, but also on the prices of its partner sources. So, in each round of the game, if one source asks higher price than what Network anticipates about it, after jointly comparing prices of all sources and their potential contribution to the overall performance, Network will buy less power from that source or even overlooks that source. On the other hand, if the price is too low, the profit obtained by Equation 12 will be uselessly low. Therefore, there is a tradeoff in setting the price. The optimization problem for source s_i or the game of seller s_i is

$$\max_{\{p_i\} > 0} U_{s_i} = (p_i - c_i) E_{s_i}. \tag{13}$$

[8]Since log is a concave function, for the sake of simplicity, we have ignored log in the formulation.

The fundamental purpose of above game is to decide the optimal price p_i for source s_i to maximize its profit U_{s_i}; the actual number of sources who will finally get selected by Network, and the corresponding power consumption $E_{s_i}, s_i \in L_s$ to maximize U_n^s. The following subsection shows the outcome of the game in detail.

1) Analysis of Source Power Allocation Game: We first examine the proposed game in detail, and obtain the closed form outcome of the game. Based on the outcome, we establish that obtained solution is the unique equilibrium called SE of the game. Then, from the properties of the game, in the following subsection, we outline a distributed price update function, and the interaction mechanism of the entities.

(i) Network/Buyer Level Analysis: Before taking decision about the amount of power it will buy from the sources, it is crucial to know the prices asked by them. Taking the partial derivative of U_n^s with respect to E_{s_i}, from Equation 11, we obtain

$$\frac{\partial U_n^s}{\partial E_{s_i}} = a \frac{\partial R'_{s,r,d}}{\partial E_{s_i}} - p_i, \quad s_i \in L_s.$$

For U_n^s being strictly concave with respect to $\{E_{s_i}\}_{s_i \in L_s}$, condition $\frac{\partial U_n^s}{\partial E_{s_i}} > 0, s_i \in L_s$ should be satisfied. This means, p_i of the source should satisfy $p_i < a \frac{\partial R'_{s,r,d}}{\partial E_{s_i}}$.

At the beginning, source s_i has knowledge of its cost c_i, which is the bare expense required for its contribution towards the overall system performance, however is unaware of the prices of other sources. In order to get its utility U_{s_i} non-negative, at the first iteration, source s_i sets its price $p_i = c_i$. If under this lowest initial price, Network is reluctant to buy power from that source, s_i will not exist in the game anymore.

Consequently, before initiating the game, Network applies some intelligence in order to sort out the sources which it will play the game with. At first, Network tentatively set $E_{s_i} = 0, s_i \in L_s$, if for some source, say s_i, it holds that $c_i \geq \left(a \frac{\partial R'_{s,r,d}}{\partial E_{s_i}} \right) |_{E_{s_i}=0}$, s_i will be disregarded by Network.

For the remaining sources in set L_s, by the first order optimality condition, the following equation must be satisfied at the optimal point.

$$\frac{\partial U_n^s}{\partial E_{s_i}} = 0, \quad s_i \in L_s. \tag{14}$$

Solving Equation 14, we can get its solution $\{E_{s_i}^*\}_{s_i \in L_s}$ shown in *Lemma 5.1*.

Lemma 5.1: The optimal power consumption by source $s_i \in L_s$ depends on the contents of the sets L_s^1 and L_s^2 which were determined by Network at the beginning of the game.

Case 1: L_s^1 is non-empty, and L_s^2 is empty.

$$E_{s_i}^*(p_i) = \begin{cases} 0 & p_i \geq p_i^{ub} \\ \sqrt{\frac{aB_{s_i}E'_{r_i}G_{s_i,r}G_{r,d}}{G_{s_i,r}^2\sigma^2(p_i - \frac{aG_{s_i,d}}{\sigma^2})}} - \frac{B_{s_i}}{G_{s_i,r}} & p_i^{lb} < p_i < p_i^{ub} \\ E_s^{max} & \frac{aG_{s_i,d}}{\sigma^2} < p_i \leq p_i^{lb} \\ \text{Undefined} & p_i \leq \frac{aG_{s_i,d}}{\sigma^2} \end{cases},$$

where $B_{s_i} = E'_{r_i}G_{r,d} + \sigma^2$, $p_i^{lb} = \frac{aB_{s_i}E'_{r_i}G_{s_i,r}G_{r,d}}{\sigma^2\left(E_s^{max}G_{s_i,r}+B_{s_i}\right)^2} + \frac{aG_{s_i,d}}{\sigma^2}$ and $p_i^{ub} = \frac{aE'_{r_i}G_{s_i,r}G_{r,d}}{B_{s_i}\sigma^2} + \frac{aG_{s_i,d}}{\sigma^2}$.

Case 2: L_s^1 is empty, and L_s^2 is non-empty.

Optimal power $E_{s_i}^*, s_i{\in}L_s^2$ of this case is obtained by solving the following optimization problem. In order to contribute non-negation utility towards the overall performance, price lower and upper bounds of the sources $s_i{\in}L_s^2$ are $p_i^{lb} = 0$ and $p_i^{ub} = \frac{aG_{s_i,d}}{\sigma^2}$, respectively.

$$\underset{\{E_{s_i}\}_{s_i\in L_s^2}}{\arg\max} \sum_{s_i\in L_s^2} \frac{aE_{s_i}G_{s_i,d}}{\sigma^2} - \sum_{s_i\in L_s^2} p_i E_{s_i}. \tag{15}$$

From the solution of this optimization problem, we observe that full power E_s^{max} is assigned to the sources following the descending order of the metric $\frac{aG_{s_i,d}}{\sigma^2} - p_i, s_i{\in}L_s^2$ until total allowable power E'_s runs out.

Case 3: Both L_s^1 and L_s^2 are non-empty.

This case is the hybrid scenario of above Case 1 and Case 2. Similar to the solution method of Case 2, we define metric $A_{s_i}, s_i{\in}L_s^1\bigcup L_s^2$ for the sources in this case.[9] The physical meaning of A_{s_i} is the profit of source s_i for the given price p_i.

$$A_{s_i} = \begin{cases} aG_{s_i,d} - p_i & s_i{\in}L_s^1 \\ aG_{s_i,d} + \frac{aG_{s_i,r}E'_{r_i}G_{r,d}}{(G_{s_i,r}+E'_{r_i}G_{r,d})} - p_i & s_i{\in}L_s^2 \end{cases}. \tag{16}$$

We sort $A_{s_i}, s_i{\in}L_s^1\bigcup L_s^2$ in the descending order. Then, following the order, we assign power among the sources in the following manner until the total allowable power E'_s runs out.

$$E_{s_i}^*(p_i) = \begin{cases} E_s^{max} & s_i{\in}L_s^1 \\ \min\left(E_s^{max}, E_{s_i^*}(p_i)\right) & s_i{\in}L_s^2 \end{cases}. \tag{17}$$

The last allocated source may not get the full power following the formula defined above. It obtains the left over power from E'_s after assigning among the sources which have relatively better value for metric A_{s_i}.

After Network announces optimal power $(E_{s_i}^*)^+$ to the sources $s_i \in L_s$, they will gradually increase the prices $p_i, s_i \in L_s$ to get possibly more benefit round by round. This will lead Network to buy decreasing amount of E_{s_i}. In order to earn maximal utility instead of being disregarded by Network, source s_i also needs to ask proper price.

(ii) Source/Seller Level Analysis: Replacing the output from *Lemma 5.1* into Equation 13, we have

$$\max_{\{E_{s_i}\}>0} U_{s_i} = (p_i - c_i)E_{s_i}^*(p_i). \tag{18}$$

Notice that this game among the sources is non-cooperative, and there exists a tradeoff between the price p_i and utility U_{s_i} of source s_i. There is an optimal price to set for all sources in order to avoid being disregarded by Network, and maximize its own utility. Optimal price depends upon the source's channel condition as well as its own price. Network only chooses the most beneficial sources for meeting up its own interest. Following the first order optimality condition, it results in

$$\frac{\partial U_{s_i}}{\partial p_i} = E_{s_i}^*(p_i) + (p_i - c_i)\frac{\partial E_{s_i}^*(p_i)}{\partial p_i} = 0, \quad s_i{\in}L_s. \tag{19}$$

Solving Equation 19, we obtain optimal price $p_i^* = p_i^*(G_{s_i,r}, G_{r,d}, G_{s_i,d}, \sigma^2), \forall s_i{\in}L_s$.

The solutions in *Lemma 5.1* and p_i^* are an equilibrium of each round in this game. The properties and convergence procedure of the equilibrium are illustrated in the following subsection.

2) Properties of Source Power Allocation Game: In this subsection, we prove the existence of a SE in this game, and prove the optimality of the SE by the following properties.

Property 1: The utility function of Network U_n^s is concave with respect to $\{E_{s_i}\}_{s_i\in L_s}$, where $E_{s_i} \geq 0$, $\forall s_i{\in}L_s$, when the prices of the source nodes are constant.

Proof: Taking the second order derivatives of U_n^s, we get

$$\frac{\partial^2 U_n^s}{\partial E_{s_i}^2} = -2a\frac{(E'_{r_i}G_{r,d} + \sigma^2)E'_{r_i}G_{s_i,r}^2G_{r,d}}{(E_{s_i}G_{s_i,r} + E'_{r_i}G_{r,d} + \sigma^2)^3}, \forall s_i{\in}L_s^1,$$

and

$$\frac{\partial^2 U_n^s}{\partial E_{s_i}\partial E_{s_j}} = 0, \quad s_i, s_j{\in}L_s^1.$$

From the above 2 equations, it is pretty much straightforward that $\frac{\partial^2 U_n^s}{\partial E_{s_i}^2}\frac{\partial^2 U_n^s}{\partial E_{s_j}^2} - (\frac{\partial^2 U_n^s}{\partial E_{s_i}\partial E_{s_j}})^2 > 0, \forall s_i{\neq}s_j$. Furthermore, U_n^s is continuous with respect to E_{s_i}. So, when $E_{s_i} \geq 0$, U_n^s is strictly concave in $\{E_{s_i}\}_{s_i\in L_s^1}$ and jointly concave as well. For Case 2, when L_s^1 is empty, U_n^s is non-differentiable with respect to $\{E_{s_i}\}_{s_i\in L_s^2}$, and the second derivative of U_n^s with respect to any $s_i{\in}L_s^2$ is 0. This concludes the concavity property of U_n^s with respect to $\{E_{s_i}\}_{s_i\in L_s^2}$ for Case 2. Case 3 is the hybrid scenario of both Cases 1 and 2. Since U_n^s is concave for both Cases 1 and 2, it is straightforward to say that U_n^s is concave with respect to E_{s_i} for Case 3.

Property 2: The optimal power consumption E_{s_i} has decreasing trend with p_i when the prices of other sources are some fixed quantity.

Proof: Taking the first order derivative, we have

$$\frac{\partial E_{s_i}^*(p_i)}{\partial p_i} = -\frac{1}{2\sigma G_{s_i,r}}\sqrt{\frac{aB_{s_i}E'_{r_i}G_{s_i,r}G_{r,d}}{(p_i - \frac{aG_{s_i,d}}{\sigma^2})^3}} < 0. \tag{20}$$

That implies, $E_{s_i}^*$ is decreasing with p_i. For Case 2, if the prices of other sources are constant, increment of p_i drives Network to buy non-increasing amount of power from source s_i. In other way, we can say that if one source increases its

[9] When the direct link SNR is better than the relayed link one, relay service is not used for that case and this makes different type of profit for different sources.

price while other sources keep their prices constant, Network will buy less power from that source.

Property 3: The utility U_{s_i} of source s_i is concave in terms of its price p_i it asks for, given that its power consumption is the optimized amount demanded from Network as calculated in *Lemma 1* and also the prices of other sources are some fixed quantity.

Proof: $E^*_{s_i}(p_i)$ is a continuous function of p_i. Since U_{s_i} is a function of $E^*_{s_i}(p_i)$ and p_i, U_{s_i} is continuous in p_i. Taking the derivatives, we obtain

$$\frac{\partial U_{s_i}}{\partial p_i} = E^*_{s_i}(p_i) + (p_i - c_i)\frac{\partial E^*_{s_i}(p_i)}{\partial p_i}, \tag{21}$$

$$\frac{\partial^2 U_{s_i}}{\partial p_i} = 2\frac{\partial E^*_{s_i}(p_i)}{\partial p_i} + (p_i - c_i)\frac{\partial^2 E^*_{s_i}(p_i)}{\partial {p_i}^2}, \tag{22}$$

where

$$\frac{\partial^2 E^*_{s_i}(p_i)}{\partial {p_i}^2} = \frac{3\sqrt{B_{s_i}}}{4\sigma G_{s_i,r}}\sqrt{\frac{aE'_{r_i}G_{s_i,r}G_{r,d}}{(p_i - \frac{aG_{s_i,d}}{\sigma^2})^5}}.$$

We know $p_i > c_i$. Furthermore, we have observed in Case 1 that if $p_i < \frac{aG_{s_i,d}}{\sigma^2}$, Network buys undefined power. Therefore, $c_i > \frac{aG_{s_i,d}}{\sigma^2}$ and the following statement is true. $\sqrt{\frac{1}{(p_i - \frac{aG_{s_i,d}}{\sigma^2})^3}} > \sqrt{\frac{1}{\frac{(p_i - \frac{aG_{s_i,d}}{\sigma^2})^5}{(p_i - c_i)^2}}}$.

Because of the above statement and U_{s_i} is continuous with respect to p_i, from Equation 22, we can conclude that $\frac{\partial^2 U_{s_i}}{\partial p_i} < 0$, which justifies the concavity property of U_{s_i} with respect to p_i for Case 1. For other cases, it is straightforward to conclude the concavity property of U_{s_i} with respect to p_i.

Remark 1: Source Selection procedure by Network described in Section IV-A1(i) is sufficient.

Proof: This is because, any source is rejected by Network at the beginning by this rule, however mistakenly taken by Network to play the game with, eventually that source will be rejected. The proof of this statement is as follows. Suppose that the source rejection criterion is applied for some source node s_i, i.e., $\frac{\partial U^s_n}{\partial E_{s_i}} < 0$, when $E_{s_i} = 0$ and $p_i = c_i$. And, Network does not exclude source s_i and in the following price update process, all source nodes gradually increase their prices to get more utilities. To prove that the new resulting $E^*_{s_i}(c_i + \delta) < 0$, it suffices to prove that $\sum_{s_j \in L_s} \frac{\partial E_{s_i}}{\partial p_j} \leq 0$, i.e., $\frac{\partial E_{s_i}}{\partial p_i} \leq 0$ since we know that $\frac{\partial E_{s_i}}{\partial p_j} = 0, s_j \in [L_s|s_i]$. *Property 2* already has proved $\frac{\partial E_{s_i}}{\partial p_i} \leq 0$.

On the other hand, for Case 1, if any source node s_i satisfies the source rejection criteria at the beginning, s_i is not rejected by Network in the final outcome of the game. Since optimal power $E^*_{s_i}(p_i)$ is a function of only source s_i's price, it does not get affected if other sources increment their prices. *Property 3* says that $\frac{\partial U_{s_i}}{\partial p_i} \geq 0$. Assume that $\bar{p_i}$ is the price for which source s_i obtains 0 power. Hence, when source s_i increments its price from c_i to some price, say p^{new}_i, this new price must be less than $\bar{p_i}$. This is because, in order to satisfy *Property 3*, it will ask such price which will increase its utility instead of obtaining 0 utility (achieved at price $\bar{p_i}$). However, for Cases 2 and 3, since Network assigns power among the sources in a greedy manner based on some metric, there is possibility that some source gets rejected in the final outcome of the game due to the total power constraint.

Theorem 1: $\{E^*_{s_i}\}_{s_i \in L_s}$ and $\{p^*_i\}_{s_i \in L_s}$ are the SE for the source power allocation game.

Proof: Having obtained the prices $\{p^*_i\}_{s_i \in L_s}$ from the sources, due to *Property 1*, $U^s_n\left(\{E^*_{s_i}(p^*_i)\}_{s_i \in L_s}\right) \geq U^s_n\left(\{E_{s_i}(p^*_i)\}_{s_i \in L_s}\right)$. That implies, $\{E^*_{s_i}(p^*_i)\}_{s_i \in L_s}$ is the optimal response strategy for Network and the SE of the game. When the optimal response is released by Network, source s_i keeps increasing its price p_i until it reaches to p^*_i. According to *Property 3*, $U_{s_i}(p^*_i, E^*_{s_i}(p^*_i)) \geq U_{s_i}(p_i, E^*_{s_i}(p_i))$. Therefore, p^*_i is the optimal response for source s_i and the SE of the game.

3) Iterative Source Price Update Function: The sources increase their utilities by incrementing their prices from acceptable lower quantity, c_i (cost of power for delivering data) towards the optimal ones. The price update function of the sources can be designed as follows. In each iteration of the price update procedure until the convergence happens, for the sources $s_i \in L^1_s$ in Case 1, it applies that

$$\frac{\partial U_{s_i}}{\partial p_i} = \frac{\partial}{\partial p_i}\left[(p_i - c_i)E^*_{s_i}(p_i)\right] = E^*_{s_i}(p_i) + (p_i - c_i)\frac{\partial E^*_{s_i}(p_i)}{\partial p_i} \geq 0.$$

By *Property 2*, we know that $\frac{\partial E^*_{s_i}(p_i)}{\partial p_i} < 0$. After rearranging, the above equation appears to

$$p_i \leq c_i - E^*_{s_i}(p_i)\left[\frac{\partial E^*_{s_i}(p_i)}{\partial p_i}\right]^{-1}. \tag{23}$$

Here, it is important to note that the value of $\frac{\partial E^*_{s_i}(p_i)}{\partial p_i}$ is negative before p_i rises to p^*_i. For the sake of simplicity, the price update procedure can be represented in vector form, $\mathbf{p} \leq \mathbf{I}(\mathbf{p})$, where $\mathbf{p} = \{p_i\}_{s_i \in L^1_s}$; $\mathbf{I}(\mathbf{p}) = \{I_i(\mathbf{p})\}_{s_i \in L^1_s}$, where $I_i(\mathbf{p})$ is the price update function for source s_i. Consequently, each iteration of the price update algorithm can be expressed as $\mathbf{p}(t+1) = \mathbf{I}(\mathbf{p}(t))$. $\mathbf{I}(\mathbf{p})$ is a standard function, and it has the similar properties as a standard function has. Because of these properties, the authors in [34] have proved that starting from some initial power vector $\mathbf{p}$, after n iterations, $\mathbf{I}^n(\mathbf{p})$ produces unique fixed prices. The properties of the standard function $\mathbf{I}(\mathbf{p})$ have been proved in *Appendix A.1*. For Case 2, $E^*_{s_i}(p_i), \forall s_i \in L^2_s$ is non-differentiable with respect to p_i. Therefore, the price update function for this case is $\mathbf{p}(t+1) = \mathbf{p}(t) + \boldsymbol{\delta}$. Whereas, for Case 3, the sources $\forall s_i \in L^1_s$ follow the similar price update procedure as Case 1, and the sources $\forall s_i \in L^2_s$ follow the procedure in Case 2.

B. Game Theoretical Relay Power Allocation

In order to enhance the performance of the network further, we present the formulation of another Stackelberg game in order to distribute relay power E'_r among the sources in set L^1_s.

1) Sources/Buyers: We first model the sources as followers who aim to get most benefits at the least possible

payment. The utility function of source $s_i, s_i \in L_s^1$ can be defined by

$$U_{r_i} = \eta \left(\frac{E_{s_i}^* G_{s_i,d}}{\sigma^2} + \frac{E_{s_i}^* G_{s_i,r} E_{r_i} G_{r,d}}{\sigma^2 \left(E_{s_i}^* G_{s_i,r} + E_{r_i} G_{r,d} + \sigma^2 \right)} \right) - \lambda E_{r_i} \quad (24)$$

η is the gain per unit of SNR, and λ denotes the price per unit of power sold by the centralized node "Network". E_{r_i} can be explained as the amount of relay power source s_i would like to buy from Network when the price λ is announced.

2) Network/Seller: Network is modeled as a leader who aims to maximize its revenue by setting a proper price. Constant c is introduced to denote the cost per unit of power. The utility function of Network is defined as

$$U_n^r = (\lambda - c) \sum_{s_i \in L_s^1} E_{r_i}^*(\lambda). \quad (25)$$

λ has the same meaning as in Equation 24. In order to earn profit, the price must be higher than the cost, which means $\lambda > c$. $E_{r_i}^*(\lambda), s_i \in L_s^1$ depends not only on source s_i's channel condition, but also depends on the global price. If Network asks a higher price than what the source expects, the source will buy less power. On the other hand, if the price is too low, profit obtained by Equation 25 will be unnecessarily small. So, there is a tradeoff for setting the price. A proper price can distribute the total allocated relay power among the sources optimally.

1) Analysis of Relay Power Allocation Game: In this section, we first show that given a price λ, there exists an unique SE of each source game. Then based on this, we prove, there is a unique optimal equilibrium for the whole Stackelberg game. Moreover, we design a price update function for Network and prove its convergence to the unique equilibrium in the following subsection.

(i) Source Level Analysis: Each source node ($s_i \in L_s^1$) determines how much power it should buy to maximize its utility. According to Equation 24, the power source s_i will buy under the price λ can be determined by solving the equation $\frac{\partial U_{r_i}}{\partial E_{r_i}} = 0$.

$$E_{r_i}^*(\lambda) = \begin{cases} 0 & \text{if } \lambda \geq \lambda_{r_i}^{ub} \\ \frac{1}{G_{r,d}} \left[\sqrt{\frac{\eta B_{r_i} E_{s_i}^* G_{s_i,r} G_{r,d}}{\lambda \sigma^2}} - B_{r_i} \right] & \text{if } \lambda_{r_i}^{ub} < \lambda < \lambda_{r_i}^{lb}, \\ E_r' & \text{if } \lambda \leq \lambda_{r_i}^{lb} \end{cases} \quad (26)$$

where $B_{r_i} = E_{s_i}^* G_{s_i,r} + \sigma^2$, $\lambda_{r_i}^{lb} = \frac{\eta B_{r_i} E_{s_i}^* G_{s_i,r} G_{r,d}}{\sigma^2 (E_r' G_{r,d} + B_{r_i})^2}$ and $\lambda_{r_i}^{ub} = \frac{\eta E_{s_i}^* G_{s_i,r} G_{r,d}}{B_{r_i} \sigma^2}$.

(ii) Network Level Analysis: Network needs to find a global optimal price so as to maximize its revenue. Given $E_{r_i}^*(\lambda), s_i \in L_s^1$, the best price can be obtained by solving the following equation

$$\frac{\partial U_n^r}{\partial \lambda} = \sum_{s_i \in L_s^1} E_{r_i}^*(\lambda) + (\lambda - c) \sum_{s_i \in L_s^1} \frac{\partial E_{r_i}^*(\lambda)}{\partial \lambda}. \quad (27)$$

Hence, the best price is given by

$$\lambda^* = f\left(\{G_{s_i,r}\}_{s_i \in L_s^1}, G_{r,d}, c, \eta, \sigma^2 \right). \quad (28)$$

Lemma 2: The optimal price is inside the interval [λ_{lb}, λ_{ub}), where $\lambda_{lb} = \max_{s_i \in L_s^1} \lambda_{r_i}^{lb}$ and $\lambda_{ub} = \max_{s_i \in L_s^1} \lambda_{r_i}^{ub}$.

Proof: We prove the lower bound by contradiction. If $\lambda < \max_{s_i \in L_s^1} \lambda_{r_i}^{lb}$, one source will definitely obtain E_r' relay power and at most one source will obtain some relay power which results in the amount of allocated relay power is more than the allowable power E_r'. This is not a feasible solution. On the other hand, if $\lambda = \max_{s_i \in L_s^1} \lambda_{r_i}^{lb}$, one source will obtain E_r' relay power and the rest others may obtain 0 relay power, which is considered as a feasible solution. Considering the feasibility of the allocated relay power among all sources, λ should be $\geq \max_{s_i \in L_s^1} \lambda_{r_i}^{lb}$.

For the upper bound, if $\lambda \geq \max_{s_i \in L_s^1} \lambda_{r_i}^{ub}$, all sources obtain 0 relay power which is also not an expected solution. Therefore, λ must be $< \max_{s_i \in L_s^1} \lambda_{r_i}^{ub}$ in order to assign some relay power among the sources.

2) Properties of Relay Power Allocation Game: In this subsection, we elaborate the properties of the relay power allocation game and prove that the solutions derived in Equations 26 and 28 are the unique optimal equilibrium for each round of this game.

Property 4: Given price λ, U_{r_i} is concave with respect to E_{r_i}, when $E_{r_i} > 0$ and $E_{r_j}, \forall s_j \in \left[L_s^1 | s_i \right]$ are fixed quantity.

Proof: Taking the second order derivative of U_{r_i} in Equation 24, we get

$$\frac{\partial^2 U_{r_i}}{\partial E_{r_i}^2} = \frac{-2\eta B_{r_i} E_{s_i}^* G_{s_i,r} G_{r,d}}{\sigma^2 (B_{r_i} + E_{r_i} G_{r,d})^3} < 0. \quad (29)$$

Moreover, U_{r_i} is continuous in E_{r_i}. So, when $E_{r_i} > 0$, U_{r_i} is concave with respect to E_{r_i}.

Property 5: The best amount of power bought by each source decreases as the price increases.

Proof: Taking the derivative of $E_{r_i}^*(\lambda)$ with respect to λ, we obtain

$$\frac{\partial E_{r_i}^*(\lambda)}{\partial \lambda} = \frac{-1}{2G_{r,d}} \sqrt{\frac{\eta B_{r_i} E_{s_i}^* G_{s_i,r} G_{r,d}}{\sigma^2 \lambda^3}} < 0. \quad (30)$$

This justifies the decrementing trend of the optimal relay power for sources $s_i \in L_s^1$ with the incrementing λ.

Property 6: If $\lambda > c$, U_n^r is concave with respect to price λ and the power consumption is the optimized purchase amount.

Proof: From Equations 27 and 30, we obtain

$$\frac{\partial^2 U_n^r}{\partial \lambda^2} = 2 \sum_{s_i \in L_s^1} \frac{\partial E_{r_i}^*(\lambda)}{\partial \lambda} + (\lambda - c) \sum_{s_i \in L_s^1} \frac{\partial^2 E_{r_i}^*(\lambda)}{\partial \lambda^2}, \quad (31)$$

$$\frac{\partial^2 E_{r_i}^*(\lambda)}{\partial \lambda^2} = \frac{3}{4G_{r,d}} \sqrt{\frac{\eta B_{r_i} E_{s_i}^* G_{s_i,r} G_{r,d}}{\sigma^2 \lambda^5}}. \quad (32)$$

Since $\sqrt{\frac{1}{\lambda^3}} > \sqrt{\frac{1}{\frac{\lambda^5}{(\lambda - c)^2}}}$, $\frac{\partial^2 U_n^r}{\partial \lambda^2} < 0$, which justifies the concavity property of U_n^r with respect to λ.

Property 7: Given the relay power price λ, $E_{r_i}^*(\lambda)$ is a non-decreasing function of $E_{s_i}^*$ and $G_{s_i,r}$.

Proof: See *Lemma 2* of [25].

Theorem 2: $\{\{E^*_{r_i}\}_{s_i \in L_s^1}, \lambda^*\}$ solved in above discussions are the SE for the proposed game.

Proof: According to *Property 4*, given λ and the power of other sources $\{E^*_{r_j}\}_{s_j \in [L_s^1 | s_i]}$ is constant, the best response of source s_i is $E^*_{r_i}(\lambda)$. And, according to *Property 6*, incrementing λ gradually increases the value of U_n^r until it reaches λ^*. It implies that

$$\begin{aligned} U_{r_i}(E^*_{r_i}(\lambda^*)) &\geq U_{r_i}(E_{r_i}(\lambda^*)), \\ U_n^r(\{E^*_{r_i}(\lambda^*)\}_{s_i \in L_s^1}) &\geq U_n^r(\{E^*_{r_i}(\lambda)\}_{s_i \in L_s^1}). \end{aligned}$$

So, $\{\{E^*_{r_i}\}_{s_i \in L_s^1}, \lambda^*\}$ are the SE of the relay power allocation Stackelberg game.

3) Iterative Relay Power Price Update Function: In order to achieve appropriate convergence of this game, Network needs to update its price correctly in each round. According to *Property 6*, U_n^r is a concave function with respect to λ. Therefore, Network increases the price λ from its initial price until the convergence happens. For the sake of obtaining non-negative utility, Network sets the initial price c. According to Equation 27 and *Property 5*, if $\frac{\partial}{\partial \lambda} U_n^r > 0$, we have

$$\lambda < c - \left(\sum_{s_i \in L_s^1} E^*_{r_i}(\lambda) \right) \left(\sum_{s_i \in L_s^1} \frac{\partial E^*_{r_i}(\lambda)}{\partial \lambda} \right)^{-1}.$$

We denote $I(\lambda)$ as

$$I(\lambda) = c - \left(\sum_{s_i \in L_s^1} E^*_{r_i}(\lambda) \right) \left(\sum_{s_i \in L_s^1} \frac{\partial E^*_{r_i}(\lambda)}{\partial \lambda} \right)^{-1}. \quad (33)$$

Hence, the price update method is $\lambda(t+1) = I(\lambda(t))$. $I(\lambda)$ fulfills the properties of a standard function which has been proved in *Appendix A.2*. Setting the initial price as c (i.e.,λ_{lb}), λ will converge to a unique equilibrium after sufficient iterations.

C. Further Discussion

Next, we will briefly discuss the possible implementation of the proposed game theoretical solution of this power allocation problem. As noted before, there is a centralized entity called ”Network” in the network. Network is responsible to lead two games in order to obtain the transmit and relay power allocation of the sources. There is a total power constraint in the network which is the sum of required power for both transmit and relay operations of the sources. In order to decide appropriate convergence of two games separately, Network needs to know how much power is separately allocated for both games. As discussed before, in order to decide the amount power for both games, Network should be aware of the complete CSI of the network. On the assumption of using block fading channel model, at the beginning of the first time slot (Entire transmit operation requires two phases or time slots), a training process is conducted for Network to obtain global CSI. Research on the efficient channel training and estimation can be found in [35], [36]. The training and estimation is performed at the relay and the destination in order to obtain the channel gains from the sources to themselves. For collecting the channel gain from the relay to the destination, another round of training and estimation is performed at the relay. Finally, all these acquired channel gains are notified to Network by the feedback method. Network can be a separate computationally powerful entity in the network or; any of the sources or the relay can play this role depending on their computational capability.

Once Network knows the allowable power for two games, first it starts the game with the sources for their transmit power. When the convergence is achieved for this game and set L_s^1 is not empty, Network initiates the second game, and continues until the convergence happens. In order to improve the performance of the network further, Network can re-distribute the power especially for the case when both sets L_s^1 and L_s^2 are non-empty. The way how Network does this is described as follows.

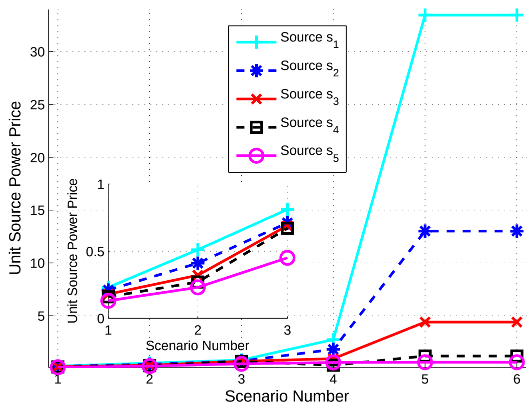
1) Sort the list L_s^2 in the descending order of $G_{s_i,d}, s_i \in L_s^2$, and denote the sorted list as OL_s^2.
2) Sort the list L_s^1 in the ascending order ot $G_{s_i,d}, s_i \in L_S^1$, and denote the sorted list as OL_s^1.
3) Take source s_i from OL_s^2 which is not assigned with full power E_s^{max}.
4) Take source s_j from OL_s^1.
5) Compute $x = \min\left(E^*_{s_j}, E^*_{r_j}\right)$ and $y = \min\left(2x, E^*_{s_i}\right)$. Add y to $E^*_{s_i}$, and subtract $y/2$ from $E^*_{s_j}$ and $E^*_{r_j}$.
6) If $E^*_{s_i} = E_s^{max}$, move s_i to the next source of list OL_s^2.
7) If $E^*_{s_j} = 0$ and $E^*_{r_j} > 0$, add $E^*_{r_j}$ to $E^*_{s_j}$.
8) If $E^*_{s_j} = 0$, move s_j to the next source of list OL_s^1.
9) If s_i or s_j does not point to a valid source, terminate this process, otherwise repeat the steps from 5.

With this implementation, we actually assume that Network is trustworthy. All sources believe that Network will not change the parameter values (e.g., CSI), however conducts all these operations in the systematic manner.

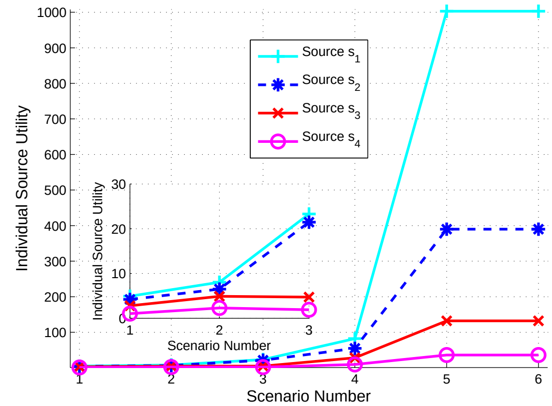
D. Game Theoretical Simulation Results

In order to evaluate the game theoretical solution, we undertake the similar network setup, channel model and the parameters as given in III-C1. Similar simulation scenario is undertaken as was taken for evaluating the performance of the centralized approach. For game theoretical solution, there are some parameters, which we set as $a = 100$, $\eta = 100$.

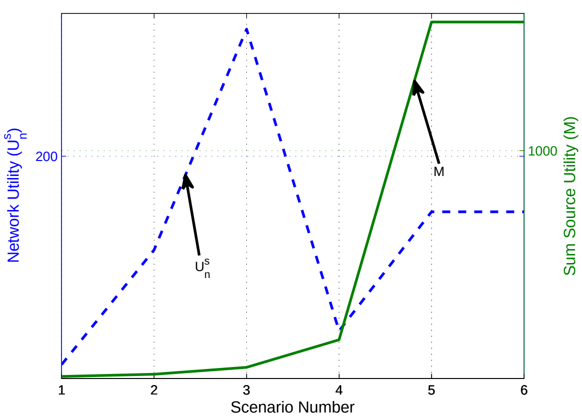
Within certain price range, the utility of each source has concavity property. Ideally, convergence of the source power allocation game should be decided by the individual source nodes. However, to make the game theoretical solution comparable with the centralized one, Network should be aware of total power constraint, i.e., the sum power sold by the sources should be equal to the total allowable source power. Therefore, if the sum of converged power is less than the total source power budget, remaining power is distributed among the sources based on their contribution towards the utility of Network node. On the other hand, if the sum of converged power is less than the total power budget, overflown power is subtracted from the sources based on their inverse utility contribution.



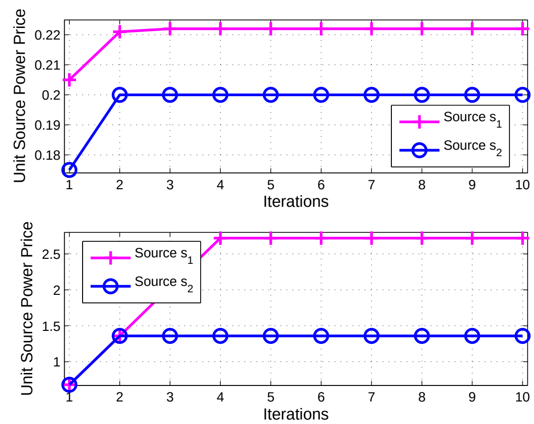
(a) Converged price comparison among all sources.



(b) Individual converged utility comparison among all sources.



(c) Converged Network utility and sum source utility comparison.



(d) Convergence for scenario 1 (up) and scenario 4 (down).

Fig. 3. Source power allocation game.

Figures 3(a), 3(b) and 3(c) compare individual converged source power price p_i, utility U_{s_i}, sum source utility M, and Network node's utility U_n^s for different scenarios explained previously. As the sources move towards the relay or the destination, prices asked by them are increased. Since their channel condition gradually improves with the increasing scenario number, they demand high price for unit source power. For scenario number 1, 2 and 3, the price of the sources is the sum of relayed and direct link contribution towards maximizing Network's utility. Whereas for other scenarios, the price is only for the direct link contribution. Since the absolute channel condition of scenarios 5 and 6 is almost same, the prices of the sources are almost equivalent for these two scenarios. In each scenario, the sources with better channel condition contribute more towards maximizing Networks's utility, and hence they demand higher price compared to others. Furthermore, the utility of individual source follows the same trend as its price since the utility is the product of its price and sold power. At the convergence point, all scenarios satisfy the same power constraint, and hence the utility of individual source is increased with the increasing price. Consequently, sum utility of all sources M is increased with the increasing scenario number. In Figure 3(b), we have skipped the utility of source s_5 because of its achieved 0 utility resulting from nearly 0 sold power. Since the channel condition of the sources is getting better with the increasing scenario number, even though M is subtracted form the first factor of Network's utility, because of using moderately larger value of a, the utility of Network is increased as well. There is a discrepancy between scenario 3 and scenario 4 in terms of the utility of Network node, this is because former one uses the relay for transmission, whereas the later one does not. Moreover, converged price of each source for scenario 4 is very close to the upper bound of its price, and hence the value of M does not deviate much from the first factor of U_n^s. However, scenarios without relay power follow the same increasing trend with the better channel quality condition because of the increasing first factor of U_n^s compared to its M. Table I shows the final outcome of the game, detailed breakdown of the power for the sources. In order to have comparison, the table also shows the power obtained from the centralized optimal solution.

At the beginning, when the game is started, Network selects the set L_s of beneficiary sources following the procedure in *Remark 1*. Taking the consideration of their own cost, the sources announce their prices. Consequently, Network calculates the amount of power it wants to buy from the sources for maximizing its own utility. Considering this interaction as one iteration, Figure 3(d) shows the convergence process for scenarios 1 and 4. For scenarios 4, 5 and 6, convergence speed depends on the step size $\delta_i, s_i \in L_s$. The larger the step size, the speedier the convergence.

Similar to the source power allocation game, for the relay power allocation game, the convergence point should be decided by the seller of the game, i.e., Network. In order to have valid comparison with the centralized solution, the sum relay power should meet some power constraint. At the convergence point of Network node, the power constraint should be satisfied. If the constraint is not satisfied at the convergence point, relay power of each source is adjusted based on its contribution towards the utility of Network node.

Figure 4(a) shows the increasing price and utility of Network node for the relay power allocation game with respect to different scenarios. Similar to the source power allocation game, the increasing scenario number implies better channel quality condition. It means, unit relay power results in more contribution towards maximizing the utility of individual source. Hence, Network demands higher relay power price while selling its power to the sources. Since the power budget is same for all scenarios, with the increasing price, the utility of Network node has increasing trend as well. Table II shows the final outcome of the game. It compares the power obtained by the relay power allocation game with the centralized optimal solution. It seems, individual source who has better channel condition buys more power compared to others. This is because, given unit price, the source with better channel condition incurs larger utility compared to those with worse ones. Moreover, the utility of individual source is the increasing function of its transmit power. Since the source with better channel condition is assigned larger transmit power from the previous game, this source is likely to be assigned with more relay power compared to others. Therefore, the larger demanded power is well adjusted balance between their utility and price.

After carefully considering the initial price, when Network announces it, the sources demand power which is the balance between their individual utility and price. Considering this interaction between the sources and Network as a single step, Figure 4(b) shows the convergence behavior of the relay power allocation game at scenario 2. The figure shows the increasing price of Network node with the increasing iteration until the convergence happens.

Taking the power presented in Table I and Table II, Figure 5 compares the throughput obtained from the game theoretical solution with the centralized optimal solution.

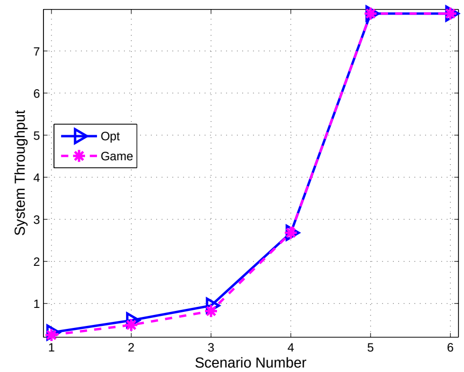


Fig. 5. System throughput comparison between optimal and game theoretical solutions.

V. Conclusion

In this paper, we have studied both transmit and relay power allocation problem in a multi-source single AF relay network. Since the existing works [11], [12] of this problem have not considered the direct link SNR in their formulation, resultant solution deviates from the optimal one if the direct link SNR is better than the relayed link one. Having noticed this, we put the missing factor in the original formulation, and have solved this using GP. Because of incorporating the direct link SNR in the problem formulation, the problem is not directly amenable to GP. Hence, we have used single condensation method in order to cast it to GP. Since this solution is computationally expensive with the growing number of sources, we also have given a suboptimal solution. In the centralized solution, all nodes work selflessly towards maximizing the network capacity. Therefore, we have proposed a game theoretical solution of this problem which models the selfish behavior of the nodes. Extensive numerical results have been provided in order to show different properties, convergence condition of both solutions. Finally, we have shown that proposed the game theoretical solution achieves comparable performance with the centralized optimal solution.

APPENDIX A

A.1 Proof of the properties of $\mathbf{I}(\mathbf{p})$

- *Positivity:* $\mathbf{I}(\mathbf{p}) > 0$. By *Property 2*, $\frac{\partial E^*_{s_i}(p_i)}{\partial p_i} < 0$. Moreover, because of practicality, $c_i > 0$, and according to *Lemma 1*, $E^*_{s_i}(p_i) \geq 0$. Therefore, following the definition in Equation 22, $I_i(p_i) \geq c_i > 0$. Hence, in this price update process, source $s_i, \forall s_i \in L^1_s$ starts increasing its price from c_i.
- *Scalability:* $\forall \alpha > 1$, $\alpha \mathbf{I}(\mathbf{p}) > \mathbf{I}(\alpha \mathbf{p})$. Subtracting $\mathbf{I}(\alpha \mathbf{p})$ from $\alpha \mathbf{I}(\mathbf{p})$ for source s_i, we have

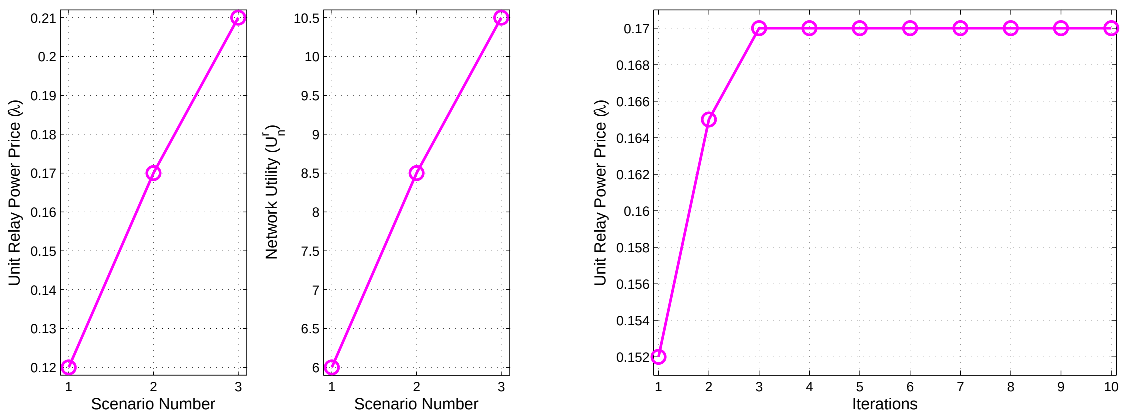
$$\alpha I_i(p_i) - I_i(\alpha p_i) = (\alpha - 1)c_i + \left[\frac{E^*_{s_i}(p_i)}{\partial E^*_{s_i}(\alpha p_i)/\partial p_i} - \frac{\alpha E^*_{s_i}(p_i)}{\partial E^*_{s_i}(p_i)/\partial p_i}\right]. \quad (34)$$

Since $\alpha > 1$, $(\alpha - 1)c_i > 0$. Then, the problem reduces to proving $\frac{E^*_{s_i}(\alpha p_i)}{\partial E^*_{s_i}(\alpha p_i)/\partial p_i} > \frac{\alpha E^*_{s_i}(p_i)}{\partial E^*_{s_i}(p_i)/\partial p_i}$. Now,

$$\frac{E^*_{s_i}(\alpha p_i)}{\partial E^*_{s_i}(\alpha p_i)/\partial p_i} = -2p_i + \frac{2c_i}{\alpha} + \frac{2\sigma\sqrt{B_{s_i}}}{\alpha\sqrt{aE'_{r_i}G_{s_i,r}G_{r,d}}} \left(\alpha p_i - \frac{aG_{s_i,d}}{\sigma^2}\right)^{3/2}, \quad (35)$$

TABLE I
SOURCE TRANSMIT POWER COMPARISON BETWEEN OPTIMAL AND GAME THEORETICAL SOLUTIONS.

Scenario Number	s_1		s_2		s_3		s_4		s_5	
	Opt	Game	Opt	Game	Opt	Game	Opt	Game	Opt	Game
1	28.29	24.32	26.46	19	15.24	16	0	10.68	0	51e-3
2	29.12	25.1	28.21	21.0	12.66	12.13	0	7.1	0	4e-3
3	30.0	27.23	30	24.4	10	11.55	0	6.80	0	71e-4
4	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	0	0
5	30.0	30.0	30.0	30.0	30.0	30.0	30.0	30.0	0	0



(a) Relay power price (left) and converged Network utility (right) comparison.

(b) Convergence for scenario 2.

Fig. 4. Relay power allocation game.

TABLE II
SOURCE RELAY POWER COMPARISON BETWEEN OPTIMAL AND GAME THEORETICAL SOLUTIONS.

Scenario Number	s_1		s_2		s_3		s_4		s_5	
	Opt	Game	Opt	Game	Opt	Game	Opt	Game	Opt	Game
1	36.19	46	13.78	4.0	0	0	0	0	0	0
2	36.96	46.3	13.031	3.7	0	0	0	0	0	0
3	32.24	43.5	17.75	6.5	0	0	0	0	0	0

$$\frac{\alpha E^*_{s_i}(p_i)}{\partial E^*_{s_i}(p_i)/\partial p_i} = -2\alpha p_i + 2\alpha c_i + \frac{2\alpha\sigma\sqrt{B_{s_i}}}{\sqrt{aE'_{r_i}G_{s_i,r}G_{r,d}}} \left(p_i - \frac{aG_{s_i,d}}{\sigma^2}\right)^{3/2}. \quad (36)$$

For the second part of Equation 34 being > 0, p_i should satisfy $p_i > \frac{\frac{aG_{s_i,d}}{\sigma^2}(\alpha - 1/\alpha)}{\alpha - 1}$, and $p_i > \frac{aG_{s_i,d}}{\sigma^2}\frac{1/\alpha - \sqrt[3]{\alpha}}{1 - \sqrt[3]{\alpha}}$. Or, in other way, $p_i > x\frac{aG_{s_i,d}}{\sigma^2}$, where $x \in \mathbb{R}$. However, x grows very slowly with the increasing value of α. Since $c_i > \frac{aG_{s_i,d}}{\sigma^2}$, and $p_i > c_i$ for Case 1, the price update function of source s_i satisfies this property.

- *Monotonicity:* If $\mathbf{p} \geq \mathbf{p}'$, then $\mathbf{I}(\mathbf{p}) \geq \mathbf{I}(\mathbf{p}')$. To satisfy this property, it is sufficient to prove $\frac{\partial \mathbf{I}(\mathbf{p})}{\partial \mathbf{p}} \geq 0$. Hence, for source s_i, we have

$$\frac{\partial I_i(p_i)}{\partial p_i} = 2\left[1 - \frac{3\sigma\sqrt{B_{s_i}}}{2\sqrt{aE'_{r_i}G_{s_i,r}G_{r,d}}}\sqrt{p_i - \frac{aG_{s_i,d}}{\sigma^2}}\right]. \quad (37)$$

For $\frac{\partial I_i(p_i)}{\partial p_i}$ being ≥ 0, p_i should be $\leq \frac{aG_{s_i,d}}{\sigma^2} + \frac{4aE'_{r_i}G_{s_i,r}G_{r,d}}{9\sigma^2 B_{s_i}}$. It is apparent that upper bound of p_i for the monotonicity property is very close to p_i^{ub} in Case 1.

A.2 Proof of the properties of I(λ)

- *Positivity:* $I(\lambda) \geq 0$. By *Property 5*, $\frac{\partial E_{r_i}(\lambda)}{\partial \lambda} < 0, \forall s_i \in L_s^1$. Furthermore, setting the cost c as λ_{lb}, $c > 0$. In Equation 26, we observe $E_{r_i}(\lambda) \geq 0, \forall s_i \in L_s^1$. Therefore, according to Equation 33, $I(\lambda) \geq c > 0$. Hence, in this price update process, Network increases price from c.
- *Monotonicity:* if $\lambda \geq \lambda'$, $I(\lambda) \geq I(\lambda')$. To satisfy this property, it is enough to prove $\frac{\partial I(\lambda)}{\partial \lambda} \geq 0$. So,

$$I(\lambda) = c + 2\left[\lambda - \frac{\sum_{s_i \in L_s^1} B_{r_i}}{\sum_{s_i \in L_s^1}\sqrt{\frac{\eta B_{r_i}E^*_{s_i}G_{s_i,r}G_{r,d}}{\sigma^2}}}\lambda^{3/2}\right], \quad (38)$$

$$\frac{\partial I(\lambda)}{\partial \lambda} = 2\left[1 - \frac{3\sum_{s_i \in L_s^1} B_{r_i}}{2\sum_{s_i \in L_s^1}\sqrt{\frac{\eta B_{r_i}E^*_{s_i}G_{s_i,r}G_{r,d}}{\sigma^2}}}\lambda^{1/2}\right]. \quad (39)$$

For $\frac{\partial I(\lambda)}{\partial \lambda} \geq 0$, it must satisfy that $\lambda \leq \frac{4}{9\sigma^2}\left(\frac{\sqrt{\eta B_{r_i} E^*_{s_i} G_{s_i,r} G_{r,d}}}{\sum_{s_i \in L^1_s} B_{r_i}}\right)^2$ which is very close to λ_{ub}.

- *Scalability:* For all $\alpha \geq 1$, $\alpha I(\lambda) \geq I(\alpha\lambda)$. We have

$$\alpha I(\lambda) = \alpha c + 2\alpha\lambda - 2\alpha \frac{\sum_{s_i \in L^1_s} B_{r_i}}{\sum_{s_i \in L^1_s} \sqrt{\frac{\eta B_{r_i} E^*_{s_i} G_{s_i,r} G_{r,d}}{\sigma^2}}} \lambda^{3/2}, \tag{40}$$

$$I(\alpha\lambda) = c + 2\alpha\lambda - 2 \frac{\sum_{s_i \in L^1_s} B_{r_i}}{\sum_{s_i \in L^1_s} \sqrt{\frac{\eta B_{r_i} E^*_{s_i} G_{s_i,r} G_{r,d}}{\sigma^2}}} \alpha\lambda^{3/2}. \tag{41}$$

Since $\alpha > 1$, and c is positive, $(\alpha - 1)c$ is always > 0. Furthermore, for the second part of $\alpha I(\lambda) - I(\alpha\lambda)$ being positive, it must satisfy that $\alpha^{3/2} \geq \alpha$, which is true for all $\alpha \geq 1$

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