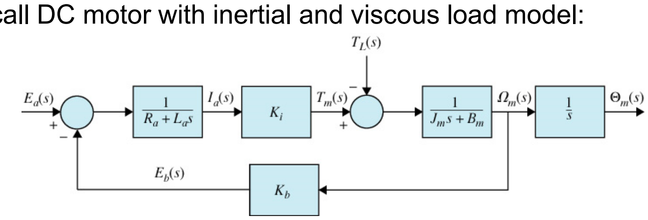
Block diagrams and signal flow graphs [Ch. 2.6,2.7,2.8]

- Objective of this set of slides
- To learn how to manipulate and simplify interconnected dynamical systems represented by block diagrams of transfer functions.

Block diagrams and signal flow graphs [Ch. 2.6,2.7,2.8]

- Recall DC motor with inertial and viscous load model:



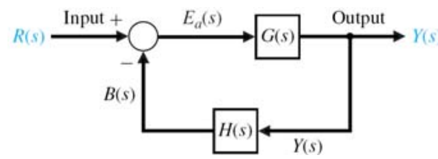
$$\text{Transfer function: } \quad H(s) = \frac{\theta_m(s)}{E_a(s)} = \frac{K_i}{s[(R_a + L_a s)(J_m s + B_m) + K_b K_i]}$$

This is a:

- Graphical representation of dynamic system relationships
- It consists of unidirectional, operational blocks representing transfer functions of components or subsystems

Closed-loop transfer function

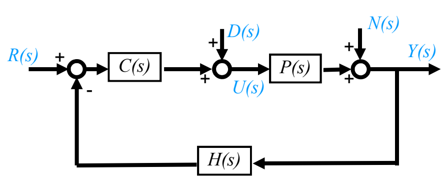
- To compute transfer function, start with the error… and follow the loop until back to the error again:

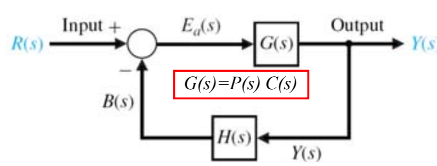


- Good practice to list signal multiplication in the order it occurs… because everything works for multi-input multi-output systems

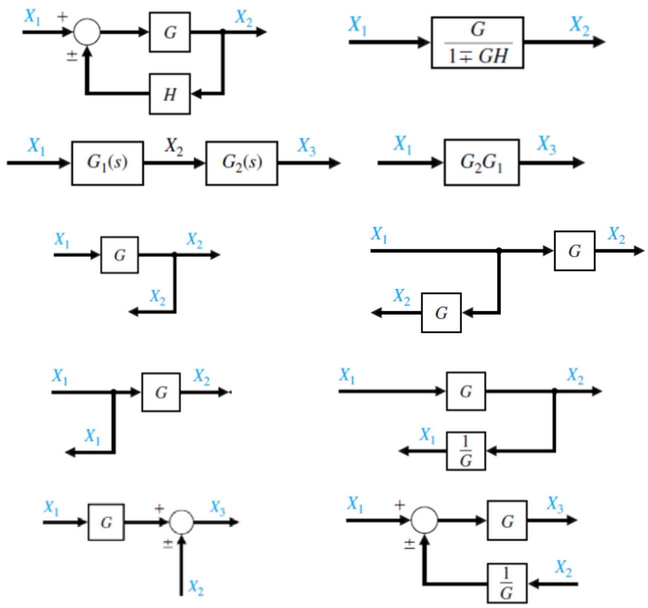
Closed-loop transfer function

- Sometimes, we are interested in other transfer functions, e.g output disturbance/noise to output, or actuator disturbance to output. Because all blocks are linear *superposition* applies…

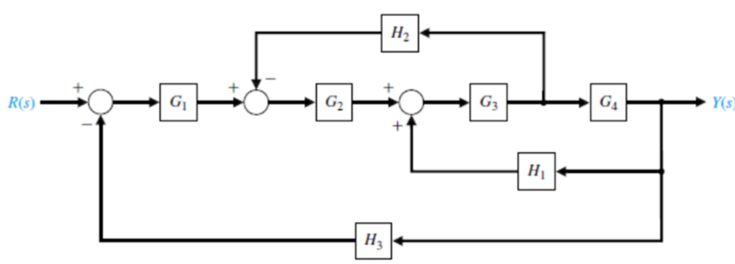


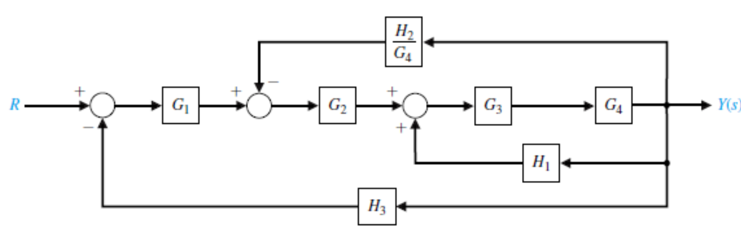


Special block diagram manipulations

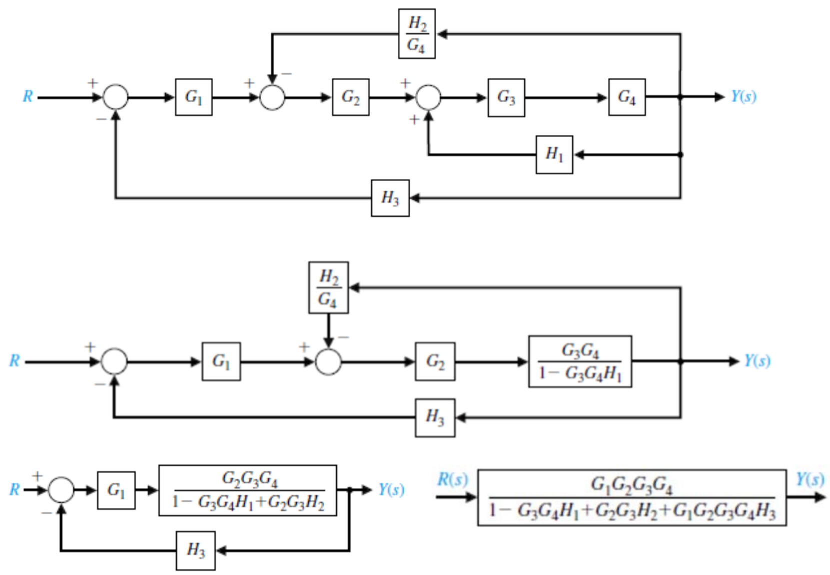


Special block diagram manipulations

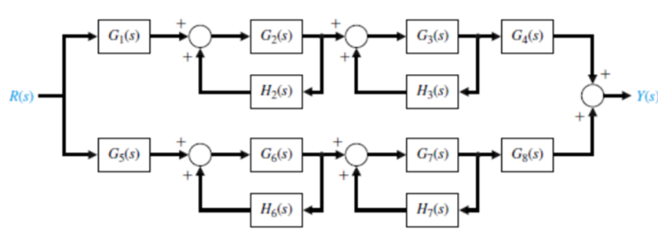


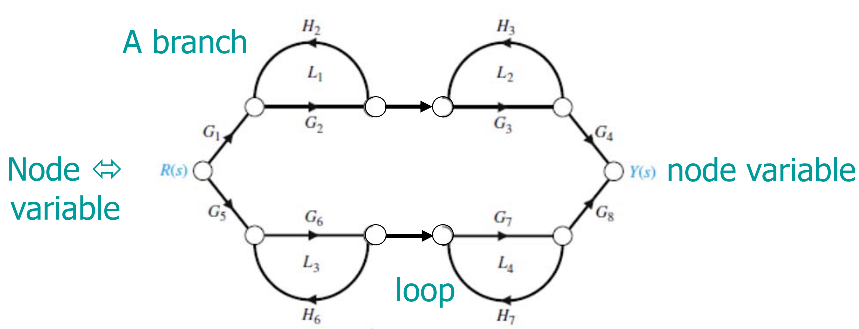


Special block diagram manipulations

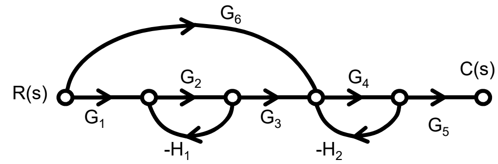


Signal Flow Graphs





Signal flow graph definitions



- **Input (Source) Node**: Has only outgoing branches: R(s)
- **Output (Sink) Node**: Has only incoming branches: C(s)
- **Path**: A branch or a continuous succession of branches traversed in the same direction from one node to another node, e.g. $G_5 G_4 G_3$, , $-H_1 G_2 G_1$,
- **Forward Path**: Path connecting a source to a sink without visiting any node more than once: $G_5 G_4 G_6$
- **Loop**: Closed path not visiting any node more than once: $-H_1 G_2$, $-H_2 G_4$
- **Path Gain**: Product of gains on path: $G_5 G_4 G_3$
- **Forward-path Gain**: Product of gains on forward path: $G_5 G_4 G_3 G_2 G_1$
- **Loop Gain**: Product of gains on a loop: $-H_1 G_2$, $-H_2 G_4$
- **Non-touching Loops**: Loops that have no node in common: $-H_1 G_2$, $-H_2 G_4$
- **Non-touching Loops and paths** : Loops and paths that have no node in common $-H_1 G_2$, $G_5 G_4 G_6$

Mason's Gain Formula

The gain between the *input node* and the *output node* is given by:

$$T = \frac{1}{\Delta} \sum M_k \Delta_k$$

$$\Delta = 1 - \sum_{n} L_n + \sum_{n \neq m,\, non-touching} L_n L_m - \sum_{n \neq m \neq p,\, non-touching} L_n L_m L_p + \ldots - \ldots$$

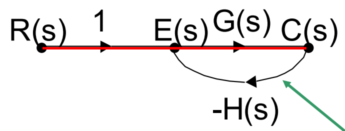
$(L_n = n^{\text{th}} \text{ loop gain})$

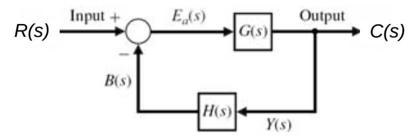
$$\begin{aligned} &= 1 - \sum \text{(all possible loop gains)} \\ &\quad + \sum \text{(all possible gain products of two non-touching loops)} \\ &\quad - \sum \text{(all possible gain products of three non-touching loops)} \\ &\quad + \sum \text{(all possible gain products of four non-touching loops)} \ldots \end{aligned}$$

$M_k = k^{th}$ forward path

$\Delta_k = \Delta$ with all loops touching k^{th} forward path M_k removed

Example 1



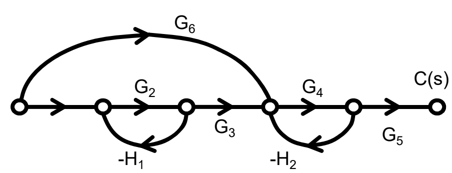


$\Delta = 1 + G(s)H(s)$ One loop –G(s)H(s)

$\Delta_1 = 1$ No loops not touching the forward path

$$T(s) = \frac{G(s)}{\Delta} = \frac{G(s)}{1 + G(s)H(s)}$$ One forward path G(s)

Example 2



$$\Delta = 1 - (-H_1G_2) - (-H_2G_4) + (-H_1G_2)(-H_2G_4)$$
$$\Delta = 1 + H_1G_2 + H_2G_4 + H_1G_2H_2G_4$$

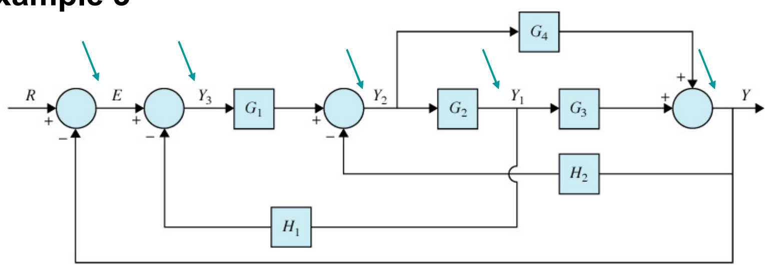
M_k	Δ_k	
$M_1 = G_1G_2G_3G_4G_5$	No loops not touching	$\Delta_1 = 1$
$M_2 = G_5G_4G_6$	One loop not touching	$\Delta_2 = 1 + H_1G_2$

$$T = \frac{1}{\Delta}\sum M_k\Delta_k = \frac{G_1G_2G_3G_4G_5 + G_5G_4G_6(1 + H_1G_2)}{1 + H_1G_2 + H_2G_4 + H_1G_2H_2G_4}$$

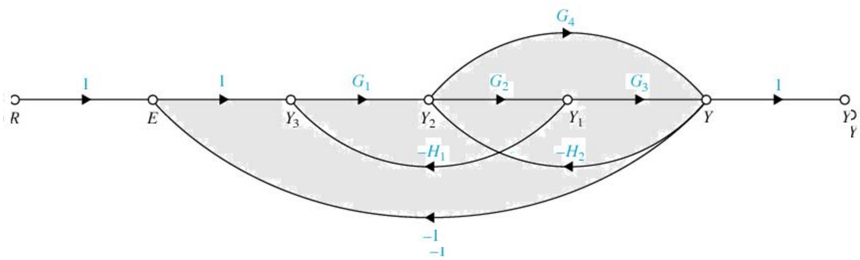
Example 3

$$\Delta = 1 - G_1G_2(-H_1) - G_2G_3(-H_2) - G_4(-H_2) - G_1G_2G_3(-1) - G_1G_4$$

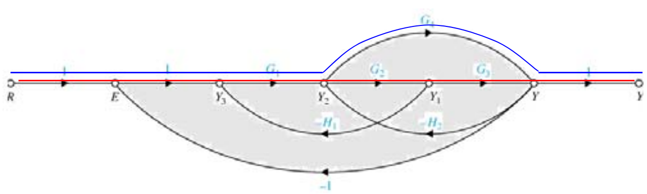
$$\Delta = 1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2 + G_1G_2G_3 + G_1G_4$$



(a)



Example 3



$$\Delta = 1 - G_1G_2(-H_1) - G_2G_3(-H_2) - G_4(-H_2) - G_1G_2G_3(-1) - G_1G_4$$

$$\Delta = 1 + G_1G_2H_1 + G_2G_3H_2 + G_4H_2 + G_1G_2G_3 + G_1G_4$$

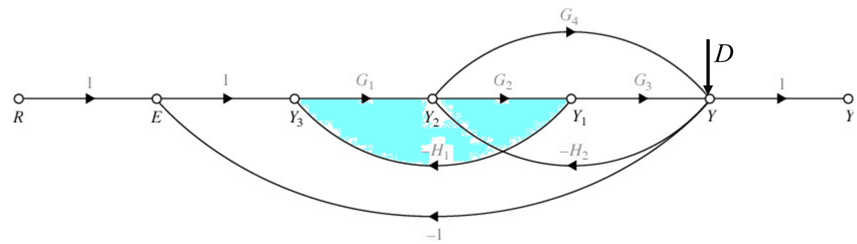
$M_1 = G_1G_2G_3$ then $\Delta_1 = 1$

$M_2 = G_1G_4$ then $\Delta_2 = 1$

Then

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{\Delta}$$

Disturbance Response



(b)

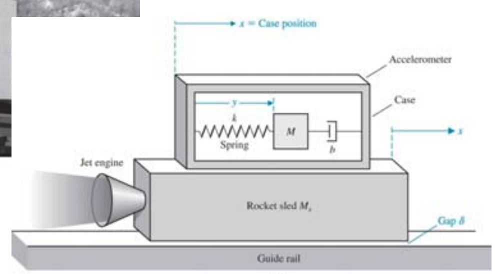
Note that Δ is unchanged but that we now have

$$M_1 = 1 \qquad \Delta_1 = 1 - (-G_1G_2H_1)$$

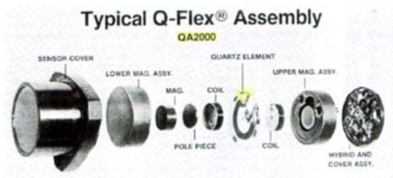
$$\frac{Y}{D} = \frac{1 + G_1G_2H_1}{\Delta}$$

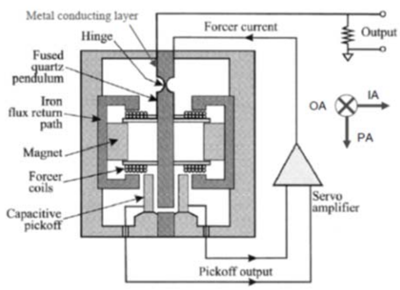
Feedback accelerometer design example

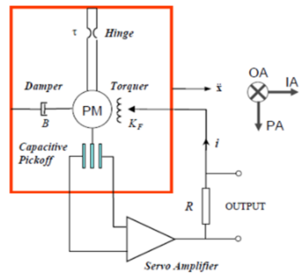

(a)



(b)







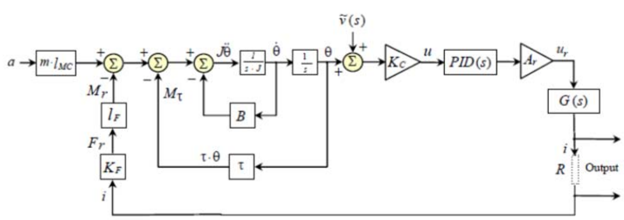


Fig. 7 *Dynamic model of the pendulous accelerometer with magnetic rebalancing*

$\frac{1}{s^2 J + sB + \tau}$: the transfer-function of the accelerometer sensor element, a damped elastic pendulum;

$PID(s) = K_{PID} \cdot \frac{s^2 T_d T_i + sT_i + 1}{sT_i(1 + s\tau_{del})}$: the transfer-function of the lag-compensation element, of type Proportional-Integral-Derivative, with K_{PID} the amplification factor of the *PID*-regulator, T_d and T_i the derivative and the integrating time constants, respectively, and τ_{del} a first-order delay term;

$G(s) = \frac{1}{R + sL}$: the equivalent Laplace admittance for the electric part of the rebalance-torquer assembly, with R the serial resistance (winding resistance neglected) and L the total coils inductivity.

$$u_{out}(s) = F_D(s) \cdot a(s) + F_N(s) \cdot \tilde{v}(s)$$

$$F_D(s)\big|_{s=0} = \frac{R \cdot m \cdot l_{MC}}{K_F \cdot l_F}$$

$$F_D(s)\big|_{s\to\infty} = 0$$

$$F_N(s)\big|_{s=0} = \frac{R \cdot \tau}{K_F \cdot l_F}$$

$$F_N(s)\big|_{s\to\infty} = 0.$$

$$u_{out}(s) = \frac{R \cdot m \cdot l_{MC}}{K_F \cdot l_F + \frac{s(s^2 J + sB + \tau)(sT_i\tau_{del} + T_i)(sL + R)}{K_C \cdot K_{PID} \cdot A_r \cdot (s^2 T_d T_i + sT_i + 1)}} \cdot a(s) + \frac{R}{\frac{K_F \cdot l_F}{s^2 J + sB + \tau} + \frac{s(sT_i\tau_{del} + T_i)(sL + R)}{K_C \cdot K_{PID} \cdot A_r \cdot (s^2 T_d T_i + sT_i + 1)}} \cdot \tilde{v}(s)$$