Cyclic Prefix Design and Allocation in Bit-loaded OFDM over Power Line Communication Channels

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***Abstract*—The usual approach when designing an orthogonal frequency division multiplexing (OFDM) system, is to dimension the length of the cyclic prefix (CP) equal to the length of a typical "bad" (i.e., long) channel impulse response, so that both inter-symbol and inter-carrier interference are avoided for almost all channel realizations. However, such an approach does not maximize system capacity. It (a) wastes channel resources for relatively short channel impulse response realizations and (b) it is not necessarily optimal to completely eliminate interference. In this paper, we study the problem of designing the CP length for OFDM systems in a capacity-optimal way. To this end, we first present optimal and simplified metrics suitable to maximize capacity. Then, we apply those metrics and propose practical resource (bit-loading and OFDM sub-channel assignment) algorithms that include CP-length adaptation. We present numerical results for the example of a power line communication (PLC) OFDM system with typical indoor PLC channels that confirm the gains achievable with the proposed CP-length adaptation.**

***Index Terms*—Orthogonal frequency division multiplexing (OFDM), cyclic prefix, resource allocation, bit-loading, power line communications.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has become the most popular transmission technology for communication over wide band channels that exhibit frequency selectivity. It has been adopted for many wireless, subscriber line, and power line communications (PLC) systems. The key feature of OFDM is the orthogonalization of the frequency selective channel into parallel sub-channels (each described by a scalar gain) through the use of a cyclic prefix (CP) and discrete Fourier transform (DFT) operations [1]. Thus, simple one-tap equalization per sub-channel can be used at the receiver, and channel capacity can be approached by practical implementation of the water filling principle using bit and power loading of OFDM sub-channels [2]. Furthermore, the partition of sub-channels among multiple users realizes a form of multiple access known as orthogonal frequency division multiple access (OFDMA) [3].

To maximize system performance, resources have to be optimally allocated. Assuming the use of finite size constellations, the resource allocation problem in OFDM(A) is also known as the bit, power, and sub-channel allocation problem [2]–[6]. The optimization and adaptation of the CP is usually not considered. Instead, the conventional approach is to use a fixed CP that is longer than the maximum channel duration such that neither inter-symbol (ISI) nor inter-carrier (ICI) interferences occur at the receiver side. However, this approach is power and bandwidth inefficient. The insertion of the CP requires additional transmission energy and introduces a loss in transmission rate of $M/(M+\mu)$, where M is the number of OFDM sub-channels and μ is the CP length in number of samples. Since the channel impulse response realizations may be different for different links and/or may vary with time, adaptation of μ to the specific channel realization is beneficial. Furthermore, the CP length does not necessarily need to be equal to the channel duration to maximize capacity[1]. That is, allowing for a controlled amount of ISI and ICI in order to reduce μ, we may overall improve performance. It is therefore reasonable to consider the optimization of the CP length.

The case of a CP interval shorter than the channel impulse response has been considered in a number of early and more recent works on OFDM, cf. e.g. [7] and references therein. Viterbo and Fazel [8] compute the interference power due to channel echoes exceeding the CP and consider the application of per sub-channel equalization and coding to mitigate the effect of interference in a digital terrestrial TV broadcasting setting. Following this work, Seoane et al. [9] provide simulative evidence that interference can well be modeled as additive white Gaussian noise (AWGN), and study the performance degradation due to interference using the suburban hilly UMTS channel model. In [7], the effect of CP length on OFDM capacity is studied for single-user wireless OFDM systems using 20 MHz bandwidth and operating at 3.7 GHz in urban and suburban environments. However, the mentioned works do not consider CP adaptation to the channel realizations. CP adaptation has been investigated in [10] for OFDM-based wireless local area network (WLAN) systems. It is suggested to choose the CP length twice the channel delay spread. As we will show in this work, this criterion is not reliable unless

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[1]In this paper, the term "capacity" is used to denote the maximal data rate achievable under some typical system constraints, e.g., a power spectral density constraint. These will be specified in Section III.

the channel attenuation, and therefore the receiver-side signal-to-noise power ratio (SNR), is taken into account.

In this paper we report a comprehensive analysis of the CP-length optimization problem. In particular, we consider joint CP-length, bit, and sub-channel allocation for single user and multi-user OFDM systems. Since our original motivation stems from OFDM for PLC, for numerical examples we choose PLC scenarios and consider parameters according to state-of-the-art systems, e.g., the HomePlug AV (HPAV) or Universal Powerline Association (UPA) specifications [11]–[13]. Due to the electromagnetic compatibility (EMC) normative [14], these systems transmit signals with a constant power spectral density (PSD) level in the used frequency band. This assumption is also made in this work. Note that, even though PLC transceivers are static, the PLC channel between two nodes is time-varying due to load-impedance variations and loads being plugged into or removed from the power grid. Hence, different channel realizations are experienced even for a fixed pair of nodes, and CP-length adaption is potentially beneficial. This said, we expect significant channel variations not being frequent with respect to data rates, and thus adaptation of the CP length is deemed practically feasible.

The remainder of this paper is organized as follows. After introducing the system model in Section II, we first show in Section III-A that assuming a power spectral density constraint at the transmitter side and signaling over a frequency selective channel, the achievable rate (capacity) is a function of the CP length. Therefore, the optimal CP length can be determined by maximizing the capacity. We consider other simplified metrics in Section III-B, namely two metrics derived from an upper and a lower capacity bound and a metric based on the channel delay spread. Then, in Section III-C we propose to adapt the CP length by choosing it from a finite set of values. The set of CP values is pre-computed from the statistical analysis of the channel, specifically the analysis of the cumulative distribution function (CDF) for the optimal CP length. In Section IV, the joint bit-loading and CP-length adaptation problem is addressed. We consider both bit-loading with distinct constellations and uniform bit-loading (identical constellations) on all the sub-channels. In Section V, we extend the idea of optimizing the CP length to the multiuser OFDM scenario (OFDMA). In Section VI, we report extensive numerical results for an HPAV like OFDM system [11] and PLC channels from a statistical channel simulator [15]. They show that significant gains can be obtained by the appropriate adaptation of the CP length to the channel conditions. Finally, Section VII concludes this work.

II. System Model

We assume an OFDM scheme (cf. e.g. [2], [6], [16]) with M sub-channels (or tones), and a CP length of $\mu = N - M$ samples, where N is the normalized sub-channel symbol period (OFDM symbol duration in samples) assuming a sampling period T. The normalized sub-carrier frequencies are defined as $f_k = k/M$, for $k = 0, 1, \ldots, M-1$. The OFDM signal is transmitted over a channel that has an equivalent discrete time complex impulse response

$$g_{\text{ch}}(n) = \sum_{p=0}^{\nu-1} \alpha_p \delta(n-p) \; , \tag{1}$$

where $\{\alpha_p\}$ denote the complex channel coefficients and $\delta(n)$ is the discrete-time delta pulse. We assume that $\nu \leq M$, so that the channel is not longer than the useful OFDM symbol duration. However, the channel duration may exceed the CP length. As a result, at the receiver, after symbol synchronization, discarding of the CP, and DFT processing, the signal for sub-channel k can be written as

$$z^{(k)}(\ell) = H^{(k)}(\mu) a^{(k)}(\ell) + I^{(k)}(\ell, \mu) + W^{(k)}(\ell) \; , \tag{2}$$

where $a^{(k)}(\ell)$ is the ℓ-th data symbol at sub-channel k, $H^{(k)}(\mu)$ is the effective channel transfer factor for the data symbol, $I^{(k)}(\ell, \mu)$ represents the ISI plus ICI term, and $W^{(k)}(\ell)$ is the noise contribution. Note that interference $I^{(k)}(\ell, \mu)$ occurs due to a loss of orthogonality when $\mu < \nu - 1$ and that we made the dependence of $H^{(k)}(\mu)$ and $I^{(k)}(\ell, \mu)$ on μ explicit. We can define the sub-channel signal-to-noise-plus-interference power ratio (SINR) as

$$\text{SINR}^{(k)}(\mu) = \frac{P_{\text{U}}^{(k)}(\mu)}{P_{\text{W}}^{(k)} + P_{\text{I}}^{(k)}(\mu)} \; , \tag{3}$$

where the useful, the interference, and the noise power terms on sub-channel k are defined as ($\mathcal{E}\{\cdot\}$ denotes the expectation operator)

$$\begin{aligned} P_{\text{U}}^{(k)}(\mu) &= |H^{(k)}(\mu)|^2 \mathcal{E}\{|a^{(k)}(\ell)|^2\}, \\ P_{\text{I}}^{(k)}(\mu) &= \mathcal{E}\{|I^{(k)}(\mu, \ell)|^2\} \; , \; P_{\text{W}}^{(k)} = \mathcal{E}\{|W^{(k)}(\ell)|^2\} \; . \end{aligned} \tag{4}$$

To evaluate the expressions in (5), we make the usual assumption that rectangular windows

$$g(n) = \frac{1}{N} \text{rect}\left(\frac{n}{N}\right) \; , \quad h(n) = \frac{\sqrt{N}}{M} \text{rect}\left(-\frac{n+\mu}{M}\right) \; , \tag{5}$$

of length N and M are used as OFDM synthesis and analysis prototype pulses, respectively, where $\text{rect}(n/M) = 1$ for $n = 0, \ldots, M-1$ and zero otherwise. Then, the cross-talk pulse-shape between sub-channels i and k can be written as ($\otimes$ denotes convolution)

$$r_{gh}^{(i,k)}(n) = (g(n) \mathrm{e}^{\mathrm{j}2\pi f_i n}) \otimes (h(n) \mathrm{e}^{\mathrm{j}2\pi f_k n}) \; . \tag{6}$$

As shown in Appendix A, under the assumption of i.i.d. zero mean data symbols, the useful signal power and the interference power are obtained as

$$P_{\text{U}}^{(k)}(\mu) = P_a^{(k)} \left| \sum_{p=0}^{\nu-1} \alpha_p r_{gh}^{(k,k)}(-p) \right|^2 \; , \tag{7}$$

$$P_{\text{I}}^{(k)}(\mu) = P_{\text{tot}}^{(k)}(\mu) - P_{\text{U}}^{(k)}(\mu) \; , \tag{8}$$

$$\begin{aligned} P_{\text{tot}}^{(k)}(\mu) &= \mathcal{E}\{|z^{(k)}(\ell)|^2\} \\ &= \sum_{q \in \mathbb{Z}} \sum_{i \in \mathbb{K}_{\text{on}}} P_a^{(i)} \left| \sum_{p=0}^{\nu-1} \alpha_p r_{gh}^{(i,k)}(qN - p) \right|^2 \; , \end{aligned} \tag{9}$$

where $P_a^{(k)} = \mathcal{E}\{|a^{(k)}(\ell)|^2\}$ is the power of the constellation in sub-channel k and $\mathbb{K}_{\text{on}} \subseteq \{0, \ldots, M-1\}$ is the set of indexes associated to the active sub-channels, i.e., $P_a^{(k)} > 0$ for $k \in \mathbb{K}_{\text{on}}$. It follows directly from (7)-(10) that if the CP is longer than the channel duration, i.e., $\mu \geq \nu - 1$, the interference power is null and the useful power term corresponds to the absolute square value of the k-th DFT output of the channel response multiplied by the power of the constellation. Therefore, increasing the CP length beyond the channel length does not increase the SINR since the interference is zero for $\mu \geq \nu - 1$.

III. Optimization of the CP Length

In this section, we present criteria for the optimization of the CP length. Throughout this work, we assume the application of a PSD mask constraint for the transmitted signal. The PSD mask determines the set of active OFDM sub-channels $\mathbb{K}_{\text{on}}$, while the remaining sub-channels are switched off (notched) in order to fulfill regulations and coexistence norms [13], [14], [17]. Furthermore, the PSD mask is such that the transmitted power is uniformly distributed over the sub-channels in $\mathbb{K}_{\text{on}}$, i.e., $P_a^{(k)} = P_a$, $\forall k \in \mathbb{K}_{\text{on}}$.

We start with the criterion for capacity-optimal CP length (Section III-A). Since the evaluation of this criterion turns out to be computationally complex, we then propose three sub-optimal metrics to select the CP length (Section III-B). Furthermore, we propose a method to design a small set of CP values over which we perform adaptation (Section III-C).

A. Channel Capacity Criterion

In order to evaluate the impact of the CP length on the system performance we consider the achievable data rate assuming parallel Gaussian channels. That is, we assume independent and Gaussian distributed input signals, which renders ISI and ICI also Gaussian (see also [9]). Furthermore, we stipulate the use of single tap sub-channel equalization, i.e., no attempt is made to suppress ISI and ICI, which is the very reason for the widespread use of OFDM. Then, the maximal data rate is

$$C(\mu) = \frac{1}{(M+\mu)T} \sum_{k \in \mathbb{K}_{\text{on}}} \log_2 \left(1 + \frac{\text{SINR}^{(k)}(\mu)}{\Gamma} \right) \text{ [bit/s]}, \tag{10}$$

where Γ represents a gap factor accounting for practical modulation and channel coding [2], [6]. For $\Gamma = 1$, $C(\mu)$ is the system capacity, and, in slight abuse of notation, we refer to the CP design such that (10) is maximized for any given $\Gamma \geq 1$ as capacity-optimal CP. That is, the capacity-optimal CP length (in number of samples) is given by

$$\mu_{\text{opt}} = \underset{0 \leq \mu < \nu}{\text{argmax}} \{C(\mu)\} \ . \tag{11}$$

The evaluation of the argument of (11) is computationally demanding because it requires the SINR and a sum-log computation for each CP value. Therefore, lower-complexity solutions are desirable.

B. Suboptimal Optimization Metrics

We introduce three suboptimal metrics suitable to adjust the CP length.

1) Capacity Bound Criteria: The first two simplifications for the optimization of the CP length are obtained by lower and upper bounding the capacity in (10). For this purpose, we assume white noise with $P_{\text{W}}^{(k)} = P_{\text{W}}$. Details of the derivations are provided in Appendix B.

The capacity can be lower bounded by

$$C(\mu) > \frac{M_{\text{on}}}{(M+\nu)T} \log_2 \left(\frac{P_{\text{U,min}}}{P_W} \right) - \frac{M_{\text{on}}}{(M+\nu)T} \log_2 \left(\frac{(M+\mu)\Gamma}{(M+\nu)M_{\text{on}}} \sum_{k \in \mathbb{K}_{\text{on}}} \left(1 + \frac{P_{\text{I}}^{(k)}(\mu)}{P_W} \right) \right) , \tag{12}$$

where $M_{\text{on}} = |\mathbb{K}_{\text{on}}|$ is the cardinality of $\mathbb{K}_{\text{on}}$ and $P_{\text{U,min}} = \min_{k \in \mathbb{K}_{\text{on}}, \mu} \{P_{\text{U}}^{(k)}(\mu)\}$. The corresponding optimal CP length is given by

$$\mu_{\text{opt}} = \underset{0 \leq \mu < \nu}{\text{argmin}} \left\{ (M+\mu) \left(M_{\text{on}} P_{\text{W}} + P_{\text{I}}(\mu) \right) \right\} \ , \tag{13}$$

where $P_{\text{I}}(\mu) = \sum_{k \in \mathbb{K}_{\text{on}}} P_{\text{I}}^{(k)}(\mu)$ denotes the total interference power.

Although the bound on capacity is not necessarily tight, metric (13), derived from this bound, yields performance close to that achieved with the optimal metric (11) as shown by numerical results in Section VI.

A capacity upper bound can be found as

$$C(\mu) \leq \frac{M_{\text{on}}}{\Gamma(M+\mu)T} \log_2(\text{e}) \overline{\text{SINR}}(\mu) \ , \tag{14}$$

where the average SINR

$$\overline{\text{SINR}}(\mu) = \frac{1}{M_{\text{on}}} \sum_{k \in \mathbb{K}_{\text{on}}} \text{SINR}^{(k)}(\mu) \tag{15}$$

has been used. The resulting CP-length criterion is

$$\mu_{\text{opt}} = \underset{0 \leq \mu < \nu}{\text{argmax}} \left\{ \frac{\overline{\text{SINR}}(\mu)}{M+\mu} \right\} \ . \tag{16}$$

The suboptimal criteria (13) and (16) have a computational advantage over (11). First, the computation of the logarithm is avoided. Second, (13) only requires the evaluation of the interference power for different values of μ instead of the computation of the SINRs as metric (11). When all M sub-channels are used, metric (13) is simple to compute since the total interference power can be evaluated as

$$P_{\text{I}}(\mu) = N P_a \sum_{p=0}^{\nu-1} |\alpha_p|^2 \left[M - (p-\mu) S_{p-\mu} + (p-\mu) S_{p-\mu-1} - \frac{(M - p S_{p-\mu-1})^2}{M} \right], \tag{17}$$

with $S_i = 1$ for $i \geq 0$ and zero otherwise.

2) Delay Spread Criterion: The third simplification that we propose for the adjustment of the CP length is based on the evaluation of the root mean square (rms) delay spread σ_{ch} for a given channel realization $g_{\text{ch}}(n)$, which is given by [18, pp. 77-78]

$$\sigma_{\text{ch}} = \sqrt{\sum_{p=0}^{\nu-1}(pT - m_{\text{ch}})^2 \alpha_p^2 \Big/ \sum_{p=0}^{\nu-1} \alpha_p^2}\,, \tag{18}$$

where

$$m_{\text{ch}} = T\sum_{p=0}^{\nu-1} p\alpha_p^2 \Big/ \sum_{p=0}^{\nu-1} \alpha_p^2\,. \tag{19}$$

Considering that the significant fraction of energy of the channel impulse response is captured within $\beta\sigma_{\text{ch}}$ for some $\beta > 0$ [18], the CP length can be adjusted according to

$$\mu_{\text{opt}} = \beta\sigma_{\text{ch}}/T\,. \tag{20}$$

This approach is very attractive for its computational simplicity. In fact, $\mu_{\text{opt}} = 2\lceil\sigma_{\text{ch}}/T\rceil$ ($\lceil.\rceil$ denotes the ceiling operator) has been used in [10] for CP-length adjustment for IEEE 802.11a OFDM systems. Nevertheless, as it is discussed in more detail in the next subsection, the factor β should be adapted as a function of SNR and thus of the attenuation introduced by the channel, in order to balance the contributions from interference (ISI and ICI) and background noise to the overall SINR.

Finally, we point out that instead of computing the delay spread of the channel realization, one could compute the duration of the window that captures most, say 95%, of the energy of the channel impulse response. This would yield a metric similar to (20).

C. Simplified Adaptation of the CP Length

The adaptation of the CP requires that for each channel realization the receiver computes the CP length according to one of the criteria above described and feeds back the selected value to the transmitter. One attractive possibility to reduce computational cost and the amount of feedback is to choose the CP length from a small set of pre-determined values. Several OFDM specifications, e.g., the power line HPAV standard [11], have opted for such an approach. To determine an appropriate set of CP values, we propose a method based on the evaluation of the CDF of the capacity-optimal CP length according to (11).

Considering the PLC channel model from [15] (more details follow in Section VI-A), which classifies channels according to their average path loss, it is found that the optimal CP length varies relatively little for members of the same class, but significantly between members of different classes. Hence, we propose, for a given channel class, to choose a single value of CP length for all channel realizations.

The specific lengths are chosen to be the value of μ for which the CDF of (11) is 99%. We denote these values as $\mu_{\text{opt,CLASS}}^{(99\%)}$, i.e., the 99th percentile of the capacity-optimal μ_{opt} (11).

Also the adaptation of β in metric (20) can be based on the CDF for μ_{opt}. That is, for each channel class, we propose to relate β to the average delay spread $\bar{\sigma}_{\text{ch,CLASS}} = \mathcal{E}\{\sigma_{\text{ch,CLASS}}\}$ via

$$\beta_{\text{CLASS}} = T\frac{\mu_{\text{opt,CLASS}}^{(99\%)}}{\bar{\sigma}_{\text{ch,CLASS}}}\,. \tag{21}$$

Since $\mu_{\text{opt,CLASS}}^{(99\%)}$ is defined as function of the class, also the weighting factor β_{CLASS} depends on the channel class. Furthermore, since path loss directly translates into SNR, β_{CLASS} is adjusted according to SNR as stipulated when introducing criterion (20). The computation of the $\mu_{\text{opt,CLASS}}^{(99\%)}$ and the corresponding β_{CLASS} when applying (20) is done off-line, and the values are stored in a look-up table. However, to apply (20) using (21), we need to know the channel class under which the system is operating. This could be obtained through the SNR estimation.

We would like to emphasize that both suggested methods, i.e., (20) using (21) and $\mu_{\text{opt,CLASS}}^{(99\%)}$, require reliable statistical channel models being available for the specific application example.

IV. Bit-Loading and CP-Length Adaptation

In this section we propose two resource allocation procedures that combine bit-loading with CP-length adaptation according to the criteria previously introduced. The procedures that we consider take into account different practical constraints. In the first one we assume that the signal constellations can vary across the sub-channels, but a constraint on the constellation size is considered. In the second one we assume that a single constellation is used across all OFDM sub-channels (uniform bit-loading). This constraint is often applied for adaptive modulation with low-rate feedback channels, cf. e.g. [19], [20].

In addition to bit-loading, OFDM also enables adaptation of the transmit powers for the sub-channels. As we have previously discussed, we assume to transmit with a constant PSD level given by the PSD constraint over the active sub-channels $\mathbb{K}_{\text{on}}$, while the other sub-channels are switched off for coexistence purposes. We note that different from conventional bit-loading for the case of an orthogonal system, notching has an effect on loading of other sub-channels in the case of CP-length adaptation, since the interference power and thus the SINR in those sub-channels changes. Likewise, if the SINR on a given sub-channel is too low for transmission and this sub-channel is switched off, the SINRs on the remaining sub-channels change. A search for which sub-channels should be switched off would be required for optimal bit-loading, which is infeasible. Therefore, for both non-uniform and uniform bit-loading we present two algorithms which take sub-channel activity into account and where the set $\mathbb{K}_{\text{on}}$ and the SINRs are updated via two bit-loading iterations. The first algorithm jointly computes the optimal CP length and bit-loading such that achievable rate is maximized. The second algorithm determines the CP length using one of the proposed criteria introduced in Section III and then it runs the bit-loading for only this CP length. Hence, it enjoys a significantly lower complexity.

A. Procedure 1: Different Constellations

We consider bit-loading with varying signal constellations across the sub-channels. The available constellation sizes are given by 2^b, where $b = 1, \ldots, b_{\max}$.

Algorithm 1.1: The first practical algorithm reads as follows.

1) Initialize the sets of used sub-channels $\mathbb{K}_{\text{on}}(\mu) \subseteq \{0, \ldots, M-1\}$ for $\mu \in \{0, \ldots, \nu-1\}$ according to the transmission mask and uniformly distribute the power among these sub-channels to meet the PSD constraint.
2) $for\ \mu = 0 : \nu - 1$
   a) Compute the $\text{SINR}^{(k)}(\mu)$ for $k \in \mathbb{K}_{\text{on}}(\mu)$, assuming the set of active sub-channels $\mathbb{K}_{\text{on}}(\mu)$.
   b) Determine the bit-loading on sub-channel k from
   $$b^{(k)}(\mu) = \min\left\{c^{(k)}(\mu), b_{\max}\right\},$$
   where $c^{(k)}(\mu) = \left\lfloor\left\lfloor \log_2\left(1 + \frac{\text{SINR}^{(k)}(\mu)}{\Gamma}\right)\right\rfloor\right\rfloor$, and $\lfloor\lfloor\cdot\rfloor\rfloor$ denotes the operation of rounding the number of bits to that associated to the nearest available constellation towards zero.
   c) $if\ (b^{(k)}(\mu) == 0)$ for some $k \in \mathbb{K}_{\text{on}}(\mu)$
   Update the set of active sub-channels $\mathbb{K}_{\text{on}}(\mu)$, i.e., $\mathbb{K}_{\text{on}}(\mu) = \{k : b^{(k)}(\mu) > 0\}$.
   Goto 2a).
   end
   end
3) Compute the optimal CP as
$$\mu_{\text{opt}} = \underset{\mu \in \{0,\ldots,\nu-1\}}{\text{argmax}} \left\{ \frac{1}{M+\mu} \sum_{k \in \mathbb{K}_{\text{on}}(\mu)} b^{(k)}(\mu) \right\}. \quad (22)$$

Algorithm 1.2: The pseudo-code for the simplified algorithm reads as follows.

1) Initialize the set of used sub-channels $\mathbb{K}_{\text{on}} \subseteq \{0, \ldots, M-1\}$ according to the transmission mask and uniformly distribute the power among these sub-channels to meet the PSD constraint.
2) Optimize the CP-length according to (11), (13), (16), (20), or via table look-up (cf. Section III-C). This step returns μ_{opt}.
3) Run steps 2a – 2c of Algorithm 1.1 assuming $\mu = \mu_{\text{opt}}$.

We note that since the SINRs of active sub-channels can only increase when other sub-channels are switched off, only one update of the sub-channel SINRs and active sub-channel set $\mathbb{K}_{\text{on}}$ is needed in Algorithms 1.1 and 1.2 (step 2c). Furthermore, Algorithm 1.2 has a significantly lower complexity than Algorithm 1.1 because the SINR computation is only required for one value of the CP length.

B. Procedure 2: Uniform Bit-loading

We now consider an OFDM system with identical constellations for all sub-channels.

Algorithm 2.1: The pseudo-code for the first practical algorithm reads as follows.

1) Initialize the sets of used sub-channels $\mathbb{K}_{\text{on}}(\mu, b) \subseteq \{0, \ldots, M-1\}$ for $\mu \in \{0, \ldots, \nu-1\}$ and $b \in \mathbb{B} = \{1, \ldots, b_{\max}\}$ according to the transmission mask, and uniformly distribute the power among these sub-channels to meet the PSD constraint.
2) $for\ \mu = 0 : \nu - 1$
   a) Set $\mathbb{K}_{\text{on}}(\mu, 0) = \mathbb{K}_{\text{on}}(\mu, 1)$.
   b) $for\ b \in \mathbb{B}$
      i) Set $\mathbb{K}_{\text{on}}(\mu, b) = \mathbb{K}_{\text{on}}(\mu, b-1)$.
      ii) Compute the $\text{SINR}^{(k)}(\mu, b)$ for $k \in \mathbb{K}_{\text{on}}(\mu, b)$, assuming the set of used sub-channels $\mathbb{K}_{\text{on}}(\mu, b)$.
      iii) Determine the maximum number of bits that can be transmitted on sub-channel k as
      $$b^{(k)} = \min\left\{c^{(k)}(\mu, b), b_{\max}\right\}, \text{where}$$
      $$c^{(k)}(\mu, b) = \left\lfloor\left\lfloor \log_2\left(1 + \frac{\text{SINR}^{(k)}(\mu, b)}{\Gamma}\right)\right\rfloor\right\rfloor.$$
      iv) Update the set of active sub-channels $\mathbb{K}_{\text{on}}(\mu, b) = \{k : b^{(k)} \geq b\}$.
   end
   end
3) Compute the optimal CP and the bits associated to the optimal constellation to be used as
$$(b_{\text{opt}}, \mu_{\text{opt}}) = \underset{(b,\mu) \in \{1,\ldots,b_{\max}\} \times \{0,\ldots,\nu-1\}}{\text{argmax}} \{R(\mu, b)\}. \quad (23)$$
where $R(\mu, b)$ is the bit-rate that can be achieved using a CP equal to μ samples and b bits per QAM symbol, i.e.,
$$R(\mu, b) = b \cdot |\mathbb{K}_{\text{on}}(\mu, b)|. \quad (24)$$

Algorithm 2.2: The pseudo-code for the simplified uniform bit-loading algorithm reads as follows.

1) Initialize the sets of used sub-channels $\mathbb{K}_{\text{on}}(b) \subseteq \{0, \ldots, M-1\}$ for $b \in \mathbb{B} = \{1, \ldots, b_{\max}\}$ according to the transmission mask, and uniformly distribute the power among these sub-channels to meet the PSD constraint.
2) Optimize the CP-length according to (11), (13), (16), (20), or via table look-up (cf. Section III-C). This step returns μ_{opt}.
3) Set $\mathbb{K}_{\text{on}}(\mu_{\text{opt}}, b) = \mathbb{K}_{\text{on}}(b)$ and run steps 2a and 2b of Algorithm 2.1 for $\mu = \mu_{\text{opt}}$.
4) Compute the optimal constellation to be used as
$$b_{\text{opt}} = \underset{b \in \{1,\ldots,b_{\max}\}}{\text{argmax}} \{b \cdot |\mathbb{K}_{\text{on}}(\mu_{\text{opt}}, b)|\}. \quad (25)$$

Algorithms 2.1 and 2.2 take into account the fact that the set of sub-channels that are switched off for a certain constellation (because they cannot sustain the associated bit-rate) necessarily contains the set of sub-channels that are switched off for the constellation of immediately lower order.

Therefore, the algorithms require only to update the set of active sub-channels in the event that more sub-channels need to be switched off. Again, Algorithm 2.2, which makes use of the CP-length criteria from Section III has considerably lower computational complexity than Algorithm 2.1.

V. Extension to OFDMA

In this section, we extend the idea of optimizing the CP length to the multiuser context. We assume a network where a central coordinator (CCo) allocates resources by collecting information regarding the network state, i.e., number of users, channel conditions of each user, rate requirement from each user request, etc. Once the CCo has collected all the information needed, it dynamically allocates the resources among the users. We focus on the downlink channel from the CCo to the N_U users of the network. Multiplexing is accomplished by partitioning the sub-channels among the users realizing OFDMA. Since the channels experienced by the users are different, the CCo allocates the sub-channels and sub-channel bits, and adjusts the CP length according to a fair principle based on maximizing aggregate rate and ensuring that all users exceed a minimum rate.

We follow the capacity-optimal approach in Section III-A. In OFDMA the capacity of the k-th sub-channel of user u, for a certain CP length, is given by

$$C^{(u,k)}(\mu) = \frac{1}{(M+\mu)\,T}\log_2\left(1+\frac{\mathrm{SINR}^{(u,k)}(\mu)}{\Gamma}\right)\ \text{[bit/s]}, \tag{26}$$

where $\mathrm{SINR}^{(u,k)}(\mu) = \frac{P_U^{(u,k)}(\mu)}{P_W^{(u,k)}+P_I^{(u,k)}(\mu)}$, and $P_U^{(u,k)}(\mu)$, $P_W^{(u,k)}(\mu)$, and $P_I^{(u,k)}(\mu)$ respectively are the useful, the noise and the interference power experienced by u-th user in the k-th sub-channel.

In order to allocate resources to the users, for a given CP length, the central manager can solve the following optimization problem that is obtained by generalizing the formulation in [21]:

$$\begin{aligned} AR(\mu) &= \max_{\underline{\alpha}} \sum_{u=1}^{N_U}\sum_{k\in\mathbb{K}_{\text{on}}} \alpha^{(u,k)} C^{(u,k)}(\mu), \\ \underline{\alpha} &= \left\{\alpha^{(u,k)},\ \text{for } k\in\mathbb{K}_{\text{on}},\ \text{and } u=1,\cdots,N_U\right\}, \\ \text{subject to } & \sum_{u=1}^{N_U}\alpha^{(u,k)} = 1, \quad k\in\mathbb{K}_{\text{on}} \\ & \sum_{k\in\mathbb{K}_{\text{on}}}\alpha^{(u,k)}C^{(u,k)}(\mu) \geq \frac{p^{(u)}}{100}\sum_{k\in\mathbb{K}_{\text{on}}} C^{(u,k)}(\mu), \\ & u=1,...,N_U \end{aligned} \tag{27}$$

where $\alpha^{(u,k)}\in\{0,1\}$ denotes the binary sub-channel index, which is equal to 1 if sub-channel k is allocated to user u, and zero otherwise, $p^{(u)}$ is the percentage of the bit-rate that the u-th user has to achieve with respect to the one that it would achieve in a single user scenario, and $AR(\mu)$ is the aggregate network rate. (27) is a binary integer programming problem. To reduce the problem complexity we consider the relaxed problem for $\alpha^{(u,k)}\in[0,1]$, which can be solved by linear programming (LP) [22]. The solution returned by LP is rounded towards the closest integer. The optimal CP is obtained via a discrete search for $\mu\in\{0,\cdots,\nu-1\}$ that maximizes the aggregate rate $AR(\mu)$.

Starting from the formulation in (27), a practical bit-loading algorithm for OFDMA is given by the following pseudo-code.

Algorithm 3:

1) Initialize the set of used sub-channels $\mathbb{K}_{\text{on}}\subseteq\{0,\ldots,M-1\}$ according to the transmission mask and uniformly distribute the power among these sub-channels to meet the PSD constraint.
2) Determine a set $\mathcal{M}\subseteq\{0,\cdots,\nu-1\}$ of CP lengths to be considered.
3) *for* $\mu\in\mathcal{M}$
   a) For each user, compute the SINRs according to (3).
   b) Solve (27) using LP.
   c) Round the coefficients given by LP, i.e., $\beta^{(u,k)}(\mu) = \text{round}\left(\alpha^{(u,k)}\right)$, to partition the sub-channels among the users.
   d) Load the bits to each user according to
   $$b^{(u,k)}(\mu) = \min\left\{\beta^{(u,k)}(\mu)c^{(u,k)}(\mu), b_{\max}\right\},$$
   for $u=1,\ldots,N_U$, $k\in\mathbb{K}_{\text{on}}$, where $c^{(u,k)}(\mu) = \left\lfloor\left\lfloor\log_2\left(1+\frac{\mathrm{SINR}^{(u,k)}(\mu)}{\Gamma}\right)\right\rfloor\right\rfloor$.
   e) If there exist at least one sub-channel k that is switched off, i.e., $b^{(u,k)}(\mu)==0\ \forall\ u$, do another bit-loading iteration as follows
      i) Define $\mathbb{K}_{\text{OFF}}(\mu)=\{k\in\mathbb{K}_{\text{on}} : b^{(u,k)}(\mu)==0\ \forall\ u=1,\ldots,N_U\}$.
      ii) For $k\in\mathbb{K}_{\text{OFF}}(\mu)$, set $\beta^{(u,k)}(\mu)=0\ \forall\ u$.
      iii) Recompute the SINRs and load the bits according to d) on the sub-channels that are switched on.
   f) Compute the aggregate network rate: $R(\mu) = \frac{1}{(M+\mu)T}\sum_{u=1}^{N_U}\sum_{k\in\mathbb{K}_{\text{on}}} b^{(u,k)}(\mu)$.

   end
4) Compute the optimal CP length as $\mu_{\text{opt}} = \text{argmax}_{\mu\in\mathcal{M}}\{R(\mu)\}$. The final sub-channel allocation is given by $\beta^{(u,k)}(\mu_{\text{opt}})$, for $u=1,\cdots,N_U$, $k\in\mathbb{K}_{\text{on}}$.

It is worth noting that this algorithm requires only one update of the SINRs and the set of active sub-channels, similar to Algorithms 1.1 and 1.2. This is because the set of active sub-channels is defined at the first step, while at the second step the SINRs on the remaining sub-channels can only increase such that no more sub-channels can be switched off. The choice for $\mathcal{M}$ is not obvious in general. One possibility is full enumeration, i.e., $\mathcal{M}=\{0,1,\ldots,\nu-1\}$, which entails relatively high complexity. The criteria from Section III are not immediately applicable as N_U different channels need to be dealt with. However, the simplified adaptation via table look-up (cf. Section III-C) is perhaps best suited. Numerical results presented in the next section confirm this suggestion.

VI. Numerical Results and Discussion

In this section, we present and discuss numerical results that illustrate the performance of OFDM transmission using

CP-length adaptation and quantify the gains achievable with the proposed metrics. We assume a constant transmit PSD mask equal to -50 dBm/Hz in the 2-28 MHz range, and zero outside as it is used in the HPAV system [11] to comply with EMC rules [14]. The AWGN has a PSD of -110 dBm/Hz which is typical for indoor PLC scenarios [15]. The considered channel model (for in-home PLC applications) is described in detail in Section VI-A. In Section VI-B we first report results for CP-length adaptation using the metrics from Section III. It will be seen that the adaptation of the CP to the channel realization yields significant improvements in the system performance. Then, in Section VI-C we show that also in the case of bit-loaded OFDM large gains are attained from CP optimization. Finally, results for the multiuser case are presented in Section VI-D.

A. Statistical Channel Model

In-home PLC channels exhibit a relatively large range of signal attenuation and delay spread. Based on the results of a measurement campaign it has been proposed to categorize transmission channels into nine classes [15]. Each class is characterized by a specific average frequency dependent path loss, which has significant impact on the corresponding channel capacity. To generate statistically representative channel frequency responses according to this classification, we herein generalize the model from [23] based on a multipath model with a finite number of components (cf. e.g. [24]) and write the channel frequency response as

$$G_{\text{ch}}(f) = \sum_{i=1}^{N_p} \left(A_0 g_i + A_1 h_i f^{K_2}\right) \mathrm{e}^{-(\gamma_0+\gamma_1 f^K)d_i} \mathrm{e}^{-\mathrm{j}2\pi f \frac{d_i}{v_p}}, \quad 0 \le f \le 37.5 \text{ MHz}, \tag{28}$$

where the number of components N_p is drawn from a Poisson process with average path rate per unit length $\Lambda = 0.2$ path/m, g_i and h_i are two independent uniformly distributed random variables in [-1,1]. The variable h_i models the frequency dependent coupling that may exist between lines in the network. The path lengths d_i follow an Erlang distribution of parameter Λ and index i. The maximum path length in the network is set to L_{max}, i.e., $d_{N_p} \le L_{max}$. The speed of propagation in the medium is v_p. The constant parameters have been chosen to generate three channel classes, denoted Classes 1, 5, and 9 according to [15], in the frequency band 0-37.5 MHz and the values are reported in Table I.

Figure 1 shows the average path loss profiles and three exemplary channel frequency responses for Classes 1, 5, and 9. Class 9 channels cause relatively little signal attenuation (the average SNR is 53.6 dB), Class 5 channels show medium attenuation (the average SNR is 35.7 dB), and Class 1 channels represent scenarios with strong signal attenuation (the average SNR is 8.8 dB). The lengths of the channel responses are truncated to 209 samples, which is about $5.57\,\mu$s and identical to the CP length used in HPAV [11]. Figure 2 shows the CDF of the rms delay spread for the three classes. We notice that the average values for the rms delay spread indicated in Figure 2 are similar to the ones obtained from measurements in [15].

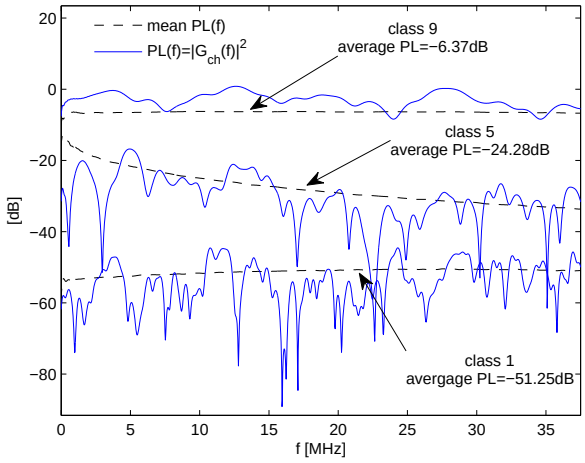

Fig. 1. Mean path loss (PL) and sample frequency response realizations for the three modeled channel classes.

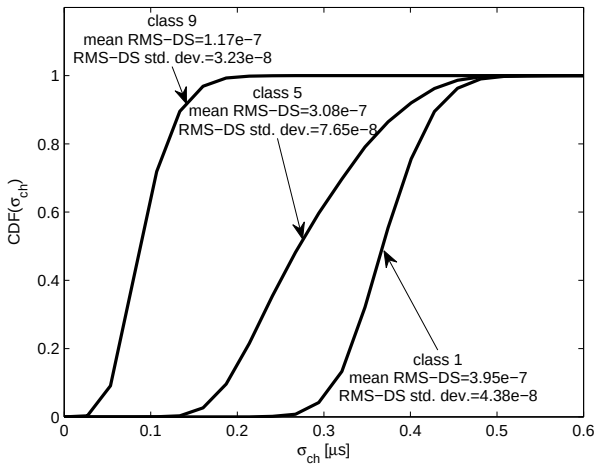

Fig. 2. CDF of the rms delay spread (RMS-DS) for the three modeled channel classes.

B. CP-Length Adaptation

The OFDM system uses $M = 384$ sub-channels[2] in the 37.5 MHz band. To satisfy the PSD mask, 266 sub-channels

[2]This is a quarter of the number of sub-channels used in HPAV. The effect of different M will be addressed in Section VI-C.

TABLE I
CHANNEL MODEL PARAMETERS FOR CHANNEL CLASSES 1, 5 AND 9.

Parameter [unit]	Class 1	Class 5	Class 9
L_{max} [m]	580	280	130
Λ [path/m]	0.2	0.2	0.2
v_p [m/s]	2e8	2e8	2e8
γ_0 [m^{-1}]	-0.0064	-0.0179	-0.0281
γ_1 [s·m^{-1}]	9.9240e-27	1.9962e-5	2.4875e-20
K	2.9843	0.3654	2.2005
K_2	0.4039	-	0.3415
A_0	2.1763e-5	0.0016	0.0108
A_1 [s^{-1}]	2.6116e-8	0	1.62e-5

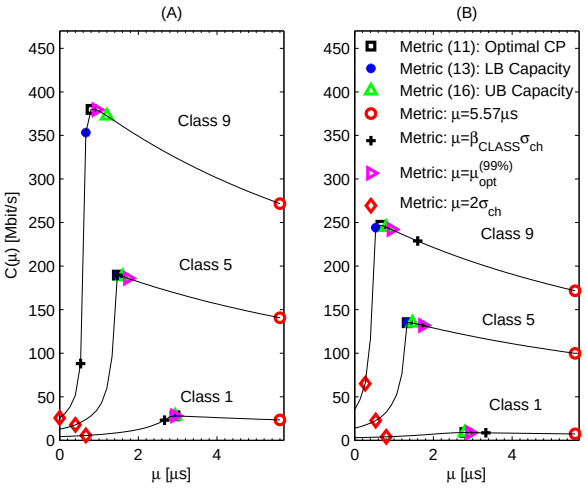

Fig. 3. Capacity $C(\mu)$ as function of the CP length μ, for different PLC channels. Markers indicate the CP length obtained from the different criteria proposed in Section III. Subplots (A) and (B) show the results for the best case and the worst case channel impulse responses, respectively.

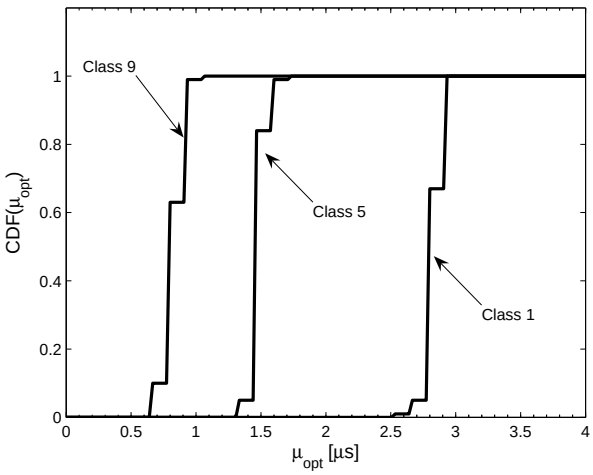

Fig. 4. CDF of capacity-optimal CP length measured for 100 channel realizations belonging to Classes 1, 5, and 9.

are active and the sub-channels at the band edges are switched off. The SNR gap Γ is fixed to 9 dB for all simulations.

Figure 3 shows the capacity $C(\mu)$ (10) as function of the CP length for different channel realizations that are representative, in terms of capacity, of the best (subplot A) and of the worst (subplot B) realization of each channel class (selected from a total of 100 realizations for each class). We also report with markers the CP lengths and the corresponding capacities obtained with the criteria proposed in Section III. We observe that, although $\mathrm{SINR}(\mu)$ monotonically increases with μ, the capacity $C(\mu)$ attains its maximum at $\mu_{\mathrm{opt}} < \max\{\mu\} = 209$ samples. Furthermore, the higher the channel attenuation (therefore the lower the SNR) the higher the value for μ_{opt} that maximizes capacity. This behavior is explained by the fact that for high SNR the performance is dominated by interference, whose mitigation requires a relatively long CP. Therefore, the capacity gains from CP shortening are expected to be more pronounced in the low SNR region, when the system is noise limited. Metrics (13) and (16) based on capacity bounds give results that are very similar to those achieved with the optimal metric. Also the adaptation based on the simplified approach described in Section III-C that uses $\mu_{\mathrm{opt,CLASS}}^{(99\%)}$ performs close to optimum. In this case, we used fixed values for the CP length as derived below.

When adaptation to the channel class is performed using the delay spread criterion (20), we observe that choosing a fixed value of β as it was advocated in [10] results in a consistently poor performance. The suggested adaptation according to (21) (the specific values for β are given below) leads to significant performance improvements. In particular, the achievable rate is close to the optimal one except for the best-case impulse response for Class 9, which is due to the rather steep shape of the rate curve in this case.

Finally, a comparison of the results in Figures 3(A) and (B) reveals that the optimal CP length is a function of both the channel class and the specific channel impulse response.

Next, we consider a set of 100 channel realizations. Figure 4 shows the measured CDF of the capacity-optimal CP length according to (11). We notice that for Class 9, the optimal CP is shorter than $0.93\,\mu$s (or 35 samples) in 99% of the cases. This value increases to $1.73\,\mu$s (or 65 samples) for the channel class 5, and to $2.93\,\mu$s (or 110 samples) for the channel class 1. Hence, we set $\mu_{\mathrm{opt},9}^{(99\%)} = 35$ for Class 9 channels, $\mu_{\mathrm{opt},5}^{(99\%)} = 65$ for Class 5 channels, and $\mu_{\mathrm{opt},1}^{(99\%)} = 110$ for Class 1 channels. Furthermore, since the average rms delay spread $\bar{\sigma}_{\mathrm{ch,CLASS}}$ for the Classes 1, 5, and 9 is respectively equal to 14.84, 11.50, and 4.42 samples, we obtain the scaling parameters $\beta_1 = 7.41$, $\beta_5 = 5.65$, and $\beta_9 = 7.92$, according to rule (21).

Given those parameters, Figures 5, 6, and 7 provide a comparison of the various CP-length criteria from Section III. We also include the capacity obtained setting the CP length equal to $5.57\mu s$. As we can see, for all channel classes, the adaptation of the CP length yields significant gains compared to a fixed CP length of $5.57\,\mu$s. For channel classes 1, 5, and 9, the capacity-optimal CP (11) respectively increases the average (over the 100 channel realizations) achievable rate by 20.7%, 32.9%, and 39.3%. Very similar results are obtained when using lower (20.5%, 31.3%, and 32.0%) and upper (20.7%, 27.6%, and 32.2%) capacity bound criteria. The delay-spread criterion (20) increases the rate by 16.6%, 18.3%, and 33.0%. Notably, the simplified adaptation with $\mu_{\mathrm{opt,CLASS}}^{(99\%)}$ results in close-to-optimal-rate gains of 20.2%, 30.7%, and 38.6% for the three channel classes.

We note that the gains depend on the number of OFDM sub-channels used for data transmission. More specifically, the larger M the lower is the impact of the CP length. This aspect will be addressed in the next section.

C. CP-Length Adaptation and Bit-Loading

We now turn our attention to the combination of CP-length adaptation with bit-loading. In addition to the system parameters specified above, we consider different number of OFDM sub-channels, namely, $M \in \{384, 768, 1536\}$ in the

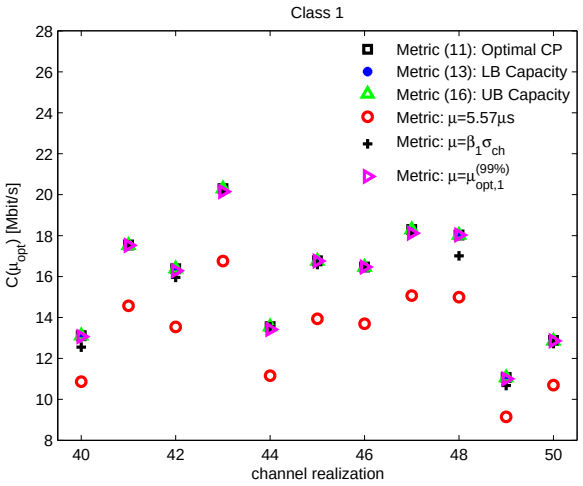

Fig. 5. Capacity $C(\mu)$ for μ optimized according to the different criteria presented in Section III as function of the channel realization belonging to Class 1. For the sake of readability, only realizations 40 to 50 are shown out of 100 realizations.

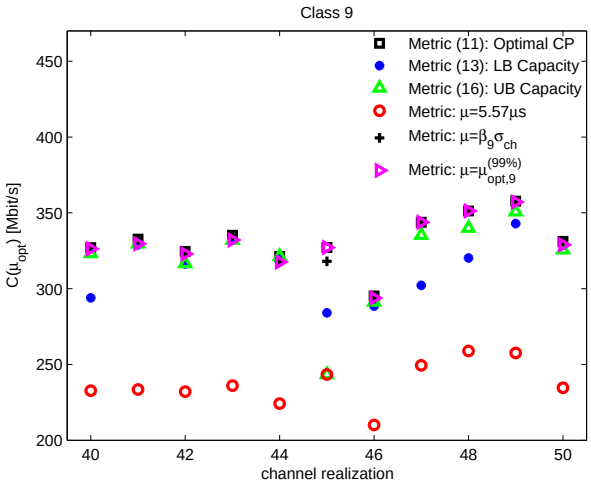

Fig. 7. Capacity $C(\mu)$ for μ optimized according to the different criteria presented in Section III as function of the channel realization belonging to Class 9. For the sake of readability, only realizations 40 to 50 are shown out of 100 realizations.

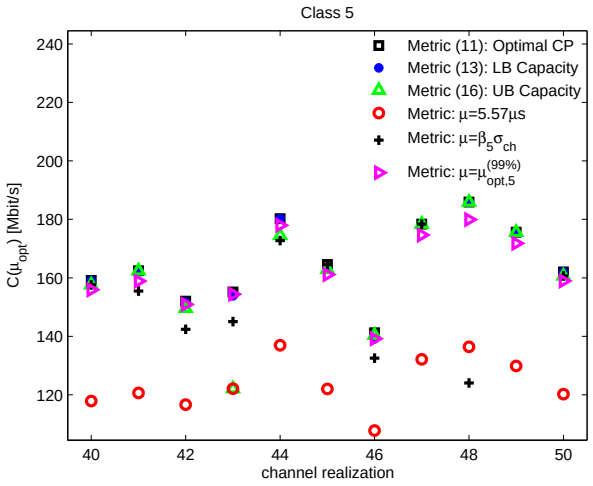

Fig. 6. Capacity $C(\mu)$ for μ optimized according to the different criteria presented in Section III as function of the channel realization belonging to Class 5. For the sake of readability, only realizations 40 to 50 are shown out of 100 realizations.

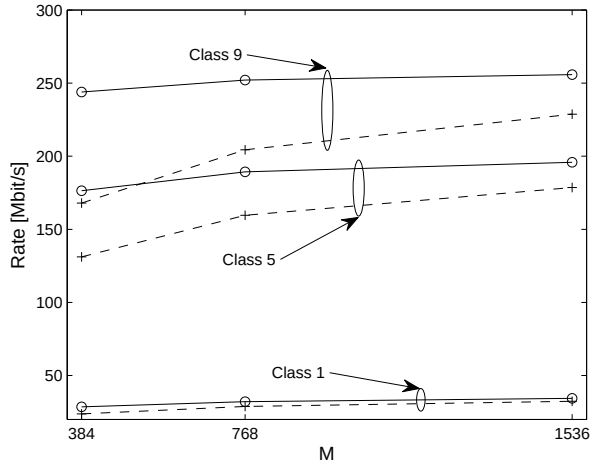

Fig. 8. Capacity $C(\mu)$ for bit loading with Algorithm 1.2 of Section IV-A and μ optimized according to the capacity-optimal criterion (11) and three different numbers of OFDM sub-channels $M \in \{384, 768, 1536\}$. For a comparison, $C(\mu)$ for bit loading and $\mu = 209$ (corresponding to $5.57\,\mu$s) is also included. The channels employed are the BeC channels of the three used classes.

0-37.5 MHz band. The number of used tones such that the PSD mask is met, follows as $M_{\text{on}} \in \{266, 532, 1065\}$, which defines $\mathbb{K}_{\text{on}}$ at the initial step of bit-loading. The constellations employed are 2-PAM and $\{4, 8, 16, 64, 256, 1024\}$-QAM, and we consider the best case (BeC) channel realizations for the three considered classes (cf. Figure 3(A)). The baseline OFDM system uses a fixed CP length of $5.57\,\mu s$.

1) Allocation of Different Constellations: Figure 8 shows the data rate achieved with Algorithm 1.2 as function of the number of OFDM sub-channels M for the BeC channels. For brevity, we assume CP optimization only using the capacity-optimal criterion (11). However, we note that very similar results are obtained with the simplified CP metrics (13), (16) and (20), as we have seen in the previous section. We observe that data rate gains range between 4 Mbit/s and 70 Mbit/s with $M = 384$ and between about 4 Mbit/s and 25 Mbit/s with $M = 1536$, which translates into significant relative rate improvements of between 6% and 39%. The smaller gains for larger M are due to a reduced impact of the CP length on data rate. It is also interesting to note that the rate for $M = 384$ and CP-length adaptation is the same or higher than the rate for $M = 1536$ and $\mu = 5.57\mu$s. Therefore, CP-length adaptation is a suitable way of lowering the implementation complexity in terms of DFT size of an OFDM system without sacrificing data rate.

2) Uniform Bit-loading: Figure 9 shows the bit rate $R(\mu, b)$ (24) as function of the CP length and the constellation used for an OFDM system with 384 sub-channels. The employed

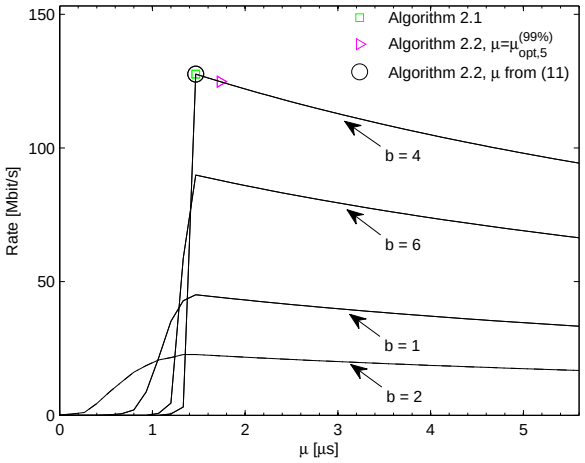

Fig. 9. Bit rate $R(\mu, b)$ from (24) as function of CP length μ and constellation size 2^b for uniform bit-loading. Markers are results from Algorithms 2.1 and 2.2 from Section IV-B. The BeC channel of Class 5 is assumed. The system uses 384 sub-channels.

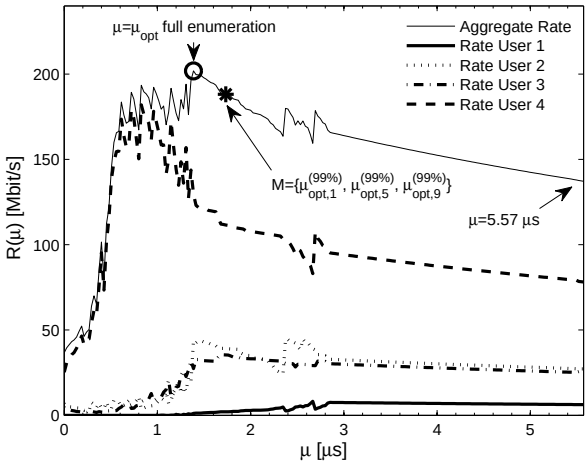

Fig. 10. Aggregate and single user rate using bit-loading as a function of the CP length. The OFDMA system has $N_U = 4$ users and $M = 384$ sub-channels. The users experience channels belonging to different classes. Markers are results from Algorithm 3 from Section V with different sets of CP length $\mathcal{M}$.

channel is the BeC channel of Class 5. In this example, we restrict the possible QAM constellations to $b \in \{1, 2, 4, 6\}$ bits per constellation point. As we can see in Figure 9, the jointly optimized CP length and constellation size obtained with Algorithm 2.1 are $\mu_{\text{opt}} = 55$ and $b_{\text{opt}} = 4$, for which a considerably enhanced rate relative to the standard choice of a CP equal to the channel length is achieved. The gain obtained with Algorithm 2.1 equals 36%. The bit rate obtained with Algorithm 2.2 and a CP length computed using the optimal metric (11) and via table look-up (cf. Section III-C) is essentially equal and very close, respectively, to that from Algorithm 2.1. This is quite remarkable and renders the low-complexity Algorithm 2.2, as well as Algorithm 1.2, attractive solutions for practical implementations. Finally, comparing the performance of Algorithm 2.2 in Figure 9 with that of Algorithm 1.2 in Figure 8, we observe that uniform bit-loading provides lower bit rate than full bit-loading.

D. CP-Length Adaptation and Resource Allocation in OFDMA

Finally, we consider OFDMA for a network of $N_U = 4$ users with a proportionally fair resource allocation, i.e., $p^{(u)} = 25$ for all users, and $M = 384$. Four different channel realizations belonging to the three channel classes presented in Section VI-A are used for the four users. The other system parameters are the same as for the single user scenario in Section VI-C. We have applied Algorithm 3 for sub-channel allocation, bit-loading, and CP-length selection. Figure 10 shows the per-user and the aggregate data rate as function of the CP length. It can be seen that the rate-maximizing CP length for the multiuser OFDMA scenario is markedly different from the default CP length of 5.57 μs. For the presented scenario, the maximal gain in aggregate rate due CP adaptation compared to a fixed CP of length 5.57μs is 44%, which has been obtained using Algorithm 3 with full enumeration of $\mathcal{M} = \{0, 1, \ldots, \nu - 1\}$. Remarkably, almost the full gain (36%) is retained if $\mathcal{M} = \{\mu^{(99\%)}_{\text{opt},1}, \mu^{(99\%)}_{\text{opt},5}, \mu^{(99\%)}_{\text{opt},9}\}$ is used, which is a promising result towards devising low-complexity resource allocation algorithms for OFDMA with CP adaption.

VII. Conclusion

In this paper, we have investigated the problem of CP-length adaptation in OFDM(A) transmission systems. We have argued that the use of a CP length adjusted to the current transmission conditions is beneficial in terms of achievable data rate. The underlying rationale is that the level of self-interference can be raised in noise-limited systems. We have considered constrained capacity as the pertinent figure of merit, and suggested a number of related, simplified criteria to select the CP length. Furthermore, we have presented four practical single-user bit-loading algorithms that take into account the CP optimization, and we have outlined extensions to the multiuser transmission scenario using OFDMA. Numerical results for typical indoor power line channels have shown significant gains due to CP-length adaptation. These gains come from (i) adjusting the CP length according to the instantanenous channel impulse response and (ii) allowing for a controlled amount of self-interference using the proposed optimization criteria.

Appendix A
Derivation of Useful and Interference Power

In this appendix, we derive the expressions in (7)-(10). To this end, we consider the received signal in the k-th sub-channel $z^{(k)}(\ell)$ in (2) for the case of zero noise, i.e., $W^{(k)}(\ell) = 0$. Considering OFDM as a DFT modulated filter

bank [6], [16] and the channel model (1) we obtain

$$z^{(k)}(\ell) = \sum_{n\in\mathbb{Z}}\sum_{m\in\mathbb{Z}}\sum_{p=0}^{\nu-1}\sum_{i\in\mathbb{K}_{\text{on}}} \alpha_p a^{(i)}(m) g(n-p-mN) \times h(\ell N-n)\mathrm{e}^{\mathrm{j}2\pi f_i(n-p)}\mathrm{e}^{-\mathrm{j}2\pi f_k n} \quad (29)$$

$$= \sum_{p=0}^{\nu-1}\sum_{m\in\mathbb{Z}}\sum_{i\in\mathbb{K}_{\text{on}}} \alpha_p a^{(i)}(m) \times r_{gh}^{(i,k)}(\ell N-mN-p)\mathrm{e}^{\mathrm{j}2\pi f_i mN}\mathrm{e}^{-\mathrm{j}2\pi f_k \ell N}, \quad (30)$$

where the window functions $g(n)$ and $h(n)$ and the cross-talk pulse-shape $r_{gh}^{(i,k)}(n)$ between sub-channels i and k are defined in (5) and (6), respectively. Making the usual assumption that data symbols in different sub-channels and OFDM frames are independent with zero mean and thus $\mathcal{E}\{a^{(i)}(m)[a^{(k)}(n)]^*\} = P_a^{(k)}\delta(i-k)\delta(n-m)$, the total signal power can be expressed as

$$P_{\text{tot}}^{(k)}(\mu) = \mathcal{E}\{|z^{(k)}(\ell)|^2\} = \sum_{p=0}^{\nu-1}\sum_{p'=0}^{\nu-1}\sum_{q\in\mathbb{Z}}\sum_{i\in\mathbb{K}_{\text{on}}} P_a^{(i)}\alpha_p\alpha_{p'}^* \times r_{gh}^{(i,k)}(qN-p)\left[r_{gh}^{(i,k)}(qN-p')\right]^*, \quad (31)$$

which has been used in (10). The useful signal power is the component of (31) for which $q = \ell - m = 0$ and $k = i$, which is the expression in (7). The remaining terms are the interference power as stated in (8).

Appendix B
Capacity Bound Criteria

In this appendix, we provide the details for the derivation of the capacity lower and upper bounds used in Section III-B. The lower bound is obtained by the application of the Bernoulli inequality [25] $(1+x)^r > 1+rx$, for $r > 1$ and $x > 0$, to the capacity formula (10), which results in

$$C(\mu) = \frac{1}{(M+\nu)T}\sum_{k\in\mathbb{K}_{\text{on}}}\log_2\left(1+\frac{\text{SINR}^{(k)}(\mu)}{\Gamma}\right)^{\frac{M+\nu}{M+\mu}} \quad (32)$$

$$> \frac{1}{(M+\nu)T}\sum_{k\in\mathbb{K}_{\text{on}}}\log_2\left(1+\frac{(M+\nu)\text{SINR}^{(k)}(\mu)}{(M+\mu)\Gamma}\right) \quad (33)$$

$$> \frac{1}{(M+\nu)T}\sum_{k\in\mathbb{K}_{\text{on}}}\log_2\left(\frac{(M+\nu)\text{SINR}^{(k)}(\mu)}{(M+\mu)\Gamma}\right) \quad (34)$$

$$= \frac{1}{(M+\nu)T}\sum_{k\in\mathbb{K}_{\text{on}}}\log_2\left(\frac{P_{\text{U}}^{(k)}(\mu)}{P_{\text{W}}}\right) - \frac{1}{(M+\nu)T}\sum_{k\in\mathbb{K}_{\text{on}}}\log_2\left(\frac{(M+\mu)\Gamma}{M+\nu}\left(1+\frac{P_{\text{I}}^{(k)}(\mu)}{P_{\text{W}}}\right)\right). \quad (35)$$

Defining $P_{\text{U,min}} = \min_{k\in\mathbb{K}_{\text{on}},\mu}\{P_{\text{U}}^{(k)}(\mu)\}$ and thus $\sum_{k\in\mathbb{K}_{\text{on}}}\log_2\left(P_{\text{U}}^{(k)}(\mu)/P_{\text{W}}\right) \geq \sum_{k\in\mathbb{K}_{\text{on}}}\log_2\left(P_{\text{U,min}}/P_{\text{W}}\right) = M_{\text{on}}\log_2\left(P_{\text{U,min}}/P_{\text{W}}\right)$, (35) can further be bounded by

$$C(\mu) > \frac{M_{\text{on}}}{(M+\nu)T}\log_2\left(P_{\text{U,min}}/P_{\text{W}}\right) - \frac{1}{(M+\nu)T}\sum_{k\in\mathbb{K}_{\text{on}}}\log_2\left(\frac{(M+\mu)\Gamma}{M+\nu}\left(1+\frac{P_{\text{I}}^{(k)}(\mu)}{P_{\text{W}}}\right)\right). \quad (36)$$

Finally, applying Jensen's inequality [26] to the second term of (36), we obtain the final result (12). It should be noted that the bound is not necessarily tight as an arbitrary factor $F > M+\nu$ could have been introduced in (32). However, the resulting criterion is independent of this factor and thus also applies to F producing the tightest possible approximation.

The application of the inequality $\log_b(x) \leq (x-1)\log_b(\mathrm{e})$, $\forall x \in \mathbb{R}^+$, $b > 1$, to $C(\mu)$ in (10) leads to the capacity upper bound

$$C(\mu) \leq \frac{1}{\Gamma(M+\mu)T}\log_2(\mathrm{e})\sum_{k\in\mathbb{K}_{\text{on}}}\text{SINR}^{(k)}(\mu)\,, \quad (37)$$

which, considering (15) is the result in (14).

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