This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2780833, IEEE Transactions on Communications

Robust Fairness Transceiver Design for a Full-Duplex MIMO Multi-Cell System

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***Abstract*—We consider optimal linear precoder and decoder designs in a multi-cell multiple-input multiple-output system, where base stations (BSs) and mobile users are both operating in full-duplex (FD) mode. Existing works on FD cellular systems focus on the maximization of overall throughput, which can result in unfairness between uplink and downlink channels depending on the self-interference power and inter-user interference levels. Therefore, to introduce fairness, in this paper, we consider the transmit and receive beamforming designs that maximize the harmonic-sum of signal-to-interference-plus-noise ratios (SINRs) in the uplink and downlink channels. We propose a low-complexity alternating optimization algorithm which converges to a stationary point. Moreover, in order to address practical system design aspects, we consider the transceiver design that enforces robustness against imperfect channel state information (CSI) while providing fair performance among the users. To this end, we formulate an optimization problem that maximizes the worst-case SINR among all users under norm-bounded CSI errors. We devise a low-complexity iterative algorithm based on alternating optimization and semidefinite relaxation techniques. Numerical results verify the advantages of incorporating FD mode into cellular systems, as well as practical issues such as CSI uncertainty and fairness performance.**

Keywords*—*Fairness, full-duplex, MIMO, multi-cell, multi-user, self interference, transceiver design.

I. Introduction

THE increasing demand for high data rates requires powerful communication technologies utilizing the spectrum more efficiently. Current half-duplex (HD) wireless communication systems employ duplexing, which uses two orthogonal channels to transmit and receive. However, full-duplex (FD) wireless communication can potentially double spectral efficiency by enabling transmission and reception at the same time and in the same frequency band. It therefore has recently drawn attention in the research community [1]-[25], as a technique to meet the spectral efficiency targets for the next generation wireless communication systems.

The limiting factor on the performance of FD systems is the strong self-interference at the front-end of the receiver created by the signal leakage from the transmit path of an FD device. Unless this self-interference is canceled satisfactorily, a radio transceiver cannot perform FD operation. Promising results from experimental research that demonstrate the feasibility of FD transmission have been presented in [1]-[6]. However, due to imperfections of radio devices, such as amplifier non-linearity, phase noise, and I/Q channel imbalance, the self-interference cannot be canceled completely. Therefore, optimization of FD transmission systems under consideration of this residual self-interference is an active research area [7]-[25].

In this paper, we make an attempt to understand the benefits that can be achieved by the use of FD-based transceivers in a multi-cell multi-user multiple-input multiple-output (MIMO) system, where all the nodes in the system operate in FD mode. In addition to the well-known interference in traditional multi-cell HD systems, from uplink users to base-stations (BSs) and from BSs to downlink users, incorporating FD empowered BSs and users introduces new sources of interference due to simultaneous transmission and reception at all nodes in the system. In particular, cellular FD systems experience 1) self-interference at all FD BSs and users, 2) interference among adjacent BSs, i.e., inter-base-station interference, and 3) inter-user interference among all the users in all cells. The additional interference notably complicates system optimization.

FD communication has been investigated for single cell systems in [7]-[13]. However, HD users are assumed and signal distortions in FD systems caused by non-ideal amplifiers, oscillators, analog-to-digital converters (ADCs), and digital-to-analog converters (DACs) are not taken into account. In [15] and [16], the authors have developed scheduling and power control algorithms for multi-cell FD networks. Furthermore, a stochastic geometry approach for performance characterization of FD multi-cell systems has been considered in [17]-[18]. In contrast to [15]-[18], which assume that all nodes are equipped with a single antenna, we consider a MIMO system and beamformer design. The authors in [19] have investigated the performance of multi-cell MIMO case based on simulations due to the complex scheduling and power control solutions. In [20], the authors have studied the degrees of freedom region under full BSs coordination. However, the proposed approach is based on a full coordination and a complex transmission method, which is hard to achieve in practice. The system performance of FD multi-cell systems in the asymptotic regime of infinitely many BS antennas has been analyzed in [21].

Beamformer design for the sum-rate maximization in FD multi-cell cellular systems has been studied in [22]. However,

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This work has been supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

when sum-rate maximization problems are considered in FD systems, as the self-interference power increases, it starts overwhelming the desired signals coming from the uplink users, which reduces the achievable rate in the uplink channel [8]. Therefore, reducing the transmit power in the uplink channel and concentrating on the downlink channel is more beneficial. In this case, uplink users are not served, i.e., all the resources are devoted for the downlink transmission, which results in unfairness. The situation is compounded in multi-cell FD systems, where as mentioned above, additional interference sources exist, which will degrade the performance of the users, especially for the ones at the cell-edge. Therefore, it is important to ensure satisfactory performance among all the users in the network.

Unfortunately, adding a fairness constraint by way of a throughput guarantee leads to the inherently difficult non-convex throughput constraints. In this case, even finding a feasible point is already a challenging task because the feasible set is non-convex and disconnected [26], [27]. In particular, the formulation and initialization of the problem with quality-of-service (QoS) constraints are subject to two feasibility issues. First, too high QoS constraints may cause the problem to become infeasible due to the restricted power budget. Secondly, it requires strictly feasible initialization, which is difficult to efficiently manage in a decentralized manner. In particular, a preliminary feasibility check must be employed for the minimum rate requirements, where feasible QoS levels are first determined. If the problem is not feasible to begin with, the algorithm might start to oscillate among a group of non-feasible rate constraints.

One possibility to achieve network-wide fairness is to maximize the minimum signal-to-interference-plus-noise ratio (SINR) among all the users in the network. This is same as equalizing the SINR performance of all users, and thus it is a strategy for enforcing the desired level of fairness in the network. The authors in [28], [29] have shown that max-min problems can be solved by a sequence of second order cone programs (SOCPs), which have a high computational complexity and require a centralized algorithm. In this paper, our goal is to design a low complexity distributed algorithm that achieves fairness. As discussed in [30], [31], maximizing the harmonic-sum of user rates, an approximation of the max-min SINR problem, prioritizes the cell-edge users, leading to increased cell-edge user throughput. Therefore, in the first part of this paper, we will propose a transceiver design that maximizes the harmonic-sum of SINRs for MIMO systems, and derive an iterative low-complexity distributed algorithm that finds local maxima of the associated non-convex optimization problem.

In the second part of the paper, we extend our design to provide resilience against inaccuracies of channel state information (CSI). Such robust FD system designs have been studied for cognitive radio transmission in [32], [33], for physical layer security in [34], [35], and for single-cell systems in [36]. However, prior work on FD multi-cell systems assumes the availability of perfect CSI at the transmitters, which is practically impossible due to the inaccurate channel estimation. To this end, the CSI errors are often modeled as Gaussian random variables [37], and the robustness can be provided in the statistical sense. Alternatively, another way to achieve robustness is by worst-case optimization, which designs the system to operate under the worst-case channel condition if the CSI uncertainty is bounded [38]-[42]. We adopt this second approach and propose a low complexity iterative algorithm based on semidefinite relaxation (SDR) technique to achieve max-min fairness.

Before proceeding we note that we are considering the most general scenario of FD BSs communicating with FD users in a multi-cell environment. The scenarios of i) FD BSs and HD users in a multi-cell, ii) FD BS and FD users in a single-cell, iii) FD BS and HD users in a single cell environments, as well as iv) HD BS and HD users can be recovered as special cases of our proposed algorithms.

A. Notations

The following notations are used in this paper. Matrices and vectors are denoted as bold capital and lowercase letters, respectively. $(\cdot)^T$ is the transpose; $(\cdot)^H$ is the conjugate transpose. $\mathbb{E}\{\cdot\}$ means the statistical expectation; $\mathbf{I}_N$ is the N by N identity matrix; $\mathbf{0}_{N\times M}$ is the N by M zero matrix; $\mathrm{tr}(\cdot)$ is the trace; $|\cdot|$ is the determinant; $\|\cdot\|_F$ is the Frobenius norm; $\mathrm{diag}\left(\mathbf{A}\right)$ is the diagonal matrix with the same diagonal elements as $\mathbf{A}$, $\mathrm{rank}\left(\mathbf{A}\right)$ denotes the rank of matrix $\mathbf{A}$, $\left[\mathbf{A}\right]_{nn}$ denotes the nth row and nth column of matrix $\mathbf{A}$. $\mathcal{CN}\left(\mu,\sigma^2\right)$ denotes a complex Gaussian distribution with mean μ and variance σ^2. $\mathbb{C}^{N\times M}$ denotes the set of complex matrices with a dimension of N by M, $\perp$ denotes the statistical independence, $\otimes$ denotes the Kronecker product, and finally $\circ$ denotes the Hadamard product.

II. System Model

In this section, we describe the system model of an FD multi-cell multi-user MIMO system as seen in Fig. 1. We consider a K cell FD system, where BS $k,\ k=1,\ldots,K$ is equipped with M_k transmit and N_k receive antennas, and serves I_k users in cell k. We denote i_k to be the ith user in cell k with M_{i_k} transmit and N_{i_k} receive antennas. We define the set of BSs as $\mathcal{K}=\{k\in\{1,\ldots,K\}\}$ and users as

$$\mathcal{I}=\{i_k \mid k\in\{1,2,\ldots,K\},\ i\in\{1,2,\ldots,I_k\}\}.$$

We also take into account the limited dynamic range (DR) at the FD nodes. Limited-DR is caused by non-ideal amplifiers, oscillators, analog-to-digital converters (ADCs), and digital-to-analog converters (DACs). We adopt the limited DR model in [14], which has also been used in [11], [12], [13], [22], [23], [24], [25] and validated in [43], [44]. Accordingly, at each receive antenna an additive white Gaussian "receiver distortion" with a variance equal to β times the power of the undistorted received signal at that antenna is applied, and at each transmit antenna an additive white Gaussian "transmitter noise" with a variance equal to κ times the power of the intended transmit signal is applied. Note that κ^{-1} and β^{-1} characterize the transmitter and receiver DR, respectively. In particular, $\kappa\left(\beta\right)$ characterizes the level of transmit (receive) imperfection. For example, $\kappa=0\left(\beta=0\right)$ corresponds to the

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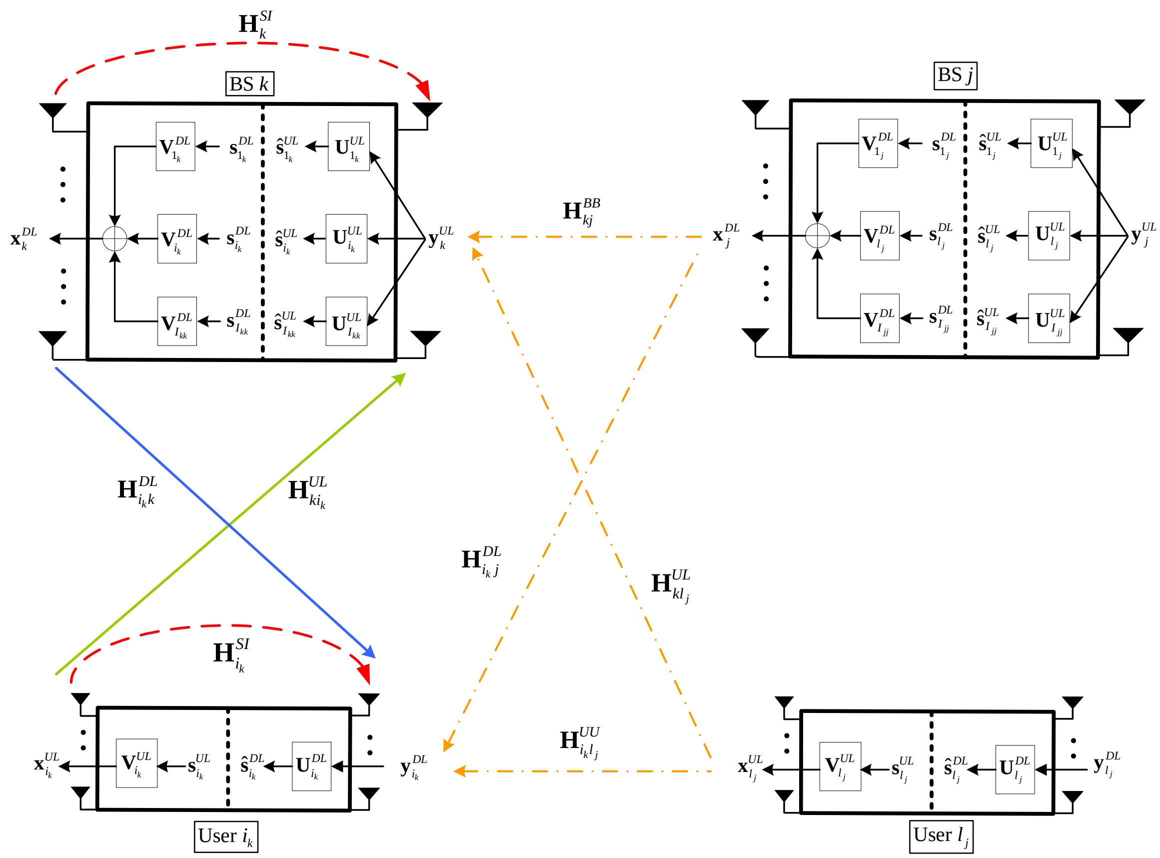

Fig. 1. Full-duplex MIMO multi-cell system. Dashed arrows denote the self-interference and the dash-dotted arrows denote the interference between different nodes.

conventional assumption of perfect transmit (receive) radio frequency (RF) chains. The quality of transmit (receive) RF chains degrades as $\kappa\,(\beta)$ increases. The experimental results in [43], [44] have shown that the independent Gaussian distortion noise model closely captures the joint effect of imperfect components in transmit and receive RF chains.

Let us denote $\mathbf{H}_{kl_j}^{\text{UL}} \in \mathbb{C}^{N_k \times M_{l_j}}$ as the channel between BS k and user l_j in the uplink, $\mathbf{H}_{i_k j}^{\text{DL}} \in \mathbb{C}^{N_{i_k} \times M_j}$ as the channel between BS j and user i_k in the downlink, $\mathbf{H}_{i_k l_j}^{\text{UU}} \in \mathbb{C}^{N_{i_k} \times M_{l_j}}$ as the channel from the user l_j to the user i_k, $\mathbf{H}_{kj}^{\text{BB}} \in \mathbb{C}^{N_k \times M_j}$ as the interference channel from the BS j to the BS k, $\mathbf{H}_k^{\text{SI}} \in \mathbb{C}^{N_k \times M_k}$ as the self-interference channel from the transmit antennas to the receive antennas of BS k, and $\mathbf{H}_{i_k}^{\text{SI}} \in \mathbb{C}^{N_{i_k} \times M_{i_k}}$ as the self-interference channel from the transmitter antennas to the receiver antennas of user i_k. We assume that CSI is only *locally* available at the transmitters, i.e., the transmitters are able to obtain the knowledge of channel coefficients which are directly connected to them [45]-[51]. Compared to the requirement of CSI for all links, i.e., global CSI, used in [29], this not only reduces signaling overhead but also permits a distributed transceiver optimization. The channel coefficients as well as the interference-plus-noise covariance matrix are estimated at the receiver of each link and fed back to the transmitters via wireless broadcast [47].

The source symbol transmitted by user i_k in the uplink channel with length $d_{i_k}^{\text{UL}}$ is denoted as $\mathbf{s}_{i_k}^{\text{UL}} \in \mathbb{C}^{d_{i_k}^{\text{UL}} \times 1}$. It is assumed that the symbols are independent and identically distributed (i.i.d.) with unit power, i.e., $\mathbb{E}\left[\mathbf{s}_{i_k}^{\text{UL}} \left(\mathbf{s}_{i_k}^{\text{UL}}\right)^H\right] = \mathbf{I}_{d_{i_k}^{\text{UL}}}$. Similarly, the transmit symbol for the user i_k in the downlink channel with length $d_{i_k}^{\text{DL}}$ is denoted by $\mathbf{s}_{i_k}^{\text{DL}} \in \mathbb{C}^{d_{i_k}^{\text{DL}} \times 1}$, with $\mathbb{E}\left[\mathbf{s}_{i_k}^{\text{DL}} \left(\mathbf{s}_{i_k}^{\text{DL}}\right)^H\right] = \mathbf{I}_{d_{i_k}^{\text{DL}}}$. Denoting the transmit beamforming matrix for the data streams of user i_k as $\mathbf{V}_{i_k}^{\text{UL}} = \left[\mathbf{v}_{i_k,1}^{\text{UL}}, \ldots, \mathbf{v}_{i_k,d_{i_k}^{\text{UL}}}^{\text{UL}}\right] \in \mathbb{C}^{M_{i_k} \times d_{i_k}^{\text{UL}}}$ in the uplink channel, and $\mathbf{V}_{i_k}^{\text{DL}} = \left[\mathbf{v}_{i_k,1}^{\text{DL}}, \ldots, \mathbf{v}_{i_k,d_{i_k}^{\text{DL}}}^{\text{DL}}\right] \in \mathbb{C}^{M_k \times d_{i_k}^{\text{DL}}}$ in the downlink channel, the transmitted signal of the user i_k and that of the

BS k can be written, respectively, as

$$\mathbf{x}_{i_k}^{\text{UL}} = \mathbf{V}_{i_k}^{\text{UL}} \mathbf{s}_{i_k}^{\text{UL}}, \tag{1}$$

$$\mathbf{x}_{k}^{\text{DL}} = \sum_{i=1}^{I_k} \mathbf{V}_{i_k}^{\text{DL}} \mathbf{s}_{i_k}^{\text{DL}}. \tag{2}$$

The signal received by the BS k and that received by the user i_k can be written, respectively, as

$$\mathbf{y}_k^{\text{UL}} = \sum_{j=1}^{K}\sum_{l=1}^{I_j} \mathbf{H}_{kl_j}^{\text{UL}} \mathbf{x}_{l_j}^{\text{UL}} + \mathbf{H}_k^{\text{SI}} \left(\mathbf{x}_k^{\text{DL}} + \mathbf{c}_k^{\text{DL}}\right) + \sum_{j=1, j\neq k}^{K} \mathbf{H}_{kj}^{\text{BB}} \mathbf{x}_j^{\text{DL}} + \mathbf{e}_k^{\text{UL}} + \mathbf{n}_k^{\text{UL}}, \tag{3}$$

$$\mathbf{y}_{i_k}^{\text{DL}} = \sum_{j=1}^{K} \mathbf{H}_{i_k j}^{\text{DL}} \mathbf{x}_j^{\text{DL}} + \mathbf{H}_{i_k}^{\text{SI}} \left(\mathbf{x}_{i_k}^{\text{UL}} + \mathbf{c}_{i_k}^{\text{UL}}\right) + \sum_{(l,j)\neq(i,k)} \mathbf{H}_{i_k l_j}^{\text{UU}} \mathbf{x}_{l_j}^{\text{UL}} + \mathbf{e}_{i_k}^{\text{DL}} + \mathbf{n}_{i_k}^{\text{DL}}, \tag{4}$$

where $\mathbf{n}_k^{\text{UL}} \in \mathbb{C}^{N_k \times 1}$ and $\mathbf{n}_{i_k}^{\text{DL}} \in \mathbb{C}^{N_{i_k} \times 1}$ denote the additive white Gaussian noise (AWGN) vector with zero mean and unit covariance matrix at the BS k and user i_k, respectively. Moreover, in (4), $\mathbf{c}_{i_k}^{\text{UL}} \in \mathbb{C}^{M_{i_k}}$ is the signal distortion at the transmitter antennas of user i_k, which models the effect of limited transmitter DR, and closely approximates the effects of additive power-amplifier noise, non-linearities in the DAC and phase noise. It is modeled as [14]

$$\mathbf{c}_{i_k}^{\text{UL}} \sim \mathcal{CN}\left(\mathbf{0}, \kappa \,\text{diag}\left(\mathbf{V}_{i_k}^{\text{UL}} \left(\mathbf{V}_{i_k}^{\text{UL}}\right)^H\right)\right), \tag{5}$$

$$\mathbf{c}_{i_k}^{\text{UL}} \perp \mathbf{x}_{i_k}^{\text{UL}}. \tag{6}$$

Finally, in (4), $\mathbf{e}_{i_k}^{\text{DL}} \in \mathbb{C}^{N_{i_k}}$ is the additive distortion at the receiver antennas of user i_k, which models the effect of limited receiver DR, and closely approximates the combined effects of additive gain-control noise, non-linearities in the ADC and phase noise. It is modeled as [14]

$$\mathbf{e}_{i_k}^{\text{DL}} \sim \mathcal{CN}\left(\mathbf{0}, \beta \text{diag}\left(\mathbf{\Phi}_{i_k}^{\text{DL}}\right)\right), \tag{7}$$

$$\mathbf{e}_{i_k}^{\text{DL}} \perp \mathbf{z}_{i_k}^{\text{DL}}, \tag{8}$$

where $\mathbf{\Phi}_{i_k}^{\text{DL}} = \text{Cov}\{\mathbf{z}_{i_k}^{\text{DL}}\}$ and $\mathbf{z}_{i_k}^{\text{DL}}$ is the undistorted received vector at the user i_k, i.e., $\mathbf{z}_{i_k}^{\text{DL}} = \mathbf{y}_{i_k}^{\text{DL}} - \mathbf{e}_{i_k}^{\text{DL}}$. In (3), $\mathbf{c}_k^{\text{DL}}$ and $\mathbf{e}_k^{\text{UL}}$ are the transmitter and receiver distortion at the BS k, which are modeled similar to (5)-(6) and (7)-(8), respectively.

Since the FD nodes know their own transmit signals and self-interference channel, the terms $\mathbf{H}_k^{\text{SI}}\mathbf{x}_k^{\text{DL}}$ and $\mathbf{H}_{i_k}^{\text{SI}}\mathbf{x}_{i_k}^{\text{UL}}$ can be cancelled from the received signal $\mathbf{y}_k^{\text{UL}}$ at the kth BS and $\mathbf{y}_{i_k}^{\text{UL}}$ at the i_kth user, respectively [14]. The self-interference free received signals are denoted as $\tilde{\mathbf{y}}_k^{\text{UL}}$ and $\tilde{\mathbf{y}}_{i_k}^{\text{UL}}$, respectively.

The received signals are processed by linear decoders, denoted as $\mathbf{U}_{i_k}^{\text{UL}} = \left[\mathbf{u}_{i_k,1}^{\text{UL}}, \ldots, \mathbf{u}_{i_k,d_{i_k}^{\text{UL}}}^{\text{UL}}\right] \in \mathbb{C}^{N_k \times d_{i_k}^{\text{UL}}}$, and $\mathbf{U}_{i_k}^{\text{DL}} = \left[\mathbf{u}_{i_k,1}^{\text{DL}}, \ldots, \mathbf{u}_{i_k,d_{i_k}^{\text{DL}}}^{\text{DL}}\right] \in \mathbb{C}^{N_{i_k} \times d_{i_k}^{\text{DL}}}$ by the BS k and the user i_k, respectively. Therefore, the estimates of data streams of user i_k in the uplink and downlink channels are given as

$$\hat{\mathbf{s}}_{i_k}^{\text{UL}} = \left(\mathbf{U}_{i_k}^{\text{UL}}\right)^H \tilde{\mathbf{y}}_k^{\text{UL}}, \quad \hat{\mathbf{s}}_{i_k}^{\text{DL}} = \left(\mathbf{U}_{i_k}^{\text{DL}}\right)^H \tilde{\mathbf{y}}_{i_k}^{\text{DL}}. \tag{9}$$

Using these estimates, the SINR values of the m-th stream associated with user i_k in the uplink and downlink channel can be expressed as in (10) and (11), respectively, shown on the following page. Here, $\mathbf{\Sigma}_{i_k}^{\text{UL}}(\mathbf{v})$ and $\mathbf{\Sigma}_{i_k}^{\text{DL}}(\mathbf{v})$ denote the covariance matrix of aggregate interference-plus-noise for the user i_k in the uplink and downlink channels, and can be approximated, under $\beta \ll 1$ and $\kappa \ll 1$, as in (12) and (13), respectively, given at the bottom of the following page. Here, the variable $\mathbf{v}$ denotes the stacked vectors of all transmit filters in the uplink and downlink channel. Note that despite the fact that $\mathbf{\Sigma}_{i_k}^{\text{UL}}(\mathbf{v})$ and $\mathbf{\Sigma}_{i_k}^{\text{DL}}(\mathbf{v})$ depend on non-local parameters such as channel matrices and pre-coding matrices at other links, these covariance matrices can be determined locally provided that there is a sufficient coherence time window within which all channel matrices and pre-coding matrices do not change [45], [46].

We note that there are three types of CSI involved in the system design: I) BS to user ($\mathbf{H}_{i_k j}^{\text{DL}}$) or user to BS ($\mathbf{H}_{kl_j}^{\text{UL}}$) channels, II) BS to BS ($\mathbf{H}_{kj}^{\text{BB}}$) channels, and III) user to user ($\mathbf{H}_{i_k l_j}^{\text{UU}}$) channels. Considering, for example in the 3rd Generation Partnership Project (3GPP) Long-Term Evolution (LTE) system, each BS broadcasts the cell-specific reference signal, including its cell identity [52]. Therefore, BS to user channels can be estimated from the received reference signal at each user. Users then report the CSI via control and/or shared channels to the BSs, which allows the estimation of type I channels [52]. The same cell-specific reference signal can be used at other BSs to estimate the type II channels [15]. The type III channels are difficult to obtain as there is no direct signaling between users. However, the channel estimation between users can be facilitated via neighbor discovery at each user through the use of sounding reference signals in 3GPP LTE system [53]. Similar mechanisms to estimate channels between users have been proposed for device-to-device communications [54]. We note that the acquisition of CSI leads to imperfection. To this end, we have proposed a transceiver design to enforce robustness against imperfect CSI. The perfect CSI scenario is the ideal case, which can be considered as a performance bound for the imperfect CSI.

To simplify the presentation, we will use the following notation in the rest of the paper:

$$\mathbf{H}_{i_k}^{\text{X}} = \begin{cases} \mathbf{H}_{ki_k}^{\text{UL}}, & \text{if X} = \text{UL}, \\ \mathbf{H}_{i_k k}^{\text{DL}}, & \text{if X} = \text{DL}. \end{cases} \tag{14}$$

III. Fairness Design under Perfect CSI

A. Problem Formulation

The problem that maximizes the minimum SINR of users can be formulated as:

$$\max_{\mathbf{v},\mathbf{u}} \min_{\forall i_k\in\mathcal{I}, m^{\mathrm{X}}\in\mathcal{M}} \gamma_{i_k,m}^{\mathrm{X}}\left(\mathbf{v},\mathbf{u}_{i_k,m}^{\mathrm{X}}\right) \quad (15)$$

$$\text{s.t.} \quad \sum_{m=1}^{d_{i_k}^{\mathrm{UL}}} \left(\mathbf{v}_{i_k,m}^{\mathrm{UL}}\right)^H \mathbf{v}_{i_k,m}^{\mathrm{UL}} \le P_{i_k},\ i_k\in\mathcal{I}, \quad (16)$$

$$\sum_{i=1}^{I_k}\sum_{m=1}^{d_{i_k}^{\mathrm{DL}}} \left(\mathbf{v}_{i_k,m}^{\mathrm{DL}}\right)^H \mathbf{v}_{i_k,m}^{\mathrm{DL}} \le P_k,\ k\in\mathcal{K} \quad (17)$$

where P_{i_k} in (16) is the transmit power constraint at the user i_k, and P_k in (17) is the total power constraint at the BS k. Here, the optimization variable $\mathbf{u}$ denotes the stacked vectors of all receive filters in the uplink and downlink channel. Furthermore, $\mathcal{M}$ denotes the set of all uplink and downlink data streams, i.e., $\mathcal{M}=\left\{m=1,\ldots,d_{i_k}^{\mathrm{X}},\ \mathrm{X}=\{\mathrm{UL},\ \mathrm{DL}\}\right\}$. The problem formulation (15)-(17) has two significant shortcomings. First, it requires an iterative search for a SOCP feasibility test [29], [55], which has a high computational complexity and necessitates a centralized implementation. Second, it ignores the overall throughput of the cell for the purpose of maintaining fairness. In the following, we address both those issues.

$$\gamma_{i_k,m}^{\mathrm{UL}}\left(\mathbf{v},\mathbf{u}_{i_k,m}^{\mathrm{UL}}\right) = \frac{\left|\left(\mathbf{u}_{i_k,m}^{\mathrm{UL}}\right)^H \mathbf{H}_{ki_k}^{\mathrm{UL}}\mathbf{v}_{i_k,m}^{\mathrm{UL}}\right|^2}{\left(\mathbf{u}_{i_k,m}^{\mathrm{UL}}\right)^H \underbrace{\left(\boldsymbol{\Sigma}_{i_k}^{\mathrm{UL}}(\mathbf{v}) + \sum_{n=1}^{d_{i_k}^{\mathrm{UL}}} \mathbf{H}_{ki_k}^{\mathrm{UL}}\mathbf{v}_{i_k,n}^{\mathrm{UL}}\left(\mathbf{v}_{i_k,n}^{\mathrm{UL}}\right)^H\left(\mathbf{H}_{ki_k}^{\mathrm{UL}}\right)^H\right)}_{\mathbf{Q}_{i_k}^{\mathrm{UL}}(\mathbf{v})} \mathbf{u}_{i_k,m}^{\mathrm{UL}} - \left|\left(\mathbf{u}_{i_k,m}^{\mathrm{UL}}\right)^H \mathbf{H}_{ki_k}^{\mathrm{UL}}\mathbf{v}_{i_k,m}^{\mathrm{UL}}\right|^2}, \quad (10)$$

$$\gamma_{i_k,m}^{\mathrm{DL}}\left(\mathbf{v},\mathbf{u}_{i_k,m}^{\mathrm{DL}}\right) = \frac{\left|\left(\mathbf{u}_{i_k,m}^{\mathrm{DL}}\right)^H \mathbf{H}_{i_kk}^{\mathrm{DL}}\mathbf{v}_{i_k,m}^{\mathrm{DL}}\right|^2}{\left(\mathbf{u}_{i_k,m}^{\mathrm{DL}}\right)^H \underbrace{\left(\boldsymbol{\Sigma}_{i_k}^{\mathrm{DL}}(\mathbf{v}) + \sum_{n=1}^{d_{i_k}^{\mathrm{DL}}} \mathbf{H}_{i_kk}^{\mathrm{DL}}\mathbf{v}_{i_k,n}^{\mathrm{DL}}\left(\mathbf{v}_{i_k,n}^{\mathrm{DL}}\right)^H\left(\mathbf{H}_{i_kk}^{\mathrm{DL}}\right)^H\right)}_{\mathbf{Q}_{i_k}^{\mathrm{DL}}(\mathbf{v})} \mathbf{u}_{i_k,m}^{\mathrm{DL}} - \left|\left(\mathbf{u}_{i_k,m}^{\mathrm{DL}}\right)^H \mathbf{H}_{i_kk}^{\mathrm{DL}}\mathbf{v}_{i_k,m}^{\mathrm{DL}}\right|^2}. \quad (11)$$

$$\begin{aligned}\boldsymbol{\Sigma}_{i_k}^{\mathrm{UL}}(\mathbf{v}) = &\sum_{(l,j)\neq(i,k)}\sum_{n=1}^{d_{l_j}^{\mathrm{UL}}} \mathbf{H}_{kl_j}^{\mathrm{UL}}\mathbf{v}_{l_j,n}^{\mathrm{UL}}\left(\mathbf{v}_{l_j,n}^{\mathrm{UL}}\right)^H\left(\mathbf{H}_{kl_j}^{\mathrm{UL}}\right)^H + \sum_{j=1,j\neq k}^{K}\sum_{l=1}^{I_j}\sum_{n=1}^{d_{l_j}^{\mathrm{DL}}} \mathbf{H}_{kj}^{\mathrm{BB}}\mathbf{v}_{l_j,n}^{\mathrm{DL}}\left(\mathbf{v}_{l_j,n}^{\mathrm{DL}}\right)^H\left(\mathbf{H}_{kj}^{\mathrm{BB}}\right)^H + \mathbf{I}_{N_k}\\ &+\sum_{l=1}^{I_k}\sum_{n=1}^{d_{l_k}^{\mathrm{DL}}} \kappa\mathbf{H}_k^{\mathrm{SI}}\mathrm{diag}\left(\mathbf{v}_{l_k,n}^{\mathrm{DL}}\left(\mathbf{v}_{l_k,n}^{\mathrm{DL}}\right)^H\right)\left(\mathbf{H}_k^{\mathrm{SI}}\right)^H + \beta\sum_{j=1}^{K}\sum_{l=1}^{I_j}\sum_{n=1}^{d_{l_j}^{\mathrm{UL}}}\mathrm{diag}\left(\mathbf{H}_{kl_j}^{\mathrm{UL}}\mathbf{v}_{l_j,n}^{\mathrm{UL}}\left(\mathbf{v}_{l_j,n}^{\mathrm{UL}}\right)^H\left(\mathbf{H}_{kl_j}^{\mathrm{UL}}\right)^H\right)\\ &+\beta\sum_{j=1,j\neq k}^{K}\sum_{l=1}^{I_j}\sum_{n=1}^{d_{l_j}^{\mathrm{DL}}}\mathrm{diag}\left(\mathbf{H}_{kj}^{\mathrm{BB}}\mathbf{v}_{l_j,n}^{\mathrm{DL}}\left(\mathbf{v}_{l_j,n}^{\mathrm{DL}}\right)^H\left(\mathbf{H}_{kj}^{\mathrm{BB}}\right)^H\right) + \beta\sum_{l=1}^{I_k}\sum_{n=1}^{d_{l_k}^{\mathrm{DL}}}\mathrm{diag}\left(\mathbf{H}_k^{\mathrm{SI}}\mathbf{v}_{l_k,n}^{\mathrm{DL}}\left(\mathbf{v}_{l_k,n}^{\mathrm{DL}}\right)^H\left(\mathbf{H}_k^{\mathrm{SI}}\right)^H\right), \quad (12)\end{aligned}$$

$$\begin{aligned}\boldsymbol{\Sigma}_{i_k}^{\mathrm{DL}}(\mathbf{v}) = &\sum_{(l,j)\neq(i,k)}\sum_{n=1}^{d_{l_j}^{\mathrm{DL}}} \mathbf{H}_{i_kj}^{\mathrm{DL}}\mathbf{v}_{l_j,n}^{\mathrm{DL}}\left(\mathbf{v}_{l_j,n}^{\mathrm{DL}}\right)^H\left(\mathbf{H}_{i_kj}^{\mathrm{DL}}\right)^H + \sum_{(l,j)\neq(i,k)}\sum_{n=1}^{d_{l_j}^{\mathrm{UL}}} \mathbf{H}_{i_kl_j}^{\mathrm{UU}}\mathbf{v}_{l_j,n}^{\mathrm{UL}}\left(\mathbf{v}_{l_j,n}^{\mathrm{UL}}\right)^H\left(\mathbf{H}_{i_kl_j}^{\mathrm{UU}}\right)^H\\ &+\kappa\mathbf{H}_{i_k}^{\mathrm{SI}}\sum_{n=1}^{d_{i_k}^{\mathrm{UL}}}\mathrm{diag}\left(\mathbf{v}_{i_k,n}^{\mathrm{UL}}\left(\mathbf{v}_{i_k,n}^{\mathrm{UL}}\right)^H\right)\left(\mathbf{H}_{i_k}^{\mathrm{SI}}\right)^H + \beta\sum_{j=1}^{K}\sum_{l=1}^{I_j}\sum_{n=1}^{d_{l_j}^{\mathrm{DL}}}\mathrm{diag}\left(\mathbf{H}_{i_kj}^{\mathrm{DL}}\mathbf{v}_{l_j,n}^{\mathrm{DL}}\left(\mathbf{v}_{l_j,n}^{\mathrm{DL}}\right)^H\left(\mathbf{H}_{i_kj}^{\mathrm{DL}}\right)^H\right)\\ &+\beta\sum_{(l,j)\neq(i,k)}\sum_{n=1}^{d_{l_j}^{\mathrm{UL}}}\mathrm{diag}\left(\mathbf{H}_{i_kl_j}^{\mathrm{UU}}\mathbf{v}_{l_j,n}^{\mathrm{UL}}\left(\mathbf{v}_{l_j,n}^{\mathrm{UL}}\right)^H\left(\mathbf{H}_{i_kl_j}^{\mathrm{UU}}\right)^H\right) + \beta\sum_{n=1}^{d_{i_k}^{\mathrm{UL}}}\mathrm{diag}\left(\mathbf{H}_{i_k}^{\mathrm{SI}}\mathbf{v}_{i_k,n}^{\mathrm{UL}}\left(\mathbf{v}_{i_k,n}^{\mathrm{UL}}\right)^H\left(\mathbf{H}_{i_k}^{\mathrm{SI}}\right)^H\right) + \mathbf{I}_{N_{i_k}} \quad (13)\end{aligned}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2780833, IEEE Transactions on Communications

B. Harmonic-Sum

In order to balance throughput for cell-edge users and overall throughput, we choose to maximize the harmonic-sum of the user throughput, which is the sum of reciprocals of the user SINRs [30], [31]. The harmonic-sum has the following properties for any set of positive numbers:

$$\min\{x_1, x_2, \ldots, x_N\} \geq \frac{1}{\sum_{i=1}^{N} \frac{1}{x_i}}, \tag{18}$$

$$\frac{1}{N}\sum_{i=1}^{N} x_i \geq \frac{N}{\sum_{i=1}^{N} \frac{1}{x_i}}, \tag{19}$$

It is easy to see from (18) and (19) that maximizing the harmonic-sum will indirectly maximize the minimum and the sum of SINRs, respectively. Therefore, harmonic-sum maximization indirectly balances the overall throughput and fairness.

The harmonic-sum objective function for multi-cell multi-stream multi-user MIMO systems can be expressed as

$$\mathrm{SINR_H} = \frac{1}{\sum_{k=1}^{K}\sum_{i=1}^{I_k}\sum_{\mathrm{X}\in\{\mathrm{UL,DL}\}}\sum_{m=1}^{d_{i_k}^{\mathrm{X}}} \frac{1}{\gamma_{i_k,m}^{\mathrm{X}}}}. \tag{20}$$

Maximizing the harmonic-sum $\mathrm{SINR_H}$ is equivalent to minimizing $1/\mathrm{SINR_H}$, and from (10)-(11), this problem can be written as

$$\min_{\mathbf{v},\mathbf{u}} \sum_{k=1}^{K}\sum_{i=1}^{I_k}\sum_{\mathrm{X}\in\{\mathrm{UL,DL}\}}\sum_{m=1}^{d_{i_k}^{\mathrm{X}}} \frac{\left(\mathbf{u}_{i_k,m}^{\mathrm{X}}\right)^H \mathbf{Q}_{i_k}^{\mathrm{X}}(\mathbf{v})\,\mathbf{u}_{i_k,m}^{\mathrm{X}} - \left|\left(\mathbf{u}_{i_k,m}^{\mathrm{X}}\right)^H \mathbf{H}_{i_k}^{\mathrm{X}} \mathbf{v}_{i_k,m}^{\mathrm{X}}\right|^2}{\left|\left(\mathbf{u}_{i_k,m}^{\mathrm{X}}\right)^H \mathbf{H}_{i_k}^{\mathrm{X}} \mathbf{v}_{i_k,m}^{\mathrm{X}}\right|^2} \tag{21}$$

$$\text{s.t.} \quad (16),\ (17), \tag{22}$$

where $\mathbf{Q}_{i_k}^{\mathrm{X}}(\mathbf{v}),\ \mathrm{X}\in\{\mathrm{UL,DL}\}$, is defined in (10) and (11). The problem (21)-(22) can be equivalently rewritten as

$$\min_{\mathbf{v},\mathbf{u}} \sum_{k=1}^{K}\sum_{i=1}^{I_k}\sum_{\mathrm{X}\in\{\mathrm{UL,DL}\}}\sum_{m=1}^{d_{i_k}^{\mathrm{X}}} \left(\left(\mathbf{u}_{i_k,m}^{\mathrm{X}}\right)^H \mathbf{Q}_{i_k}^{\mathrm{X}}(\mathbf{v})\,\mathbf{u}_{i_k,m}^{\mathrm{X}} - 1\right) \tag{23}$$

$$\text{s.t.} \quad \left|\left(\mathbf{u}_{i_k,m}^{\mathrm{X}}\right)^H \mathbf{H}_{i_k}^{\mathrm{X}} \mathbf{v}_{i_k,m}^{\mathrm{X}}\right|^2 = 1,\ i_k\in\mathcal{I},\ m\in\mathcal{M}, \tag{24}$$

$$(16),\ (17), \tag{25}$$

which can be further simplified as

$$\min_{\mathbf{V},\mathbf{U}} \sum_{k=1}^{K}\sum_{i=1}^{I_k}\sum_{\mathrm{X}\in\{\mathrm{UL,DL}\}} \mathrm{tr}\left(\left(\mathbf{U}_{i_k}^{\mathrm{X}}\right)^H \mathbf{Q}_{i_k}^{\mathrm{X}}(\mathbf{V})\,\mathbf{U}_{i_k}^{\mathrm{X}}\right) \tag{26}$$

$$\text{s.t.} \quad \left|\left(\mathbf{u}_{i_k,m}^{\mathrm{X}}\right)^H \mathbf{H}_{i_k}^{\mathrm{X}} \mathbf{v}_{i_k,m}^{\mathrm{X}}\right|^2 = 1,\ i_k\in\mathcal{I},\ m\in\mathcal{M}, \tag{27}$$

$$(16),\ (17), \tag{28}$$

where the optimization variable $\mathbf{V}\,(\mathbf{U})$ denotes all transmit (receiving) beamforming matrices in the uplink and downlink channels and $\mathbf{Q}_{i_k}^{\mathrm{X}}(\mathbf{V})$ is a function of transmit beamforming matrices, $\mathbf{V}$.

Furthermore, the phase rotation of the column vectors of the transmit beamforming matrices, $\mathbf{v}_{i_k,m}^{\mathrm{X}}$, does not affect the unit norm constraint (27). Therefore, we can replace the constraint $\left|\left(\mathbf{u}_{i_k,m}^{\mathrm{X}}\right)^H \mathbf{H}_{i_k}^{\mathrm{X}} \mathbf{v}_{i_k,m}^{\mathrm{X}}\right|^2 = 1$ by $\left(\mathbf{u}_{i_k,m}^{\mathrm{X}}\right)^H \mathbf{H}_{i_k}^{\mathrm{X}} \mathbf{v}_{i_k,m}^{\mathrm{X}} = 1$[1].

The new optimization problem can be cast as

$$\min_{\mathbf{V},\mathbf{U}} \sum_{k=1}^{K}\sum_{i=1}^{I_k}\sum_{\mathrm{X}\in\{\mathrm{UL,DL}\}} \mathrm{tr}\left(\left(\mathbf{U}_{i_k}^{\mathrm{X}}\right)^H \mathbf{Q}_{i_k}^{\mathrm{X}}(\mathbf{v})\,\mathbf{U}_{i_k}^{\mathrm{X}}\right) \tag{29}$$

$$\text{s.t.} \quad \left(\left(\mathbf{U}_{i_k}^{\mathrm{X}}\right)^H \mathbf{H}_{i_k}^{\mathrm{X}} \mathbf{V}_{i_k}^{\mathrm{X}}\right) \circ \mathbf{I}_{d_{i_k}^{\mathrm{X}}} = \mathbf{I}_{d_{i_k}^{\mathrm{X}}},\ i_k\in\mathcal{I},\ \forall \mathrm{X}, \tag{30}$$

$$\mathrm{tr}\left(\left(\mathbf{V}_{i_k}^{\mathrm{UL}}\right)^H \mathbf{V}_{i_k}^{\mathrm{UL}}\right) \leq P_{i_k},\ i_k\in\mathcal{I}, \tag{31}$$

$$\sum_{i=1}^{I_k} \mathrm{tr}\left(\left(\mathbf{V}_{i_k}^{\mathrm{DL}}\right)^H \mathbf{V}_{i_k}^{\mathrm{DL}}\right) \leq P_k,\ k\in\mathcal{K}. \tag{32}$$

Note that the objective function (29) is not jointly convex over transmit beamforming matrices in the set $\mathbf{V}$ and receiving beamforming matrices in the set $\mathbf{U}$ (since they are coupled). Therefore, we cannot apply the standard convex optimization methods to obtain the optimal solution. However, as the objective function (29) is component-wise convex over the matrices in $\mathbf{V}$ and $\mathbf{U}$, we employ an iterative algorithm that finds the efficient solutions of $\mathbf{V}$ and $\mathbf{U}$ in an alternating fashion. Particularly, we update the transmit beamforming matrices in $\mathbf{V}$ when the receiving beamforming matrices in $\mathbf{U}$ are fixed. Thereafter, we update the receiving beamforming matrices in $\mathbf{U}$ using $\mathbf{V}$ obtained at the previous step. The iterations continue until convergence or a pre-defined number of iterations is reached.

The Lagrangian function of the problem (29)-(32) can be

[1]We note that the new (restrictive) constraint contains the phase information, where the original constraint unit norm constraint does not. However, we also note that the phase information in the new constraint neither changes the objective function nor other constraints of the optimization problem. To show this, let us consider that $(\mathbf{V}_{i_k,m}^{X})^*,\ i_k\in\mathcal{I},\ m\in\mathcal{M},\ X\in\{\mathrm{UL,DL}\}$ is the optimal solution of the problem in (29)–(32) obtained using the new restrictive constraint. Since the new restrictive constraint involves a phase rotation, let us consider that there is a phase rotation, $e^{j\theta_{i_k,m}^{X}},\ i_k\in\mathcal{I},\ m\in\mathcal{M},\ X\in\{\mathrm{UL,DL}\}$ to the optimal solution if one were to use the original unit norm constraint. If we can show that the new optimal solution, $(\mathbf{V}_{i_k,m}^{X})^* e^{j\theta_{i_k,m}^{X}}$, does not change the optimization objective or other constraints, we can claim that $(\mathbf{V}_{i_k,m}^{X})^*$ is also optimal solution of the original problem. We also note that the optimization objective and power constraints involve quadratic forms of the optimization variables, $\mathbf{V}_{i_k,m}^{X}$. Therefore, once we replace them with $(\mathbf{V}_{i_k,m}^{X})^* e^{j\theta_{i_k,m}^{X}}$, the phase information $e^{j\theta_{i_k,m}^{X}}$ that is involved with $(\mathbf{V}_{i_k,m}^{X})^*$ will vanish. Therefore, we conclude that the new constraint does not impact the optimization objective nor other constraints. A similar approach has been followed to simplify optimization problems in [28], [31].

written as

$$\begin{aligned}
&\mathcal{L}\left(\mathbf{V},\mathbf{U},\boldsymbol{\lambda},\boldsymbol{\Delta}\right) \\
&=\sum_{k=1}^{K}\sum_{i=1}^{I_k}\sum_{\mathrm{X}\in\{\mathrm{UL,DL}\}}\mathrm{tr}\left(\left(\mathbf{U}_{i_k}^{\mathrm{X}}\right)^H\mathbf{Q}_{i_k}^{\mathrm{X}}\left(\mathbf{v}\right)\mathbf{U}_{i_k}^{\mathrm{X}}\right) \\
&+\sum_{i_k\in\mathcal{I}}\lambda_{i_k}^{\mathrm{UL}}\left(\mathrm{tr}\left(\left(\mathbf{V}_{i_k}^{\mathrm{UL}}\right)^H\mathbf{V}_{i_k}^{\mathrm{UL}}\right)-P_{i_k}\right) \\
&+\sum_{k=1}^{K}\lambda_{k}^{\mathrm{DL}}\left(\sum_{i=1}^{I_k}\mathrm{tr}\left(\left(\mathbf{V}_{i_k}^{\mathrm{DL}}\right)^H\mathbf{V}_{i_k}^{\mathrm{DL}}\right)-P_{k}\right) \\
&+\sum_{k=1}^{K}\sum_{i=1}^{I_k}\sum_{\mathrm{X}\in\{\mathrm{UL,DL}\}}\mathrm{tr}\left(\boldsymbol{\Delta}_{i_k}^{\mathrm{X}}\left(\mathbf{I}_{d_{i_k}^{\mathrm{X}}}-\left(\mathbf{U}_{i_k}^{\mathrm{X}}\right)^H\mathbf{H}_{i_k}^{\mathrm{X}}\mathbf{V}_{i_k}^{\mathrm{X}}\right)\right),
\end{aligned}\tag{33}$$

where $\lambda_{i_k}^{\mathrm{UL}}$ and $\lambda_{k}^{\mathrm{DL}}$ are dual variables associated with the power constraints (31) and (32), respectively, and $\boldsymbol{\Delta}_{i_k}^{\mathrm{X}}$ is a diagonal matrix with dual variables for the equality constraint (30). Here, $\boldsymbol{\lambda}$ and $\boldsymbol{\Delta}$ are the set of all dual variables for the power and equality constraints, respectively.

The optimal transmit and receiver beamforming matrices can be computed by taking the derivative of the Lagrangian function $\mathcal{L}\left(\mathbf{V},\mathbf{U},\boldsymbol{\lambda},\boldsymbol{\Delta}\right)$ with respect to $\mathbf{V}$ and $\mathbf{U}$, respectively. They can be expressed as

$$\mathbf{U}_{i_k}^{\mathrm{X}}=\left(\mathbf{H}_{i_k}^{\mathrm{X}}\mathbf{V}_{i_k}^{\mathrm{X}}\left(\mathbf{V}_{i_k}^{\mathrm{X}}\right)^H\left(\mathbf{H}_{i_k}^{\mathrm{X}}\right)^H+\boldsymbol{\Sigma}_{i_k}^{\mathrm{X}}\left(\mathbf{v}\right)\right)^{-1}\mathbf{H}_{i_k}^{\mathrm{X}}\mathbf{V}_{i_k}^{\mathrm{X}}\tilde{\boldsymbol{\Delta}}_{i_k}^{\mathrm{X}}\tag{34}$$

$$\mathbf{V}_{i_k}^{\mathrm{X}}=\left(\bar{\lambda}_{i_k}^{\mathrm{X}}\mathbf{I}_{\bar{M}_{i_k}^{\mathrm{X}}}+\mathbf{X}_{i_k}^{\mathrm{X}}\left(\mathbf{U}\right)\right)^{-1}\left(\mathbf{H}_{i_k}^{\mathrm{X}}\right)^H\mathbf{U}_{i_k}^{\mathrm{X}}\left(\boldsymbol{\Delta}_{i_k}^{\mathrm{X}}\right)^H,\tag{35}$$

where $\mathbf{X}_{i_k}^{\mathrm{X}}\left(\mathbf{U}\right)$ is defined in (36)-(37) at the bottom of this page and $\bar{\lambda}_{i_k}^{\mathrm{X}}$ in (38) at the bottom of the following page. Here, $\boldsymbol{\Delta}_{i_k}^{\mathrm{X}}$ and $\tilde{\boldsymbol{\Delta}}_{i_k}^{\mathrm{X}}$ are the diagonal matrices used to scale the transmit and receive beamforming matrices, respectively to ensure that equality constraint (30) is satisfied. By plugging the optimal receive and transmit beamforming matrices in (34) and (35) into (30), respectively, the optimal scaling matrices can be computed as

$$\boldsymbol{\Delta}_{i_k}^{\mathrm{X}}=\left(\mathbf{I}_{d_{i_k}^{\mathrm{X}}}\circ\left(\left(\mathbf{U}_{i_k}^{\mathrm{X}}\right)^H\mathbf{H}_{i_k}^{\mathrm{X}}\left(\bar{\lambda}_{i_k}^{\mathrm{X}}\mathbf{I}_{\bar{M}_{i_k}^{\mathrm{X}}}+\mathrm{X}_{i_k}^{\mathrm{X}}\left(\mathbf{U}\right)\right)^{-1}\times\left(\mathbf{H}_{i_k}^{\mathrm{X}}\right)^H\mathbf{U}_{i_k}^{\mathrm{X}}\right)\right)^{-1},\tag{39}$$

$$\tilde{\boldsymbol{\Delta}}_{i_k}^{\mathrm{X}}=\left(\mathbf{I}_{d_{i_k}^{\mathrm{X}}}\circ\left(\left(\mathbf{V}_{i_k}^{\mathrm{X}}\right)^H\left(\mathbf{H}_{i_k}^{\mathrm{X}}\right)^H\left(\mathbf{H}_{i_k}^{\mathrm{X}}\mathbf{V}_{i_k}^{\mathrm{X}}\left(\mathbf{V}_{i_k}^{\mathrm{X}}\right)^H\left(\mathbf{H}_{i_k}^{\mathrm{X}}\right)^H+\boldsymbol{\Sigma}_{i_k}^{\mathrm{X}}\left(\mathbf{v}\right)\right)^{-1}\mathbf{H}_{i_k}^{\mathrm{X}}\mathbf{V}_{i_k}^{\mathrm{X}}\right)\right)^{-1}.\tag{40}$$

The first Lagrange multiplier $\boldsymbol{\Delta}_{i_k}^{\mathrm{X}}$ is updated using the closed-form expression in (39). The second Lagrange multiplier $\bar{\lambda}_{i_k}^{\mathrm{X}}$, $\mathrm{X}=\{\mathrm{UL},\ \mathrm{DL}\}$ in (35) and associated with the power constraints in (31) and (32) should be configured so that the power constraints are satisfied. Since the power constraints in (31) and (32) are monotonically decreasing functions of $\lambda_{i_k}^{\mathrm{UL}}$ and $\lambda_{k}^{\mathrm{DL}}$, respectively [31], they can be obtained via the bisection method. If the values of the Lagrange multipliers $\bar{\lambda}_{i_k}^{\mathrm{X}}$ are negative, we assign $\bar{\lambda}_{i_k}^{\mathrm{X}}$ as zeros.

The iterative alternating algorithm for solving the optimization problem (29)-(32) is given in Algorithm 1. In Appendix, it is proved that the iterative proposed algorithm converges to a theoretical limit.

Although there exist various definitions of *local* CSI, in this paper, with *local* CSI, we assume that the transmitters are able to obtain the knowledge of channel coefficients which are directly connected to them. The same definition has also been adopted and commonly used in other papers [45]-[51].

In particular, we assume each transmitter has perfect knowledge of the channel matrices only between itself and all receivers. This information can be obtained easily by overhearing signaling packets at the medium-access-layer (MAC) layer. For example, in the IEEE 802.11n scheme, assuming the channel reciprocity, a transmitter can estimate the channel between itself and the unintended receiver by capturing the "Clear-to-Send" message, which contains a training sequence from an unintended receiver [50], [56]. Since the optimal

$$\begin{aligned}
\mathbf{X}_{i_k}^{\mathrm{UL}}\left(\mathbf{U}\right)=&\sum_{j=1}^{K}\sum_{l=1}^{I_j}\left(\left(\mathbf{H}_{ji_k}^{\mathrm{UL}}\right)^H\mathbf{U}_{l_j}^{\mathrm{UL}}\left(\mathbf{U}_{l_j}^{\mathrm{UL}}\right)^H\mathbf{H}_{ji_k}^{\mathrm{UL}}+\beta\left(\mathbf{H}_{ji_k}^{\mathrm{UL}}\right)^H\mathrm{diag}\left(\mathbf{U}_{l_j}^{\mathrm{UL}}\left(\mathbf{U}_{l_j}^{\mathrm{UL}}\right)^H\right)\mathbf{H}_{ji_k}^{\mathrm{UL}}\right) \\
&+\kappa\mathrm{diag}\left(\left(\mathbf{H}_{i_k}^{\mathrm{SI}}\right)^H\mathbf{U}_{i_k}^{\mathrm{DL}}\left(\mathbf{U}_{i_k}^{\mathrm{DL}}\right)^H\mathbf{H}_{i_k}^{\mathrm{SI}}\right)+\beta\left(\mathbf{H}_{i_k}^{\mathrm{SI}}\right)^H\mathrm{diag}\left(\mathbf{U}_{i_k}^{\mathrm{DL}}\left(\mathbf{U}_{i_k}^{\mathrm{DL}}\right)^H\right)\mathbf{H}_{i_k}^{\mathrm{SI}} \\
&+\sum_{(l,j)\neq(i,k)}\left(\left(\mathbf{H}_{l_ji_k}^{\mathrm{UU}}\right)^H\mathbf{U}_{l_j}^{\mathrm{DL}}\left(\mathbf{U}_{l_j}^{\mathrm{DL}}\right)^H\mathbf{H}_{l_ji_k}^{\mathrm{UU}}+\beta\left(\mathbf{H}_{l_ji_k}^{\mathrm{UU}}\right)^H\mathrm{diag}\left(\mathbf{U}_{l_j}^{\mathrm{DL}}\left(\mathbf{U}_{l_j}^{\mathrm{DL}}\right)^H\right)\mathbf{H}_{l_ji_k}^{\mathrm{UU}}\right),
\end{aligned}\tag{36}$$

$$\begin{aligned}
\mathbf{X}_{i_k}^{\mathrm{DL}}\left(\mathbf{U}\right)=&\sum_{j=1,j\neq k}^{K}\sum_{l=1}^{I_j}\left(\left(\mathbf{H}_{jk}^{\mathrm{BB}}\right)^H\mathbf{U}_{l_j}^{\mathrm{UL}}\left(\mathbf{U}_{l_j}^{\mathrm{UL}}\right)^H\mathbf{H}_{jk}^{\mathrm{BB}}+\beta\left(\mathbf{H}_{jk}^{\mathrm{BB}}\right)^H\mathrm{diag}\left(\mathbf{U}_{l_j}^{\mathrm{UL}}\left(\mathbf{U}_{l_j}^{\mathrm{UL}}\right)^H\right)\mathbf{H}_{jk}^{\mathrm{BB}}\right) \\
&+\sum_{l=1}^{I_k}\left(\kappa\mathrm{diag}\left(\left(\mathbf{H}_{k}^{\mathrm{SI}}\right)^H\mathbf{U}_{l_k}^{\mathrm{UL}}\left(\mathbf{U}_{l_k}^{\mathrm{UL}}\right)^H\mathbf{H}_{k}^{\mathrm{SI}}\right)+\beta\left(\mathbf{H}_{k}^{\mathrm{SI}}\right)^H\mathrm{diag}\left(\mathbf{U}_{l_k}^{\mathrm{UL}}\left(\mathbf{U}_{l_k}^{\mathrm{UL}}\right)^H\right)\mathbf{H}_{k}^{\mathrm{SI}}\right) \\
&+\sum_{j=1}^{K}\sum_{l=1}^{I_j}\left(\left(\mathbf{H}_{l_jk}^{\mathrm{DL}}\right)^H\mathbf{U}_{l_j}^{\mathrm{DL}}\left(\mathbf{U}_{l_j}^{\mathrm{DL}}\right)^H\mathbf{H}_{l_jk}^{\mathrm{DL}}+\beta\left(\mathbf{H}_{l_jk}^{\mathrm{DL}}\right)^H\mathrm{diag}\left(\mathbf{U}_{l_j}^{\mathrm{DL}}\left(\mathbf{U}_{l_j}^{\mathrm{DL}}\right)^H\right)\mathbf{H}_{l_jk}^{\mathrm{DL}}\right).
\end{aligned}\tag{37}$$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TCOMM.2017.2780833, IEEE Transactions on Communications

transmit beamforming matrix given in (35) is a function of these *local* channels, we can conclude that only *local* CSI is required at each node. In addition, since the computation of transmit beamforming matrix of one user in (35) does not require any information about transmit beamforming matrices of other users, the proposed method fits for distributed implementation. Therefore, each user can compute the optimal transmit beamforming matrices independently when the BSs share the receive beamforming matrices and CSI. On the other hand, Max-Min SINR algorithm [29] does not allow independent processing to compute the transmit beamformer at each user. In particular, a centralized scheduler first collects all MIMO channels of all links (global CSI) by employing multihop routing to receive the CSI feedback from the nodes located far from the centralized scheduler, and then calibrates, computes and distributes the optimum filtering matrices of all links. The complexity of the centralized algorithm increases substantially as the number of links increases and it comes at the cost of signaling overhead. Different from this, the proposed distributed scheme requires each link to collect only *local* CSI, and thus possesses improved scalability and less complexity.

C. Complexity Analysis

Assuming the same number of transmit antennas (M), receive antennas (N) and same number of data streams (d) at each node, in this section, we will compare the computational complexity of the proposed algorithm with those of the weighted-sum-rate [45], [57] and Max-Min SINR [29] algorithms, which also employ an alternating iterative algorithm.

The Max-Min SINR algorithm employs a bisection algorithm to compute the highest minimum SINR value and in each iteration, a SOCP problem is solved. This can be solved using an iterative interior point method requiring $\mathcal{O}\left(M^2 d^3 |\mathcal{I}|^3\right)$ calculations per iteration of the interior point method. The complexity of the weighted-sum-rate algorithm is analyzed in [45]. We summarize the complexity of these algorithms in Table I. Since in the next generation wireless communication systems, the number of users, $|\mathcal{I}|$, is expected to outnumber the number of transmit/receive antennas, it is seen from Table I that overall computational complexity of Max-Min SINR algorithm is much higher than that of the proposed algorithm.

Algorithm 1 Harmonic-Sum Maximization Algorithm.

Set the iteration number $n = 0$ and initialize the transmit beamforming matrices $\mathbf{V}_{i_k}^{\mathrm{X},[0]},\ i_k \in \mathcal{I},\ \mathrm{X} \in \{\mathrm{UL,DL}\}$.
Compute the receive beamforming matrices $\mathbf{U}_{i_k}^{\mathrm{X},[n]}$ from (34) and (40).
repeat
    $n \leftarrow n+1$.
    Configure $\bar{\lambda}_{i_k}^{\mathrm{X}}$ with an initial value.
    for $l = 1, \ldots$ **do**
        Update the transmit beamforming matrices $\mathbf{V}_{i_k}^{\mathrm{X},[n]}$, $\forall (i,k,\mathrm{X})$ using (35) and (39).
        Update $\bar{\lambda}_{i_k}^{\mathrm{X}}$ numerically using bisection search.
    end for
    Compute the receiving matrices $\mathbf{U}_{i_k}^{\mathrm{X},[n]},\ \forall (i,k,\mathrm{X})$ from (34) and (40).
until convergence of the objective function in (29), or a predefined number of iterations is reached.

IV. Fairness Design under Imperfect CSI

In this section, we will study the fairness problem under imperfect CSI scenario. We characterize the imperfect CSI using the norm-bounded deterministic (or worst-case) model, where the instantaneous channel lies in a known set of possible values [38]-[42]. In particular, it is expressed as

$$\mathbf{H} \in \mathcal{H} = \left\{\tilde{\mathbf{H}} + \mathbf{\Lambda} : \|\mathbf{\Lambda}\|_F \leq \epsilon\right\}, \tag{41}$$

where $\tilde{\mathbf{H}}$, $\mathbf{\Lambda}$, and ϵ denote the nominal value of the CSI, the channel error matrix, and the uncertainty bound, respectively. Here, $\mathbf{H}$ represents all the channels in FD multi-cell system.

With the imperfect CSI, the max-min optimization problem can be written as[2]

$$\max_{\mathbf{v},\mathbf{u}} \min_{\forall i_k \in \mathcal{I}, m^X \in \mathcal{M}} \min_{\|\mathbf{\Lambda}\|_F \leq \epsilon} \gamma_{i_k,m}^{\mathrm{X}}\left(\mathbf{v}, \mathbf{u}_{i_k,m}^{\mathrm{X}}\right) \tag{42}$$

$$\text{s.t.} \sum_{m=1}^{d_{i_k}^{\mathrm{UL}}} \left(\mathbf{v}_{i_k,m}^{\mathrm{UL}}\right)^H \mathbf{v}_{i_k,m}^{\mathrm{UL}} \leq P_{i_k},\ i_k \in \mathcal{I}, \tag{43}$$

$$\sum_{i=1}^{I_k} \sum_{m=1}^{d_{i_k}^{\mathrm{DL}}} \left(\mathbf{v}_{i_k,m}^{\mathrm{DL}}\right)^H \mathbf{v}_{i_k,m}^{\mathrm{DL}} \leq P_k,\ k \in \mathcal{K}. \tag{44}$$

To simplify the presentation, we will use the result in [58, Lemma 1] to express $\min_{\|\mathbf{\Lambda}\|_F \leq \epsilon}\ \gamma_{i_k,m}^{\mathrm{X}}$ as

$$\begin{aligned}\tilde{\gamma}_{i_k,m}^{\mathrm{X}}\left(\mathbf{v}, \mathbf{u}_{i_k,m}^{\mathrm{X}}\right) &\triangleq \min_{\|\mathbf{\Lambda}\|_F \leq \epsilon}\ \gamma_{i_k,m}^{\mathrm{X}}\left(\mathbf{v}, \mathbf{u}_{i_k,m}^{\mathrm{X}}\right) \\ &= \frac{\left(\mathbf{u}_{i_k,m}^{\mathrm{X}}\right)^H \mathbf{E}_{i_k,m}^{\mathrm{X}}(\mathbf{v})\, \mathbf{u}_{i_k,m}^{\mathrm{X}}}{\left(\mathbf{u}_{i_k,m}^{\mathrm{X}}\right)^H \mathbf{F}_{i_k,m}^{\mathrm{X}}(\mathbf{v})\, \mathbf{u}_{i_k,m}^{\mathrm{X}}},\end{aligned} \tag{45}$$

[2] Note that under perfect CSI case, solving harmonic-sum problem instead of max-min problem results in a distributed and a low complexity algorithm. On the other hand, for the imperfect CSI case, the solution of both max-min and harmonic sum problems are centralized, which requires the use of SDR technique, and thus harmonic-sum metric does not have much improvement over max-min metric in terms of complexity when there is a channel uncertainty. Therefore, in this section, we will only focus on the max-min problem.

$$\left(\bar{\lambda}_{i_k}^{\mathrm{X}},\ \bar{N}_{i_k}^{\mathrm{X}},\ \bar{M}_{i_k}^{\mathrm{X}}\right) = \begin{cases} \left(\lambda_{i_k},\ N_k,\ M_{i_k}\right) & \text{if } \mathrm{X} = UL, \\ \left(\lambda_k, N_{i_k},\ M_k\right) & \text{if } \mathrm{X} = DL. \end{cases} \tag{38}$$

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TABLE I. COMPARISON OF COMPUTATIONAL COMPLEXITY

Proposed Algorithm	Weighted-Sum-Rate	Max-Min SINR
$\|\mathcal{I}\| N \left(Nd + NM + M^2\right)$ + $M^3 + Nd^2 + MNd + d^3 + d^2M$	$\|\mathcal{I}\| N \left(d^2 + Nd + NM + M^2\right)$ + $M^3 + MNd + d^3 + d^2M$	$M^2 d^3 \|\mathcal{I}\|^3$

where $\mathbf{E}^{\rm X}_{i_k,m}$ and $\mathbf{F}^{\rm X}_{i_k,m}$ are defined as

$$\mathbf{E}^{\rm X}_{i_k,m}\left(\mathbf{v}\right) = \tilde{\mathbf{H}}^{\rm X}_{i_k}\mathbf{v}^{\rm X}_{i_k,m}\left(\tilde{\mathbf{H}}^{\rm X}_{i_k}\mathbf{v}^{\rm X}_{i_k,m}\right)^H - \epsilon^2\left\|\mathbf{v}^{\rm X}_{i_k,m}\right\|^2\mathbf{I}_{\bar{N}^{\rm X}_{i_k}}, \quad (46)$$

$$\begin{aligned}\mathbf{F}^{\rm X}_{i_k,m}\left(\mathbf{v}\right) = {} & \tilde{\mathbf{\Sigma}}^{\rm X}_{i_k}\left(\mathbf{v}\right) + \tilde{\mathbf{H}}^{\rm X}_{i_k}\sum_{l=1}^{d^{\rm X}_{i_k}}\mathbf{v}^{\rm X}_{i_k,l}\left(\mathbf{v}^{\rm X}_{i_k,l}\right)^H\left(\tilde{\mathbf{H}}^{\rm X}_{i_k}\right)^H \\ & - \tilde{\mathbf{H}}^{\rm X}_{i_k}\mathbf{v}^{\rm X}_{i_k,m}\left(\tilde{\mathbf{H}}^{\rm X}_{i_k}\mathbf{v}^{\rm X}_{i_k,m}\right)^H \\ & - \epsilon^2\left\|\mathbf{v}^{\rm X}_{i_k,m}\right\|^2\mathbf{I}_{\bar{N}^{\rm X}_{i_k}} + \epsilon^2\Theta^{\rm X}_{i_k}\mathbf{I}_{\bar{N}^{\rm X}_{i_k}}. \end{aligned} \quad (47)$$

Here, $\tilde{\mathbf{\Sigma}}^{\rm X}_{i_k}\left(\mathbf{v}\right)$ is obtained by replacing the channel matrices $\mathbf{H}$ in $\mathbf{\Sigma}^{\rm X}_{i_k}\left(\mathbf{v}\right)$ given in (12) and (13) with the estimated ones $\tilde{\mathbf{H}}$, and $\Theta^{\rm X}_{i_k}$ is expressed as

$$\begin{aligned}\Theta^{\rm UL}_{i_k} = {} & \sum_{j=1}^{K}\sum_{l=1}^{I_j}\left(1+\beta\right)\left\|\mathbf{V}^{\rm UL}_{l_j}\right\|_F^2 + \sum_{l=1}^{I_k}(\kappa+\beta)\left\|\mathbf{V}^{\rm DL}_{l_k}\right\|_F^2 \\ & + \sum_{j=1, j\neq k}^{K}\sum_{l=1}^{I_j}\left(1+\beta\right)\left\|\mathbf{V}^{\rm DL}_{l_j}\right\|_F^2, \end{aligned} \quad (48)$$

$$\begin{aligned}\Theta^{\rm DL}_{i_k} = {} & \sum_{j=1}^{K}\sum_{l=1}^{I_j}\left(1+\beta\right)\left\|\mathbf{V}^{\rm DL}_{l_j}\right\|_F^2 + (\kappa+\beta)\left\|\mathbf{V}^{\rm UL}_{i_k}\right\|_F^2 \\ & + \sum_{(l,j)\neq(i,k)}\left(1+\beta\right)\left\|\mathbf{V}^{\rm UL}_{l_j}\right\|_F^2. \end{aligned} \quad (49)$$

Using the simplified SINR definition in (45) and epigraph form with the slack variable γ, the problem (42)-(44) can be rewritten as

$$\min_{\mathbf{v},\mathbf{u},\gamma} \quad -\gamma \quad (50)$$

$$\text{s.t.} \quad \sum_{m=1}^{d^{\rm UL}_{i_k}}\left(\mathbf{v}^{\rm UL}_{i_k,m}\right)^H\mathbf{v}^{\rm UL}_{i_k,m} \le P_{i_k},\ i_k\in\mathcal{I}, \quad (51)$$

$$\sum_{i=1}^{I_k}\sum_{m=1}^{d^{\rm DL}_{i_k}}\left(\mathbf{v}^{\rm DL}_{i_k,m}\right)^H\mathbf{v}^{\rm DL}_{i_k,m} \le P_k,\ k\in\mathcal{K}, \quad (52)$$

$$\tilde{\gamma}^{\rm X}_{i_k,m}\left(\mathbf{v},\mathbf{u}^{\rm X}_{i_k,m}\right) \ge \gamma,\ i_k\in\mathcal{I},\ m\in\mathcal{M}. \quad (53)$$

Because of the minimum SINR constraints in (53), the optimization problem (50)-(53) is non-convex, and thus we iteratively compute the transmit and receive beamforming matrices to monotonically improve the minimum SINR.

A. Receive Beamforming Matrices Design

The receiver beamforming matrices optimization problem to maximize the minimum SINR among all users' data streams under fixed transmit beamforming matrices can be solved independently, since the SINR terms in (10) and (11) depend on a single stream receive filter. Therefore, the optimal receiver beamforming matrices can be computed by solving the following problem:

$$\max_{\mathbf{u}^{\rm X}_{i_k,m}} \quad \tilde{\gamma}^{\rm X}_{i_k,m}\left(\mathbf{v},\mathbf{u}^{\rm X}_{i_k,m}\right). \quad (54)$$

Using [59, Appendix E] and [60], the optimal solution of (54) can be given as

$$\mathbf{u}^{\rm X}_{i_k,m} = \frac{\left(\mathbf{F}^{\rm X}_{i_k,m}\left(\mathbf{v}\right)\right)^{-1/2}\mathbf{w}^{\rm X}_{i_k,m}}{\left\|\left(\mathbf{F}^{\rm X}_{i_k,m}\left(\mathbf{v}\right)\right)^{-1/2}\mathbf{w}^{\rm X}_{i_k,m}\right\|}, \quad (55)$$

where $\mathbf{w}^{\rm X}_{i_k,m}$ is the principle eigenvector of $\left(\mathbf{F}^{\rm X}_{i_k,m}\left(\mathbf{v}\right)\right)^{-1/2}\mathbf{E}^{\rm X}_{i_k,m}\left(\mathbf{v}\right)\left(\mathbf{F}^{\rm X}_{i_k,m}\left(\mathbf{v}\right)\right)^{-1/2}$, and $\mathbf{E}^{\rm X}_{i_k,m}\left(\mathbf{v}\right)$ and $\mathbf{F}^{\rm X}_{i_k,m}\left(\mathbf{v}\right)$ are defined in (46) and (47), respectively.

B. Transmit Beamforming Matrices Design

To solve the transmit beamforming matrices design problem under fixed receiver beamforming matrices, we apply the inverse relationship between max-min fairness and power minimization problems proposed for broadcast and multicast channels in [28, Theorem 3], [39], and [61, Claim 3], respectively.

Denoting $\tilde{P} = \{P_{i_k},\ i_k\in\mathcal{I},\ P_k,\ k\in\mathcal{K}\}$, and $\rho_{i_k} = P_{i_k}/\tilde{P},\ i_k\in\mathcal{I}$ and $\rho_k = P_k/\tilde{P},\ k\in\mathcal{K}$, the problem (50)-(53) can be rewritten as

$$\mathcal{P}\left(\tilde{\mathcal{P}}\right) = \min_{\mathbf{v},\mathbf{u},\gamma} \quad -\gamma \quad (56)$$

$$\text{s.t.} \quad \sum_{m=1}^{d^{\rm UL}_{i_k}}\left(\mathbf{v}^{\rm UL}_{i_k,m}\right)^H\mathbf{v}^{\rm UL}_{i_k,m} \le \rho_{i_k}\tilde{P},\ i_k\in\mathcal{I}, \quad (57)$$

$$\sum_{i=1}^{I_k}\sum_{m=1}^{d^{\rm DL}_{i_k}}\left(\mathbf{v}^{\rm DL}_{i_k,m}\right)^H\mathbf{v}^{\rm DL}_{i_k,m} \le \rho_k\tilde{P}, k\in\mathcal{K}, \quad (58)$$

$$\tilde{\gamma}^{\rm X}_{i_k,m}\left(\mathbf{v},\mathbf{u}^{\rm X}_{i_k,m}\right) \ge \gamma, i_k\in\mathcal{I}, m\in\mathcal{M}. \quad (59)$$

Now consider the power minimization problem below:

$$\mathcal{Q}\left(\gamma\right) = \min_{\mathbf{v},\mathbf{u},\tau} \quad \tau \quad (60)$$

$$\text{s.t.} \quad \sum_{m=1}^{d^{\rm UL}_{i_k}}\left(\mathbf{v}^{\rm UL}_{i_k,m}\right)^H\mathbf{v}^{\rm UL}_{i_k,m} \le \rho_{i_k}\tau,\ i_k\in\mathcal{I}, \quad (61)$$

$$\sum_{i=1}^{I_k}\sum_{m=1}^{d^{\rm DL}_{i_k}}\left(\mathbf{v}^{\rm DL}_{i_k,m}\right)^H\mathbf{v}^{\rm DL}_{i_k,m} \le \rho_k\tau, k\in\mathcal{K}, \quad (62)$$

$$\tilde{\gamma}^{\rm X}_{i_k,m}\left(\mathbf{v},\mathbf{u}^{\rm X}_{i_k,m}\right) \ge \gamma,\ i_k\in\mathcal{I},\ m\in\mathcal{M}. \quad (63)$$

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Using a minimum SINR constraint γ^* in (63), assume that the optimal solution of the problem $\mathcal{Q}(\gamma^*)$ in (60)-(63) is $\mathbf{v}^*, \mathbf{u}^*, \tau^*$. It was shown in [28, Theorem 3] that $\mathbf{v}^*, \mathbf{u}^*, \gamma^*$ is the optimal solution of the problem $\mathcal{P}(\tau^*)$ in (56)-(59), and thus we can solve the problem $\mathcal{Q}(\gamma)$ to solve the problem $\mathcal{P}\left(\tilde{\mathcal{P}}\right)$, and vice versa. Under the fixed receiver beamforming matrices, the problem $\mathcal{Q}(\gamma)$ is a quadratically constrained quadratic program (QCQP), which can be solved through SDR techniques. Let $\tilde{\mathbf{V}}^{\mathrm{X}}_{i_k,m} = \mathbf{v}^{\mathrm{X}}_{i_k,m}\left(\mathbf{v}^{\mathrm{X}}_{i_k,m}\right)^H$, then the transmit beamforming matrices design problem can be written as

$$\min_{\tilde{\mathbf{V}},\tau} \quad \tau \tag{64}$$

$$\text{s.t.} \quad \sum_{m=1}^{d^{\mathrm{UL}}_{i_k}} \mathrm{tr}\left\{\tilde{\mathbf{V}}^{\mathrm{UL}}_{i_k,m}\right\} \le \rho_{i_k}\tau,\ i_k \in \mathcal{I}, \tag{65}$$

$$\sum_{i=1}^{I_k}\sum_{m=1}^{d^{\mathrm{DL}}_{i_k}} \mathrm{tr}\left\{\tilde{\mathbf{V}}^{\mathrm{DL}}_{i_k,m}\right\} \le \rho_k\tau,\ k \in \mathcal{K}, \tag{66}$$

$$\tilde{\gamma}^{\mathrm{X}}_{i_k,m}\left(\tilde{\mathbf{V}}, \mathbf{u}^{\mathrm{X}}_{i_k,m}\right) \ge \gamma,\ i_k \in \mathcal{I},\ m \in \mathcal{M}, \tag{67}$$

$$\tilde{\mathbf{V}}^{\mathrm{X}}_{i_k,m} \succeq \mathbf{0},\ i_k \in \mathcal{I},\ m \in \mathcal{M}, \tag{68}$$

$$\mathrm{rank}\left(\tilde{\mathbf{V}}^{\mathrm{X}}_{i_k,m}\right) = 1,\ i_k \in \mathcal{I},\ m \in \mathcal{M}, \tag{69}$$

where $\tilde{\mathbf{V}}$ denotes all matrices $\tilde{\mathbf{V}}^{\mathrm{X}}_{i_k,m},\ i_k \in \mathcal{I},\ m \in \mathcal{M}$, and $\tilde{\gamma}^{\mathrm{X}}_{i_k,m}\left(\tilde{\mathbf{V}}, \mathbf{u}^{\mathrm{X}}_{i_k,m}\right)$ is obtained by replacing $\mathbf{v}^{\mathrm{X}}_{i_k,m}\left(\mathbf{v}^{\mathrm{X}}_{i_k,m}\right)^H$ in (10) and (11) with $\tilde{\mathbf{V}}^{\mathrm{X}}_{i_k,m}$. The problem (64)-(69) is still non-convex because of the rank constraint in (69). By dropping this constraint, the problem (64)-(69) can be solved through semidefinite programming (SDP) techniques. If the optimal solution $\mathbf{V}^{\mathrm{X}}_{i_k,m}$ has rank 1, then it is also an optimal solution for (60)-(63), and the optimal $\mathbf{v}^{\mathrm{X}}_{i_k,m}$ can be obtained through rank-one decomposition. Otherwise, one can apply Gaussian randomization [62] to obtain the approximate solution of the beamforming vectors. The steps of the proposed robust algorithm are given in Algorithm 2, and since each step of the Algorithm 2 increases the minimum SINR, Algorithm 2 converges.

Algorithm 2 Robust Fairness Algorithm.

1: Initialize the transmit beamforming vectors $\mathbf{v}^{\mathrm{X}}_{i_k,m},\ i_k \in \mathcal{I},\ m \in \mathcal{M}$ to ensure the power constraints are satisfied.
2: **repeat**
3: Compute the receive beamforming vectors $\mathbf{u}^{\mathrm{X}}_{i_k,m},\ i_k \in \mathcal{I},\ m \in \mathcal{M}$ from (55).
4: Update the target minimum SINR from (45).

$$\gamma^* = \min_{i_k\in\mathcal{I},\ m\in\mathcal{M}} \tilde{\gamma}^{\mathrm{X}}_{i_k,m}. \tag{70}$$

5: Compute the transmit beamforming vectors $\mathbf{v}^{\mathrm{X}}_{i_k,m},\ i_k \in \mathcal{I},\ m \in \mathcal{M}$ by solving $\mathcal{Q}(\gamma^*)$ through (64)-(69).
6: Scale the transmit beamforming vectors $\mathbf{v}^{\mathrm{X}}_{i_k,m},\ i_k \in \mathcal{I},\ m \in \mathcal{M}$ to ensure the power constraints are satisfied.
7: Update the target minimum SINR

$$\gamma^* = \min_{i_k\in\mathcal{I},\ m\in\mathcal{M}} \tilde{\gamma}^{\mathrm{X}}_{i_k,m}. \tag{71}$$

8: **until** convergence of the minimum SINR, γ^*.

V. Numerical Results and Discussions

In this section, we numerically investigate the proposed algorithms for a MIMO FD multi-user multi-cell system. We choose the simulation parameters from the 3GPP LTE specifications [63]. In particular, we consider small cell scenarios, since small cells are considered to be suitable for deployment of FD technology due to low transmit powers, short transmission distances as well as low mobility [8], [64]. We simulate an outdoor multi-cell scenario with three Pico cells within a hexagonal macrocell. For simplicity, we set the same number of transmit and receive antennas at each BS, i.e., $M_k = N_k = N,\ k \in \mathcal{K}$ and each users, i.e., $M_{i_k} = N_{i_k} = M,\ i_k \in \mathcal{I}$. The BSs are assumed to have $N = 4$ transmit and receive antennas. There are 2 users in each cell, each equipped with $M = 2$ transmit and receive antennas[3]. Furthermore, we assume that each user sends a single data stream in both uplink (UL) and downlink (DL) directions. The maximum transmit power for each BS and mobile user is set to 24dBm and 23dBm, respectively. We choose right singular matrices initialization, and average the results over 1000 independent channel realizations. The uncertainty sizes are related to the quality of channels, e.g., the radius of uncertainty regions can be set to $\epsilon = s\|\tilde{\mathbf{H}}\|_F,\ s \in [0,1)$ [41]. The channels between BS and users are assumed to experience the path loss model for line-of-sight (LOS) and non-line-of-sight (NLOS) communications according to the probability

$$P_{\mathrm{LOS}} = 0.5 - \min(0.5, 5\exp(-0.156/d)) + \min(0.5, 5\exp(-d/0.03)), \tag{72}$$

where d is the distance between BS and users in km. Table II summarizes the simulation parameters for the transmission channels.

The uplink channel gain between user i_k and BS k is given by $\mathbf{H}^{\mathrm{UL}}_{ki_k} = \sqrt{\kappa^{\mathrm{UL}}_{i_k}}\tilde{\mathbf{H}}^{\mathrm{UL}}_{ki_k}$, where $\tilde{\mathbf{H}}^{\mathrm{UL}}_{ki_k}$ denotes the small scale fading following a complex Gaussian distribution with zero mean and unit variance, and $\kappa^{\mathrm{UL}}_{i_k} = 10^{(-\mathrm{Z}/10)},\ Z \in \{\mathrm{LOS}, \mathrm{NLOS}\}$ represents the large scale fading consisting of path loss and shadowing, where LOS and NLOS are calculated from a specific path loss model given in Table II. The channel between BS and downlink users, and between uplink users and downlink users are defined similarly. For the self-interference channel, we adopt the model from [2], in which the self-interference channel at BS k is distributed as $\mathbf{H}^{\mathrm{SI}}_k \sim \mathcal{CN}\left(\sqrt{\frac{K_R}{1+K_R}}\tilde{\mathbf{H}}^{\mathrm{SI}}_k, \frac{1}{1+K_R}\mathbf{I}_{N_k} \otimes \mathbf{I}_{M_k}\right)$, where K_R is the Rician factor and $\tilde{\mathbf{H}}^{\mathrm{SI}}_k$ is a deterministic matrix[4]. We apply the above networks parameters in the following, unless stated for a specific figure otherwise.

[3]Note that although the BS k has $N_k + M_k$ antennas in total, similar to [9], [14], we assume that only N_k (M_k) antennas can be used for transmission (reception) in HD mode. The same holds for mobile users.

[4]Similar to [8], without loss of generality, we set $K_R = 1$ and $\tilde{\mathbf{H}}^{\mathrm{SI}}_k$ to be the matrix of all ones for all experiments.

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TABLE II. SIMULATION PARAMETERS FOR MULTI-CELL

Parameter	Settings
Cell Radius	40m
Minimum Distance between BSs	40m
Carrier Frequency	2GHz
Bandwidth	10MHz
Thermal Noise Density	-174dBm/Hz
Noise Figure	BS: 13dB, User: 9dB
Path Loss (dB) between BS and users (d in km)	LOS: $103.8 + 20.9\log_{10} d$ NLOS: $145.4 + 37.5\log_{10} d$
Path Loss (dB) between users (d in km)	$98.45 + 20\log_{10} d,\ d \leq 50$m $175.78 + 40\log_{10} d,\ d > 50$m
Path Loss (dB) between BSs (d in km)	LOS: $89.5 + 16.9\log_{10} d,\ d < 2/3$km, LOS: $101.9 + 40\log_{10} d,\ d \geq 2/3$km, NLOS: $169.36 + 40\log_{10} d$
Shadowing Standard Deviation between BS and users	10dB
Shadowing Standard Deviation between BSs	6dB
Shadowing Standard Deviation between users	12dB

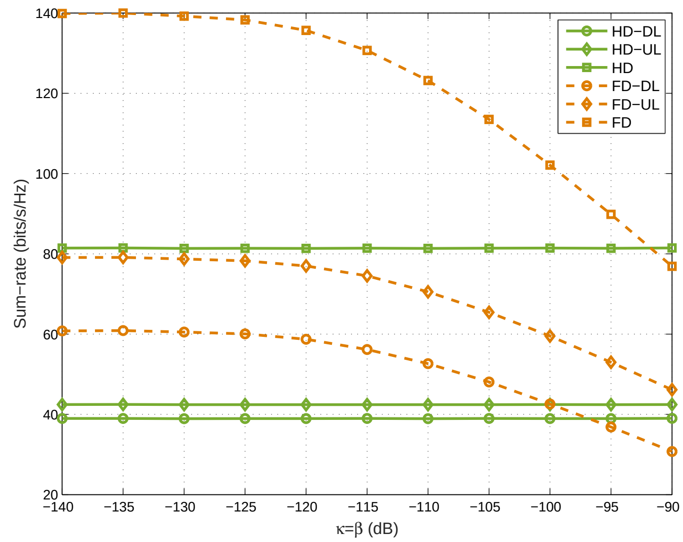

Fig. 3. Total sum-rates achieved for FD and HD setups with perfect CSI.

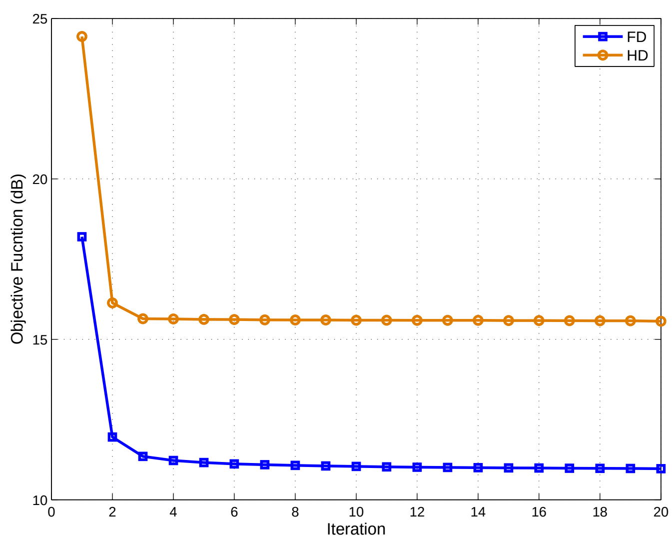

Fig. 2. Convergence of the objective function in (29) for perfect CSI design with $\kappa = \beta = -120$dB.

A. Perfect CSI Results

The proposed fairness design presented in Section III is based on an iterative update of the design parameters. The iterative nature of the algorithm is to ensure that a local optimal solution is obtained. Hence, it is of interest to observe the convergence behavior of this algorithm. To this end, Fig. 2 shows the convergence of the objective function in (29) for both FD and HD operations. As expected, the strictly non-increasing behavior of the optimization objective is observed as the number of iterations increases. Moreover, we observe that HD setup converges with relatively fewer iterations with the penalty of worse performance. This is because the FD design needs to consider additional interference terms in the design process, which may contribute to the slower convergence of the algorithm.

Fig. 3 provides a comparison of the sum rates achieved by FD and HD systems. The HD transmission design is a special case of our proposed algorithm, which can be obtained by ignoring the additional interference for the UL and DL transmissions, as mentioned in Section I. Also, we assume that each BS in HD operation serves the same number of downlink and uplink users to guarantee fairness among them as in the FD system. As we see from Fig. 3, both FD and HD transmissions achieve the same rates at around $\kappa = \beta \approx -92$dB. However, FD outperforms HD transmission when $\kappa = \beta < -95$ dB, which has been achieved by a recent advanced self-interference cancellation techniques reported in [4]. We also note that the spectral efficiency gain for FD over HD transmission varies with different κ and β values. This is due to the fact that the higher transmitter (receiver) distortion, represented by κ (β) corresponds to larger residual self-interference. Therefore, with smaller values of κ (β), we obtain a higher spectral efficiency gain. In particular, going from $\kappa = \beta = -100$dB to $\kappa = \beta = -120$dB, the spectral efficiency gain over HD operation improves from 25% to 65%, respectively.

In order to understand the impact of inter-cell (and inter-user interference), we study the system sum-rate with respect to distance between BSs. Unlike the results presented above, in this case we keep the positions of users fixed with respect to its BS location. Therefore, when the distance between the BSs increases, the inter-user distance among the users in different small-cells also increases. As we see from Fig. 4, as the distance between the BSs increases, the interference

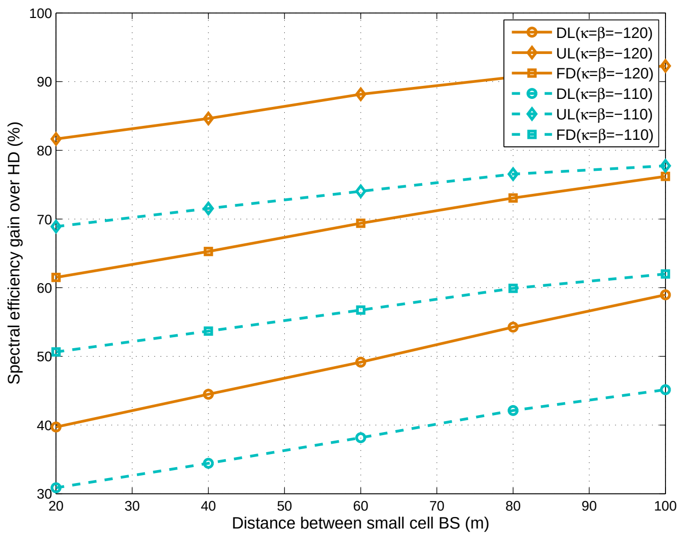

Fig. 4. Total sum-rates achieved for FD and HD setups with perfect CSI and varying distance between BSs. The parameters representing transceiver distortion are chosen as $\kappa = \beta = -110$dB and $\kappa = \beta = -120$dB

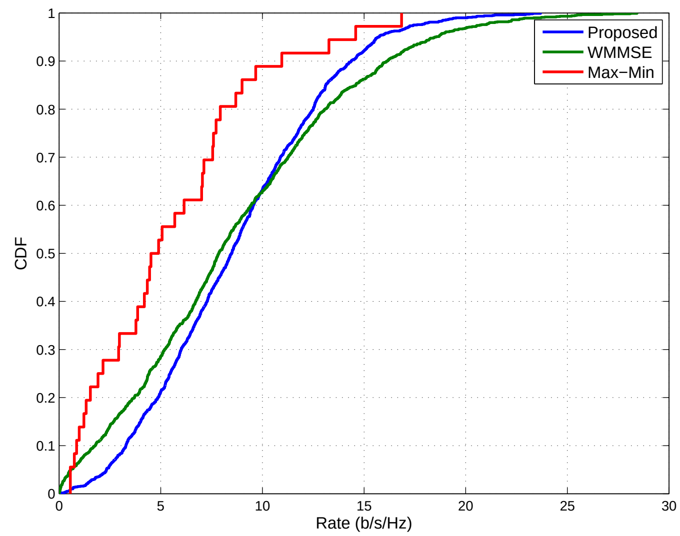

Fig. 5. Comparison of CDFs of individual user rate among the proposed, Max-Min SINR fairness and sum-rate maximizing WMMSE designs with $\kappa = \beta = -80$ dB.

(i.e., inter-cell and inter-user) decreases, which leads to an increase in total sum-rates and improve the spectral efficiency gains over the HD transmission, irrespective of transceiver distortions. In particular, with an increase of BS distance from 40m to 100m, the spectral efficiency gain over HD setup increases from from 65% to 76%, respectively. The relatively small increase in spectral efficiency over HD setup indicates that the system performance is largely dominated by residual self-interference inherent to FD system. We also note that as the FD setup experiences less transceiver distortion, it provides higher spectral efficiency gain over HD setup. This is apparent from the superior spectral efficiency gain obtained for $\kappa = \beta = -120$dB over that of $\kappa = \beta = -110$dB.

Next we compare the proposed algorithm with the sum-rate maximization and Max-Min SINR fairness algorithms proposed in [22] and [27], respectively. The weighted minimum mean squared error (WMMSE) algorithm in [22] tries to maximize the total sum-rate of the network by way of minimizing the WMMSE of each user [45], and the Max-Min fairness algorithm optimizes the minimum SINR of the users. Fig. 5 shows the cumulative density function (CDF) of the three algorithms. The rate along the x-axis of this figure denotes the total rate obtained by each user in the uplink and downlink transmissions. In particular, the results show that the Max-Min fairness algorithm provides the highest fairness to the users at the lower individual rate, i.e., below 1 b/s/Hz, since it maximizes the minimum SINR. Compared to WMMSE and proposed algorithms, the users are less likely to be served by the Max-Min SINR design when the data rate increases, e.g., beyond 2 b/s/Hz. In contrast, it is more likely that a user with higher individual rate will be served by WMMSE design since it maximizes the total throughput. Nonetheless, compared to the WMMSE design, a user is more likely to be served by the proposed design if it experiences a rate, for instance below 9 b/s/Hz. Furthermore, we note that the LTE user throughput requirement is specified at two points: at the average and at the fifth-percentile of the user distribution (where 95 percent of the users have better performance) [65]. The fifth-percentile users correspond to ones operating on the cell edge [66]. As it can be observed from this figure that the proposed algorithm almost surely provides a better fifth-percentile user throughput when compared to the WMMSE design, and also against the Max-Min SINR fairness algorithm when the individual rate is above 1 b/s/Hz.

B. Imperfect CSI Results

After observing the gains offered by FD transmission over HD setup with perfect CSI, in this section we show the performance with imperfect CSI. As discussed in Section IV, we consider the norm-bounded CSI uncertainty to obtain worst-case fairness among mobile users. This is to say, knowing the boundary of the uncertainty region denoted by ϵ, we provide the worst-case transceiver design to obtain fairness among mobile users.

We first study the convergence behavior of the proposed algorithm with imperfect CSI, as presented in Section IV. Fig. 6 shows the convergence of the proposed algorithm for both FD and HD setups over multiple design parameters. The result is obtained by averaging the convergence behavior of the network, over several channel realizations. As expected, we observe strictly non-decreasing behavior of the optimization objective, i.e., improved minimum SINR in terms of associated rate, as the number of iteration increases. Furthermore, as in the case of perfect CSI, it is observed that HD setup converges

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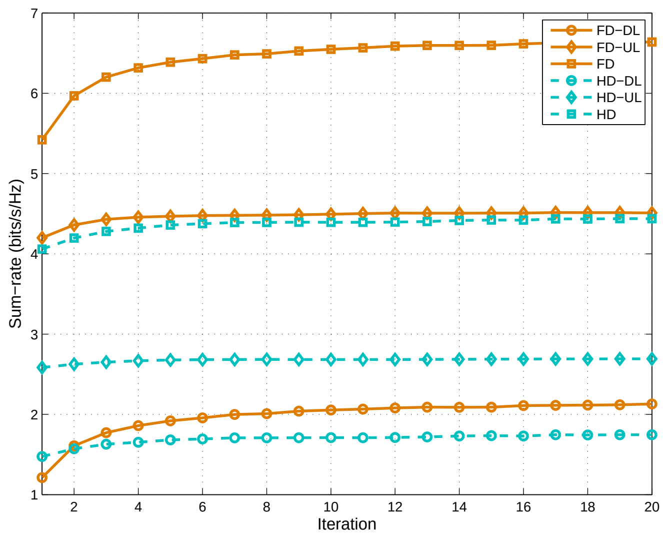

Fig. 6. Convergence of minimum SINR i.e., improved associated rate in the presence of imperfect CSI with $s = 0.02$ and $\kappa = \beta = -120$dB.

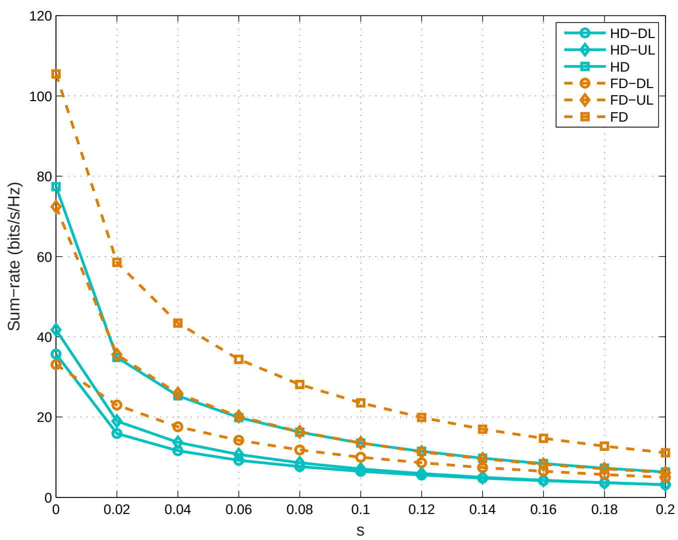

Fig. 7. Total sum-rates achieved for FD and HD setups with varying CSI uncertainties and $\kappa = \beta = -120$dB.

with relatively fewer optimization iterations compared to its FD counterpart.

In Fig. 7, we present the sum-rate performances for both FD and HD setups against the measure of CSI uncertainty, s. As we see from this figure, as the boundary of the CSI uncertainty, i.e., s increases, the sum-rate decreases for both FD and HD systems. It is perceivable, since larger s means higher CSI uncertainty, hence lower rate. Furthermore, we observe that for increasing CSI uncertainty, the performance of FD setup falls more rapidly than its HD counterpart. For example, for the same decrease in CSI quality from $s = 0$ to $s = 0.2$, the sum-rate difference between FD and HD setups goes from 28.5 b/s/Hz to 5 b/s/Hz. This is due to the fact that the FD system involves a larger number of channels, i.e., self-interference and inter-user interference channels; and thus increased CSI uncertainty with its transmission, which degrades its performance more than that of the HD transmission. Hence, the performance of FD system is more susceptible to increasing s in comparison to an HD setup.

Next we compare the performance of robust and non-robust FD setups with various design parameters. The objective of this result is to show the performance gain that can be harnessed through a robust design in the presence of CSI uncertainty. To this end, we make the following observations from Fig. 8. We observe that with small CSI uncertainty i.e., $s = 0.1$, as the κ and β increase, the sum-rate decreases sharply. This is to say, in this regime the transceiver distortion is a more limiting factor on the sum-rate performance than the CSI uncertainty. On the other hand, with larger s, i.e., $s = 0.2$, the rate of decrease for total sum-rate with increasing κ and β is relatively smaller, i.e., the sum-rate curves flatten out for the FD setup. This indicates that the CSI uncertainty is a more limiting factor on the sum-rate performance than the transceiver distortion. In addition, we observe that with lower κ and β, the difference between robust and non-robust design is noticeable. However, with increasing κ and β, the difference in performance becomes smaller as the system performance is dominated by transceiver distortion rather than CSI uncertainty in this region. Finally, we observe that as the CSI uncertainty increases, the difference between the sum-rate performance of the robust and non-robust designs also increases.

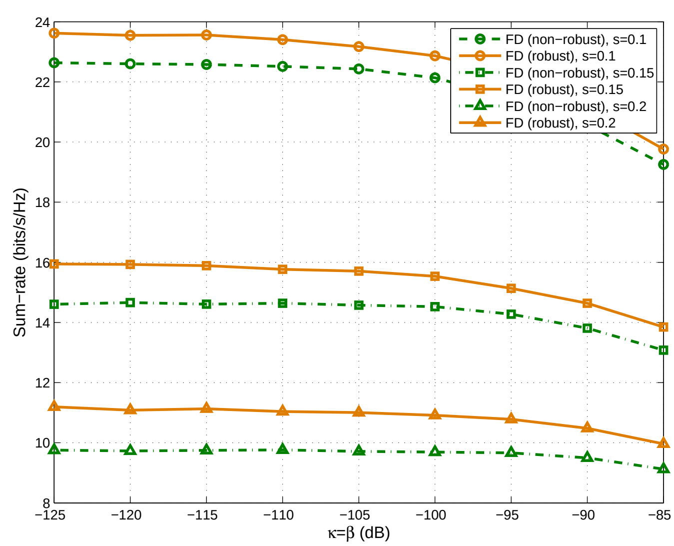

Fig. 8. Total sum-rates achieved for robust and non-robust FD setups with varying CSI uncertainties and transceiver distortions.

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VI. Conclusion

In this work, we have studied the transmit and receive beamforming designs in order to address the fairness problem in FD MIMO multi-cell systems under the assumption of limited-DR. To this end, we consider design approaches with both perfect and imperfect CSI. Since the globally optimal solution is difficult to obtain due to the non-convex nature of the problems, we resort to alternating minimization to obtain locally optimal solutions. The optimization objective is found to be converging in a few iterations. The simulation results suggest that the FD transmission provides better sum-rate performance when compared to that of HD transmission with low to moderate transceiver distortion. Furthermore, we notice that the spectral efficiency gain of FD transmission increases with increasing distance between the small cell BSs due to reduced inter-cell interference. Also, our results reveal that the proposed design offers a better fairness among the cell-edge users, in comparison to WMMSE and in some cases, also against Max-Min fairness based designs. In dealing with practical design issues, our study demonstrates that the proposed robust design similarly provides improved sum-rate performance when compared to HD transmission in the presence of bounded CSI uncertainty. In comparison to a non-robust design, the results demonstrates that a higher sum-rate is achievable with the aid of a robust design in the presence of CSI uncertainty.

Appendix

The proof is similar to the analysis in [31]. Let $f\left(\mathbf{V},\mathbf{U}\right)$ be the optimization objective (29). Then, for any feasible value of $\mathbf{V}$ and $\mathbf{U}$ (i.e., constraints are satisfied), the Lagrangian $\mathcal{L}\left(\mathbf{V},\mathbf{U},\boldsymbol{\lambda},\boldsymbol{\Delta}\right)$ in (33) is equal to $f\left(\mathbf{V},\mathbf{U}\right)$. Since $\mathcal{L}\left(\mathbf{V},\mathbf{U},\boldsymbol{\lambda},\boldsymbol{\Delta}\right)$ is convex for $\mathbf{V}$ when all other variables are fixed, a feasible optimal precoding matrix $\mathbf{V}^{*,[n]}$ at the nth iteration will be the minimum of the objective with respect to a given receive filter $\mathbf{U}^{[n]}$, i.e.,

$$\mathcal{L}\left(\mathbf{V}^{*,[n]},\mathbf{U}^{[n]},\boldsymbol{\lambda},\boldsymbol{\Delta}\right)=\min_{\mathbf{V}}\quad \mathcal{L}\left(\mathbf{V},\mathbf{U}^{[n]},\boldsymbol{\lambda},\boldsymbol{\Delta}\right). \quad (73)$$

The same observation can be made for the receive filter, $\mathbf{U}$, i.e.,

$$\mathcal{L}\left(\mathbf{V}^{[n]},\mathbf{U}^{*,[n+1]},\boldsymbol{\lambda},\boldsymbol{\Delta}\right)=\min_{\mathbf{U}}\quad \mathcal{L}\left(\mathbf{V}^{[n]},\mathbf{U},\boldsymbol{\lambda},\boldsymbol{\Delta}\right). \quad (74)$$

Combining observations made in (73) and (74), we can make the following inequality statement:

$$\begin{aligned}\mathcal{L}\left(\mathbf{V}^{*,[n+1]},\mathbf{U}^{*,[n+1]},\boldsymbol{\lambda},\boldsymbol{\Delta}\right) &\leq \mathcal{L}\left(\mathbf{V}^{*,[n]},\mathbf{U}^{*,[n+1]},\boldsymbol{\lambda},\boldsymbol{\Delta}\right)\\ &\leq \mathcal{L}\left(\mathbf{V}^{*,[n]},\mathbf{U}^{[n]},\boldsymbol{\lambda},\boldsymbol{\Delta}\right) \quad (75)\end{aligned}$$

If we iterate between computing optimal precoding matrix and receive filter, the inequality in (75) guarantees that the Lagrangian is always updated with an equal or smaller value. The Lagrangian with any feasible value of $\mathbf{V}$ and $\mathbf{U}$ is lower bounded by zero; therefore, the algorithm guarantees that the objective converges to some limit value.

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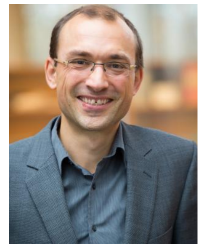

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