Fair Allocation of Subcarrier and Power in an OFDMA Wireless Mesh Network

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Abstract—This paper presents a new fair scheduling scheme for orthogonal frequency-division multiple-access-based wireless mesh networks (WMNs), which fairly allocates subcarriers and power to mesh routers (MRs) and mesh clients to maximize the Nash bargaining solution fairness criterion. In WMNs, since not all the information necessary for scheduling is available at a central scheduler (e.g., MR), it is advantageous to involve the MR and as many mesh clients as possible in distributed scheduling based on the limited information that is available locally at each node. Instead of solving a single global control problem, we hierarchically decouple the subcarrier and power allocation problem into two subproblems, where the MR allocates groups of subcarriers to the mesh clients, and each mesh client allocates transmit power among its subcarriers to each of its outgoing links. We formulate the two subproblems by nonlinear integer programming and nonlinear mixed integer programming, respectively. A simple and efficient solution algorithm is developed for the MR's problem. Also, a closed-form solution is obtained by transforming the mesh client's problem into a time-division scheduling problem. Extensive simulation results demonstrate that the proposed scheme provides fair opportunities to the respective users (mesh clients) and a comparable overall end-to-end rate when the number of mesh clients increases.

Index Terms—Distributed control, fairness, orthogonal frequency-division multiple-access (OFDMA), resource management, wireless mesh network (WMN).

I. INTRODUCTION

DVANCES in information and communications technologies over the past decade have caused a remarkable increase of traffic demand especially for Internet access, which requirements for high data rate, quality-of-service (QoS) support and reliability have motivated the development of advanced wireless networks. Recently, wireless mesh networks (WMNs) have emerged as a promising technology for next-generation wireless networks [1], [2] with flexible and reconfigurable architectures. To realize the potential of this new technology, research is rapidly progressing to develop high-performance techniques for advanced radio transmissions, medium access control, and routing. Because of the challenges presented by the multihop communications over WMNs, techniques to improve basic network performance, such as end-to-end transmission delay, end-to-end data rate and fairness, remain

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important subjects of investigation. Recently, adaptive resource management for multiuser orthogonal frequency-division multiple-access (OFDMA) systems has attracted enormous research interest [3]–[9]. OFDMA is a very promising solution to provide a high-performance physical layer for emerging wireless networks, as it is based on OFDM and inherits its desirable properties of immunity to intersymbol interference and frequency-selective fading.

In [3], the authors studied how to minimize the total transmit power, while satisfying a minimum rate constraint for each user. The problem was formulated as an integer programming problem and a continuous-relaxation-based suboptimal solution method was studied. In [4], computationally inexpensive methods for power allocation and subcarrier assignment were developed, which are shown to achieve comparable performance, but do not require intensive computation. However, these approaches did not provide a *fair* opportunity for users so that some users may be dominant in resource occupancy even when the others starve.

In [5], the authors proposed a fair scheduling scheme to minimize the total transmit power by allocating subcarriers to the users, and then to determine the number of bits transmitted on each subcarrier. Also, they developed suboptimal solution algorithms by using the linear programming technique and the Hungarian method. In [6], the authors formulated a combinatorial problem to jointly optimize the subcarrier and power allocation, subject to the constraint that resources are to be allocated to user according to predetermined fractions. By using the constraint, the resources can be fairly allocated. A novel scheme to fairly allocate subcarrier, rate, and power for multiuser OFDMA systems was proposed in [8], which introduces a new fairness criterion for generalized proportional fairness based on Nash bargaining solutions (NBSs) and coalitions. This study is very different from the previous OFDMA scheduling studies in the sense that the resource allocation is performed with a game-theoretic decision rule. They proposed a very fast near-optimal algorithm using the Hungarian method and showed by simulations that their fair scheduling scheme provides a similar overall rate to that of a rate-maximizing scheme. However, the studies reviewed above have focused on wireless single-hop networks.

In this paper, we propose a new fair scheduling scheme, called distributed hierarchical scheduling, for a WMN that is connected to the Internet via a mesh router (MR), and which consists of a group of mesh clients (MCs) that can potentially relay each other's packets to/from the MR. Hierarchical scheduling has been widely used in computer systems [10], [15], and computer networks [11].

The goal of the proposed hierarchical scheduling scheme is to fairly allocate the subcarriers and power to multiple users;

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i.e., the MCs. To achieve this goal in a both reliable and efficient manner, we employ hierarchical decoupling for the WMN scheduling between the MR level and the MC level. At the MR level (Level-0 scheduling performed by the MR), the objective is to fairly allocate subcarriers to MCs so that the NBS fairness criterion is maximized. At the MC level (Level-1 scheduling performed by each MC in a distributed manner) the objective is to fairly allocate the total transmit power to the available subcarriers assigned to each outgoing link so that the NBS fairness criterion is maximized. Distributed hierarchical scheduling is very useful for WMNs because in practice it is difficult if not impossible to obtain perfect information about every node and every link in the WMN at a central location (e.g., MR). The proposed hierarchical decoupling method allows fair scheduling to be contemplated based on the use of a suitable subset of the network information at the MR for Level-0 scheduling, and local information available at each MC for Level-1 scheduling. Since a limited subset of network information can be provided to the MR more frequently, the scheduling input based on this is potentially more reliable than full network information that is hard to update, thus resulting in more reliable scheduling. Using distributed hierarchical scheduling also reduces the signaling overhead and distributes the processing loads of scheduling among the network nodes so that both the signaling and scheduling efficiencies can be improved.

In the hierarchically decoupled scheduling problem, we formulate the first problem (Level-0 scheduling) as a nonlinear integer programming problem and develop an efficient near-optimal solution algorithm. Also, we formulate the second problem (Level-1 scheduling) as a nonlinear mixed integer programming problem. To solve the latter problem more efficiently, we transform this problem into a time-division scheduling problem and develop a closed-form solution for subcarrier and power allocation. Extensive simulation results demonstrate that the proposed scheme provides fair opportunities to the MCs and an overall end-to-end rate comparable to existing max-rate schemes [8] when the number of users increases.

The remainder of this paper is organized as follows. Section II describes the models considered in this paper and presents the proposed hierarchical scheduling method. We mathematically formulate the fair scheduling problems and develop efficient solution methods for Level-0 and Level-1 scheduling, respectively, in Sections III and IV. Simulation results are presented and discussed in Section V. Section VI concludes this paper.

II. MODEL DESCRIPTION AND PROPOSAL

A. Network Model

We consider an OFDMA WMN that consists of N nodes: one MR and (N-1) MCs. Fig. 1 illustrates the network model considered in this paper. The set of nodes is denoted by $\mathcal{N} =$ $\{0, 1, \dots, N-1\}$, where node 0 represents the MR that delivers traffic between the MCs and the Internet. Some MCs, denoted by set \mathcal{N}_0 , are located in the effective radio coverage of the MR whereas the others, denoted by set $\mathcal{N}_1 (= \mathcal{N} - \mathcal{N}_0)$, are located outside the MR's effective radio coverage and need to access



Fig. 1. Wireless mesh network model. $\mathcal{N}=\{0,1,2,3,4,5\},\mathcal{N}_0=\{0,1,2,3\},\mathcal{N}_1=\{4,5\}$

the Internet via multihops (e.g., MCs 4 and 5 in Fig. 1). Table I presents the notation used in this paper.

B. Routing Model

MC $i \in \mathcal{N}_0$ may alternately access the Internet via multihops if there exists a path to the MR (node 0) that cost less than a direct transmission to the MR. Also, MC i may have multiple paths to the MR (e.g., paths $5 \rightarrow 2 \rightarrow 0$ and $5 \rightarrow 3 \rightarrow 0$ in Fig. 1). A multipath routing policy, where a portion of traffic of MC *i* is delivered to the destination via a neighboring MC and another portion is delivered via a different neighboring MC, is useful comparing to single-path routing because multipath routing can provide more flexibility in network resource allocation, potentially increasing the network capacity. For example, suppose that neither MC 2 nor MC 3 in Fig. 1 has enough capacity to deliver the traffic from MC 5 but the sum of their capacity is sufficient. Another possible scenario is that both MC 2 and MC 3 have enough capacity to deliver the traffic from MC 5 but the use of multipath routing saves the overall delivery cost compared with single-path routing. Such scenarios are very common in OFDMA WMNs because of the strict concavity of the Shannon capacity of each subcarrier. The routing information is denoted by a matrix (y_{ij}) , where y_{ij} is the fraction of traffic that MC i sends to MC j (in the case of j = 0, MC 0 denotes MR) so that $\sum_{\forall j} y_{ij} = 1$.

C. Radio Transmission Model

There are a total of C subcarriers in the system. Each subcarrier has a bandwidth of W. The channel gain of subcarrier k on link (i, j), which connects MC i to MC j, is denoted by G_{ij}^k and the transmit power of MC i on subcarrier k is denoted by p_i^k . Each MC i has a transmit power limit of \bar{p}_i and a minimal rate requirement of R_i . M-ary quadrature amplitude modulation (MQAM) is adopted in our system. We consider a slow-fading channel such that the channel is stationary (channel gain is constant) within each OFDM frame. This is consistent with contemporary applications of OFDMA WMNs.

D. Model and Scope of the Proposed Hierarchical Scheduling

There are extensive studies on routing algorithms, in general wired and wireless networks [12]–[14]. Developing routing algorithms is not the focus of this paper. Instead, this paper focuses on developing fair resource-allocation algorithms in an OFDMA WMN given that routing has been determined. The

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TABLE I	
NOTATION	

item	description
\mathcal{A}_i	$\{j: y_{ij} \neq 0\}$
x_{ij}^k	$x_{ij}^k = 1$ if subcarrier k is used in link $(i, j), 0$ otherwise; $\sum_{k \in C_i} \sum_{j \in A_i} x_{ij}^k = x_i$
r_i	rate allocated to MC i
$ar{R}_i$	minimum rate required by MC i
R_i	minimum rate required for delivering traffic, including forwarding traffic, by MC <i>i</i> ; $R_i \equiv \bar{R}_i + \sum_{l \in \{l: y_{li} \neq 0\}} R_l y_{li}$

resources to schedule are defined as a set of subcarriers, and the total transmit power available at each node. We consider the following model for distributed hierarchical scheduling, where the MR only performs a rough scheduling with limited information (because not all the information is always available) and the MCs performs more refined scheduling with full information that is available locally.

- 1) For MR's Scheduling:
- Scheduling input: the traffic demand of each MC in terms of the data rate R_i ; the average channel gain of all outgoing links at MC i, \bar{G}_i (abbreviated information).
- The role of MR: to determine the number of subcarriers to be assigned to each node. However, the MR does not allocate power levels on the respective subcarriers because the MR does not know G_{ij}^k . Instead, power allocation is performed by the MC.¹
- 2) For MC i's Scheduling:
- Scheduling input: the traffic demand for each outgoing link (i, j) in terms of the data rate R_{ij} ; the channel gain of subcarrier k on its outgoing link $(i, j) G_{ij}^k$ (full information).
- *The role of MC*: to allocate its available subcarriers that have been assigned to it by the MR and to allocate its available transmit power among these subcarriers.

III. LEVEL-0 SCHEDULING: SUBCARRIER ALLOCATION

A. Scheduling at Mesh Router

In Level-0 scheduling, the MR determines how many subcarriers to allocate to the respective MCs (nodes $i = 1, \dots, N-1$). The scheduling inputs are traffic demand vector $(R_i)^t$ and the channel gain vector $(\bar{G}_i)^t$. The MR does not need any routing information for Level-0 scheduling. The scheduling method consists of two phases (refer to Fig. 2): 1) the MC optimizes the vector $x^* = (x_i^*)^t$, which element x_i^* is the decision variable denoting the number of subcarriers to allocate to MC *i*, using only information available at the MR and 2) the MR randomly generates the set of subcarriers for each MC *i*, say C_i , $i \in \mathcal{N} - \{0\}$, using the vector x^* ($x_i^* = |C_i|$). The main reason that the MR randomly determines C_i based on the vector x_i^* is that exact and complete information needed for deterministic channel assignments to the MCs is not usually available at the MR. The scheduling output, $C_i, i \in \mathcal{N} - \{0\}$, is sent to the respective nodes via multihops if necessary.

We formulate the Level-0 scheduling as a nonlinear integer programming problem and suggest a simple and efficient solution algorithm in the following two subsections.

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Phase I: Find an Optimal Real-Number Solution

Step 1 (Find an optimal real-number solution vector \bar{x}^*)

If \lambda^* < 0, then "(P1) is infeasible" (Stop); // see (19)

\bar{x}_i^* := g_i^{-1}(\lambda^*); // \bar{x}^* = (\bar{x}_1^*, \cdots, \bar{x}_{N-1}^*)^t

Phase 2: Find a Near-Optimal Integer Solution (\tilde{x}_i)^t Using (\bar{x}_i^*)^t

Step 2 (Find an initial integer solution)

\tilde{x}_i := [\bar{x}_{i=1}^*]; // \tilde{x} = (\tilde{x}_1, \cdots, \tilde{x}_{N-1})^t

s := \sum_{i=1}^{N-1} (\bar{x}_i^* - \tilde{x}_i);

Step 3 (Improve the integer solution vector \tilde{x})

Choose k such that

\frac{\partial f_1}{\partial x_k}|_{x=\tilde{x}} \ge \frac{\partial f_1}{\partial x_i}|_{x=\tilde{x}} for all i \neq k (random tie-breaking),

then \tilde{x}_k := \tilde{x}_k + 1, s := s - 1;

While s \ge 1, repeat Step 3;

x^* := \tilde{x} (Stop)
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Fig. 2. Iterative steepest-ascent direction search for (P1) performed by MR.

B. Problem Formulation for Mesh Router

Each MR solves problem (P1) as defined below to determine $x = (x_1, \ldots, x_{N-1})^t$, given R_i and \bar{G}_i , and sends the number of subcarrier x_i that is allocated to MC *i* to the respective MC

(P1)

$$\operatorname{Max} f_0(x) = \prod_{i=1}^{N-1} \{ r_i(x_i) - R_i \}$$
(1)

subject to

$$r_i(x_i) \ge R_i, i \in \mathcal{N} - \{0\}$$

$$(2)$$

$$\sum_{i=1}^{N} x_i \le C$$

 x_i : non-negative integer (3)

where $r_i(x_i)$, the rate realized at MC *i* from allocation x_i^* , is given by

$$r_i(x_i) \equiv x_i W \log_2\left(1 + a\bar{G}_i \frac{\bar{p}_i}{x_i}\right). \tag{4}$$

Since we consider MQAM in this paper [8], we have

$$a \approx -\frac{1.5}{\sigma^2 \cdot \log(5 \cdot \text{BER})}$$
 (5)

where σ^2 is the thermal noise power at the receiver over the bandwidth of each subcarrier, and BER is the desired bit-error rate. We assume that σ^2 is constant at all receivers over all subcarrier channels.

Proposition 1—Proportional Fairness: Suppose that $R_i = 0$. Given that the optimal solution to problem (P1) exists, the optimal solution provides proportionally fair resource allocation.

¹The spatial reuse of radio resources is not considered in this paper.

Proof: The objective function $f_0(x)$ can be rewritten in terms of the rate vector r as

$$f_1(r) = \prod_{i=1}^{N-1} (r_i - R_i)$$
(6)

where $r = (r_1, \ldots, r_{N-1})^t$. Given that R_i is zero, the objective function is reduced to

$$f_1(r) = \prod_{i=1}^{N-1} r_i.$$
 (7)

Taking advantage of the strictly concave increasing property of a logarithm function, we can transform the problem (P1) into the same problem with the objective function $\log(f_1(r))$. Since the optimal solution of $(P1), x^*$, determines the optimal rate vector $r^* = (r_1^*, \ldots, r_{N-1}^*)^t$, the following condition holds at $r = r^*$:

$$\sum_{i} \left. \frac{\partial \log f_1(r)}{dr_i} \right|_{r_i = r_i^*} (r_i - r_i^*) = \sum_{i} \frac{r_i - r_i^*}{r_i^*} \le 0.$$
(8)

This is because movement along any direction $(r - r^*)$ at the optimum rate vector r^* cannot improve the objective function. Thus, the rate allocation r^* is *proportionally fair* [16].

C. Solution Method

The problem (P1) is combinatorial in nature. By relaxing the integer control variables into real variables, we can obtain the unique global-optimal real-number solution. Then, a near-optimal integer solution can be searched around this real-number solution.

Taking advantage of the strictly increasing property of a logarithm function, we can transform the problem (P1) into the same problem with the objective function $\log(f_0(x))$. First, we define a Lagrangian function $L : \mathbb{R}_0^N \mapsto \mathbb{R}$ as

$$L(x,\lambda) \equiv \log(f_0(x)) - \lambda \cdot \left(\sum_{i=1}^{N-1} x_i - C\right)$$
(9)

where $x = (x_1, \ldots, x_{N-1})^t$, λ is the associated Lagrange multiplier, \mathbb{R} and \mathbb{R}_0 denote the set of real numbers and the set of non-negative real numbers, respectively. Let

$$g_i(x_i) \equiv \frac{\partial \log(f_0(x))}{\partial x_i} \tag{10}$$

and let $\delta_i \equiv a \bar{G}_i \bar{p}_i$, then we have

$$g_i(x_i) = \frac{\log_2(1+\delta_i/x_i) - \delta_i/(x_i+\delta_i)/\log 2}{x_i W \log_2(1+\delta_i/x_i) - R_i}.$$
 (11)

Proposition 2: Suppose that the maximum transmit power of MC *i* is positive, i.e., $\bar{p}_i > 0$. Then, $\log(f_0(x))$ is strictly increasing for all x_i 's such that $x_iW\log_2(1 + \frac{\delta_i}{x_i}) > R_i$ for $i = 1, \dots, N - 1$. **Proof:** For all x_i 's such that $x_iW\log_2(1 + (\delta_i)/(x_i)) > R_i$, the denominator of $g_i(x_i)$ in (11) is positive. Thus, it is sufficient for us to show that the numerator is positive for $x_i > x_i^{\min}$, where $x_i^{\min} > 0$ is the unique solution of $x_iW\log_2(1 + (\delta_i)/(x_i)) = R_i$. Consider a function

$$\rho(z) \equiv \log(1+z) - \frac{z}{1+z} (z>0).$$
(12)

Since the first derivative is positive for z > 0, i.e.,

$$\frac{d\rho(z)}{dz} = \frac{z}{(1+z)^2} > 0 \tag{13}$$

 $\rho(z)$ is strictly increasing $\forall z > 0$, and $\rho(z) > \rho(0) = 0$. Thus, $\forall x_i > 0$, the numerator of $g_i(x_i)$ in (11) is reduced to

$$\frac{1}{\log 2} \left\{ \log \left(1 + \frac{\delta_i}{x_i} \right) - \frac{\delta_i}{x_i + \delta_i} \right\} = \frac{1}{\log 2} \cdot \rho \left(\frac{\delta_i}{x_i} \right) > 0.$$
(14)

This results in the numerator of $g_i(x_i) > 0$ for $x_i > 0$ and $x_i > x_i^{\min}$. This completes the proof.

From the Karush–Kuhn–Tucker (KKT) theorem [14], [17], we have

$$\frac{\partial L(x,\lambda)}{\partial x_i} = 0, \text{ i.e., } g_i(x_i) = \lambda, \quad \forall i.$$
(15)

By differentiating $g_i(x_i)$ with respect to x_i , we can simply verify that $(dg_i)/(dx_i) < 0$ for all $x_i > x_i^{\min}$, i.e., $g_i(x_i)$ is a strictly decreasing function.² Thus, the inverse function of $g_i(x_i), g_i^{-1}(x_i)$, exists for $x_i > 0$. Because $g_i^{-1}(\lambda) = x_i, \forall i$, we have

$$\sum_{i=1}^{N-1} g_i^{-1}(\lambda) = C.$$
 (16)

$$h(\lambda) \equiv \sum_{i=1}^{N-1} g_i^{-1}(\lambda). \tag{17}$$

Proposition 3: An inverse function exists for $h(\lambda)$.

Proof: From the fact that
$$(ag_i)/(ax_i) < 0$$
, we have

$$\frac{dh}{d\lambda} = \sum_{i=1}^{N-1} \frac{dg_i^{-1}}{d\lambda} \\
= \sum_{i=1}^{N-1} \left\{ 1 \left/ \left(\frac{dg_i}{dx_i} \right) \right\} < 0$$
(18)

namely, $h(\lambda)$ is a strictly decreasing function. This completes the proof.

From Proposition 3, we obtain

$$\lambda^* = h^{-1}(C) \tag{19}$$

²Recall that x_i^{\min} is the unique solution of $x_iW\log_2(1+(\delta_i)/(x_i))=R_i$, which is positive.

and finally

$$x_i^* = g_i^{-1}(\lambda^*), \forall i.$$
(20)

This is the optimal real-number solution of (P1). The near-optimal algorithm to find an integer solution based on this optimal real-number solution is presented as Phase 2 in Fig. 2.

Even though the integer solution obtained by the proposed algorithm (Fig. 2) is based on searching the neighbors of the optimal real-number solution for an integer solution, the optimality of the integer solution is not always guaranteed. To show the performance of the proposed algorithm, we evaluate the gap between the optimal real-number solution, which is obtained by the proposed algorithm in Phase 1 of Fig. 2, and our integer solution. The experimental results on the gap are shown in Fig. 4, where it is observed that among 2000 randomly configured test problems, more than 96% of them have a gap smaller than 0.002 (0.2%).

IV. LEVEL-1 SCHEDULING: SUBCARRIER AND POWER ALLOCATION

A. Scheduling of Mesh Client

In Level-1 scheduling, each MC i performs scheduling to determine which subcarriers (out of the subcarriers allocated to node i) to allocate to the respective links $(i, j), j \in A_i$, and how much power to allocate to the respective subcarriers. The scheduling inputs for MC i are: the set of allocated subcarriers C_i , a vector traffic demands, including forwarding traffic, from MC i to MCs $j \in A_i, (R_{ij})^t$ $(R_{ij} \equiv R_i \cdot y_{ij})$, and the channel gains of the respective links $(i, j), j \in \mathcal{A}_i$, on subcarriers $k \in \mathcal{C}_i, G_{ij}^k$. The objective of Level-0 scheduling is to fairly allocate subcarriers to the respective MCs, whereas that of Level-1 scheduling is to fairly allocate the subcarriers, which are allocated to each MC, to the respective links and to allocate the powers for the respective subcarriers. We formulate an optimization problem for the Level-1 scheduling and suggest an alternative scheduling problem with an exact solution method in the following sections.

B. Problem Formulation for Mesh Client i

Each MC $i \in \mathcal{N} - \{0\}$ should solve problem (P2) to determine: 1) the number of subcarriers, $\sum_{k \in \mathcal{C}_i} x_{ij}^k$ to allocate to each link (i, j), where $x_{ij}^k = 1$ if link (i, j) is assigned subcarrier k and $x_{ij}^k = 0$ otherwise, and 2) the power p_i^k for each subcarrier k assigned to link (i, j). Problem (P2) is solved at MC i using the current values of \mathcal{C}_i $(x_i^* = |\mathcal{C}_i|)$ obtained from the MR, and R_{ij} 's and G_{ij}^k 's obtained locally. This problem can be considered as a generalized case of proportionally fair resource allocation

$$(P2)$$

$$\operatorname{Max} \prod_{j \in \mathcal{A}_i} \left\{ \sum_{k \in \mathcal{C}_i} x_{ij}^k W \log_2 \left(1 + a G_{ij}^k p_i^k \right) - R_{ij} \right\}$$

$$(21)$$

 TABLE II

 COMPARISON OF (P2) AND (P3)

item	(P2)	(P3)
problem type	non-linear programming*	non-linear programming
integer variable	included	none
real variable	included	included
complexity	high	low^\dagger
feasibility check	difficult	simple
achievable rate	≤	\$

*: non-linear mixed integer programming.

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†: a compact form of the optimal solution exists if (P3) is feasible; it is not necessary for each MC to optimize subcarrier assignment.

 \ddagger : this is because there is no granularity in (P3).

abject to

$$\sum_{k \in \mathcal{C}_i} x_{ij}^k W \log_2 \left(1 + a G_{ij}^k p_i^k \right) \ge R_{ij}, j \in \mathcal{A}_i \qquad (22)$$

$$\sum_{k \in \mathcal{C}_i} \sum_{j \in \mathcal{A}_i} x_{ij}^k \le x_i^* \tag{23}$$

$$\sum_{j \in \mathcal{A}_i} x_{ij}^k \le 1, k \in \mathcal{C}_i \tag{24}$$

$$\sum_{k \in C_i} p_i^k \le \bar{p}_i, i \in \mathcal{N} - \{0\}$$

$$x_{ij}^k : \text{binary integer}$$

$$p_i^k : \text{non-negative real-number.}$$
(25)

C. Solution Method—An Alternative Approach

As shown above, (P2) is a nonlinear mixed integer programming problem. To improve the tractability of the solution technique, such as criteria for checking feasibility and methods for finding the optimal solution in real-time computations, we suggest an alternate problem formulation (P3) and solution method that can give better performance in terms of both the computational efficiency and the optimum objective value. (P3) given next is a nonlinear real-number programming problem. The alternate problem formulation is possible not due to forced relaxation but due to the benefit of time allocation scheduling where the control variable of allocation time is a real-number. The characteristics of (P2) and (P3) are compared in Table II.

In the alternate problem formulation (P3), the problem (P2) is reduced to a time allocation problem in which the allocated time fraction τ_j is optimized, where τ_j is defined as the fraction of time when link $(i, j), j \in A_i$, occupies the full capacity that MC *i* can achieve using all the subcarriers assigned to it

(P3)

$$\operatorname{Max} \prod_{j \in \mathcal{A}_{i}} \left\{ \tau_{j} \cdot \sum_{k \in \mathcal{C}_{i}} W \log_{2} \left(1 + a G_{ij}^{k} p_{i}^{k} \right) - R_{ij} \right\}$$
(26)

subject to

$$\tau_j \cdot \sum_{k \in \mathcal{C}_i} W \log_2 \left(1 + a G_{ij}^k p_i^k \right) \ge R_{ij}, j \in \mathcal{A}_i$$
 (27)

$$\sum_{j \in \mathcal{A}_i} \tau_j \le 1 \tag{28}$$

$$\sum_{k \in \mathcal{C}_i} p_i^k \le \bar{p}_i, i \in \mathcal{N} - \{0\}$$

$$\tau_j, p_i^k : \text{non-negative real-number.}$$
(29)

The value 1 on the right-hand side of (28) is the normalized scheduling interval of this time allocation scheduling. For any desired scheduling interval T, the optimal amount of time at MC *i* scheduled for link (i, j) is $\tau_i^* \cdot T$.

Proposition 4-Necessary and Sufficient Condition for Op*timal Power Allocation:* Suppose that (P3) is feasible. For any given feasible τ_i , the power allocation vector $p_i^* = (p_i^{*})^t$ is optimal if and only if p_i^* is the (not necessarily unique) maximizer of $\sum_{k \in C_i} W \log_2(1 + aG_{ij}^k p_i^k)$ subject to the constraint $\sum_{k \in \mathcal{C}_i} p_i^k \leq \overline{p}_i.$

Proof: Note that all the subcarriers that have been allocated to MC i are used during each time interval τ_i . The amount of resource assigned to each respective link (i, j)'s is dependent on τ_i 's only. This completes the proof. The following shows more details.

 (\Leftarrow) For any given feasible τ_i , if p_i^* is the maximizer of $\sum_{k \in C_i} W \log_2(1 + aG_{ij}^k p_i^k)$ subject to the constraint $\sum_{k \in C_i} p_i^k \leq \overline{p}_i$, then it is also the maximizer of $\{\tau_j \cdot$

Proposition 5-Optimum Power Allocation: Suppose that (P3) is feasible. For any given MC *i*, the elements of the optimal power allocation vector $p_i^* = (p_i^{k*})^t$ for (P3) are given by

$$p_i^{k*} = \left(\frac{1}{|\mathcal{C}_i|} \cdot \left\{\bar{p}_i + \sum_{l \in \mathcal{C}_i} \frac{1}{aG_{ij}^l}\right\} - \frac{1}{aG_{ij}^k}\right)^+, k \in \mathcal{C}_i$$
(30)

where $x^+ \equiv \max(x, 0)$.

Proof: The optimal power allocation vector is obtained by maximizing $\{\sum_{k \in C_i} W \log_2(1 + aG_{ij}^k p_i^k)\}$ subject to $\sum_{k \in \mathcal{C}_i} p_i^k \leq \overline{p}_i$. Let $L : \mathbb{R}_0^{|\mathcal{C}_i|+1} \mapsto \mathbb{R}$ be

$$L(p_i, \lambda) = \sum_{k \in \mathcal{C}_i} W \log_2 \left(1 + a G_{ij}^k p_i^k \right) - \lambda \\ \times \left(\sum_{k \in \mathcal{C}_i} p_i^k - \bar{p}_i \right) \quad (31)$$

where $p_i = (p_i^k)^t$ and λ is the Lagrange multiplier. Then

$$\frac{\partial L}{\partial p_i^k} = \frac{WaG_{ij}^k}{\log 2 \cdot \left(1 + aG_{ij}^k p_i^k\right)} - \lambda, k \in \mathcal{C}_i$$
(32)

$$\frac{\partial L}{\partial \lambda} = \sum_{k \in \mathcal{C}_i} p_i^k - \bar{p}_i.$$
(33)

Employing KKT conditions [17] yields

$$p_i^{k*} = \left(\frac{W}{\lambda^*} - \frac{1}{aG_{ij}^k}\right)^+ \tag{34}$$

where

$$\frac{1}{\lambda^*} = \frac{1}{W|\mathcal{C}_i|} \left\{ \bar{p}_i + \sum_{l \in \mathcal{C}_i} \frac{1}{aG_{ij}^l} \right\}.$$
(35)

Note that the existence of the optimum power allocation given by Proposition 5 does not imply the feasibility of (P3). We establish the feasibility of (P3) as follows.

Corollary 1—Feasibility Criterion for (P3): For any given MC i, (P3) is feasible if and only if

$$\sum_{j \in \mathcal{A}_i} \frac{R_{ij}}{\sum_{k \in \mathcal{C}_i} W \log_2(1 + aG_{ij}^k p_i^{k*})} \le 1$$
(36)

where p_i^* is the optimal power allocation vector which elements are given by (30).

Proof: With the optimal power allocation vector p_i^* , the rest of constraints (27) and (28) result in this criterion.

Proposition 6-Optimum Time Allocation: Suppose that (P3) is feasible. For any given MC *i*, the elements of the optimal time allocation vector $\tau^* = (\tau_i^*)^t$ for (P3) are given by

$$\tau_j^* = \frac{1}{|\mathcal{A}_i|} \cdot \left\{ 1 - \sum_{l \in \mathcal{A}_i} \frac{R_{il}}{D^*} \right\} + \frac{R_{ij}}{D^*}, j \in \mathcal{A}_i \qquad (37)$$

where

$$D^* = \sum_{k \in \mathcal{C}_i} W \log_2(1 + aG_{ij}^k p_i^{k*}).$$
(38)

Proof: Here, we take logarithm on the objective function of (P3). Let $L: \mathbb{R}_{0}^{|\mathcal{A}_{i}|+1} \mapsto \mathbb{R}$ be

$$L(\tau,\lambda) = \sum_{j \in \mathcal{A}_i} \log(\tau_j D^* - R_{ij}) - \lambda \left(\sum_{j \in \mathcal{A}_i} \tau_j - 1\right)$$
(39)

where $\tau = (\tau_i)^t$ and λ is the Lagrange multiplier. Then

$$\frac{\partial L}{\partial \tau_j} = \frac{D^*}{\tau_j D^* - R_{ij}} - \lambda, j \in \mathcal{A}_i \tag{40}$$

$$\frac{\partial L}{\partial \lambda} = \sum_{j \in \mathcal{A}_i} \tau_j - 1. \tag{41}$$

Employing KKT conditions [17] yields

$$\tau_j^* = \frac{1}{\lambda^*} + \frac{R_{ij}}{D^*} \tag{42}$$



Fig. 3. An exact algorithm for (P3) performed by MC $i \in \mathcal{N} - \{0\}$.



Fig. 4. Gap performance of the proposed algorithm for (P1). Gap is less then 0.2% (0.002) for 96.6% of randomly configured 2000 test problems for BER = 2,5 dB.

where

$$\frac{1}{\lambda^*} = \frac{1}{|\mathcal{A}_i|} \left\{ 1 - \sum_{l \in \mathcal{A}_i} \frac{R_{il}}{D^*} \right\}.$$
 (43)

Based on the above properties of (P3), we summarize the exact solution algorithm for the problem in Fig. 3.

V. SIMULATION RESULTS

In order to evaluate the performance of the proposed schemes, we consider network scenarios with two MCs and multiple MCs in simulations. Two different optimization criteria (end-to-end maximal rate and generalized proportional fairness) are compared. Here, the end-to-end maximal rate scheme is to maximize the overall end-to-end rate with the same constraint of (P1) [8], whereas the generalized proportional fairness denotes the fair scheme proposed in this paper.

We simulated an OFDMA WMN³ with 128 subcarriers over a 3.2-MHz band. Each transmitter is synchronized with respect to the receiver's clock reference to make the tones orthogonal. The maximal power at MC *i* is $\bar{p}_i = 50$ mW, the thermal noise power is $\sigma^2 = 10^{-11}$ W, and two desired BERs: 10^{-2} and 10^{-5} are considered. The propagation loss exponent is three.

In the two MC scenario, the three nodes are located on the same line. The distance between MC 1 and the MR is fixed at



Fig. 5. Each user's end-to-end rate (Mb/s).



Fig. 6. Fairness comparison. In this two-MC simulation setup, the end-to-end rate of MC 1 is $r_1 - r_2$, whereas that of MC 2 is r_2 .

 $D_1 = 100$ m, whereas D_2 , the distance between MC 1 and MC 2 varies from 110 to 300 m. Each MC requires a minimum rate of $\bar{R}_i = 100$ kb/s $\forall i$. For $i, j \in \mathcal{N} = \{0, 1, 2\}$, the routing information is fixed as: $y_{10} = y_{21} = 1$; $y_{ij} = 0$, otherwise. The routing information in the reverse direction is $(y_{ij})^t$.

In Fig. 5, the end-to-end rates of both MCs for the proposed scheme and maximal rate scheme are shown versus D_2 . The end-to-end rate of each MC includes only traffic generated by that MC, but not forwarded traffic from adjacent MCs. For the maximal rate scheme, the MCs closer to the MR has a higher rate and the rate increases as D_2 increases. This is because allocating as much resources as possible to the MC that has a better channel gain increases the overall rate. For the proposed scheme, the difference between the rates of two MCs is smaller than the difference observed in the maximal rate scheme. However, the difference increases as D_2 increases. This is because the marginal utility in the case of allocating more resources is decreasing in accordance with the fast decrease of channel gain at speed D_2^{-3} , i.e., the fast decrease of channel gain due to decrease in D_2 cannot be compensated with allocating more resources to MC 2.

In addition, the fairness ratio between the two nodes' end-toend rates is shown in Fig. 6, where the end-to-end rates of MC 1 and 2 are $r_1 - r_2$ and r_2 , respectively. The fairness ratio between the end-to-end rates is given by $(r_1 - r_2)/r_2$. For the maximal

³Most parameters except those related to single hopping are taken to be the same values used in [8].



Fig. 7. Overall end-to-end rate (Mb/s).



Fig. 8. Each user's rate (Mb/s) versus D_2 .

rate scheme, it is observed that the fairness ratio changes greatly at the rate of about +3.1 per 10 m as D_2 increases and that the ratio is much greater than unity. This shows that the maximal rate scheme is very unfair. Even though allocating resources as much as possible to the MC that has a better channel gain increases the overall rate, this results in the MC with a better channel gain being dominant in resource occupancy, causing the other MC(s) to starve. For the proposed scheme, the ratio $(r_1 - r_2)/r_2$ changes very slowly at the rate of changes about $+2.7 \times 10^{-2}$ per 10 m, which is less than 1% of the rate of change in the maximal rate scheme. This shows the fairness of the proposed resource allocation scheme.

Fig. 7 shows the overall (total) end-to-end rate of the two MCs for the maximal rate and proposed schemes versus D_2 . It is observed that the maximal rate scheme achieves almost constant overall rate. This means that the scheme does not care for MC 2 even though the MC starves under bad channel conditions, which shows that the maximal rate scheme is very unfair. In contrast, the proposed scheme trades off the overall rate for fair support of MC 2 under bad channel conditions.

In Fig. 8, the rates of both MCs for the proposed and maximal rate schemes are shown versus D_2 . Here, the rate of each MC includes both traffic generated by the MC, as well as forwarded traffic coming from neighboring MCs. For the maximal



Fig. 9. Overall rate (Mb/s) $r_1 + r_2$.



Fig. 10. Efficiency of resource utilization (%). The fraction of overall end-to-end rates out of overall rates.

rate scheme, the MC closer to the MR has a higher rate, and the difference increases, with an upper bound, as the distance D_2 increases. For the proposed scheme, however, the rate of MC 1 decreases as the distance of MC 2 from the MR D_2 increases. That is, the rates of both MC 1 and MC 2 decrease as the average channel gain decreases.

In Fig. 9, we show the overall (total) rate of the two MCs for two schemes versus D_2 . Note that the rate shown in this figure includes all the rates in all the links. For example, suppose that 20 b/s is allocated to link (1, 0) and 10 b/s is to link (2, 1). Then, the overall rate mentioned here is 20+10 = 30 b/s, which is not necessarily equal to the overall end-to-end rate. In the figure, we observe that the overall rate of the proposed scheme is greater than that of the maximal rate scheme. Note that the maximal rate scheme does not maximize the overall rate but it maximizes the overall end-to-end rate. This shows that a greater overall rate does not always imply a greater overall end-to-end rate in multihop networks. Thus, the overall rate is not a performance metric that has significant implications in WMNs. Here, we just consider this metric to show the efficiency of the schemes. In this paper, we define the efficiency as the relative ratio of the overall end-to-end rate to the overall rate that a network consumes. Fig. 10 shows the efficiency of two schemes versus D_2 , where we observe that the efficiency increases as D_2 increases.



Fig. 11. Efficiency versus fairness.



Fig. 12. Overall end-to-end rate (Mb/s) versus number of MCs.

This is because the rate of MC 2 gets smaller as it gets farther from MC 1, making the rate of decrease in the overall rate greater than the rate of decrease in the overall end-to-end rate. Fig. 11 shows the fairness index versus the efficiency for two schemes. The efficiency of the maximal rate scheme is greater than that of the proposed scheme at the cost of a large degree of unfairness.

Next, we set up the simulations with a total of 19 MCs to evaluate the proposed scheme at BER = 10^{-2} . We consider a circular cell with a 150 m radius. The MR is located at the center of the cell, 6 MCs (called *near MCs*) are randomly located within the cell and the other 12 MCs (called *far MCs*) are randomly located within the co-center circle of radius 300 m but outside the cell so that the 12 far MCs cannot access the MR without relaying. Also, the far MCs may not access the MR if there are no relaying MCs near themselves. To evaluate the schemes in a multihop environment, the routing matrices for the respective layouts of MCs are chosen to form minimum spanning trees with the MR at the root. The other settings are the same as those of the two MC scenarios.

Fig. 12 shows the overall end-to-end rate versus the number of MCs connected to the MR (regardless of the number of hops). It is observed that two schemes have better performance when the

number of connected MCs increases. This is due to multiuser diversity. The performance improvement decreases gradually. The proposed scheme has comparable performance to that of the maximal rate scheme. Also, the performance gap between the proposed scheme and the maximal rate scheme decreases when the number of connected users increases. This is because both schemes search for feasible solutions which do not violate the constraint that the allocated rate must be greater than or equal to the minimum rate requirement $r_i \geq R_i$. As the number of connected MCs increases, the feasible region of rate allocation decreases and, finally, it converges to a single point $r^* = R^*$. If there are further MCs trying to join the WMN, then the two problems (maximal rate problem and fairness problem) become infeasible. This demonstrates that the maximal rate scheme and the proposed scheme go to the same rate allocation vector $r^* =$ R^* as far as the problems are feasible, making the gap converge to zero.

VI. CONCLUSION

In this paper, we have proposed a new fair scheduling scheme for WMNs using OFDMA, with the objective of fairly allocating subcarriers and power so that the NBS fairness criterion is maximized. Instead of solving a single global control problem, the subcarrier and power allocation problem has been decoupled into two subproblems that can be solved hierarchically in a distributed manner. This decoupling is useful and necessary because not all the information necessary for centralized scheduling is available at the MR in a WMN. By decoupling the global control problem, we have proposed a distributed fair scheduling scheme, where the MR solves only the subcarrier allocation problem online based on the limited available information, and each MC solves the reduced subcarrier and power allocation problem based on the limited available information online. We have formulated the two problems into nonlinear integer programming problems. Also, we have developed simple and efficient solution algorithms for the MR's problem and developed a closed form solution by transforming the MC's problem into a time-division scheduling problem. Extensive simulation results have demonstrated that the proposed scheme provides fair opportunities to the MCs and an overall end-to-end rate comparable to maximal rate scheduling when the number of MCs increases.

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