Benchmark: Flight Envelope Protection in Autonomous Quadrotors

Shahab Kaynama and Claire J. Tomlin

EECS, UC Berkeley

CPSWeek – ARCH 2014

Context and Origin

- Civilian applications of quadrotors:
  - Monitoring and exploration of hazardous areas;
  - Search and rescue;
  - Delivery of antibiotics in underprivileged countries;
  - HD filming of sports events; etc.
- Actuator limitations + aerodynamical-structural constraints = **flight envelope**
- Respect **hard** state and input constraints, despite potentially conflicting performance objectives and unknown (bounded) disturbances
- Enforcing constraint satisfaction ⇒ ensuring flight envelope protection and safety

[Photos courtesy of Digital Trends and Amazon]

Model Description

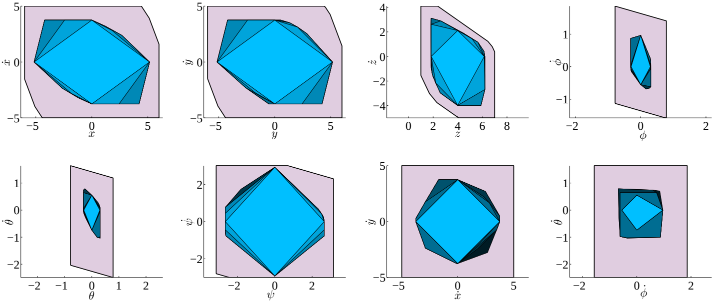
- 12D nonlinear ODE (normalized equations of motion) + 4D control input
- $$\left\{\begin{array}{l}\text{States: positions, velocities, angles, angular velocities}\\ x = \begin{bmatrix} \mathrm{x} & \mathrm{y} & \mathrm{z} & \dot{\mathrm{x}} & \dot{\mathrm{y}} & \dot{\mathrm{z}} & \phi & \theta & \psi & \dot{\phi} & \dot{\theta} & \dot{\psi} \end{bmatrix}^\top \\ \text{Inputs: thrust and angular accelerations} \\ u = \begin{bmatrix} u_1 & \ddot{\phi} & \ddot{\theta} & \ddot{\psi} \end{bmatrix}^\top \end{array}\right\}$$ All constrained!
- Agile yet under-actuated (6 degrees of freedom, 4 actuators)
- Can be linearized (e.g. about hover mode)

Verification Problem

- Use this high-dim benchmark to assess any algorithm that
  1. Computes an **under-approximation** of the viability kernel
     (states from which all constraints can be satisfied over a given horizon)
  2. Synthesizes the corresponding control signals
- Quality indicators: Conservatism; accuracy; convergence; termination
- Recommended frameworks: Sampled-data or continuous time
  - Due to agility, discrete time may be acceptable if high sampling frequency
- Variants: Robustness against uncertainty/disturbance; simplified dynamics

Example of One Such Algorithm

- Sampling-based approach; scalable and efficient; synthesizes safety controls
- **Sampled-data** framework; analysis around hover mode



Selected projection plots; under-approx (blue), over-approx (lavender).

See HSCC talk by J. Gillula and first author's website for more detail.

Benchmark: Flight Envelope Protection in Autonomous Quadrotors

Shahab Kaynama and Claire J. Tomlin

www.eecs.berkeley.edu/∼kaynama