A Suboptimal Discrete-Time Predictive Current Controller for a Voltage-Source Inverter

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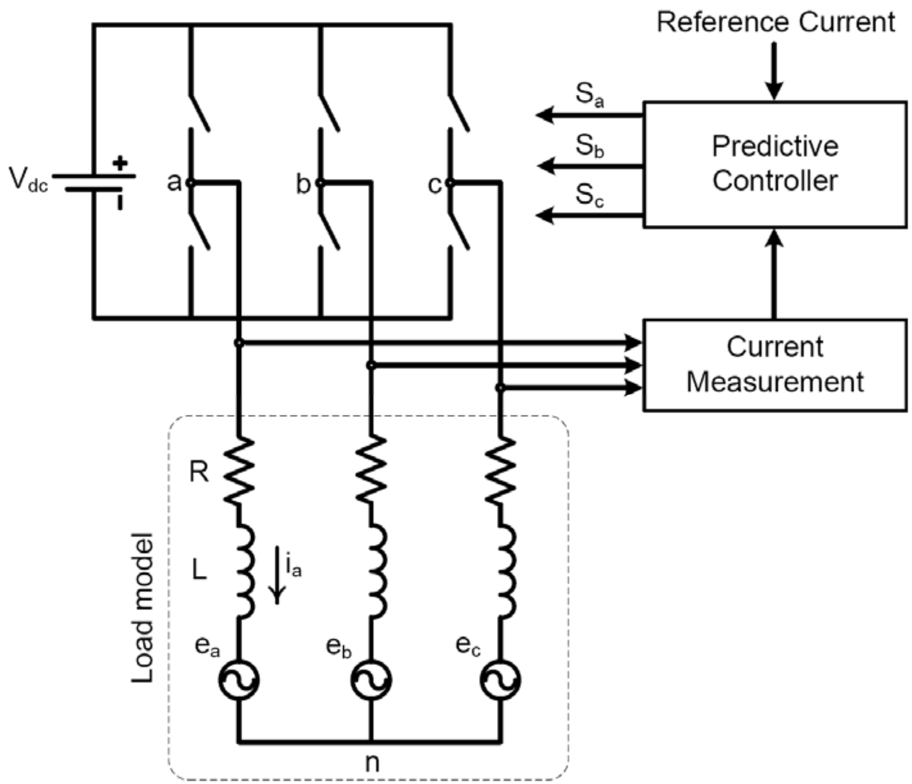
Introduction

- Voltage-Source Inverter (VSI)
- Predictive current control of VSI's
  - Model-based to adaptive

    Wu, Dewan, Slemon (1990); Wu, et al. (1991); Habetler (1993); Kükrer (1996); Malesani, Mattavelli, Buso (1999); Kennel, Linder, Linke (2001); Bode, et al. (2003); Moon, Kim, Youn (2003); Abu-Rub, et al. (2004); Mattavelli, Spiazzi, Tenti (2005); Kojabadi, et al. (2006); Wu, Lehn (2006); Saggini, et al. (2007); Nasiri (2007); Rodriguez, et al. (2007); Mohamed, El-Saadany (2007); Cortes, et al. (2008); …
- Discrete-time predictive control

Introduction

- VSI under predictive current control



Introduction

- Rodriguez, et al. (2007)
  – Very effective, computationally simple

Predict load current for all switching states

↓

Choose the one that minimizes a cost function

↓

Apply corresponding signal to switches

Introduction

- Our contribution

  Compute optimal deadbeat control

  ↓

  Approximate with physically-realizable input

  ↓

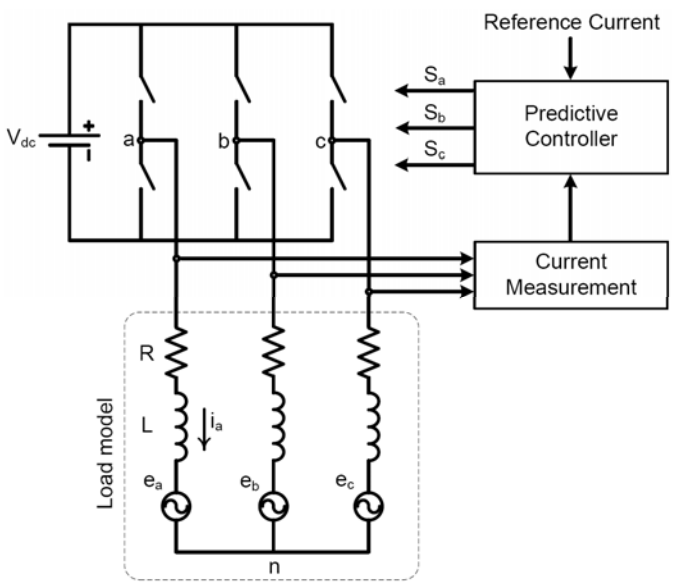
  Apply corresponding signal to switches

  - Simple to implement
  - Compensates for delay
  - Feedback–feedforward
  - Better back-EMF prediction improves performance and robustness

Outline

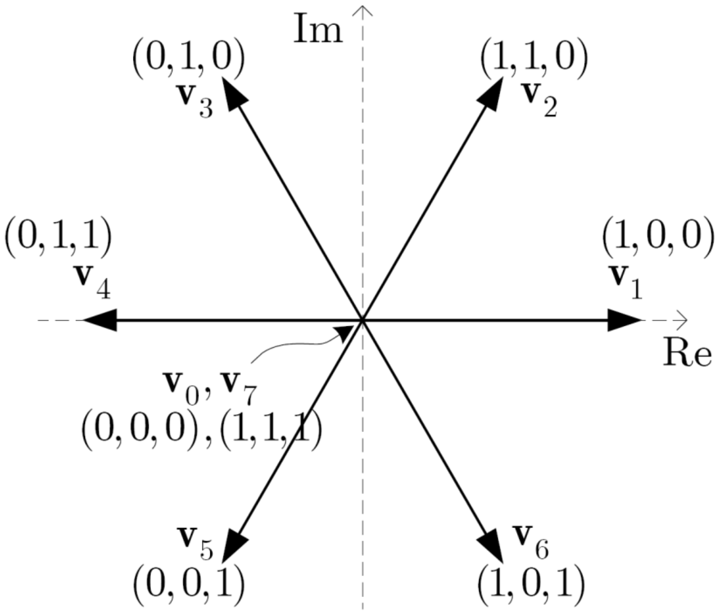
- Discrete-time load model
- Proposed Control Strategy
  - Optimal deadbeat control
  - Suboptimal input selection
  - Back-EMF prediction
  - Robustness
- Simulation results

Discrete-Time Model

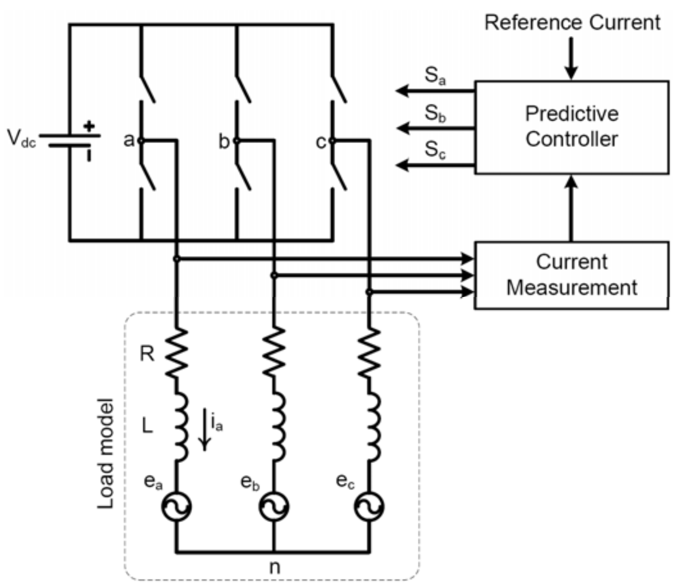


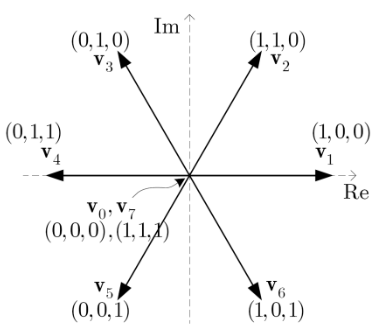
$$\mathbf{v} = \frac{2}{3}\left(v_{an} + \boldsymbol{\eta} v_{bn} + \boldsymbol{\eta}^2 v_{cn}\right), \quad \boldsymbol{\eta} := e^{j2\pi/3}$$

$$\mathbf{v}_\ell = \begin{cases} \frac{2}{3} V_{dc} e^{j(\ell-1)\pi/3} & \text{for} \quad \ell = 1, \dots, 6 \\ \mathbf{0} & \text{for} \quad \ell = 0, 7 \end{cases}$$



Discrete-Time Model





$$\mathbf{v} = \frac{2}{3}\left(v_{an} + \boldsymbol{\eta} v_{bn} + \boldsymbol{\eta}^2 v_{cn}\right), \quad \boldsymbol{\eta} := e^{j2\pi/3}$$

$$\mathbf{v}_\ell = \begin{cases} \frac{2}{3} V_{\mathrm{dc}} e^{j(\ell-1)\pi/3} & \text{for} \quad \ell = 1, \dots, 6 \\ \mathbf{0} & \text{for} \quad \ell = 0, 7 \end{cases}$$

$$\mathbf{i} = \frac{2}{3}\left(i_a + \boldsymbol{\eta} i_b + \boldsymbol{\eta}^2 i_c\right)$$

$$\mathbf{e} = \frac{2}{3}\left(e_a + \boldsymbol{\eta} e_b + \boldsymbol{\eta}^2 e_c\right)$$

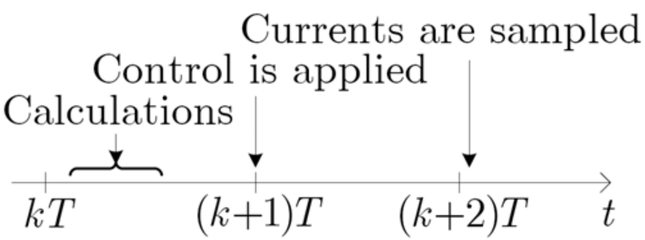
$$\mathbf{v} = R\mathbf{i} + L\frac{d\mathbf{i}}{dt} + \mathbf{e}$$

$d\mathbf{i}/dt \approx \frac{1}{T}\left(\mathbf{i}(k+1) - \mathbf{i}(k)\right)$ *(forward Euler method)*

$$\mathbf{i}(k+1) = A\mathbf{i}(k) + B\left\{\mathbf{v}(k) - \mathbf{e}(k)\right\}$$
$$A = 1 - TR/L \text{ and } B = T/L$$

Proposed Predictive Controller
(Deadbeat Control)

- Goal: $\Delta_i := \mathbf{i} - \mathbf{i}^*$



$$\mathbf{v}(k) = z^{-1}\mathbf{u}^*(k+1)$$

$$\mathbf{u}^*(k+1) = \frac{1}{B}\Big[\mathbf{i}^*(k+2) - A\Big(A\mathbf{i}(k) + B\{\mathbf{v}(k) - z^{-1}\mathbf{e}(k+1)\}\Big)\Big] + \mathbf{e}(k+1).$$

Proposed Predictive Controller
(Deadbeat Control)

$$\mathbf{u}^*(k+1) = \frac{1}{B}\Big[\mathbf{i}^*(k+2) - A\Big(A\mathbf{i}(k) + B\{\mathbf{v}(k) - z^{-1}\mathbf{e}(k+1)\}\Big)\Big] + \mathbf{e}(k+1).$$

$$\mathbf{i}_p^*(k+2) = 6\mathbf{i}^*(k) - 8\mathbf{i}^*(k-1) + 3\mathbf{i}^*(k-2)$$

$$\mathbf{e}_p(k+1) = 6\mathbf{e}(k-1) - 8\mathbf{e}(k-2) + 3\mathbf{e}(k-3)$$

$$\mathbf{e}(k-1) = \frac{1}{B}\Big[A\mathbf{i}(k-1) - \mathbf{i}(k)\Big] + \mathbf{v}(k-1)$$

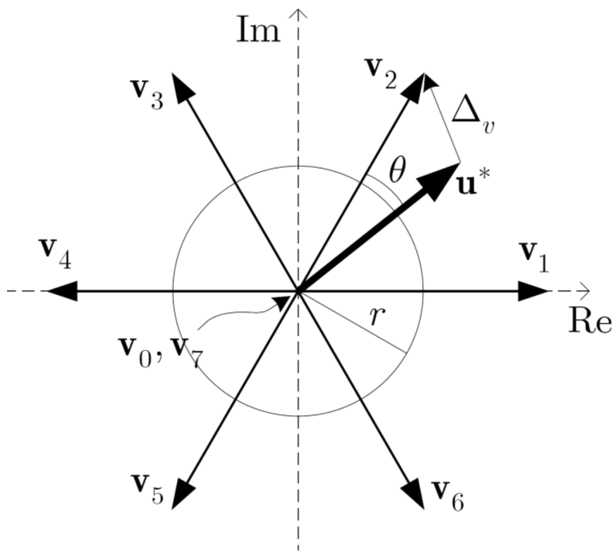
Proposed Predictive Controller
(Deadbeat Control)

$$\mathbf{u}^*(k+1) = \frac{1}{B}\Big[\mathbf{i}^*(k+2) - A\Big(A\mathbf{i}(k) + B\{\mathbf{v}(k) - z^{-1}\mathbf{e}(k+1)\}\Big)\Big] + \mathbf{e}(k+1).$$

$$\mathbf{i}(k+1) = A\mathbf{i}(k) + B\{z^{-1}\mathbf{u}^*(k+1) - \mathbf{e}(k)\}$$
$$= \mathbf{i}^*(k+1)$$

Proposed Predictive Controller
(Suboptimal Voltage Vector Selection)

$$(\mathbf{v}_\ell|\mathbf{u}^*) : \ell = \begin{cases} \underset{\sigma\in\{1,\dots,6\}}{\arg\min} \left|\cos^{-1}\left(\frac{\langle \mathbf{u}^*, \mathbf{v}_\sigma\rangle}{|\mathbf{u}^*|\cdot|\mathbf{v}_\sigma|}\right)\right|, & |\mathbf{u}^*| > r \\ 0, & |\mathbf{u}^*| \leq r \end{cases}$$



- With $r = 0.5|\mathbf{v}_\ell|$, we minimize $|\Delta_v| := |\mathbf{v}_\ell - \mathbf{u}^*|$.
- $r = 0.4|\mathbf{v}_\ell|$ results in lower Total Harmonic Distortion (THD).

Proposed Predictive Controller
(Suboptimal Voltage Vector Selection)

$$(\mathbf{v}_\ell|\mathbf{u}^*) : \ell = \begin{cases} \underset{\sigma\in\{1,\ldots,6\}}{\arg\min} \left| \cos^{-1}\left(\dfrac{\langle \mathbf{u}^*, \mathbf{v}_\sigma \rangle}{|\mathbf{u}^*| \cdot |\mathbf{v}_\sigma|}\right) \right|, & |\mathbf{u}^*| > r \\ 0, & |\mathbf{u}^*| \leq r \end{cases}$$

$$\begin{aligned} \mathbf{i}(k+1) &= A\mathbf{i}(k) + B\{z^{-1}(\mathbf{u}^*(k+1) \\ &\qquad + \Delta_v(k+1)) - \mathbf{e}(k)\} \\ &= \mathbf{i}^*(k+1) + B\Delta_v(k) \end{aligned}$$

Proposed Predictive Controller
(Back-EMF Prediction)

- Closed-loop equation with Lagrange back-EMF prediction:

$$\mathbf{i}(k+1) = \frac{1}{F_0(z)}\mathbf{i}^*(k+1) + \frac{D_0(z)}{F_0(z)}\mathbf{v}(k)$$

where

$$F_0(z) = \frac{1}{B}E_0(z)z^{-1} - \left(\frac{A}{B}E_0(z) - A^2\right)z^{-2}$$
$$D_0(z) = -B + \left(E_0(z) - BA\right)z^{-1}$$
$$E_0(z) = \left(Bz^{-1} + BAz^{-2}\right)\underline{\left[6 - 8z^{-1} + 3z^{-2}\right]}$$

Proposed Predictive Controller
(Back-EMF Prediction)

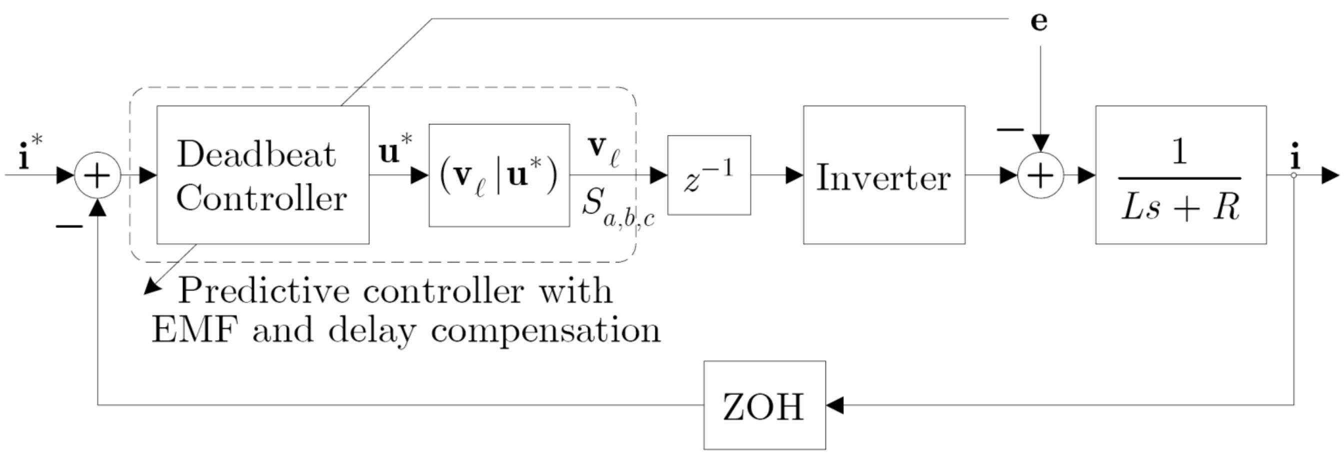
- Replace with a 4-tap, two-step-ahead, finite impulse response (FIR) prediction filter:

$$\mathbf{e}_p(k+1) = \sum_{n=0}^{3} \alpha_n \mathbf{e}(k-n-1)$$

- Significantly better closed-loop response

Proposed Predictive Controller (Robustness)

- Feedforward strategy rejects input disturbance



- Disturbance to partially encapsulate uncertainty. Better back-EMF prediction = increased robustness

Proposed Predictive Controller

(Robustness)

$$\mathbf{u}^*(k+1) = \frac{1}{\widetilde{B}}\Big[\mathbf{i}^*(k+2) - \widetilde{A}\Big(\widetilde{A}\mathbf{i}(k) + \widetilde{B}\{\mathbf{v}(k) - z^{-1}\mathbf{e}(k+1)\}\Big)\Big] + \mathbf{e}(k+1)$$

$\widetilde{A} := 1 - T\widetilde{R}/\widetilde{L}$ and $\widetilde{B} := T/\widetilde{L}$ (*blind estimates*)

$$\mathbf{e}_p(k+1) = \Gamma(z)\widetilde{\mathbf{e}}(k-1)$$

$$\widetilde{\mathbf{e}}(k-1) = \frac{1}{\widetilde{B}}\Big[\widetilde{A}\mathbf{i}(k-1) - \mathbf{i}(k)\Big] + \mathbf{v}(k-1)$$

$\Gamma(z) = \sum_{n=0}^{3} \alpha_n z^{-n}$ (*for an FIR prediction*)

Proposed Predictive Controller
(Robustness)

- Closed-loop equation under uncertainty:

$$\mathbf{i}(k+1) = \frac{B}{\widetilde{B}F(z)}\mathbf{i}^*(k+1) + \frac{D(z)}{F(z)}\mathbf{v}(k)$$

where

$$F(z) = \frac{1}{\widetilde{B}}E(z)z^{-1} - \Big(\frac{\widetilde{A}}{\widetilde{B}}E(z) - \frac{BA^2}{\widetilde{B}}\Big)z^{-2}$$

$$D(z) = -B + \left(E(z) - B\widetilde{A}\right)z^{-1}$$

$$E(z) = \left(Bz^{-1} + B\widetilde{A}z^{-2}\right)\underline{\Gamma(z)}.$$

Simulations

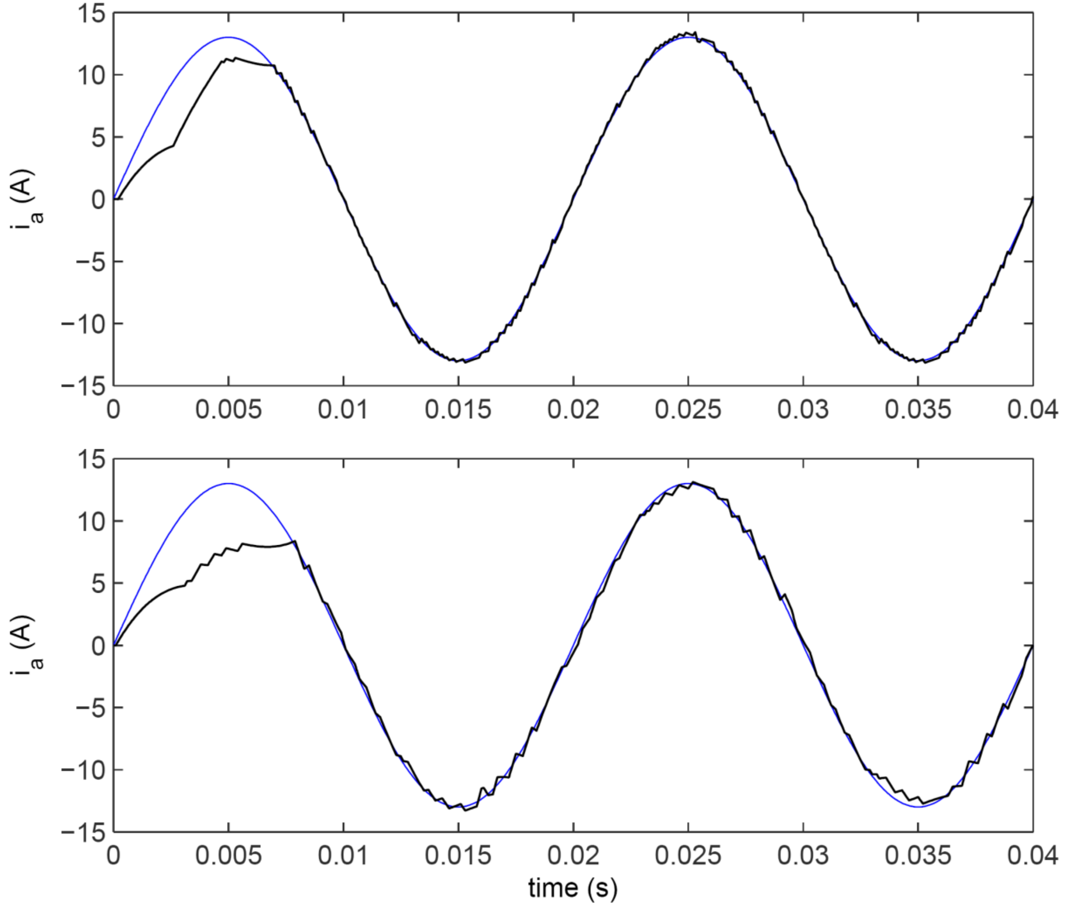
- Nominal configurations

$$\text{Case 1:} \quad R = 0.5\,\Omega,\ L = 10\,\text{mH},\ V_{\text{dc}} = 100\,\text{V}$$
$$\text{Case 2:} \quad R = 10\,\Omega,\ L = 10\,\text{mH},\ V_{\text{dc}} = 500\,\text{V}.$$

- Track a 13 A, 50 Hz reference current. Unmeasurable disturbance (34 V, 50 Hz).
- Results:
  - Phase-*a* load current (Compare with Rodriguez, et al. (2007))
  - Effect of back-EMF prediction on performance
  - Robustness (Compare with Rodriguez, et al. (2007))

Simulation

(Phase-*a* Load Current. Case 1, T=100μs)

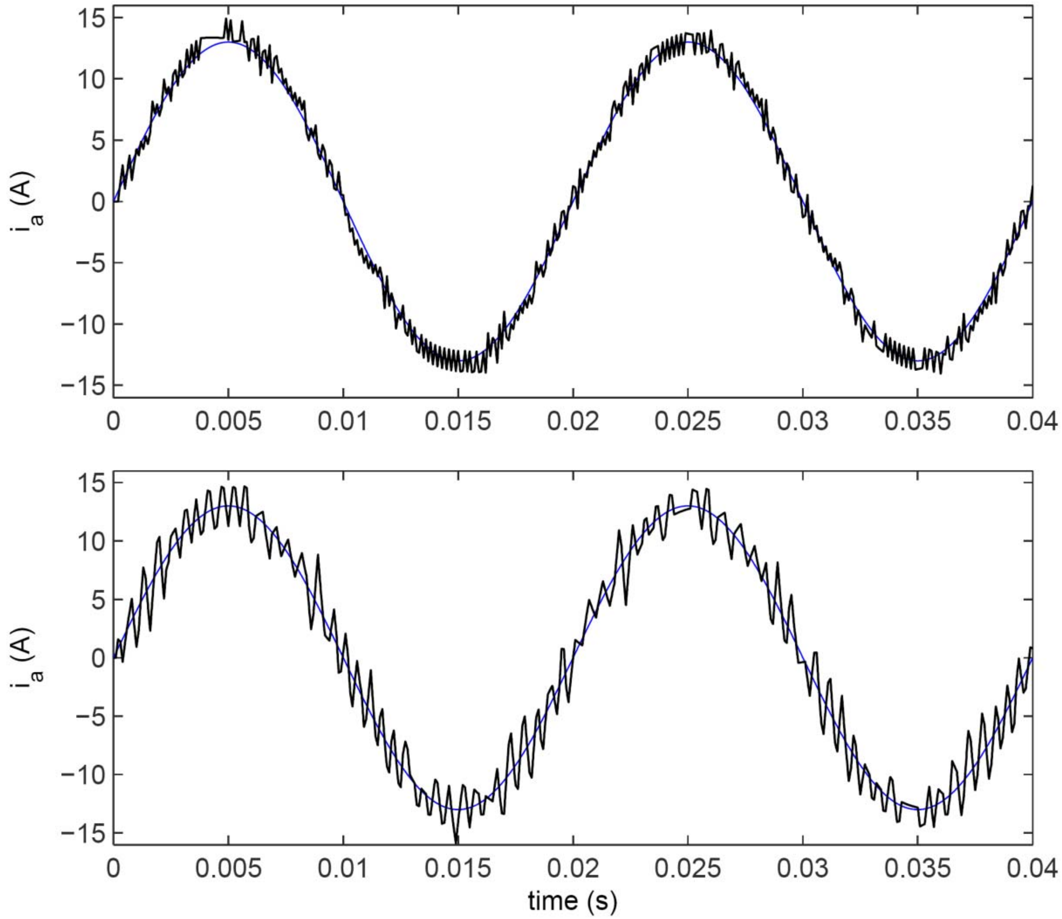


(Proposed)

(Rodriguez, et al. 07)

Simulation

(Phase-*a* Load Current. Case 2, T=100μs)



(Proposed)

(Rodriguez, et al. 07)

Simulation

(Phase-*a* Load Current)

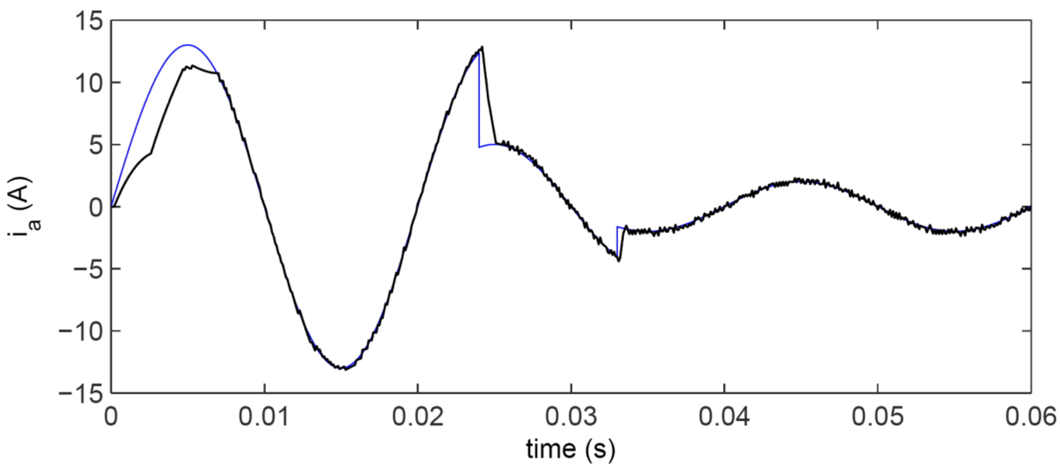
Output current THD when using the proposed predictive Controller vs. the algorithm in Rodriguez, et al. 2007

	THD using the proposed controller	**THD using the controller in [15]**
Case 1 ($T = 100\,\mu$s)	1.47%	3.23%
Case 2 ($T = 100\,\mu$s)	6.68%	15.44%
Case 1 ($T = 20\,\mu$s)	0.33%	0.71%
Case 2 ($T = 20\,\mu$s)	1.41%	3.54%

Up to 60% reduction in THD

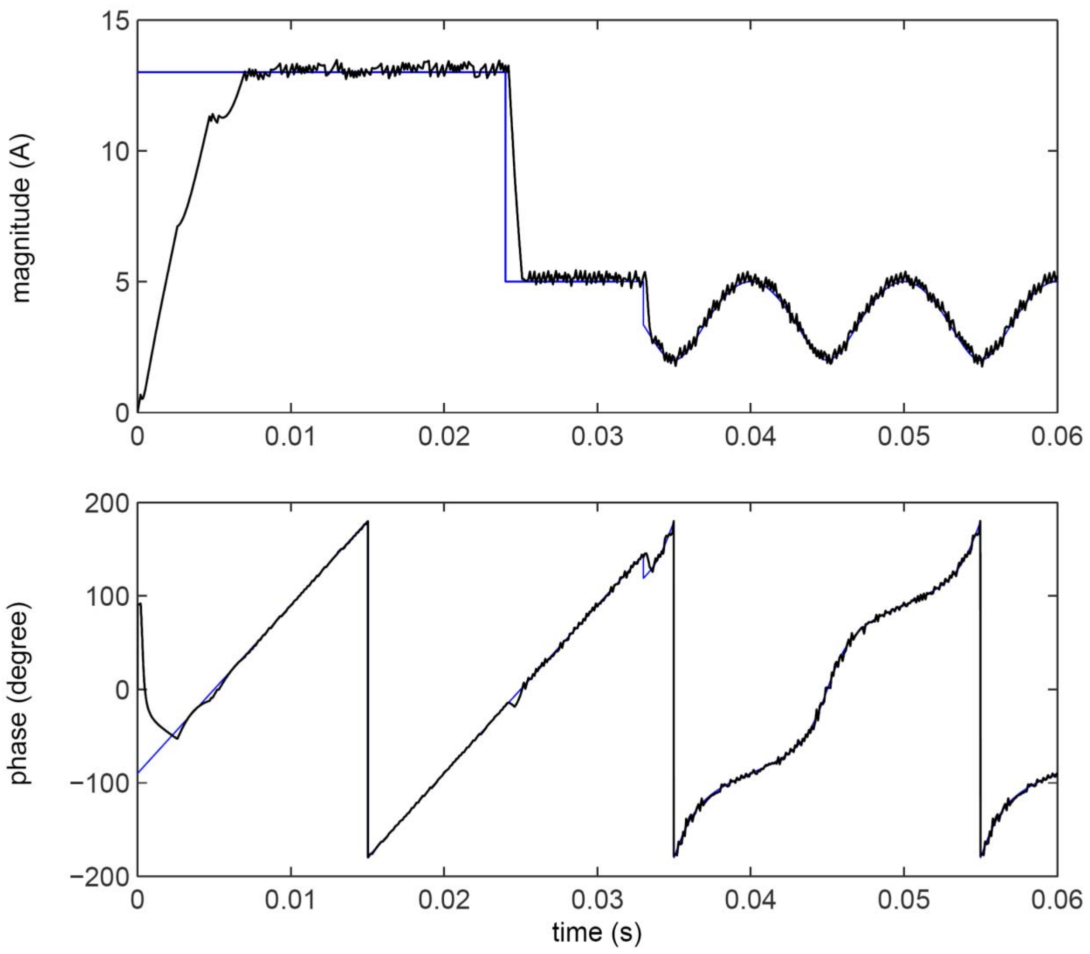
Simulation

(Phase-*a* Load Current: Step-Change. Case 1, T=100μs)



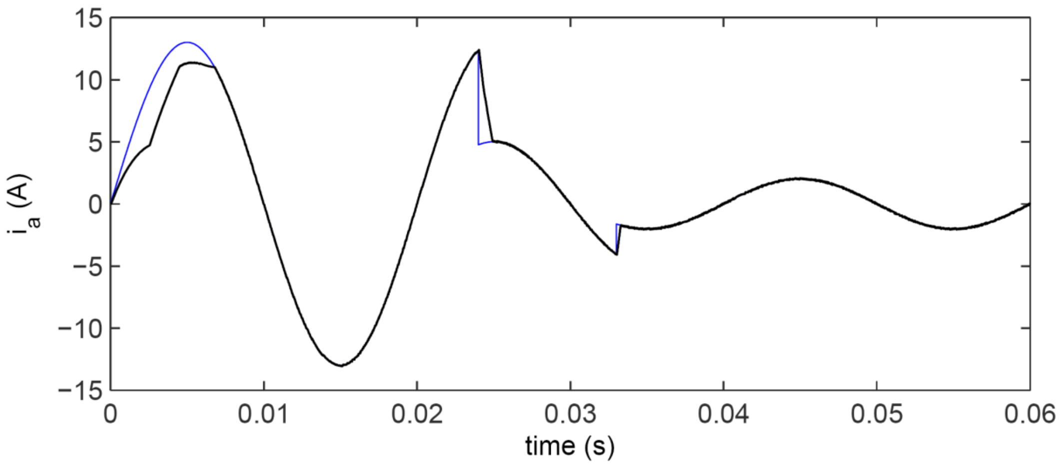
Simulation

(Phase-*a* Load Current: Step-Change. Case 1, T=100μs)



Simulation

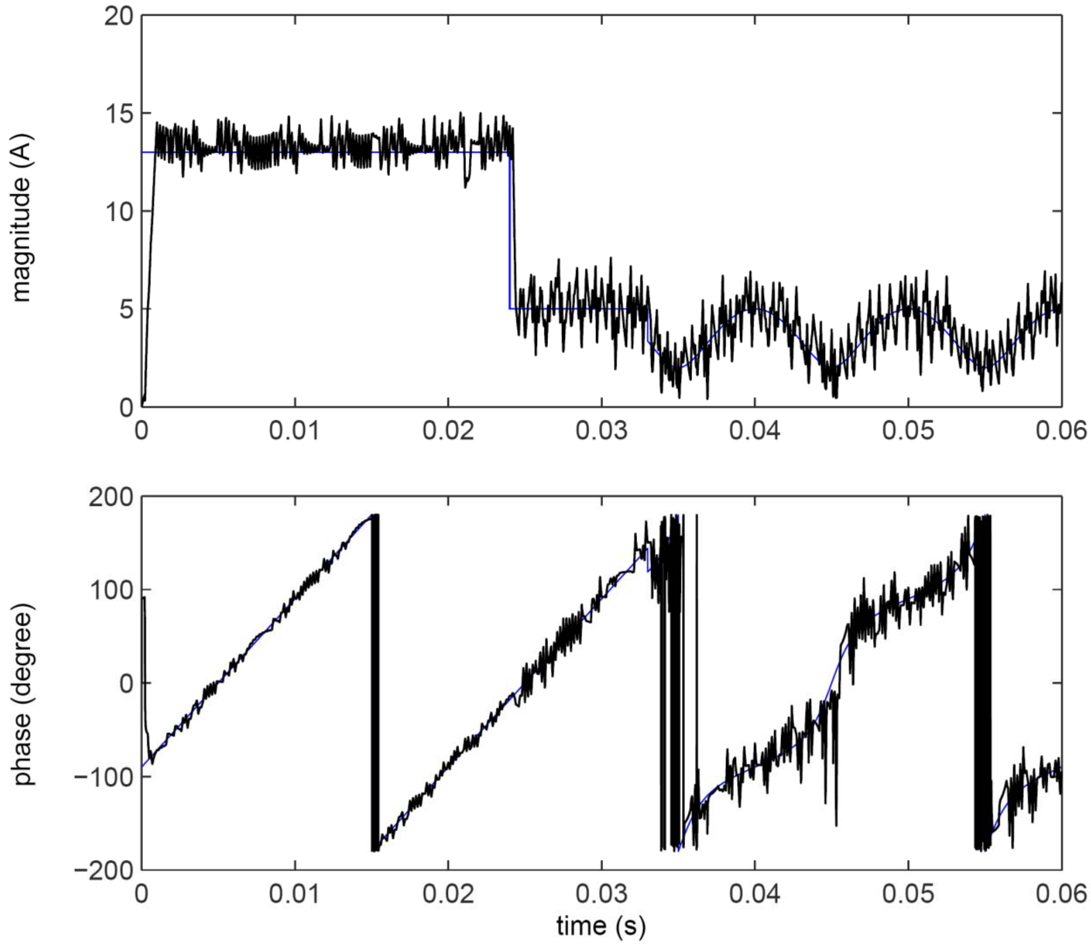
(Phase-*a* Load Current: Step-Change. Case 1, T=20μs)



Simulation

(Back-EMF Prediction & Performance. Case 2, T=100µs)

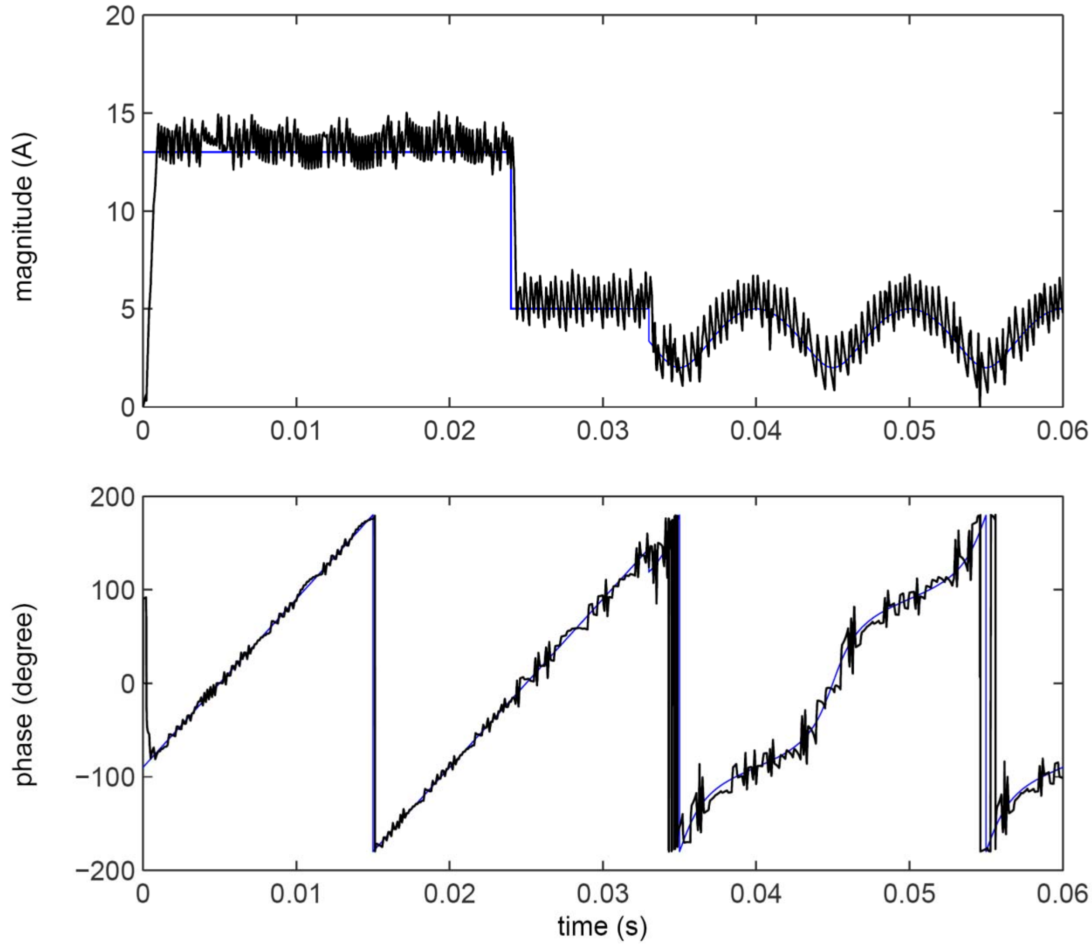
(Lagrange prediction)



Simulation

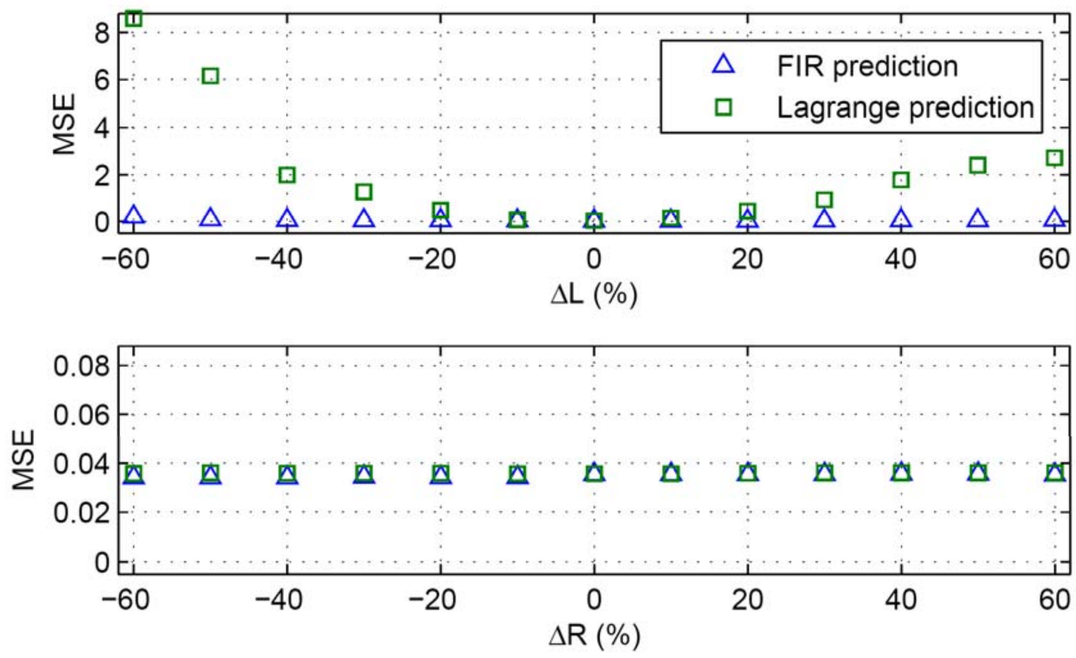
(Back-EMF Prediction & Performance. Case 2, T=100μs)

(FIR prediction)



Simulation

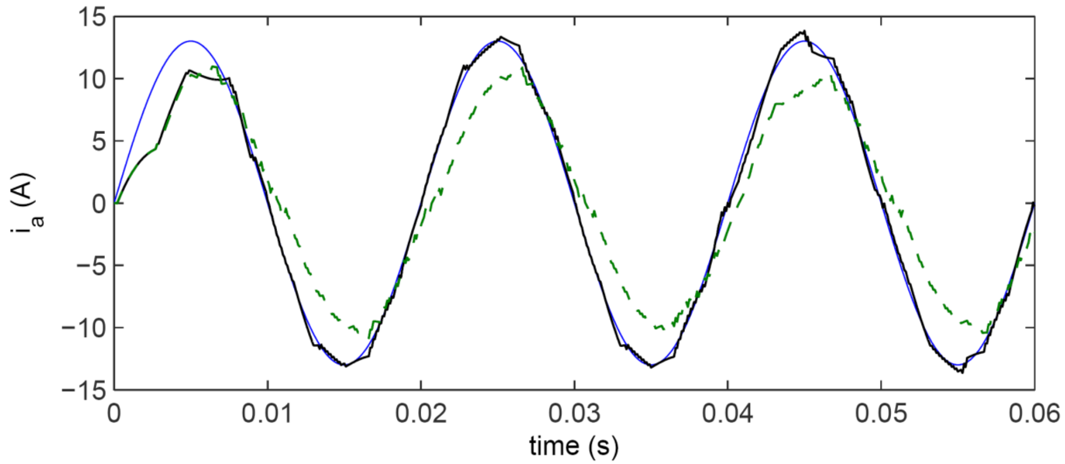
(Robustness: Parameter Mismatch. Case 1, T=100μs)



Simulation

(Robustness: Parameter Mismatch. Case 1, T=100μs)

(FIR vs. Lagrange prediction)

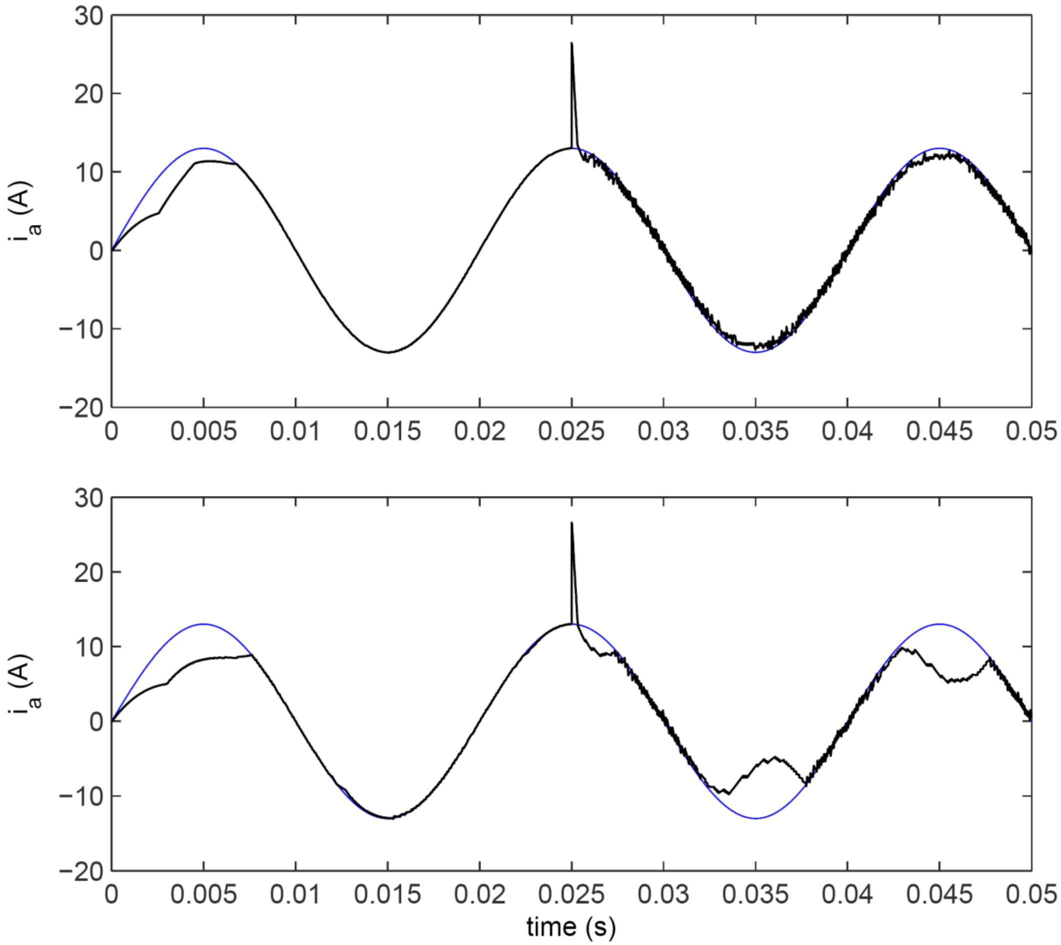


ΔL= –60%

Simulation

(Robustness: Parameter Perturbation. Case 1, T=20μs)

L dropped to 20%, *R* raised by 80%



(Proposed)

(Rodriguez, et al. 07)

Summary

- Forward Euler method to obtain a discretized model
- Simple predictive controller based on an optimal deadbeat policy and a suboptimal approximation of it
- Modifiable to reduce output ripples
- Compensates for delay, rejects disturbance
- Feedback-feedforward framework
- Prediction of back-EMF (disturbance) and its effect on performance and robustness
- Future work: Modification to control so active and reactive components of output current are decoupled

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