The University of British Columbia
Department of Electrical & Computer Engineering

Step by Step Eigenvalue Analysis with EMTP Discrete Time Solutions

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Presentation Outline

- Introduction
- Framework
- New methodology
- Test cases
- Conclusions

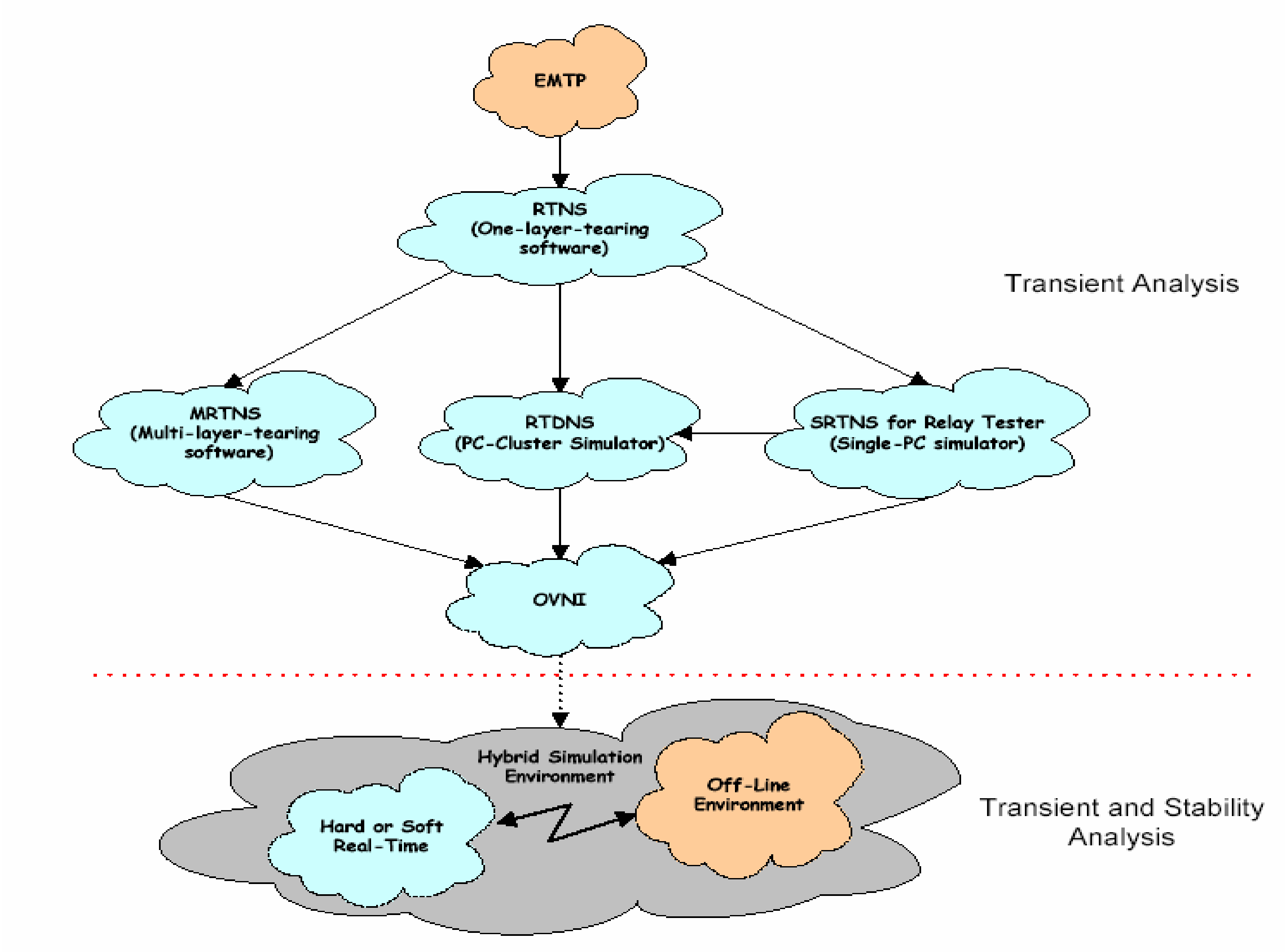
Motivation

- Evolution of power systems
  - Generation capacity 171GW (‘60), 1049 GW (‘06)
- De-regulated environment (only US)
  - Capacity margin
    25% (‘05)
    18.8% (‘06)
    15.5% (‘08)
  - Transmission lines
    High Voltage
    237,009 km (‘93)
    255,250 km (‘02)
  - Revenue all sectors
    198.2b $ (‘93)
    270.4b $ (‘04)

Motivation (cont.)

- Previous operational paradigm
  - Reliability oriented
- New operational paradigm
  - Revenue oriented
- Power systems stability analysis
  - Off-line vs. real-time simulation
- Research opportunity
  - Dynamic location of limits
  - Support allocation of investment
  - Locally coordinated operation

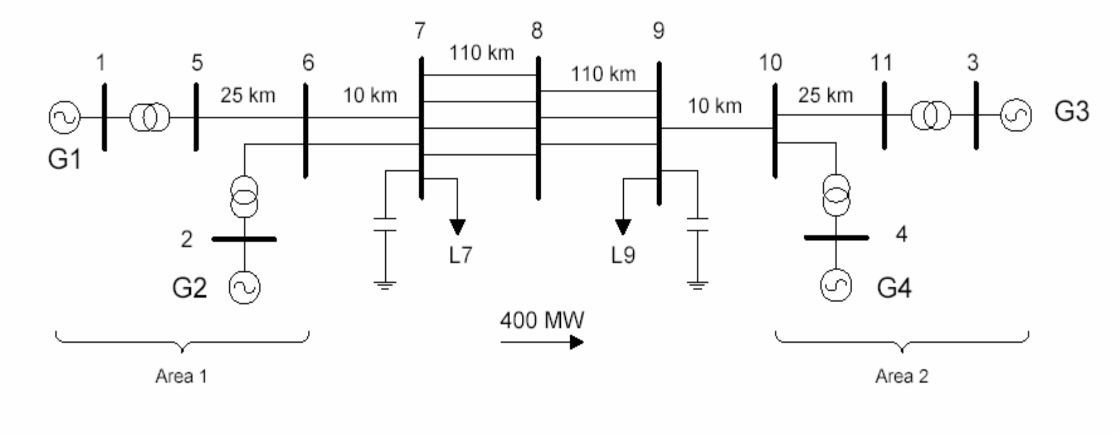
UBC power systems group simulation projects: Integration and evolution



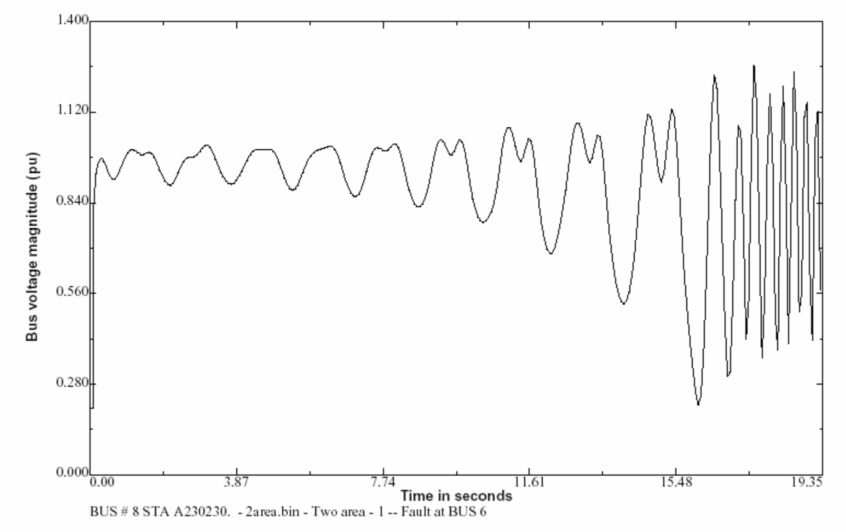
Framework

- The problem
  - Limitation of current Transient Stability analysis tools
    - Fast time-domain (FTD)
    - Prony spectral analysis
    - Transient rotor angle analysis
- EMTP extended capabilities
  - Step by step trajectory analysis
  - Non-linearities modelling
  - Modelling accuracy

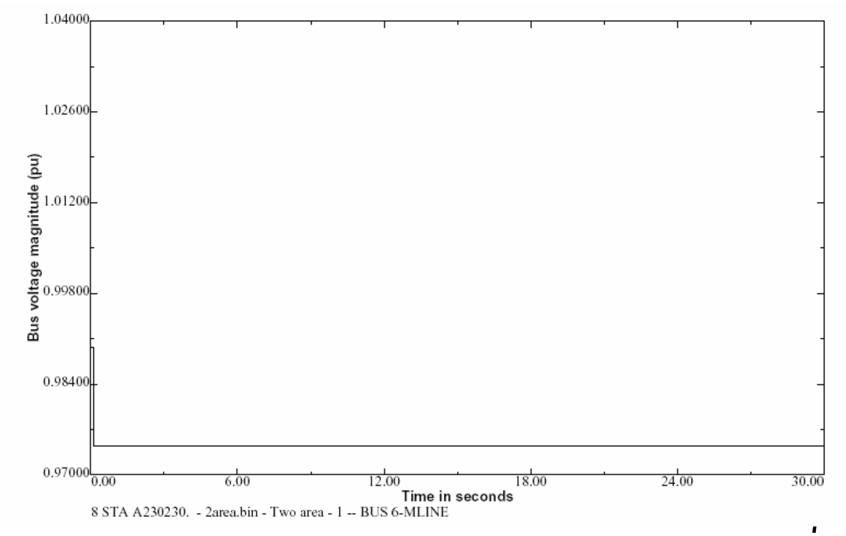
Framework (cont.)



(TSAT) Trapezoidal, 10 ms



FTD

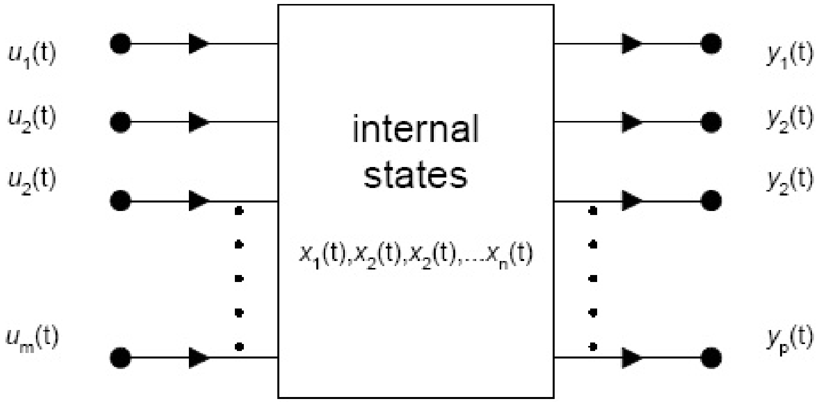


New Methodology

Eigenvalue analysis from EMTP solution

State-space formulation continuous time

Multiple input/multiple output



and for the single input/output discrete system

$$\dot{x}(t) = Ax(t) + Bu(t)$$ } *Dynamics of the system*

$$y(t) \;=\; C\,x(t) + D\,u(t)$$ } *Output of the system*

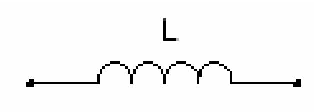
Discrete state-space formulation

Classical form – Forward Euler

$$
\begin{aligned}
x(t) &= A\,x(t-\Delta t) + B\,u(t-\Delta t) \quad \Big\} \quad \textit{Dynamics of the system} \\
y(t) &= C\,x(t) + D\,u(t) \quad \Big\} \quad \textit{Output of the system}
\end{aligned}
$$

EMTP solution

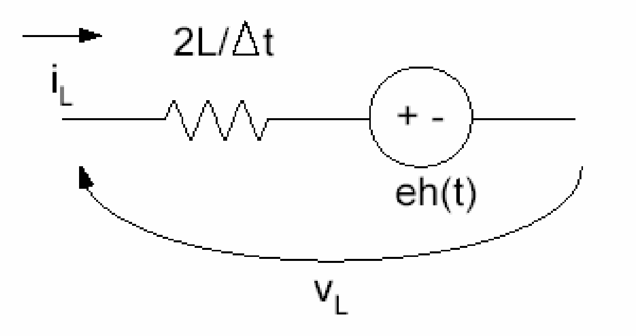
$$V_L(t) = L\frac{di_L(t)}{dt}$$



Trapezoidal rule

$$\frac{V_L(t) + V_L(t-h)}{2}h = L[i_L(t) - i_L(t-h)]$$

$$V_L(t) = \underbrace{\frac{2L}{h}}_{Requiv}\ i_L(t) + \underbrace{[-V_L(t-h) - \frac{2L}{h}i_L(t-h)]}_{history}$$



Discrete state-space from EMTP

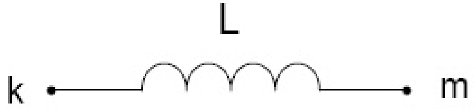
Output equation

$$\left[v(t)\right] = \left[G\right]^{-1}\left[h(t)\right] + \left[G\right]^{-1}\left[u(t)\right]$$

Dynamic part

$$\left[h(t+\Delta t)\right] = \left[A\right]\left[h(t)\right] + \left[B\right]\left[u(t)\right]$$

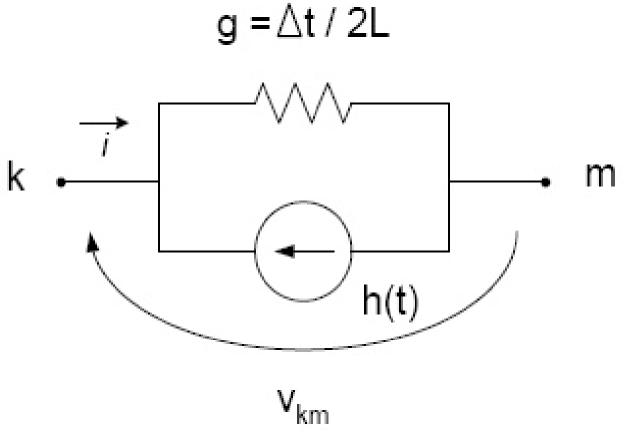
Discrete time state-space equation of basic elements - Inductor



branch voltage and current

$$v_{km}(t) = v_k(t) - v_m(t)$$

$$i(t) = gv(t) - h(t)$$



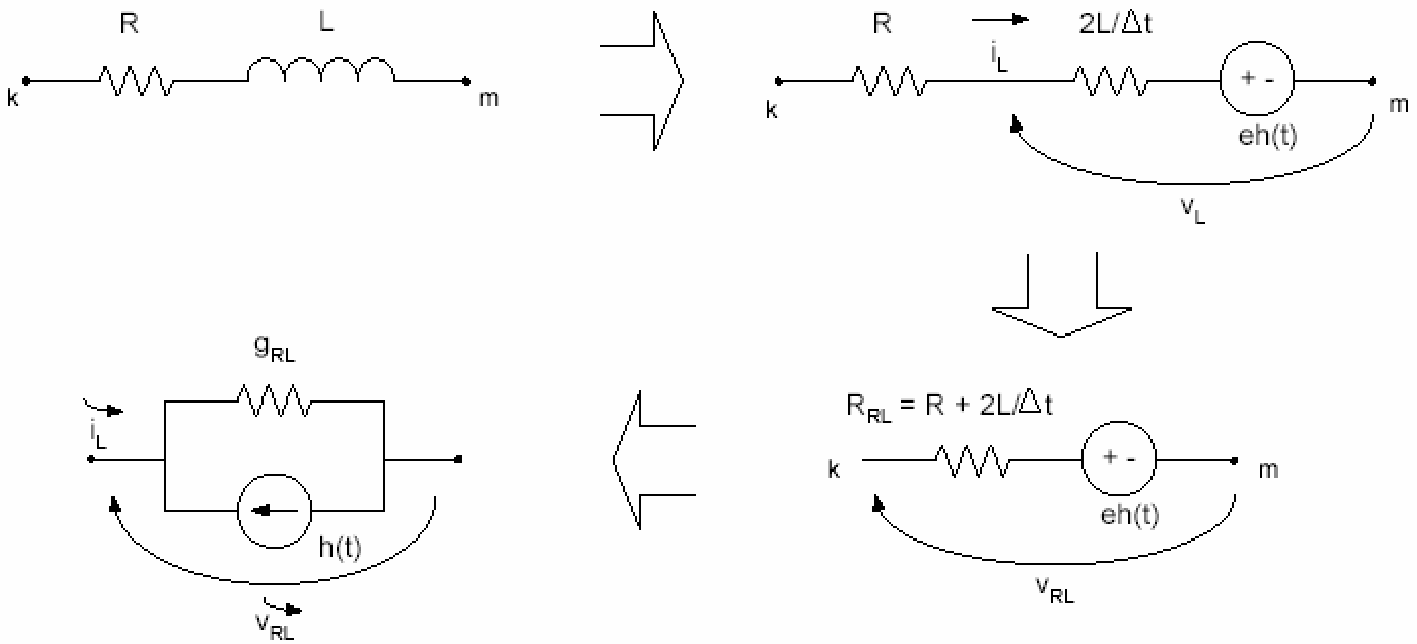
updating formula

$$h(t) = -gv(t - \Delta t) - i(t - \Delta t)$$

thus, the discrete state-space equation of a self inductor

$$\underbrace{h(t)}_{new\ state} = \underbrace{h(t - \Delta t)}_{old\ state} + \underbrace{(-2g)v_{km}(t - \Delta t)}_{forcing\ function}$$

Discrete time state-space equation of basic elements – Series RL and RC



RL discrete equivalent

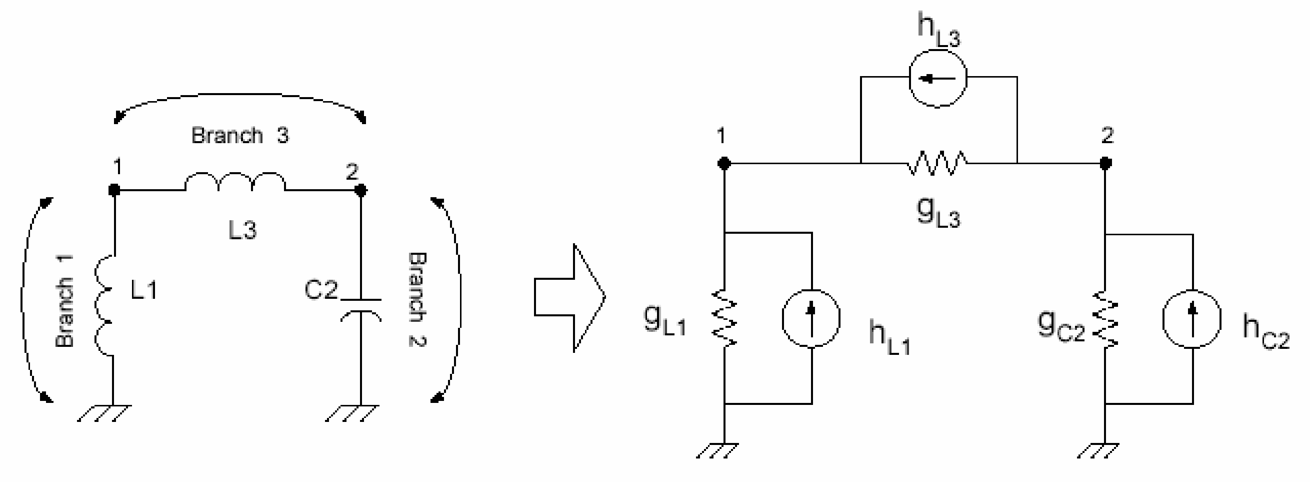
$$h_{km}(t) = h_{km}(t-\Delta t) - 2g_{RL}v_{km}(t-\Delta t)$$

$$g_{RL} = \frac{1}{R + \frac{2L}{\Delta t}}$$

$$h_{km}(t) = -h_{km}(t-\Delta t) + 2g_{RC}v_{km}(t-\Delta t)$$

$$g_{RC} = \frac{1}{R + \frac{\Delta t}{2C}}$$

Treatment of series branches



$$\begin{bmatrix} g_{L1}+g_{L3} & -g_{L3} \\ -g_{L3} & g_{C2}+g_{L3} \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} h_{L1}(t)+h_{L3}(t) \\ h_{C2}(t)-h_{L3}(t) \end{bmatrix}$$

branch history terms

$$\begin{bmatrix} \widehat{h}_1(t) \\ \widehat{h}_2(t) \\ \widehat{h}_3(t) \end{bmatrix} = \begin{bmatrix} k_{\wedge 1} & 0 & 0 \\ 0 & k_{\wedge 2} & 0 \\ 0 & 0 & k_{\wedge 3} \end{bmatrix} \begin{bmatrix} \widehat{h}_1(t-\Delta t) \\ \widehat{h}_2(t-\Delta t) \\ \widehat{h}_3(t-\Delta t) \end{bmatrix} + \begin{bmatrix} g_{\wedge 1} & 0 & 0 \\ 0 & g_{\wedge 2} & 0 \\ 0 & 0 & g_{\wedge 3} \end{bmatrix} \begin{bmatrix} \widehat{v}_1(t-\Delta t) \\ \widehat{v}_2(t-\Delta t) \\ \widehat{v}_3(t-\Delta t) \end{bmatrix}$$

Treatment of series branches (cont.)

Where
$$\begin{cases} k_{\wedge 1} = 1 & g_{\wedge 1} = \frac{-\Delta t}{L_1} \\ k_{\wedge 2} = -1 & g_{\wedge 2} = \frac{4C_2}{\Delta t} \\ k_{\wedge 3} = 1 & g_{\wedge 3} = \frac{-\Delta t}{L_3} \end{cases}$$

$$\begin{bmatrix} \widehat{v}_1(t-\Delta t) \\ \widehat{v}_2(t-\Delta t) \\ \widehat{v}_3(t-\Delta t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1(t-\Delta t) \\ v_2(t-\Delta t) \end{bmatrix}$$

and the relationship between branch and node voltages is given by the incidence matrix

$$\begin{bmatrix} \widehat{h}_1(t) \\ \widehat{h}_2(t) \\ \widehat{h}_3(t) \end{bmatrix} = \underbrace{\begin{bmatrix} k_{\wedge 1} & 0 & 0 \\ 0 & k_{\wedge 2} & 0 \\ 0 & 0 & k_{\wedge 3} \end{bmatrix}}_{[k_\wedge]} \begin{bmatrix} \widehat{h}_1(t-\Delta t) \\ \widehat{h}_2(t-\Delta t) \\ \widehat{h}_3(t-\Delta t) \end{bmatrix} + \underbrace{\begin{bmatrix} g_{\wedge 1} & 0 & 0 \\ 0 & g_{\wedge 2} & 0 \\ 0 & 0 & g_{\wedge 3} \end{bmatrix}}_{[g_\wedge]} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}}_{[L]} \underbrace{\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}}_{[G^{-1}]} \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}}_{[L^t]} \begin{bmatrix} \widehat{h}_1(t-\Delta t) \\ \widehat{h}_2(t-\Delta t) \\ \widehat{h}_3(t-\Delta t) \end{bmatrix}$$

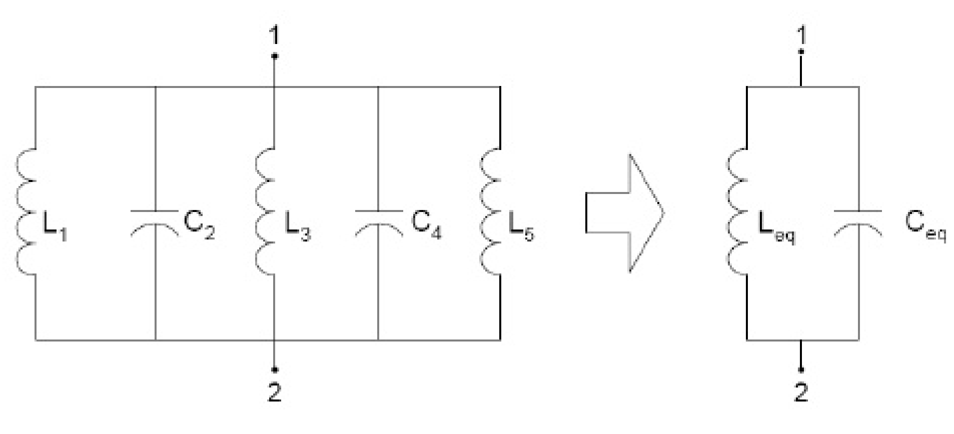
$$\left[\widehat{h}(t)\right] = \left[k_\wedge\right] \left[\widehat{h}(t-\Delta t)\right] + \left[g_\wedge\right] \left[L\right] \left[G\right]^{-1} \left[L\right]^{t} \left[\widehat{h}(t-\Delta t)\right]$$

the Transition matrix [A] is then given by

$$\boxed{\left[A\right]^{d} = \left[k_\wedge\right] + \left[g_\wedge\right] \left[L\right] \left[G^{-1}\right] \left[L^{t}\right]}$$

Treatment of parallel branches

We can keep the identity of each component or treat them as a new equivalent aggregated parallel RLC.



$$\begin{bmatrix} \widehat{h}_1(t) \\ \widehat{h}_2(t) \\ \widehat{h}_3(t) \\ \widehat{h}_4(t) \\ \widehat{h}_5(t) \end{bmatrix} = \begin{bmatrix} k_{\wedge 1} & 0 & 0 & 0 & 0 \\ 0 & k_{\wedge 2} & 0 & 0 & 0 \\ 0 & 0 & k_{\wedge 3} & 0 & 0 \\ 0 & 0 & 0 & k_{\wedge 4} & 0 \\ 0 & 0 & 0 & 0 & k_{\wedge 5} \end{bmatrix} \begin{bmatrix} \widehat{h}_1(t-\Delta t) \\ \widehat{h}_2(t-\Delta t) \\ \widehat{h}_3(t-\Delta t) \\ \widehat{h}_4(t-\Delta t) \\ \widehat{h}_5(t-\Delta t) \end{bmatrix} + \begin{bmatrix} g_{\wedge 1} & 0 & 0 & 0 & 0 \\ 0 & g_{\wedge 2} & 0 & 0 & 0 \\ 0 & 0 & g_{\wedge 3} & 0 & 0 \\ 0 & 0 & 0 & g_{\wedge 4} & 0 \\ 0 & 0 & 0 & 0 & g_{\wedge 5} \end{bmatrix} \begin{bmatrix} \widehat{v}_1(t-\Delta t) \\ \widehat{v}_2(t-\Delta t) \\ \widehat{v}_3(t-\Delta t) \\ \widehat{v}_4(t-\Delta t) \\ \widehat{v}_5(t-\Delta t) \end{bmatrix}$$

$$\begin{bmatrix} \widehat{v}_1(t-\Delta t) \\ \widehat{v}_2(t-\Delta t) \\ \widehat{v}_3(t-\Delta t) \\ \widehat{v}_4(t-\Delta t) \\ \widehat{v}_5(t-\Delta t) \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1(t-\Delta t) \\ v_2(t-\Delta t) \end{bmatrix}$$

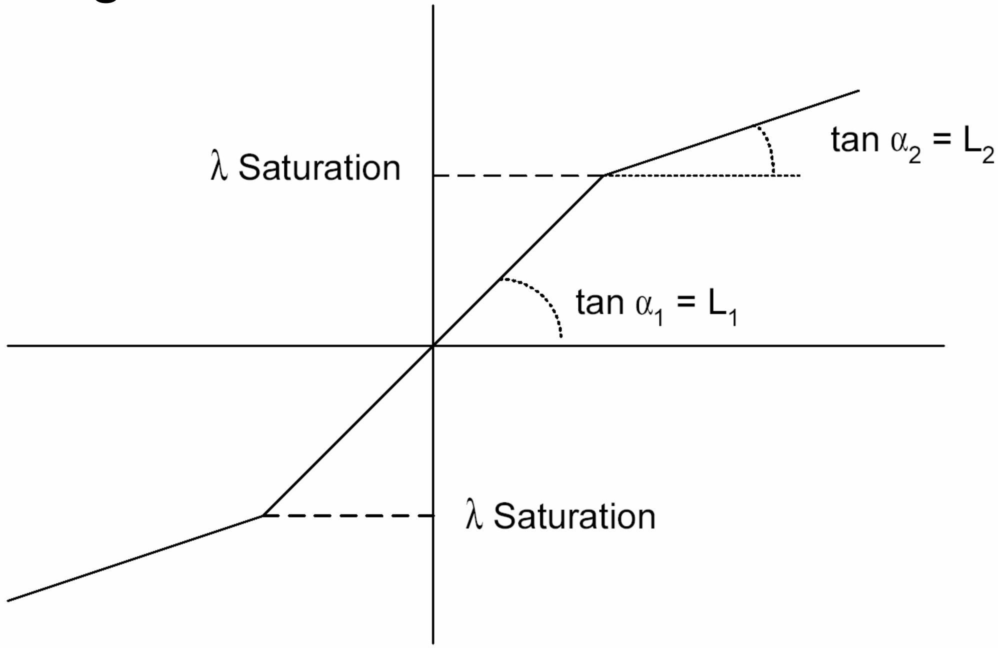
The general formulas are maintained and we define [A] as for the single branch case

$$\left[A\right] = \left[k_{\wedge}\right] + \left[g_{\wedge}\right]\left[L\right]\left[G^{-1}\right]\left[L^t\right]$$

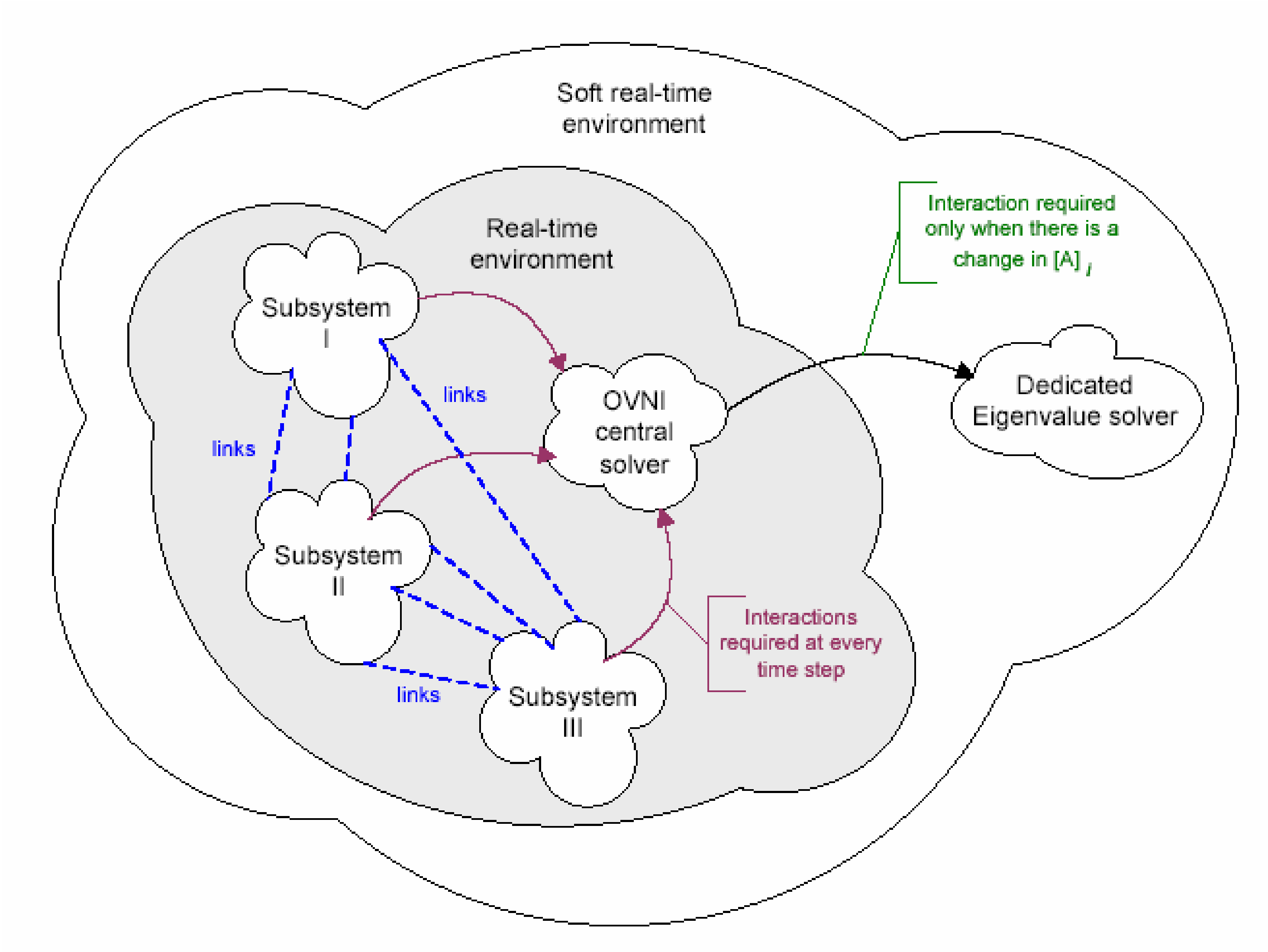
Treatment of Non-linear elements

- Non-linear elements can be made up of piecewise linear segments.

- A change of piecewise segment corresponds
  to a new set
  of eigenvalue.



Hybrid real-time/soft real-time simulator layout



Discrete to continuous time mapping

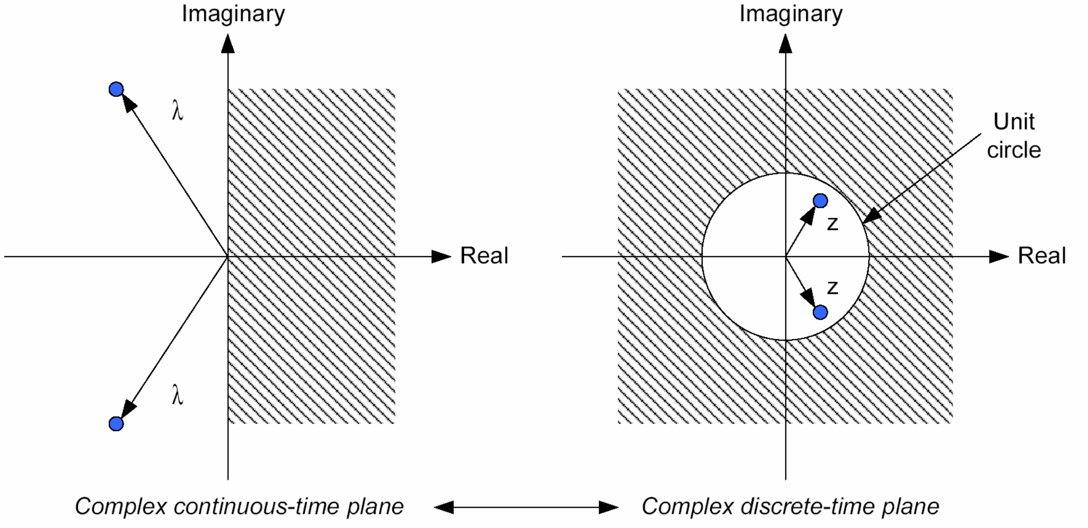
$$\dot{x}(t) = A^c x(t)$$

$$x(t + \Delta t) = A^d x(t)$$

For Trapezoidal

$$A^d = \frac{\Omega I + A^c}{\Omega I - A^c}$$

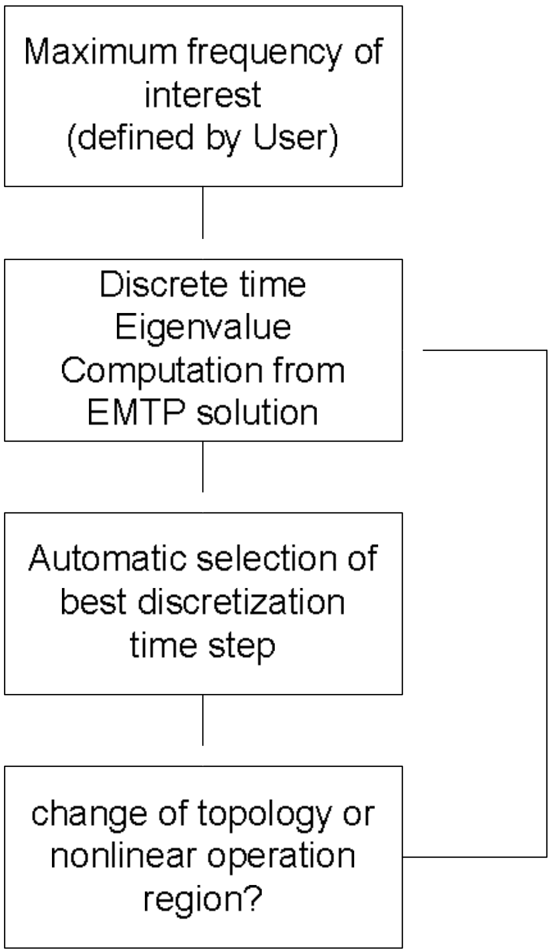
where $\Omega = 2/\Delta t$.



The continuous time eigenvalues can be reconstructed from the discrete ones by

$$\lambda_i = \Omega \frac{z_i - 1}{z_i + 1}$$

Automatic EMTP time step selection scheme



Discretization time considerations

- Linearization of differential equations: Nyquist frequency.
- Non-linear elements: small time step for accurate representation of region change.
- As long as eigenvalue frequency is below the Nyquist freq. reconstructed cont. time eigenvalues are “exact”

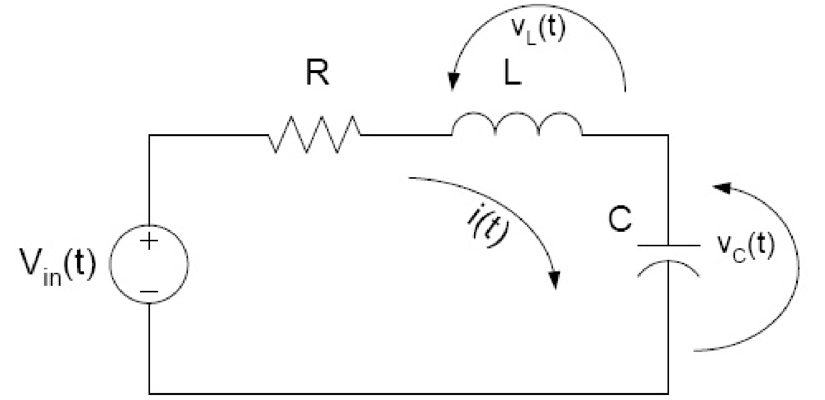
Test cases

Comparison of state-space formulation between continuous and discrete time domains

continuous time state-space system equations

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t) + D\,u(t)$$



selecting $V_R(t)$ and $V_C(t)$ as states

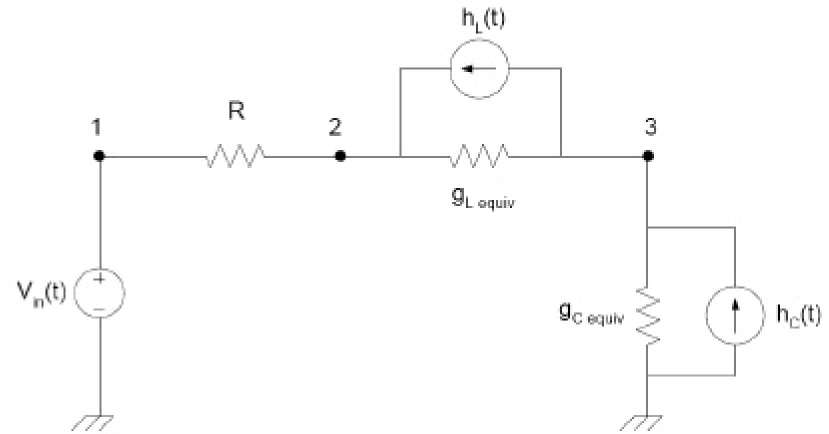
$v_{in}(t)$ as input and $V_L(t)$ as output

$$\underbrace{\begin{bmatrix} \frac{dv_R(t)}{dt} \\ \frac{dv_C(t)}{dt} \end{bmatrix}}_{future\ states} = \underbrace{\begin{bmatrix} \frac{-R}{L} & \frac{-R}{L} \\ \frac{-1}{RC} & 0 \end{bmatrix}}_{[A]} \underbrace{\begin{bmatrix} v_R(t) \\ v_C(t) \end{bmatrix}}_{states} + \underbrace{\begin{bmatrix} \frac{R}{L} \\ 0 \end{bmatrix}}_{[B]} \underbrace{v_{in}(t)}_{input}$$

$$\underbrace{v_L(t)}_{output} = \underbrace{\begin{bmatrix} -1 & -1 \end{bmatrix}}_{[C]} \underbrace{\begin{bmatrix} v_R(t) \\ v_C(t) \end{bmatrix}}_{states} + \underbrace{1}_{[D]} \underbrace{v_{in}(t)}_{input}$$

Discrete time state-space system equation

$$\begin{bmatrix} \frac{1}{R} & \frac{-1}{R} & 0 \\ \frac{-1}{R} & (\frac{1}{R} + \frac{-\Delta t}{2L}) & \frac{-\Delta t}{2L} \\ 0 & \frac{\Delta t}{2L} & (\frac{\Delta t}{2L} + \frac{2C}{\Delta t}) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} i_1 \\ h_L \\ -h_L + h_C \end{bmatrix}$$



$$\begin{bmatrix} v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} (\frac{1}{R} + \frac{\Delta t}{2L}) & \frac{-\Delta t}{2L} \\ \frac{-\Delta t}{2L} & (\frac{\Delta t}{2L} + \frac{2C}{\Delta t}) \end{bmatrix}^{-1} \begin{bmatrix} h_L + \frac{v_1}{R} \\ -h_L + h_C \end{bmatrix}$$

the branch histories

$$\begin{bmatrix} \widehat{h}_1 \\ \widehat{h}_2 \\ \widehat{h}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \widehat{h'}_1 \\ \widehat{h'}_2 \\ \widehat{h'}_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{-\Delta t}{L} & 0 \\ 0 & 0 & \frac{4C}{\Delta t} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} (\frac{1}{R} + \frac{\Delta t}{2L}) & \frac{-\Delta t}{2L} \\ \frac{-\Delta t}{2L} & (\frac{\Delta t}{2L} + \frac{2C}{\Delta t}) \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \widehat{h'}_1 \\ \widehat{h'}_2 \\ \widehat{h'}_3 \end{bmatrix}$$

the discrete transition matrix [A] for the RLC series computed from the nodal eq.

$$\begin{bmatrix} A \end{bmatrix}^d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{-\Delta t}{L} & 0 \\ 0 & 0 & \frac{4C}{\Delta t} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} (\frac{1}{R} + \frac{\Delta t}{2L}) & \frac{-\Delta t}{2L} \\ \frac{-\Delta t}{2L} & (\frac{\Delta t}{2L} + \frac{2C}{\Delta t}) \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

Continuous time eigenvalues

$$[A] = \begin{bmatrix} -20 & -20 \\ 40 & 0 \end{bmatrix} \qquad [\lambda] = \begin{bmatrix} -10.0000 + j26.4575 \\ -10.0000 - j26.4575 \end{bmatrix}$$

Discrete time eigenvalues

$$[A]^d = \begin{bmatrix} 0 & 0 & 0 \\ 1.99799^{-3} & 9.97998^{-1} & -3.99599^{-6} \\ 1.99799^{-3} & 1.99799 & 9.99996^{-1} \end{bmatrix} \qquad [z]^d = \begin{bmatrix} 9.98997^{-1} + j2.64310^{-3} \\ 9.98997^{-1} - j2.64310^{-3} \\ 0 \end{bmatrix}$$

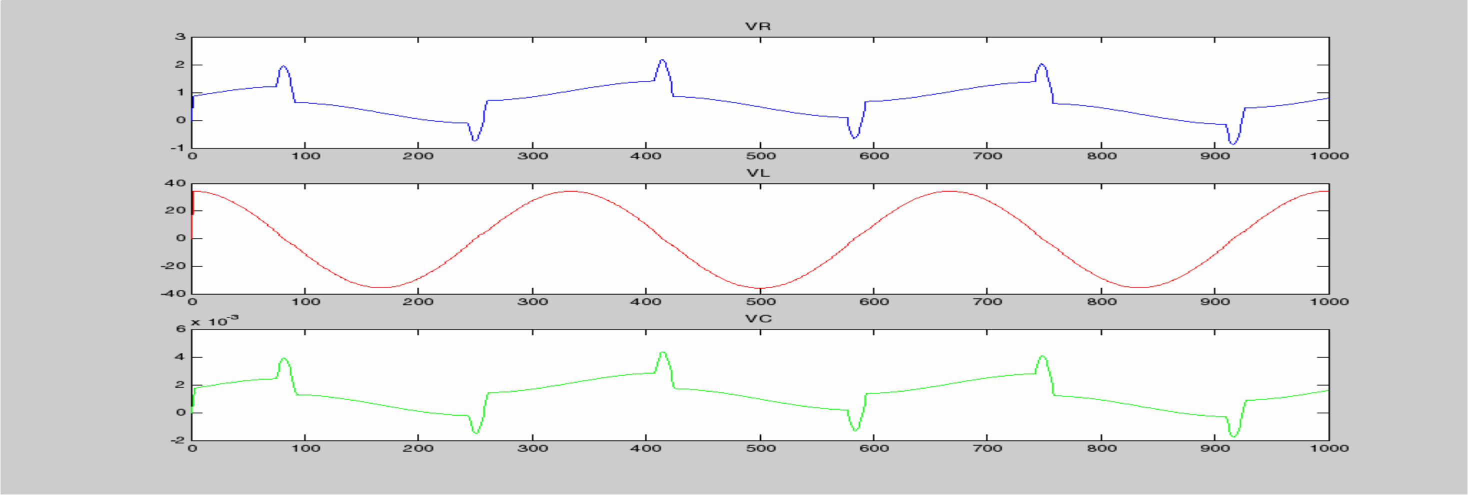
Reconstructed Continuous time eigenvalues

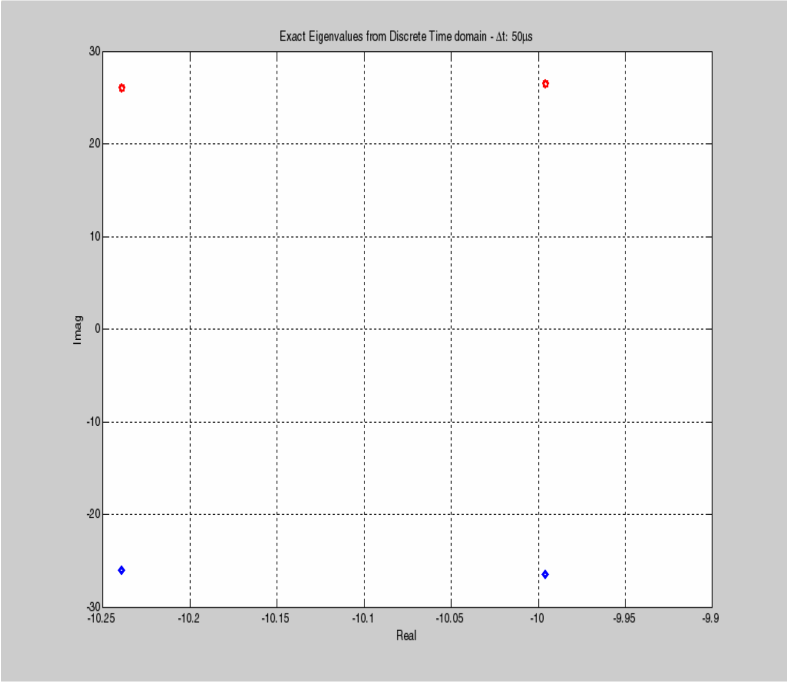
$$\lambda_i = \frac{2}{\Delta t} \frac{z_i - 1}{z_i + 1}$$

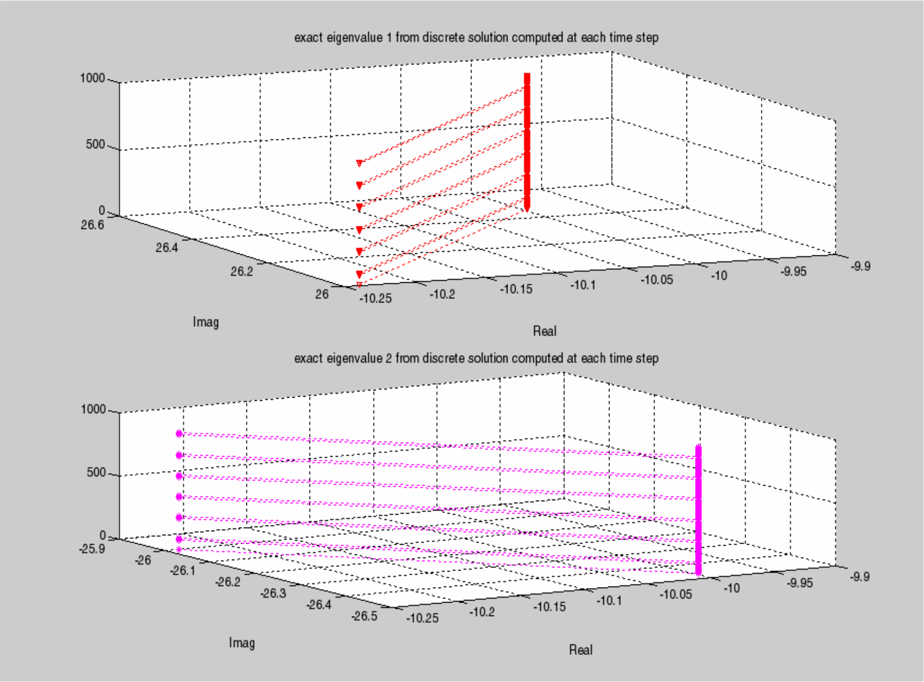
$$[\lambda]^c_{reconstructed} = \frac{2}{100\mu s} \begin{bmatrix} -0.00050 + j0.00132 \\ -0.00050 - j0.00132 \end{bmatrix}$$

$$= \begin{bmatrix} -10.0000 + j26.4575 \\ -10.0000 - j26.4575 \end{bmatrix}$$

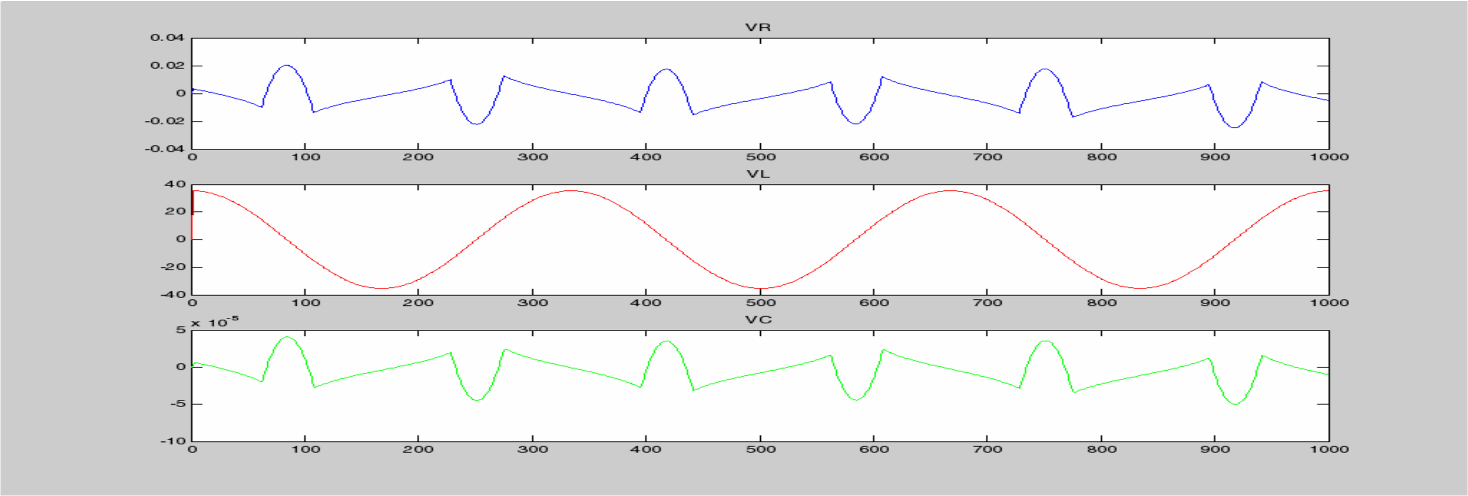
Eigenvalue trajectory of a RLC series with a non lineal L

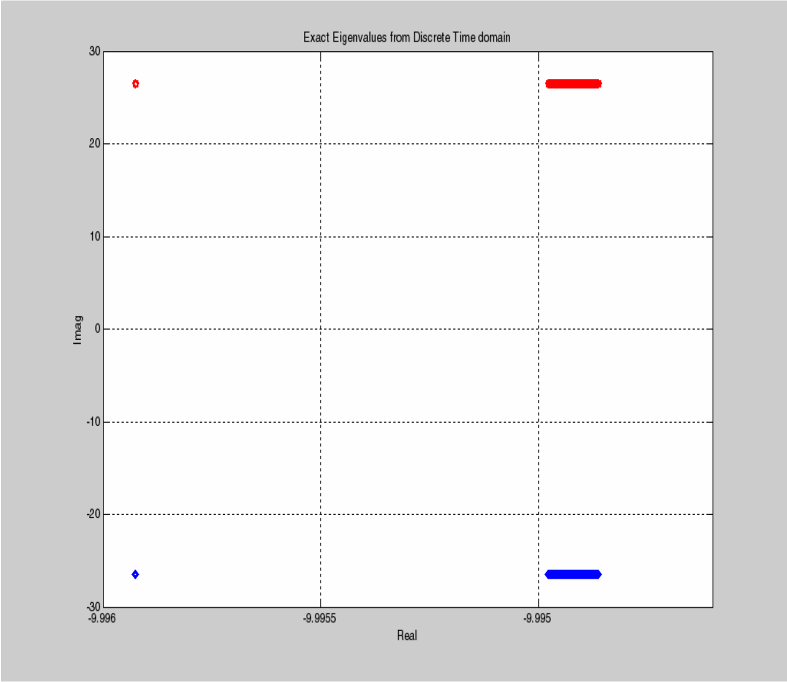


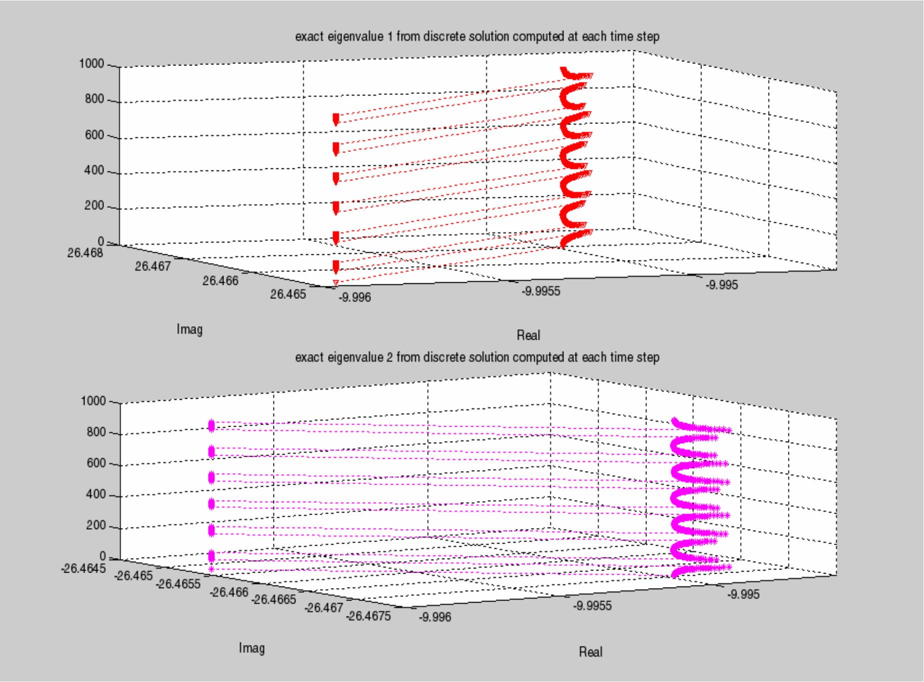




Eigenvalue trajectory of a RLC series with a non lineal L (cont.)



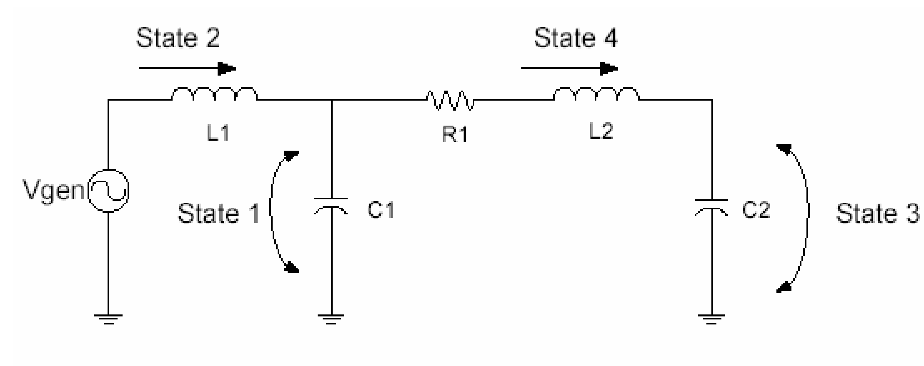




Identification of segmentation areas - Latency application

Continues time domain

$$\begin{cases} \dot{x}_1 = \frac{1}{C_1}(v_{L1} - v_{L2}) \\ \dot{x}_2 = \frac{1}{L_1}(V_{gen} - v_{C1}) \\ \dot{x}_3 = \frac{1}{C_2} v_{L2} \\ \dot{x}_4 = \frac{1}{L2}(v_{C1} - v_{C2} + R_1 v_{L2}) \end{cases}$$



$$A^c = \begin{bmatrix} 0 & \frac{1}{C_1} & 0 & \frac{-1}{C_1} \\ \frac{-1}{L_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_2} \\ \frac{1}{L_2} & 0 & \frac{-1}{L_2} & \frac{-R_1}{L_2} \end{bmatrix}$$

Continues time eigenvalues

$$\lambda^c = \left[\begin{array}{c|c} -4.99950\text{x}10^4 + j1.00379\text{x}10^6 & \lambda_1 \\ -4.99950\text{x}10^4 - j1.00379\text{x}10^6 & \lambda_2 \\ -4.99801 + j9.94988\text{x}10^4 & \lambda_3 \\ -4.99801 - j9.94988\text{x}10^4 & \lambda_4 \end{array}\right]$$

Identification of segmentation areas - Latency application (cont.)

Discrete time domain

$$G = \begin{bmatrix} \frac{\Delta t}{2L_1} & \frac{-\Delta t}{2L_1} & 0 & 0 \\ \frac{-\Delta t}{2L_1} & (\frac{1}{R_1} + \frac{\Delta t}{2L_1} + \frac{2C_1}{\Delta t}) & \frac{-1}{R_1} & 0 \\ 0 & \frac{-1}{R_1} & (\frac{1}{R_1} + \frac{\Delta t}{2L_2}) & \frac{-\Delta t}{2L_2} \\ 0 & 0 & \frac{-\Delta t}{2L_2} & (\frac{\Delta t}{2L_2} + \frac{2C_2}{\Delta t}) \end{bmatrix} \quad g_{\wedge} = \begin{bmatrix} \frac{-\Delta t}{L_1} & 0 & 0 & 0 \\ 0 & \frac{2C_1}{\Delta t} & 0 & 0 \\ 0 & 0 & \frac{-\Delta t}{L_2} & 0 \\ 0 & 0 & 0 & \frac{4C_2}{\Delta t} \end{bmatrix} \quad k_{\wedge} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad L = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left[A\right]^d = \left[k_{\wedge}\right] + \left[g_{\wedge}\right]\left[L\right]\left[G^{-1}\right]\left[L^t\right]$$

$$A^d = \begin{bmatrix} 9.99999\text{x}10^{-1} & 1.25000\text{x}10^{-15} & 1.24999\text{x}10^{-15} & 7.81467\text{x}10^{-29} \\ -2.00000 & 9.99999\text{x}10^{-1} & 1.99999 & 1.25034\text{x}10^{-13} \\ 1.24999\text{x}^{10}-15 & -1.24999\text{x}10^{-15} & 9.99999\text{x}10^{-1} & 1.24999\text{x}10^{-13} \\ -1.24999\text{x}^{10}-15 & 1.24999\text{x}10^{-15} & -1.99999 & 9.99999\text{x}10^{-1} \end{bmatrix}$$

Discrete time eigenvalues

$$\lambda^d = \begin{bmatrix} 9.99999\text{x}10^{-1} + j4.97494\text{x}10^{-8} \\ 9.99999\text{x}10^{-1} + j4.97494\text{x}10^{-8} \\ 9.99999\text{x}10^{-1} + j5.01896\text{x}10^{-7} \\ 9.99999\text{x}10^{-1} - j5.01896\text{x}10^{-7} \end{bmatrix}$$

Reconstructed Continues time eigenvalues

$$\lambda^c_{reconstructed} = \begin{bmatrix} -4.99752 + j9.94988\text{x}10^4 \\ -4.99752 - j9.94988\text{x}10^4 \\ -4.99950\text{x}10^4 + j1.00379\text{x}10^6 \\ -4.99950\text{x}10^4 - j1.00379\text{x}10^6 \end{bmatrix}$$

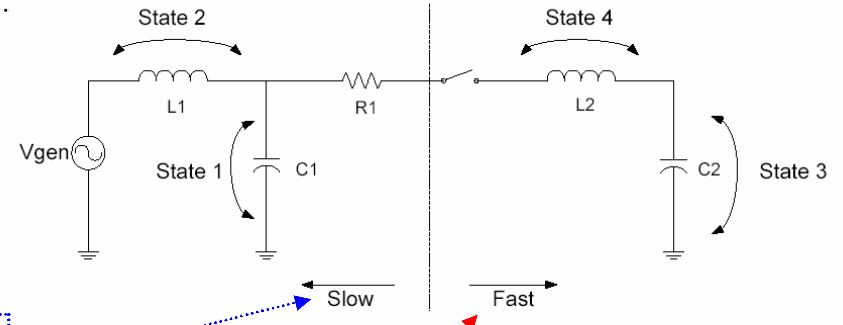
Identification of segmentation areas - Latency application (cont.)

$$\widetilde{p_i} = \begin{bmatrix} \phi_{1i}\varphi_{i1} \\ \phi_{2i}\varphi_{i2} \\ \vdots \\ \phi_{ni}\varphi_{ik} \end{bmatrix}$$

where,

$\phi_{ki} = k^{th}$ entry of the right eigenvector ϕ_i

$\varphi_{ik} = k^{th}$ entry of the left eigenvector φ_i.

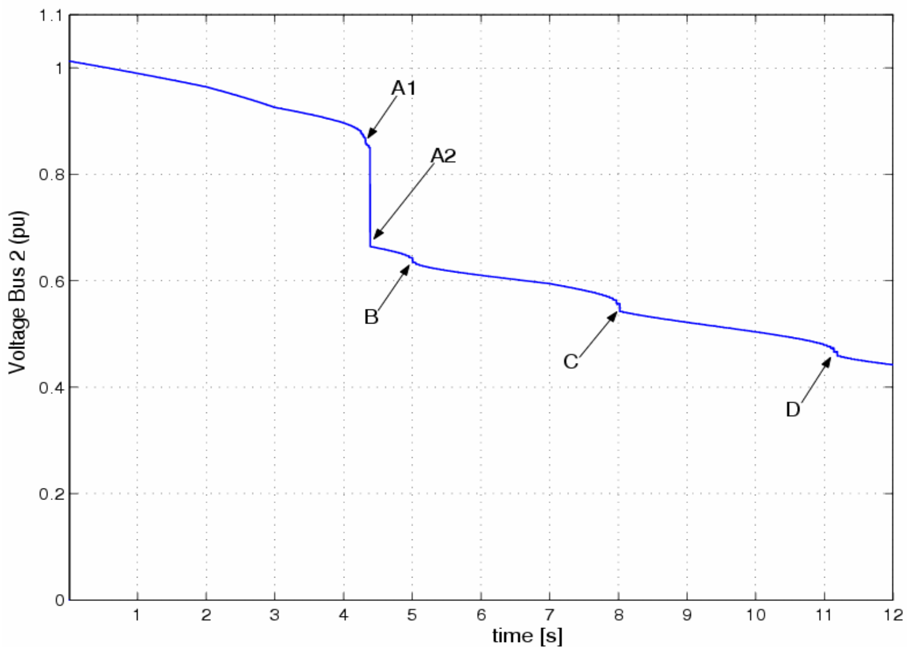


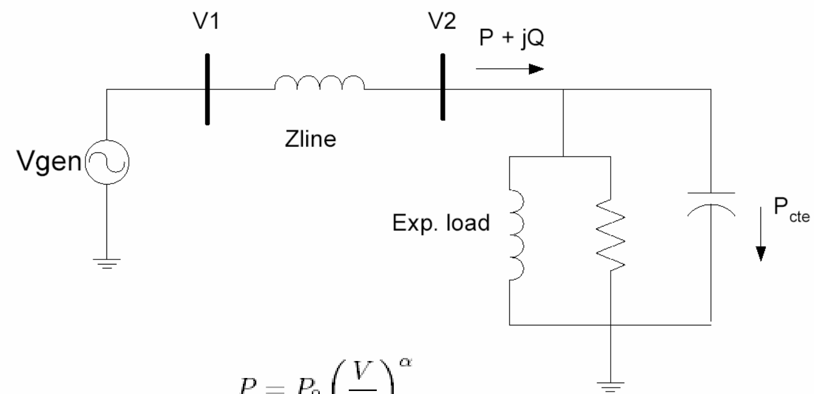
$P^c_{reconstructed} =$

$\lambda^c_{3reconstructed}$	$\lambda^c_{4reconstructed}$	
$4.9995\text{x}10^{-1} + j2.6108\text{x}10^{-5}$	$4.9995\text{x}10^{-1} - j2.6108\text{x}10^{-5}$	x_2
$4.9495\text{x}10^{-1} + j7.5560\text{x}10^{-5}$	$4.9495\text{x}10^{-1} - j7.5560\text{x}10^{-5}$	$\dot{x}_1$
$4.9970\text{x}10^{-5} - j9.9168\text{x}10^{-7}$	$4.9970\text{x}10^{-5} + j9.9168\text{x}10^{-7}$	$\dot{x}_4$
$5.0474\text{x}10^{-3} - j1.0067\text{x}10^{-4}$	$5.0474\text{x}10^{-3} + j1.0067\text{x}10^{-4}$	$\dot{x}_3$

$\lambda^c_{2reconstructed}$	$\lambda^c_{1reconstructed}$	
$4.9470\text{x}10^{-5} - j7.5406\text{x}10^{-6}$	$4.9470\text{x}10^{-5} + j7.5406\text{x}10^{-6}$	$\dot{x}_2$
$5.0479\text{x}10^{-3} - j2.6137\text{x}10^{-4}$	$5.0479\text{x}10^{-3} + j2.6137\text{x}10^{-4}$	$\dot{x}_1$
$4.9995\text{x}10^{-1} + j2.4910\text{x}10^{-2}$	$4.9995\text{x}10^{-1} - j2.4910\text{x}10^{-2}$	$\dot{x}_4$
$4.9495\text{x}10^{-1} - j2.4641\text{x}10^{-2}$	$4.9495\text{x}10^{-1} + j2.4641\text{x}10^{-2}$	$\dot{x}_3$

Voltage collapse of a radial system

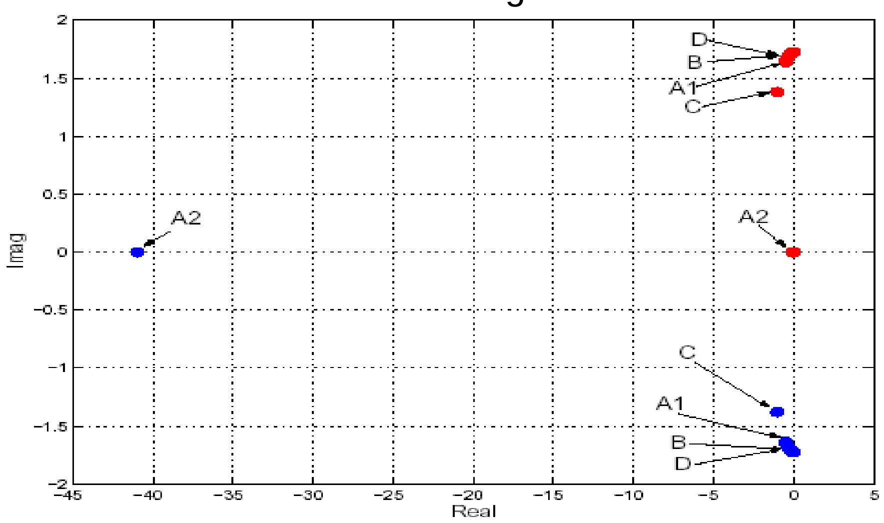




$$P = P_0 \left(\frac{V}{V_0} \right)^{\alpha}$$

$$Q = Q_0 \left(\frac{V}{V_0} \right)^{\beta}$$

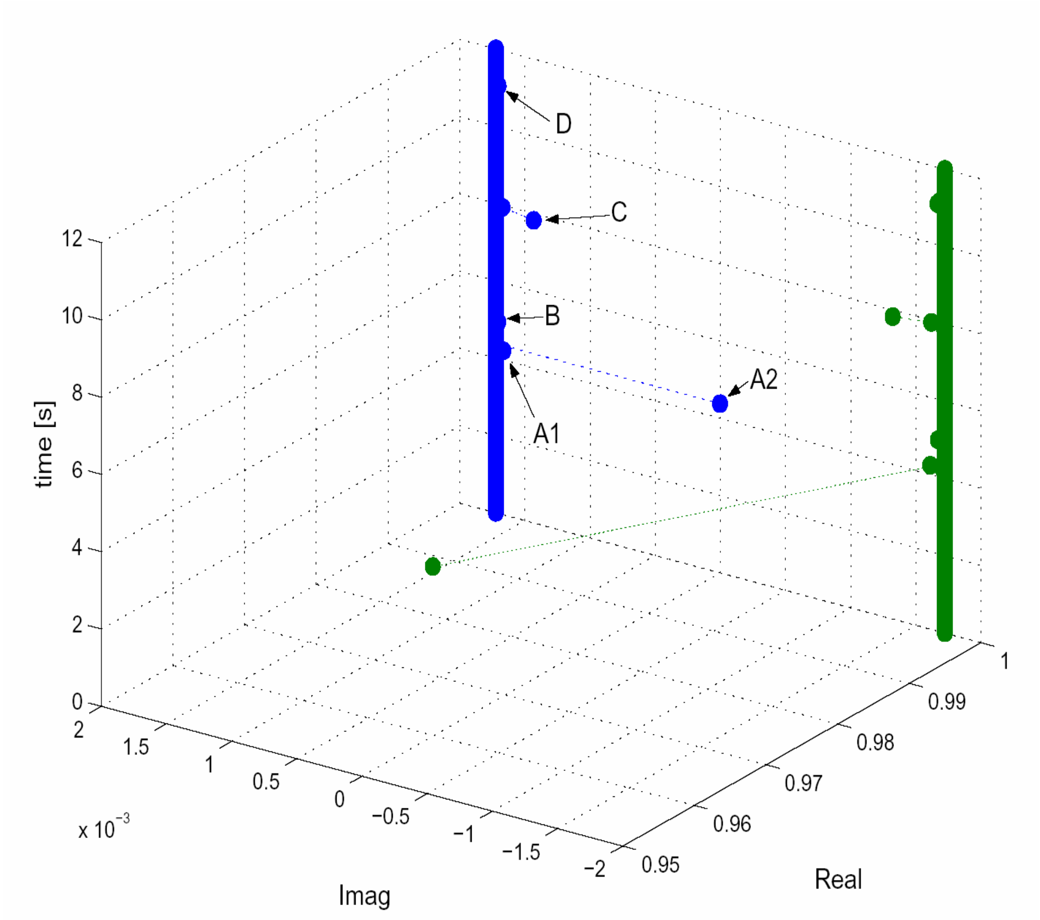
Continuous time eigenvalues



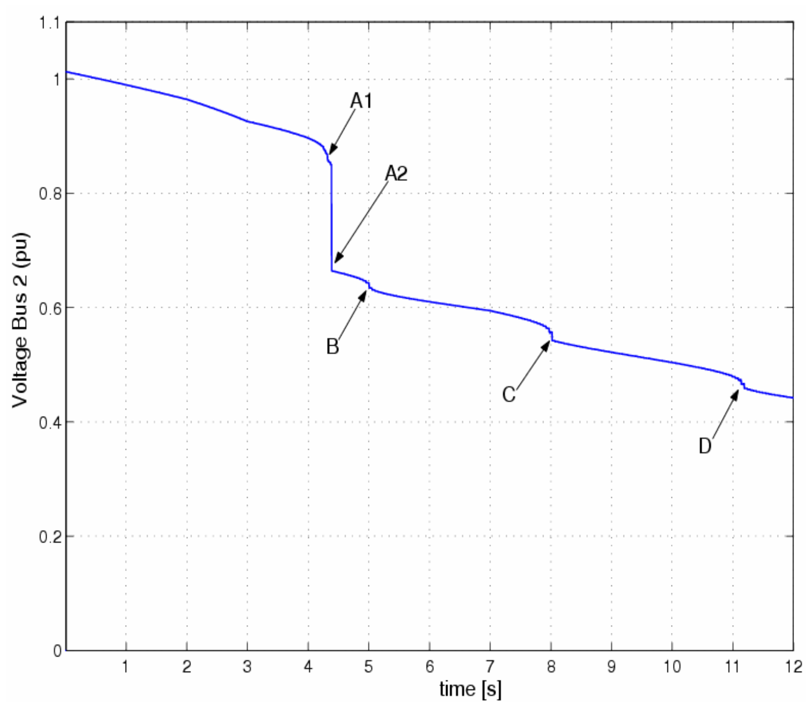
Load increment profile

Time Interval [s]	P_t	Q_t
[0-2]	0.5	0.2
[2-3]	0.7	0.3
[3-4]	1.8	0.1
[4-5]	1.8	0.12
[5-6]	1.9	0.1
[6-12]	1.55	0.2

Voltage collapse of a radial system (cont.)

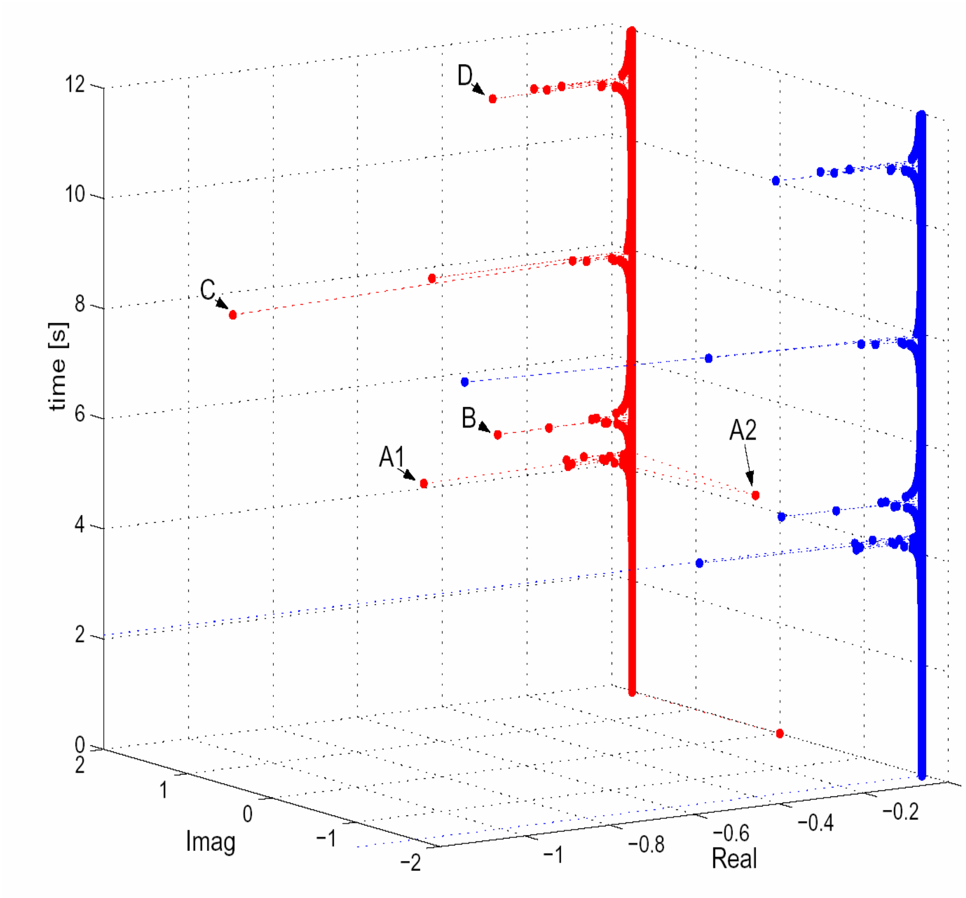


Discrete time eigenvalues

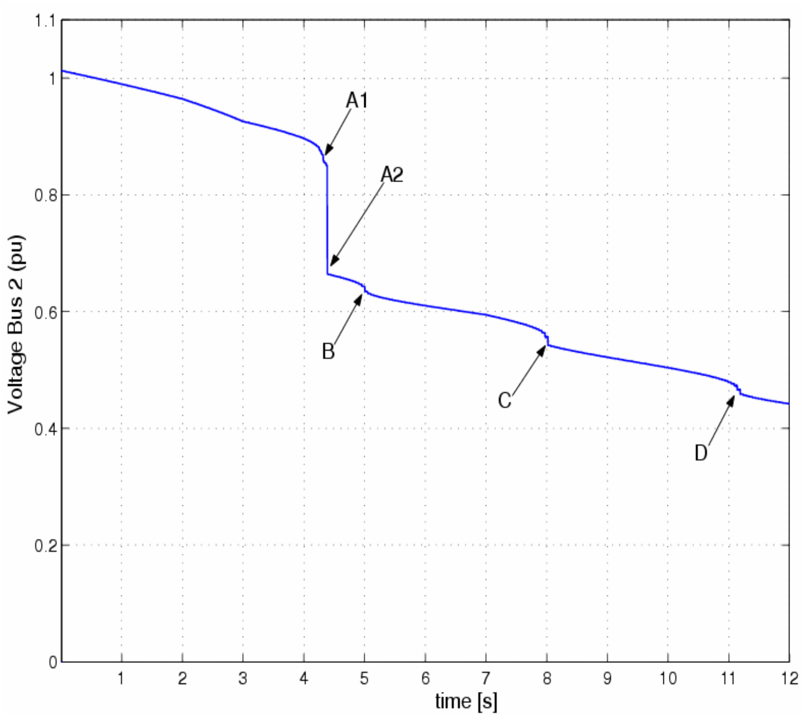


Voltage collapse

Voltage collapse of a radial system (cont.)

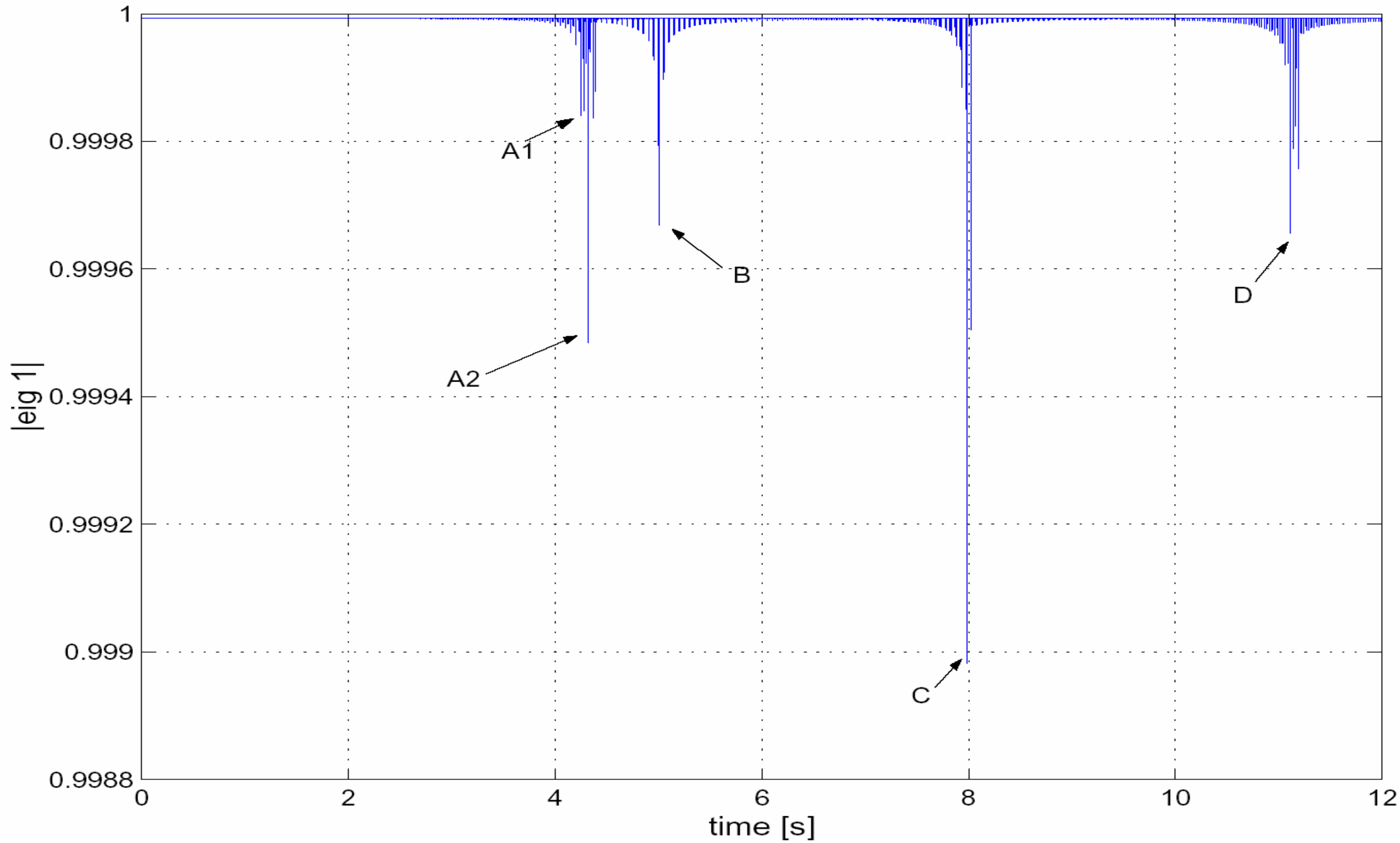


Reconstructed Continuous time eigenvalues



Voltage collapse

Voltage collapse of a radial system (cont.)



Anticipation of voltage drop from eigenvalue trajectory

250ms (A1) ; 300ms (B) ; 200ms (C) ; 450ms (D)

40-60 ms 500kV interrupter operation / 120-200 ms DAG 1000 km (optic/microwave)

Conclusions

Research Contributions

The description and implementation of a new and original power system stability assessment methodology that identifies the system's eigenvalues trajectories in a real-time EMTP solution incorporating the effect of switching and non-linear behaviour.

Research Contributions (cont.)

Advantages of Discrete state-space formulation from EMTP

- Trajectory tracking of non-linear elements eigenvalues moment by moment.
- In the context of OVNI, the capability of identifying suitable network partitioning schemes for application of multi-step integration solution in a hybrid power system simulator environment.
- Visualization of eigenvalues trajectories in discrete time domain for the purpose of assessing power system's dynamic behaviour.
- Automatic selection of discretization step from discrete time eigenvalue information
- Extension of EMTP capabilities to perform transient and voltage stability studies

Possible application extensions & future work

- Distributed intelligent control solutions based on embedded OVNI and eigenvalue trajectories.
- Integration of discrete state space eigenvalue methodology with Latency.
- Discrete state space eigenvalue methodology within UBC's OVNI-NET simulator for stability analysis and determination of segmentation schemes.
- Discrete state space eigenvalue methodology within UBC - JIIRP's I2Sim simulator for identification of trajectories of critical interdependencies among Critical Infrastructures.
- Development of new Visualization tools to provide simplified information about stability system trajectory to control center operators.

Thank you