Distributed Transmit Power Allocation for Multihop Cognitive-Radio Systems

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***Abstract*—In this paper, we consider a relay-assisted wideband cognitive-radio (CR) system under the assumption that the frequency band chosen by the CR relay network for unlicensed spectrum usage overlaps with one or more bands dedicated to primary (e.g., licensed) narrowband links. Our objective is to optimize the performance of the CR system while limiting the interference in direction of the primary receivers, without requiring any adaptation of the transmitted signal spectra at the cognitive nodes. To this end, we study appropriate transmit power allocation (TPA) strategies among the cognitive relays. We first investigate the optimal centralized (OC) TPA solution and show that it can be formulated as a linear program. Since the OC-TPA solution requires a considerable amount of information exchange between the cognitive nodes, we develop two distributed TPA schemes, namely (i) a fully decentralized (FD) TPA scheme and (ii) a distributed feedback-assisted (DFA) TPA scheme. The FD-TPA scheme aims at maximizing the output signal-to-interference-plus-noise ratio (SINR) at the destination node of the CR network according to a best-effort strategy. It requires neither feedback information from the destination node nor an exchange of channel state information between the cognitive relays. The DFA-TPA scheme, on the other hand, utilizes feedback information from the destination node, in order to achieve a pre-defined target output SINR value, while minimizing the overall transmit power spent by the relays. Analytical and simulation-based performance results illustrate that notable performance improvements compared to non-cooperative transmission (i.e., without relay assistance) are achieved by the proposed schemes, especially when more than two hops are considered. In particular, the proposed distributed TPA schemes typically perform close to the OC-TPA solution.**

***Index Terms*—Cognitive radio, relaying, transmit power allocation, performance analysis.**

I. Introduction

The concept of cognitive radio (CR) has recently attracted considerable interest in the wireless communications community [1]-[3]. Traditionally, radio spectrum usage has been organized according to fixed frequency plans defined through government licenses. However, spectrum occupancy measurements have shown that within confined geographical areas significant amounts of licensed spectrum are typically underutilized [4]. As a central feature, CR systems are envisioned to take advantage of unused or only partially occupied bands in an adaptive, dynamic, and unlicensed ('secondary') fashion, thus allowing for a more efficient spectrum utilization [5]. To this end, CR systems will require spectrum-sensing capabilities [6], [7], based on which they adjust key transmission parameters such as frequency bands and radiated transmit power. For example, CR capabilities will be relevant for ultra-wideband (UWB) radio systems[1] [9], which have been approved by regulatory bodies around the world for unlicensed spectrum usage in (parts of) the 3.1-10.6 GHz band [10]. In this paper, we focus on wideband (or UWB) CR networks consisting of a possibly large number of low-power transceivers for short-range transmission (on the order of a couple of meters). Such a setup is relevant for wireless sensor networks (WSNs) employed for monitoring and control tasks, as well as for future personal area networks (PANs), e.g., for wireless exchange of multimedia content between laptops/personal computers and peripheric devices. In order to achieve connectivity and to guarantee a certain quality of service for such networks, relaying techniques appear to be an attractive choice. Available relays can either be dedicated cognitive relays, which do not disseminate any data of their own, or temporarily inactive cognitive devices that act as relays to assist the current source–destination link.

The literature on relaying techniques with explicit incorporation of CR concepts is still relatively sparse. In [11], the cognitive idea was used to design spectrally efficient relaying protocols. Instead of allocating dedicated time slots to the relays, it was proposed that the relays sense for silent source node periods and use the corresponding vacant time slots for relaying. In [12], the possibility was considered that unlicensed cognitive relays might assist a primary source–destination link, so as to reduce the required number of primary retransmissions and exploit the resulting idle times for secondary transmissions within the CR network. In [13], relay-assisted CR systems using smart-antenna techniques for interference reduction were investigated. In [14], the outage performance of relay-assisted CR systems was considered, under the constraint that the individual relays operate in unused frequency bands only ('spectrum holes'). For a similar scenario, efficient relaying protocols have been proposed in [15], [16]. Finally, in [17], [18] the scenario was investigated that the frequency band chosen by the CR network for unlicensed spectrum usage is not completely unoccupied, but accommodates one or more active

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[1] So-called detection-and-avoidance techniques will become mandatory for UWB radio systems in, for example, Europe [8] and Japan.

primary links. In particular, different transmission techniques were developed that limit the interference from the CR network to the primary receivers.

In this paper, we consider a wideband relay-assisted multihop CR system consisting of a cognitive source–destination node pair and multiple cooperating cognitive relays. Similar to [17], [18], we assume that the frequency band chosen by the CR relay network overlaps with one or more bands dedicated to primary narrowband links and address the problem of optimal transmit power allocation (TPA) among the cognitive relays. Our objective is to optimize the performance of the CR system while limiting the interference experienced by the primary receivers, without requiring an adaptation of the transmitted signal spectra at the cognitive nodes. First, we investigate the optimal centralized (OC) TPA solution and show that it can be formulated as a linear program. As will be seen, a major drawback of the OC-TPA solution is that it requires a considerable amount of information exchange between the cognitive nodes (similar to the transmission technique proposed in [18]), which might be costly and difficult to achieve in practice. We therefore develop two distributed TPA schemes: (i) a fully decentralized (FD) TPA scheme and (ii) a distributed feedback-assisted (DFA) TPA scheme. The FD-TPA scheme aims at maximizing the output signal-to-interference-plus-noise ratio (SINR) at the destination node according to a best-effort strategy, so as to establish a quick connection between source and destination. It requires neither feedback information from the destination node nor an exchange of channel state information between the cognitive relays. If feedback information from the destination node is available, the DFA-TPA scheme is able to utilize this feedback, in order to achieve a pre-defined target output SINR value, while minimizing the overall transmit power spent by the relays. A thorough performance analysis as well as simulation-based performance results illustrate that notable improvements compared to non-cooperative transmission are achieved by the proposed schemes, especially if more than two hops are considered. Moreover, it is demonstrated that the proposed distributed TPA schemes typically perform close to the OC solution. It is worth noting that, in principle, the proposed distributed TPA schemes could be employed in any relay-assisted wireless system which aims to limit its interference to other systems. However, our schemes seem to fit best into a CR framework, where functionalities like spectrum sensing and radio-scene analysis are intrinsic pre-requisites [2].

The remainder of this paper is organized as follows: In Section II, the system model and the optimization problem under consideration are introduced. Starting from the OC solution, the distributed TPA schemes are developed in Section III. Analytical and simulation-based performance results are presented in Sections IV and V, respectively, and the benefits of the proposed TPA schemes in comparison with non-cooperative transmission are highlighted. Finally, some conclusions are provided in Section VI.

II. System Model and Problem Formulation

A. Assumptions

We consider a wideband (or UWB) CR network consisting of a (large) number of low-power cognitive transceiver nodes. Throughout this paper, we will focus on a single point-to-point link between a cognitive source node S and a cognitive destination node D. To this end, we assume that an appropriate network protocol is employed, which manages the communication within the CR network and selects the current source–destination node pair S–D, e.g., based on the transmission buffer states of all cognitive transceivers. In the following, we assume that the selected source–destination node pair S–D is assisted by N_r cognitive relay nodes R_i ($i=1,\dots,N_\mathrm{r}$), which are either dedicated relays or close-by, temporarily inactive cognitive transceivers. Throughout this paper, we assume that the frequency band chosen by the CR relay network for unlicensed spectrum usage (fully) overlaps with N_p active primary narrowband point-to-point links, which may, for example, represent wireless local area network (WLAN) links in the vicinity of the CR relay network. A couple of further assumptions employed throughout this paper are listed below:

- *CR relay network:* The CR relay network is assumed to be based on code-division multiple access (CDMA). The N_r relay nodes are assumed to employ mutually orthogonal spreading codes while being perfectly synchronized in time and frequency. We assume that the source node transmits a large number of short messages using a low duty cycle (as it is typical for, e.g., WSN applications), so that multihop transmissions can be accommodated in the time domain without causing a critical rate loss. For simplicity and practical relevance, all nodes within the CR network are assumed to employ a single omni-directional antenna. The maximum transmit powers available at the source node and the relay nodes are in the following denoted as $P_{\mathrm{S},\max}$ and $P_{\mathrm{R}_i,\max}$ ($i=1,\dots,N_\mathrm{r}$), respectively.
- *Primary links:* The index set associated with all primary transmitters is denoted as $\mathbb{I}_\mathrm{p}^{(\mathrm{tx})} \subseteq \{1,\dots,2N_\mathrm{p}\}$ ($|\mathbb{I}_\mathrm{p}^{(\mathrm{tx})}| := N_\mathrm{p}$). The remaining primary nodes, denoted by index set $\mathbb{I}_\mathrm{p}^{(\mathrm{rx})} = \{1,\dots,2N_\mathrm{p}\} \backslash \mathbb{I}_\mathrm{p}^{(\mathrm{tx})}$, are assumed to be receiving. We assume that $\mathbb{I}_\mathrm{p}^{(\mathrm{tx})}$ and $\mathbb{I}_\mathrm{p}^{(\mathrm{rx})}$ remain fixed during the entire transmission of the CR network. The bandwidths B_{U_j} occupied by the primary nodes U_j ($j=1,\dots,2N_\mathrm{p}$) are assumed to be small compared to the bandwidth B_CR of the CR system. The bandwidth ratio for primary node U_j is denoted by $\rho_j := B_{\mathrm{U}_j}/B_\mathrm{CR} < 1$, and the maximum sum interference power tolerated by the jth primary receiver ($j \in \mathbb{I}_\mathrm{p}^{(\mathrm{rx})}$) is denoted by ξ_j. Finally, the average transmit powers of the primary transmitters U_j ($j \in \mathbb{I}_\mathrm{p}^{(\mathrm{tx})}$) are denoted as P_{U_j}. We assume that $P_{\mathrm{U}_j} \gg P_{\mathrm{S},\max}$ and $P_{\mathrm{U}_j} \gg P_{\mathrm{R}_i,\max}$ for all i, j.
- *Channel model:* We assume quasi-static channel conditions. The channel impulse response (CIR) of a certain link $\mathrm{X}\to\mathrm{Y}$ from one node X to another node Y ($\mathrm{X},\mathrm{Y} \in \{\mathrm{S},\mathrm{D},\mathrm{R}_1,\dots,\mathrm{R}_{N_\mathrm{r}},\mathrm{U}_1,\dots,\mathrm{U}_{2N_\mathrm{p}}\}$) is denoted as $\mathbf{h}_{\mathrm{X},\mathrm{Y}} := [\,h_{\mathrm{X},\mathrm{Y}}^{(0)},\dots,h_{\mathrm{X},\mathrm{Y}}^{(L_{\mathrm{X},\mathrm{Y}})}\,]^\mathrm{T}$, where $L_{\mathrm{X},\mathrm{Y}}$ denotes the corresponding channel memory length. Moreover, we define the channel energy $\alpha_{\mathrm{X},\mathrm{Y}} := \sum_{l=0}^{L_{\mathrm{X},\mathrm{Y}}} |h_{\mathrm{X},\mathrm{Y}}^{(l)}|^2$. Since the bandwidths B_{U_j} are assumed to be comparatively small, all links associated with the primary nodes are for simplicity modeled with a channel memory length of

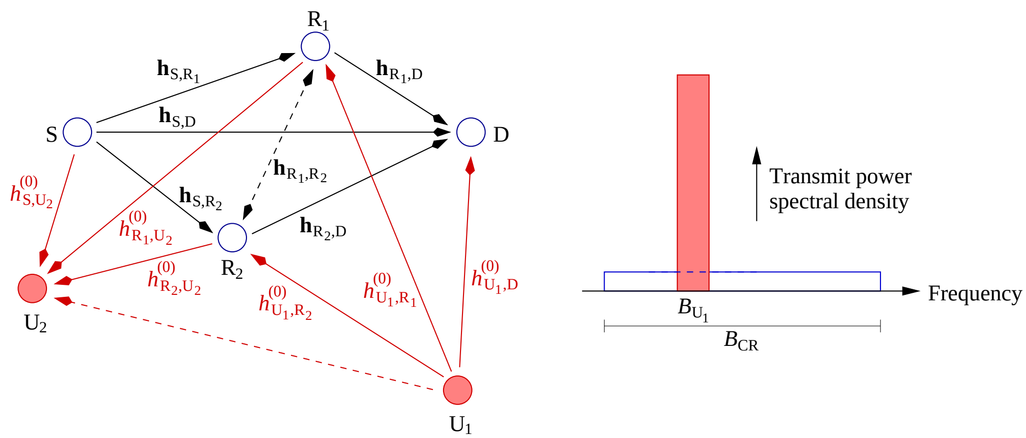

Fig. 1. Left: System model under consideration, for the example of two hops, two relays ($N_{\mathrm{r}}=2$), and a single active primary link $\mathrm{U}_1 \rightarrow \mathrm{U}_2$. Right: Corresponding interference scenario in the frequency domain.

zero. The variance of the additive white Gaussian noise (AWGN) process at cognitive node Y after despreading (assumed identical for all cognitive nodes) is denoted as $\sigma^2_{\mathrm{n,Y}} := N_0 F B_{\mathrm{CR}}/N_{\mathrm{sp}}$, where N_0 represents the single-sided noise power spectral density, F the receiver noise figure, and N_{sp} the spreading length used by the CR system [19, Ch. 13.2].

- *Mutual interference:* It is assumed that the interference caused by a transmitting cognitive node X appears at the primary receivers U_j ($j \in \mathbb{I}^{(\mathrm{rx})}_{\mathrm{p}}$) as AWGN with variance $\rho_j P_{\mathrm{X}} \alpha_{\mathrm{X,U}_j}$. When receiving, the cognitive nodes are assumed to employ simple despreading for interference suppression (rather than more sophisticated filtering techniques). Correspondingly, the primary interference power appears at cognitive node Y as AWGN with variance $\sigma^2_{\mathrm{i,Y}} := \frac{1}{N_{\mathrm{sp}}} \sum_{j\in \mathbb{I}^{(\mathrm{tx})}_{\mathrm{p}}} P_{\mathrm{U}_j} \alpha_{\mathrm{U}_j,\mathrm{Y}}$ [19, Ch. 13.2].
- *Side information:* We assume that the maximum transmit powers $P_{\mathrm{R}_i,\max}$ ($i=1,...,N_{\mathrm{r}}$) of the cognitive relays, the average transmit powers P_{U_j} ($j \in \mathbb{I}^{(\mathrm{tx})}_{\mathrm{p}}$) of the primary transmitters, as well as the maximum sum interference powers ξ_j and the parameters ρ_j ($j=1,...,2N_{\mathrm{p}}$) are known throughout the CR network. This appears to be reasonable, since due to the fixed frequency plans associated with primary spectrum usage it is known which systems will operate in the frequency band under consideration. The destination node D is assumed to have perfect knowledge of the CIRs $\mathbf{h}_{\mathrm{S,D}}$ and $\mathbf{h}_{\mathrm{R}_i,\mathrm{D}}$ associated with the source–destination link $\mathrm{S}\rightarrow\mathrm{D}$ and the relay–destination links $\mathrm{R}_i \rightarrow \mathrm{D}$ ($i=1,...,N_{\mathrm{r}}$). Similarly, each relay node R_i is assumed to have perfect knowledge of the CIR $\mathbf{h}_{\mathrm{S,R}_i}$ and the CIRs $\mathbf{h}_{\mathrm{R}_{i'},\mathrm{R}_i}$ associated with the links from the other relays $\mathrm{R}_{i'}$ ($i' \neq i$) to itself. In practice, this will require an initial training phase, before the actual transmission phase can start. Furthermore, it is assumed that based on built-in radio-scene-analysis functionalities the source node and the relays are aware of the channel energies $\alpha_{\mathrm{S,U}_j} = |h^{(0)}_{\mathrm{S,U}_j}|^2$ and $\alpha_{\mathrm{R}_i,\mathrm{U}_j} = |h^{(0)}_{\mathrm{R}_i,\mathrm{U}_j}|^2$ associated with their own links in direction of the primary nodes U_j ($j=1,...,2N_{\mathrm{p}}$), respectively. This will require an initial radio-scene analysis phase, while the different primary nodes U_j are sensed to be transmitting [2], [6], [7]. In this context, we assume that the primary links employ time-division duplex (TDD)[2] and that the primary transmitters and receivers change their roles every now and then. Moreover, it is assumed that $\alpha_{\mathrm{S,U}_j} = \alpha_{\mathrm{U}_j,\mathrm{S}}$ and $\alpha_{\mathrm{R}_i,\mathrm{U}_j} = \alpha_{\mathrm{U}_j,\mathrm{R}_i}$ for all indices $i=1,...,N_{\mathrm{r}}$ and $j=1,...,2N_{\mathrm{p}}$, which is reasonable for primary systems operating in a TDD mode. Finally, we assume that $\sigma^2_{\mathrm{i,D}}$ and $\sigma^2_{\mathrm{n,D}}$ (or $\sigma^2_{\mathrm{i,D}}+\sigma^2_{\mathrm{n,D}}$) are perfectly known at the destination node.
- *Control signaling:* The multihop relaying protocol introduced in the following subsection requires the relays and the destination node to broadcast some control information in the form of short acknowledgment (ACK) signals. Throughout this paper, we assume that ACK signals are sufficiently protected using some low-rate channel code, so that they can be received reliably throughout the CR network.

The system model under consideration is illustrated in Fig. 1, for two hops, $N_{\mathrm{r}}=2$ relays, and one active primary link.

B. Multihop Relaying Protocol

The employed transmission protocol consists of $(N_{\max}+1)$ orthogonal time slots, where $N_{\max}$ denotes a pre-defined maximum number of relaying phases. Within the first time slot, the source node S broadcasts a message to the relay nodes R_1,...,$\mathrm{R}_{N_{\mathrm{r}}}$ and the destination node D, while the transmit power P_{S} is adjusted such that all interference constraints are met, i.e., $\rho_j\, P_{\mathrm{S}}\, \alpha_{\mathrm{S,U}_j} \leq \xi_j$ for all $j \in \mathbb{I}^{(\mathrm{rx})}_{\mathrm{p}}$:

$$P_{\mathrm{S}} := \min \left\{ P_{\mathrm{S},\max},\ \min_{j\in \mathbb{I}^{(\mathrm{rx})}_{\mathrm{p}}} \left\{ \frac{\xi_j}{\rho_j\, \alpha_{\mathrm{S,U}_j}} \right\} \right\}. \qquad (1)$$

The destination node and each relay node are assumed to employ a Rake receiver, which performs optimal maximum-ratio

[2]TDD is becoming increasingly popular and has been adopted as the only or one possible option in, e.g., IEEE 802.11 WLAN systems and the Chinese third-generation (3G) cellular standard TD-SCDMA (standing for time-division and synchronous CDMA) [20, Ch. 24], [21]. Moreover, TDD is likely to be employed in future fourth-generation (4G) systems, due to its flexibility with regard to asymmetric data traffic in uplink and downlink direction [22].

combining (MRC) of the signal received from the source node. In the following, let γ_{D} denote the (overall) MRC output SINR per information bit at the destination node. If γ_{D} exceeds a certain pre-defined target SINR value $\gamma_{\mathrm{D,target}}$ after completion of the source transmission phase, i.e.,

$$\gamma_{\mathrm{D}} = \gamma_{\mathrm{S}\to\mathrm{D}} := \frac{P_{\mathrm{S}}\,\alpha_{\mathrm{S,D}}}{\sigma_{\mathrm{i,D}}^2 + \sigma_{\mathrm{n,D}}^2} \geq \gamma_{\mathrm{D,target}}, \tag{2}$$

the destination node broadcasts a short ACK signal to inform the source node and the relay nodes that an additional relaying phase is not required. Otherwise, the relaying process is initiated. All relays that have received the message from the source node with an MRC output SINR per information bit of

$$\gamma_{\mathrm{R}_i} = \gamma_{\mathrm{S}\to\mathrm{R}_i} := \frac{P_{\mathrm{S}}\,\alpha_{\mathrm{S,R}_i}}{\sigma_{\mathrm{i,R}_i}^2 + \sigma_{\mathrm{n,R}_i}^2} \geq \gamma_{\mathrm{th}}, \tag{3}$$

where γ_{th} denotes some threshold SINR value, are assumed to decode the message without any errors. These $N'_{\mathrm{r},1} \leq N_{\mathrm{r}}$ relays then broadcast a short ACK signal, so as to inform the other relays and the destination node that they will participate in the upcoming relaying phase.[3] In the following, let $N'_{\mathrm{r},n}$ denote the number of relays participating in the nth relaying phase ($1 \leq n \leq N_{\max}$), and let $\mathbb{I}_{\mathrm{r},n} \subseteq \{1, ..., N_{\mathrm{r}}\}$ denote the corresponding index set. Moreover, we define $\gamma_{\mathrm{R}\to\mathrm{D},n} := \sum_{i\in\mathbb{I}_{\mathrm{r},n}} \gamma_{\mathrm{R}_i\to\mathrm{D}}$, where $\gamma_{\mathrm{R}_i\to\mathrm{D}} := P_{\mathrm{R}_i}\,\alpha_{\mathrm{R}_i,\mathrm{D}}/(\sigma_{\mathrm{i,D}}^2 + \sigma_{\mathrm{n,D}}^2)$ and P_{R_i} denotes the transmit power of relay R_i. Throughout this paper, we assume that each available relay forwards the message from the source node at most once, so as to save battery power. Within each relaying phase, the transmit powers P_{R_i} of the participating relays have to be chosen such that the interference constraints ξ_j are met at all primary receivers U_j, $j \in \mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})}$. Now, within the first relaying phase ($n = 1$, second time slot) the $N'_{\mathrm{r},1}$ relays re-encode the message using the orthogonal spreading codes and simultaneously retransmit it, and the destination node performs optimal MRC of the corresponding received signals, respectively. Thus, we have $\gamma_{\mathrm{D}} = \gamma_{\mathrm{S}\to\mathrm{D}} + \gamma_{\mathrm{R}\to\mathrm{D},1}$. While the $N'_{\mathrm{r},1} \leq N_{\mathrm{r}}$ relays are retransmitting the message from the source node during the first relaying phase ($n = 1$), the remaining $N_{\mathrm{r}} - N'_{\mathrm{r},1}$ relays can improve their own MRC output SINR γ_{R_i} by combining the corresponding received signals, respectively (similar to the destination node). Those $N'_{\mathrm{r},2} \leq N_{\mathrm{r}} - N'_{\mathrm{r},1}$ yet inactive relays, which meet the threshold SINR γ_{th} after the initial $N'_{\mathrm{r},1}$ relays have completed their transmissions, first broadcast a short ACK signal, so as to inform the other nodes within the CR network (similar to the first relaying phase). Then they simultaneously retransmit the successfully decoded message from the source node within a second relaying phase ($n = 2$, third time slot). The second relaying phase is in turn utilized by the destination node, in order to improve the overall MRC output SINR γ_{D}, as well as by the still inactive relays to improve their own MRC output SINRs γ_{R_i}. If the target MRC output SINR $\gamma_{\mathrm{D,target}}$ at the destination node is reached after the n_0th relaying phase ($n_0 \in \{1, ..., N_{\max} - 1\}$), i.e.,

$$\gamma_{\mathrm{D}} = \gamma_{\mathrm{S}\to\mathrm{D}} + \sum_{n=1}^{n_0} \gamma_{\mathrm{R}\to\mathrm{D},n} \geq \gamma_{\mathrm{D,target}}, \tag{4}$$

the relaying process is stopped by the destination node broadcasting a corresponding ACK signal. Otherwise, the above process is repeated until either all relays have once forwarded the message from the source node to the destination node or the pre-defined maximum number of relaying phases, $N_{\max}$, is reached. Note that since we have assumed quasi-static channel conditions during a large number of transmitted source messages, the set of relay nodes participating in a particular relaying phase will stay the same over the entire time horizon under consideration.

C. Optimization Problem

In practice, wireless relays are often simple devices with a limited battery power. Correspondingly, our design goal is to accomplish the pre-defined target output SINR $\gamma_{\mathrm{D,target}}$ at the destination node, while minimizing the transmit powers spent by the relay nodes. Therefore, within the nth relaying phase the transmit powers P_{R_i} of the participating relays shall be adjusted such that the sum transmit power $P_{\mathrm{R,ov},n} := \sum_{i\in\mathbb{I}_{\mathrm{r},n}} P_{\mathrm{R}_i}$ is minimized, under the constraints that

(a) the overall MRC output SINR γ_{D} at the destination node is larger than or equal to $\gamma_{\mathrm{D,target}}$ (if possible)
(b) the sum interference power experienced by each primary receiver U_j ($j \in \mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})}$) within that relaying phase remains smaller than the pre-defined maximum interference power ξ_j,
(c) the transmit power of each active relay R_i ($i \in \mathbb{I}_{\mathrm{r},n}$) does not exceed the maximum value $P_{\mathrm{R}_i,\max}$.

In the sequel, let $\gamma_{\mathrm{D},n}$ denote the MRC output SINR at the destination node that is accomplished after n relaying phases ($\gamma_{\mathrm{D},0} := \gamma_{\mathrm{S}\to\mathrm{D}}$). Assuming that $\gamma_{\mathrm{D},n-1} < \gamma_{\mathrm{D,target}}$, the optimal (centralized) transmit power allocation (TPA) strategy thus results from the following linear program:[4]

$$\begin{aligned} \text{minimize} \quad & P_{\mathrm{R,ov},n} = \sum_{i\in\mathbb{I}_{\mathrm{r},n}} P_{\mathrm{R}_i} \\ \text{subject to} \quad & \gamma_{\mathrm{R}\to\mathrm{D},n} = \frac{1}{\sigma_{\mathrm{i,D}}^2 + \sigma_{\mathrm{n,D}}^2} \sum_{i\in\mathbb{I}_{\mathrm{r},n}} P_{\mathrm{R}_i}\,\alpha_{\mathrm{R}_i,\mathrm{D}} \\ & \qquad\qquad \geq \gamma_{\mathrm{D,target}} - \gamma_{\mathrm{D},n-1}; \\ & \rho_j \sum_{i\in\mathbb{I}_{\mathrm{r},n}} P_{\mathrm{R}_i}\,\alpha_{\mathrm{R}_i,\mathrm{U}_j} \leq \xi_j \ \text{ for all } j \in \mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})}; \\ & P_{\mathrm{R}_i} \leq P_{\mathrm{R}_i,\max} \ \text{ for all } i \in \mathbb{I}_{\mathrm{r},n}. \end{aligned} \tag{5}$$

The above optimization problem is illustrated in Fig. 2 (left hand side), for the case of $|\mathbb{I}_{\mathrm{r},n}| = 2$ active relays. Obviously, the existence of a feasible solution cannot always be guaranteed. In the scenario depicted in Fig. 2, for example, the feasible region will be empty if $P_{\mathrm{R}_1,\max} < c'_1$ and $P_{\mathrm{R}_2,\max} < c'_2$ or if

[3] Since the relays are equipped with orthogonal spreading codes, one-bit ACK signals are sufficient for identification of the participating relays.

[4] Ideally, it would be desirable to conduct a joint optimization for all relaying phases. However, since the TPA in relaying phase n influences the set of active relays in relaying phase $n+1$, such a joint optimization is not directly feasible and defies the design of a simple power allocation scheme as pursued in this paper. Correspondingly, within the scope of this paper we focus on the optimization of each individual relaying phase.

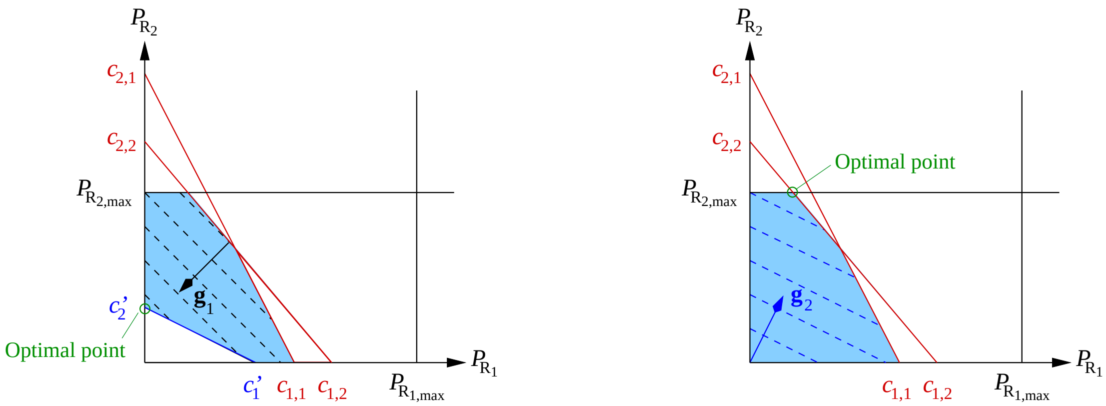

Fig. 2. Graphical illustration of the optimization problems under consideration, for the example of $|\mathcal{I}_{\mathrm{r},n}|=2$ active relays and $|\mathcal{I}_{\mathrm{p}}^{(\mathrm{rx})}|=2$ primary receivers. Left: Optimization problem (5). Right: Optimization problem (6). The feasible regions for P_{R_1} and P_{R_2} are shaded. The level curves of the respective objective functions are marked by dashed lines. Parameter $c_{i,j}$ is given by $c_{i,j} = \xi_j/(\rho_j\, \alpha_{\mathrm{R}_i,\mathrm{U}_j})$ and parameter c_i' is given by $c_i'=(\gamma_{\mathrm{D,target}}-\gamma_{\mathrm{D},n-1}) \times (\sigma_{\mathrm{i,D}}^2+\sigma_{\mathrm{n,D}}^2)/\alpha_{\mathrm{R}_i,\mathrm{D}}$. Moreover, the corresponding gradient vectors for optimization problem (5) and optimization problem (6) are given by $\mathbf{g}_1=[-1,\ -1]^{\mathrm{T}}$ and $\mathbf{g}_2=[\alpha_{\mathrm{R}_1,\mathrm{D}},\ \alpha_{\mathrm{R}_2,\mathrm{D}}]^{\mathrm{T}}/(\sigma_{\mathrm{i,D}}^2+\sigma_{\mathrm{n,D}}^2)$, respectively.

$c_1' > \min\{c_{1,1}, c_{1,2}\}$ and $c_2' > \min\{c_{2,1}, c_{2,2}\}$. The solution of (5) can be found by means of well-established linear programming techniques.[5] Note, however, that the standard Simplex algorithm [24, Ch. 4] cannot be used in this case, because the trivial solution $P_{\mathrm{R}_i}=0$ for all $i\in\mathcal{I}_{\mathrm{r},n}$ does not lie in the feasible region. Yet, there are interior-point algorithms [25] or numerical methods such as the Big-M method and the Two-Phase Simplex method [24, Ch. 4] that can be employed instead.

If a feasible solution of (5) does not exist, i.e., the target output SINR $\gamma_{\mathrm{D,target}}$ cannot be accomplished within the current relaying phase, it is useful to pursue a best-effort strategy in order to maximize the MRC output SINR increment $\gamma_{\mathrm{R}\to\mathrm{D},n}$ under the given constraints (in anticipation of meeting the target SINR in the next relaying phase). Correspondingly, we turn to the following optimization problem in this case:

$$\begin{aligned} \text{maximize} \quad & \gamma_{\mathrm{R}\to\mathrm{D},n} = \frac{1}{\sigma_{\mathrm{i,D}}^2+\sigma_{\mathrm{n,D}}^2}\sum_{i\in\mathcal{I}_{\mathrm{r},n}} P_{\mathrm{R}_i}\,\alpha_{\mathrm{R}_i,\mathrm{D}} \qquad (6)\\ \text{subject to} \quad & \rho_j \sum_{i\in\mathcal{I}_{\mathrm{r},n}} P_{\mathrm{R}_i}\,\alpha_{\mathrm{R}_i,\mathrm{U}_j} \le \xi_j \ \text{ for all } j\in\mathcal{I}_{\mathrm{p}}^{(\mathrm{rx})};\\ & P_{\mathrm{R}_i} \le P_{\mathrm{R}_i,\max} \ \text{ for all } i\in\mathcal{I}_{\mathrm{r},n}. \end{aligned}$$

The above optimization problem is illustrated in Fig. 2 (right hand side), again for $|\mathcal{I}_{\mathrm{r},n}|=2$ active relays. Obviously, for (6) a feasible solution can always be found (e.g., by using the standard Simplex algorithm).

It is worth pointing out that there is a conceptional difference between the above optimization problems and other transmit power optimization problems considered in the literature. For example, in [23] the transmit powers of a set of N links are optimized based on the SINR observed on each link, where all links influence each other, i.e., each link has an impact on the considered objective function. In our problem setup, the secondary network is supposed to be transparent to the primary links. Correspondingly, only the transmit powers within the secondary network are included in the optimization, whereas the transmit powers associated with the primary links are fixed. This leads to a certain asymmetry in the problem setup, which is relevant for the development of the distributed TPA schemes in Section III, but is absent in the related literature.

[5] For specific classes of linear programs, closed-form solutions can be found. A pre-requisite for this is that all inequality constraints involved can be guaranteed to be met with equality in the optimum solution [23]. For the linear programs considered here, this pre-requisite is not valid.

D. Numerical Example

In order to illustrate the above problem setup, we consider a UWB CR system with two active relays, $B_{\mathrm{CR}}=500$ MHz, and $N_{\mathrm{sp}}=20$. In accordance with the Federal Communications Commission (FCC) spectral mask specified for UWB devices [10],[26], we set the maximum transmit powers of the CR nodes to $P_{\mathrm{X,max}}=37$ nW ($\mathrm{X}\in\{\mathrm{S},\mathrm{R}_1,\mathrm{R}_2\}$). We assume that one active primary WLAN link $\mathrm{U}_1\to\mathrm{U}_2$ is found in the vicinity of the CR relay network, with parameters $B_{\mathrm{U}_{1,2}}=20$ MHz, $\rho_{1,2}=0.04$, and $P_{\mathrm{U}_1}=40$ mW [20, Ch. 24]. The channel energy $\alpha_{\mathrm{X,Y}}$ associated with a certain link $\mathrm{X}\to\mathrm{Y}$ ($\mathrm{X},\mathrm{Y}\in\{\mathrm{S},\mathrm{D},\mathrm{R}_1,\mathrm{R}_2,\mathrm{U}_1,\mathrm{U}_2\}$) is modeled according to $\alpha_{\mathrm{X,Y}}:=L_0\cdot(d_0/d_{\mathrm{X,Y}})^p$, where $d_{\mathrm{X,Y}}$, L_0, and p denote the distance between node X and node Y, the reference attenuation for a distance of $d_0=1$ m, and the path-loss exponent, respectively. In the following, we set $L_0=-50$ dB and $p=2$ [27]. Moreover, we assume $d_{\mathrm{R}_1,\mathrm{D}}=7$ m, $d_{\mathrm{R}_2,\mathrm{D}}=4$ m, $d_{\mathrm{U}_1,\mathrm{D}}=50$ m, and $d_{\mathrm{R}_1,\mathrm{U}_2}=d_{\mathrm{R}_2,\mathrm{U}_2}=15$ m.

Suppose, we want to hide the signals of the cognitive relays below the noise level of the primary WLAN receiver, which is around -95 dBm in practice (assuming $N_0=-174$ dBm/Hz and a noise figure of 6 dB) [20, Ch. 3.2]. To this end, we want to choose the transmit powers of the relays such that the interference power observed at the primary WLAN receiver does not exceed, say, $\xi_2=-100$ dBm. If both relays employ the maximum transmit power $P_{\mathrm{R}_i,\max}=37$ nW ($i=1,2$), the resulting interference power at the primary WLAN receiver is -98.8 dBm. Correspondingly, at least one of the relays needs to choose a transmit power smaller than $P_{\mathrm{R}_i,\max}$. Now suppose, we require an SINR increment of $\gamma_{\mathrm{R}\to\mathrm{D},n}=3$ dB, in order to achieve the target SINR value $\gamma_{\mathrm{D,target}}$ at the destination node. For the considered example, one obtains $\sigma^2_{\mathrm{i,D}} + \sigma^2_{\mathrm{n,D}} = -80.76\,\mathrm{dBm}$ (again assuming a noise figure of 6 dB). Correspondingly, the transmit powers of the relays should be chosen such that in the $\{P_{\mathrm{R}_1}, P_{\mathrm{R}_2}\}$-plane the point $(P_{\mathrm{R}_1}, P_{\mathrm{R}_2})$ lies either on or above the straight line going through the points $\mathcal{P} = (0\,\mathrm{W},\, 26.8\,\mathrm{nW})$ and $\mathcal{P}' = (82.0\,\mathrm{nW},\, 0\,\mathrm{W})$. Similar to the scenario depicted in Fig. 2 (left hand side), the optimal point that minimizes the sum transmit power of the relays is given by $\mathcal{P}$. The corresponding interference power experienced by the primary WLAN receiver is $-103.22\,\mathrm{dBm}$. Therefore, $\mathcal{P}$ represents a feasible solution of the optimization problem (5).

III. Distributed Transmit Power Allocation Schemes

In order to solve the optimization problem(s) introduced in Section II-C, a central network node C is required (e.g., the destination node or one of the relays) which needs to be aware of *all* channel energies $\alpha_{\mathrm{R}_i,\mathrm{D}}$ and $\alpha_{\mathrm{R}_i,\mathrm{U}_j}$ $(i \in \mathbb{I}_{\mathrm{r},n},\ j \in \mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})})$. After computing the optimal solution, node C would then forward the resulting transmit power levels to the participating relay nodes. Obviously, this requires a significant amount of signaling overhead, since each relay node R_i needs to communicate its own channel energies $\alpha_{\mathrm{R}_i,\mathrm{D}}$ and $\alpha_{\mathrm{R}_i,\mathrm{U}_j}$ $(j \in \mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})})$ to the central node C. This might be costly and difficult to acquire in practice, especially when a larger number of relays is available. In the following, we will develop two distributed transmit power allocation (TPA) schemes, which do not require any further exchange of channel information between the cognitive nodes: (i) a fully decentralized (FD) scheme and (ii) a distributed feedback-assisted (DFA) scheme.

As pointed out by one of the reviewers, an alternative approach to find distributed solutions for convex or linear optimization problems is to employ the dual decomposition method [28]. The basic principle is to decompose the primary optimization problem into several sub-problems, which are coupled by a so-called master problem. Thus, each network node only needs to solve a (local) sub-problem. The master problem is finally solved by a special master node. A drawback of the dual decomposition method is that it requires the exchange of dual variables between network nodes – often in an iterative fashion. This leads to a considerable amount of feedback information, even if the individual nodes require only local channel information. In contrast to this, the distributed TPA schemes developed here require very little interaction between the involved network nodes. Although our approach is somewhat more heuristic than the dual decomposition method, it appears to be justified by the near-optimum performance of the proposed schemes, as verified by the analysis in Section IV and the numerical results presented in Section V.

A. Fully Decentralized (FD) TPA Scheme

Assuming that no feedback information from the destination node is available, our design goal can only be along the lines of optimization problem (6), i.e., the MRC output SINR increment $\gamma_{\mathrm{R}\to\mathrm{D},n}$ shall be maximized according to a best-effort strategy, taking the given interference and maximum transmit power constraints into account. By this means, a quick connection between source and destination node can be established. Recall that the interference caused by cognitive relay R_i appears at primary receiver U_j as AWGN with variance $\rho_j P_{\mathrm{R}_i} \alpha_{\mathrm{R}_i,\mathrm{U}_j}$. Now, based on the ACK signals broadcasted by the relays (cf. Section II-B), the number $N'_{\mathrm{r},n}$ of relays participating in the nth relaying phase is known throughout the CR network. Moreover, each relay node R_i is assumed to be aware of the channel energies $\alpha_{\mathrm{R}_i,\mathrm{U}_j}$ associated with its own links in direction of the primary receivers U_j $(j \in \mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})})$.[6] Correspondingly, each relay can adjust its transmit power level according to

$$P_{\mathrm{R}_i} = \min\left\{ P_{\mathrm{R}_i,\max},\ \min_{j\in\mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})}} \left\{ \frac{\xi_j}{\rho_j\, N'_{\mathrm{r},n}\, \alpha_{\mathrm{R}_i,\mathrm{U}_j}} \right\} \right\} \tag{7}$$

(similar to (1)). By this means, it can be guaranteed that each primary receiver experiences a sum interference power of at most ξ_j, without any further interaction between the relays. For example, in the special case of a single primary receiver $(|\mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})}| = 1)$ and $\xi_1/(\rho_1 N'_{\mathrm{r},n} \alpha_{\mathrm{R}_i,\mathrm{U}_1}) \le P_{\mathrm{R}_i,\max}$ for all $i \in \mathbb{I}_{\mathrm{r},n}$, each relay will cause an interference power of exactly $\xi_1/N'_{\mathrm{r},n}$. Moreover, due to the outer minimization in (7) it is guaranteed that the maximum transmit power available at each relay is not exceeded.

B. Distributed Feedback-Assisted (DFA) TPA Scheme

Different from the FD-TPA scheme, the DFA-TPA scheme aims to approach the target SINR value $\gamma_{\mathrm{D,target}}$ rather than exceeding it in the last relaying phase. The DFA-TPA scheme is similar to the optimal centralized (OC) solution discussed at the beginning of this section. In the DFA-TPA scheme, the destination node assumes the role of the central node, however, without having complete knowledge of all channel energies. In particular, since the destination node knows only the channel energies $\alpha_{\mathrm{R}_i,\mathrm{D}}$ $(i \in \mathbb{I}_{\mathrm{r},n})$, cf. Section II-A, it requires estimates $\tilde{\alpha}_{\mathrm{R}_i,\mathrm{U}_j}$ of all channel energies $\alpha_{\mathrm{R}_i,\mathrm{U}_j}$ associated with the links from the active relays to the primary receivers $(j \in \mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})})$. Employing these estimates, the destination node can then determine an approximation of the OC solution based on (5) and (6) and feed back the resulting transmit power levels P'_{R_i} to the participating relay nodes (using the corresponding spreading codes in conjunction with a low-rate channel code), which then readjust their transmit power levels accordingly. The challenge here is to guarantee that $\tilde{\alpha}_{\mathrm{R}_i,\mathrm{U}_j} \ge \alpha_{\mathrm{R}_i,\mathrm{U}_j}$ holds for all $i \in \mathbb{I}_{\mathrm{r},n}$ and $j \in \mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})}$, so that the resulting DFA solution always meets the interference constraints posed by the original optimization problem.

The basic idea for obtaining the required estimates $\tilde{\alpha}_{\mathrm{R}_i,\mathrm{U}_j}$ is as follows: Initially, the active relay nodes start with a slightly modified version of the FD-TPA solution (7), according to

$$\begin{aligned} P_{\mathrm{R}_i} \quad &:= \quad \min\left\{ P_{\mathrm{R}_i,\max},\ \min_{j\in\mathbb{I}_{\mathrm{p}}^{(\mathrm{rx})}} \left\{ \frac{\theta}{N'_{\mathrm{r},n}\, \alpha_{\mathrm{R}_i,\mathrm{U}_j}} \right\} \right\} \\ &= \quad \min\left\{ P_{\mathrm{R}_i,\max},\ \frac{\theta}{N'_{\mathrm{r},n}\, \alpha_{\mathrm{R}_i,\mathrm{U},\max}} \right\}, \end{aligned} \tag{8}$$

[6] In Section V, we will relax this assumption to a certain extend.

TABLE I
COMPARISON OF DISTRIBUTED TRANSMIT POWER ALLOCATION (TPA) SCHEMES (†SEE SECTION III, ‡SEE SECTIONS IV AND V).

TPA Scheme	**Design goal†**	**Feedback from destination†**	**Outage performance‡**	**Energy consumption‡**
Fully decentralized (FD)	maximize output SINR increment at destination	no feedback	close to optimal centralized solution	rather high
Distributed feedback-assisted (DFA)	aim at target output SINR at destination	feedback of real-valued transmit powers	close to optimal centralized solution	low

where

$$\theta := \frac{\min_{j\in I\!I_{\rm p}^{(\rm rx)}}\{\xi_j\}}{\max_{j\in I\!I_{\rm p}^{(\rm rx)}}\{\rho_j\}}$$

and

$$\alpha_{{\rm R}_i,{\rm U},\max} := \max_{j\in I\!I_{\rm p}^{(\rm rx)}}\{\alpha_{{\rm R}_i,{\rm U}_j}\}.$$

Note that (7) and (8) coincide in the case of a single primary receiver or multiple congenerous primary receivers (i.e., $\xi_j =: \xi$ and $\rho_j =: \rho$ for all indices $j \in I\!I_{\rm p}^{(\rm rx)}$). Moreover, it is easy to prove that the transmit powers $P_{{\rm R}_i}$ according to (8) are always smaller than or equal to the transmit powers of the original FD-TPA solution (7). Therefore, it is still guaranteed that each primary receiver experiences a sum interference power of at most ξ_j. The destination node then measures the corresponding MRC output SINRs $\gamma_{{\rm R}_i\to{\rm D}} = P_{{\rm R}_i}\,\alpha_{{\rm R}_i,{\rm D}}/(\sigma^2_{\rm i,D}+\sigma^2_{\rm n,D})$ associated with the individual relays ${\rm R}_i$ ($i\in I\!I_{{\rm r},n}$). Having knowledge of the channel energies $\alpha_{{\rm R}_i,{\rm D}}$ and the variances $\sigma^2_{\rm i,D}$ and $\sigma^2_{\rm n,D}$, the destination node then determines the transmit power level $P_{{\rm R}_i}$ of each participating relay.

Now, based on (8) it is known that

$$P_{{\rm R}_i} \le \frac{\theta}{N'_{{\rm r},n}\,\alpha_{{\rm R}_i,{\rm U},\max}},$$

which holds with equality if and only if $P_{{\rm R}_i} < P_{{\rm R}_i,\max}$. Therefore, for each relay ${\rm R}_i$ the destination node can compute a worst-case estimate for the corresponding value $\alpha_{{\rm R}_i,{\rm U},\max}$ according to $\tilde{\alpha}_{{\rm R}_i,{\rm U},\max} = \theta/(N'_{{\rm r},n}P_{{\rm R}_i}) \ge \alpha_{{\rm R}_i,{\rm U},\max}$. Moreover, since it is known that $\alpha_{{\rm R}_i,{\rm U},\max} \ge \alpha_{{\rm R}_i,{\rm U}_j}$ for all $j\in I\!I_{\rm p}^{(\rm rx)}$, the destination node can employ the same worst-case estimate for all values $\alpha_{{\rm R}_i,{\rm U}_j}$ ($j\in I\!I_{\rm p}^{(\rm rx)}$), i.e.,

$$\tilde{\alpha}_{{\rm R}_i,{\rm U}_j} := \tilde{\alpha}_{{\rm R}_i,{\rm U},\max} = \frac{\theta}{N'_{{\rm r},n}\,P_{{\rm R}_i}} \ge \alpha_{{\rm R}_i,{\rm U}_j}. \tag{9}$$

In particular, if $P_{{\rm R}_i} < P_{{\rm R}_i,\max}$ and only a single primary receiver is present ($|I\!I_{\rm p}^{(\rm rx)}| = 1$), the destination node is always able to retrieve the true value of $\alpha_{{\rm R}_i,{\rm U}_1}$ (as $\alpha_{{\rm R}_i,{\rm U}_1} = \alpha_{{\rm R}_i,{\rm U},\max}$ and $\alpha_{{\rm R}_i,{\rm U},\max} = \theta/(N'_{{\rm r},n}P_{{\rm R}_i})$). The destination node has now obtained suitable estimates $\tilde{\alpha}_{{\rm R}_i,{\rm U}_j}$ for all indices $i\in I\!I_{{\rm r},n}$ and $j\in I\!I_{\rm p}^{(\rm rx)}$. The DFA-TPA solution can thus be determined based on (5) and (6), while replacing the parameters $\alpha_{{\rm R}_i,{\rm U}_j}$ with the corresponding estimates $\tilde{\alpha}_{{\rm R}_i,{\rm U}_j}$, and the resulting transmit power levels $P'_{{\rm R}_i}$ are fed back to the participating relay nodes. For simplicity, we assume that the transmit powers at the relays can be re-adjusted in an instantaneous fashion. In other words, the acquisition time required for conducting measurements at the destination node and sending the feedback information to the relays is assumed to be small compared to the time horizon under consideration. In practice, the TPAs within earlier relaying phases should first reach their steady state values, before the transmit powers for later relaying phases are adjusted.

The main characteristics of the FD-TPA and the DFA-TPA schemes are summarized in Table I.

IV. PERFORMANCE ANALYSIS

It is desirable to have analytical expressions that allow us to assess the performance of the proposed distributed TPA schemes and highlight their advantage over non-cooperative transmission (i.e., without relay assistance). However, evaluating the performance of the proposed TPA schemes while taking a wide variety of channel conditions into account requires averaging over a large number of random variables. Our objective is therefore to reduce the number of random variables involved to a minimum by performing most of the averaging steps analytically. We achieve this goal for the case of the FD-TPA scheme and a single relaying phase ($N_{\max}=1$) and derive an expression for the cumulative distribution function (CDF) of the overall MRC output SINR $\gamma_{\rm D}$ at the destination node, while treating the channel energies $\alpha_{\rm S,D}$, $\alpha_{{\rm S},{\rm R}_i}$, $\alpha_{{\rm R}_i,{\rm D}}$, $\alpha_{{\rm S},{\rm U}_j}$, $\alpha_{{\rm R}_i,{\rm U}_j}$ ($j\in I\!I_{\rm p}^{(\rm rx)}$), $\alpha_{{\rm U}_j,{\rm R}_i}$, and $\alpha_{{\rm U}_j,{\rm D}}$ ($j\in I\!I_{\rm p}^{(\rm tx)}$), cf. Fig. 1, as statistically independent random variables. As will be seen, the analysis turns out to be rather involved, which is mainly due to the considered transmission protocol, according to which relaying is performed only if the pre-defined target SINR value $\gamma_{\rm D,target}$ is not accomplished by the source–destination link alone. As a result, our final expression requires the numerical evaluation of integrals, since closed-form expressions do not seem feasible.

In the following, we consider a single active primary link ($|I\!I_{\rm p}^{(\rm tx)}| = |I\!I_{\rm p}^{(\rm rx)}| = 1$) for simplicity. For convenience, we drop the index $n=1$ for the relaying phase in the sequel. The index set associated with the active relays is denoted as $I\!I_{\rm r} := \{i_1, ..., i_{|I\!I_{\rm r}|}\} \subseteq \{1, ..., N_{\rm r}\}$, and the primary transmitter and receiver are denoted as ${\rm U}_{\rm tx}$ and ${\rm U}_{\rm rx}$, respectively. Finally, the bandwidth ratio associated with ${\rm U}_{\rm tx}$ and ${\rm U}_{\rm rx}$ is denoted as ρ and the maximum tolerated sum interference power as $\xi_{\rm rx}$. As an example, all transmission links ${\rm X}\to{\rm Y}$ are assumed to be subject to quasi-static Rayleigh fading, i.e., the channel coefficients $h^{(l)}_{\rm X,Y}$ ($l=0, ..., L_{\rm X,Y}$) are independent complex Gaussian

TABLE II
OVERVIEW OF SPECIAL NOTATIONS USED FOR THE ANALYTICAL RESULTS IN SECTION IV.

Notation	Definition
$w_{l,\mathrm{X,Y}}$	$w_{l,\mathrm{X,Y}} = \prod_{\substack{l'=0 \\ l'\neq l}}^{L_{\mathrm{X,Y}}} \frac{\sigma^2_{l,\mathrm{X,Y}}}{\sigma^2_{l,\mathrm{X,Y}} - \sigma^2_{l',\mathrm{X,Y}}}$
$\bar{\gamma}_{l,\mathrm{X}\to\mathrm{Y}}$	$\bar{\gamma}_{l,\mathrm{X}\to\mathrm{Y}} = P_\mathrm{X}\, \sigma^2_{l,\mathrm{X,Y}} \left(\sigma^2_{\mathrm{i,Y}} + \sigma^2_{\mathrm{n,Y}}\right)^{-1}$
$\mu_{\mathrm{S,R}_i}$	$\mu_{\mathrm{S,R}_i} = \alpha_{\mathrm{S,R}_i} \left(\sigma^2_{\mathrm{i,R}_i} + \sigma^2_{\mathrm{n,R}_i}\right)^{-1}$
$C_{1,\mathrm{Y}}$	$C_{1,\mathrm{Y}} = N_\mathrm{sp} (P_{\mathrm{U_{tx}}} \sigma^2_{0,\mathrm{U_{tx}},\mathrm{Y}})^{-1}$
C_{2,l,R_i}	$C_{2,l,\mathrm{R}_i} = \gamma_\mathrm{th} \left(P_\mathrm{S}\, \sigma^2_{l,\mathrm{S,R}_i}\right)^{-1}$
K_{l,R_i}	$K_{l,\mathrm{R}_i} = \mathrm{e}^{-C_{2,l,\mathrm{R}_i}\, \sigma^2_{\mathrm{n,R}_i}} \left(1 + C_{2,l,\mathrm{R}_i}/C_{1,\mathrm{R}_i}\right)^{-1}$
K_α	$K_\alpha = \left(\sum_{l=0}^{L_{\mathrm{S,D}}} w_{l,\mathrm{S,D}} \exp\left(-\gamma_{\mathrm{D,target}}/\bar{\gamma}_{l,\mathrm{S}\to\mathrm{D}}\right)\right)^{-1}$
$\varphi_{\mathrm{X,Y}}$	$\varphi_{\mathrm{X,Y}} = \sum_{l=0}^{L_{\mathrm{X,Y}}} w_{l,\mathrm{X,Y}}$
K_β	$K_\beta = K_\alpha \left(K_\alpha \varphi_{\mathrm{S,D}} - 1\right)^{-1}$
$\tilde{w}_{l,\mathrm{R}_i,\mathrm{D}}$	$\tilde{w}_{l,\mathrm{R}_i,\mathrm{D}} = \prod_{\substack{i'\in \mathbb{I}_\mathrm{r}^{(\kappa)} \\ (i',l')\neq(i,l)}} \prod_{l'=0}^{L_{\mathrm{R,D}}} \frac{\bar{\gamma}_{l,\mathrm{R}_i\to\mathrm{D}}}{\bar{\gamma}_{l,\mathrm{R}_i\to\mathrm{D}} - \bar{\gamma}_{l',\mathrm{R}_{i'}\to\mathrm{D}}}$
$v_{l,\lambda,i}$	$v_{l,\lambda,i} = \frac{1}{C_{3,l,\lambda,\mathrm{R}_i}} \frac{w_{l,\mathrm{S,D}}}{\bar{\gamma}_{l,\mathrm{S}\to\mathrm{D}}} \frac{\tilde{w}_{\lambda,\mathrm{R}_i,\mathrm{D}}}{\bar{\gamma}_{\lambda,\mathrm{R}_i\to\mathrm{D}}}$
$C_{3,l,\lambda,\mathrm{R}_i}$	$C_{3,l,\lambda,\mathrm{R}_i} = \bar{\gamma}^{-1}_{\lambda,\mathrm{R}_i\to\mathrm{D}} - \bar{\gamma}^{-1}_{l,\mathrm{S}\to\mathrm{D}}$
$C_{4,\mathrm{X}}$	$C_{4,\mathrm{X}} = \xi_\mathrm{rx} \left(\rho\, \sigma^2_{0,\mathrm{X,U_{rx}}}\right)^{-1}$

random variables with zero mean and variance $\sigma^2_{l,\mathrm{X,Y}}$. We normalize all channel energies $\alpha_{\mathrm{X,Y}}$ with respect to a certain reference link length d_ref. Correspondingly, the channel variances $\sigma^2_{l,\mathrm{X,Y}}$ are modeled as $\sigma^2_{l,\mathrm{X,Y}} := \tilde{\sigma}^2_{l,\mathrm{X,Y}}\,(d_\mathrm{ref}/d_{\mathrm{X,Y}})^p$, where $\sum_{l=0}^{L_{\mathrm{X,Y}}} \tilde{\sigma}^2_{l,\mathrm{X,Y}} = 1$.

With regard to the relaying process, we distinguish the following two cases:

(α) Event $\mathcal{E}^{(\alpha)}$: The source node is able to accomplish the desired target SINR $\gamma_{\mathrm{D,target}}$ on its own (cf. (2)), i.e., relaying is not required ($\gamma_{\mathrm{S}\to\mathrm{D}} \geq \gamma_{\mathrm{D,target}}$).

(β) Event $\mathcal{E}^{(\beta_\kappa)}$: The source node is not able to accomplish the target SINR $\gamma_{\mathrm{D,target}}$ on its own, and a relaying process with $N'_\mathrm{r} \leq N_\mathrm{r}$ active relays is initiated. The number of all possible index sets $\mathbb{I}_\mathrm{r} \subseteq \{1, ..., N_\mathrm{r}\}$ of active relays is given by $\psi := \sum_{i=0}^{N_\mathrm{r}} \binom{N_\mathrm{r}}{i}$. The κth index set is denoted as $\mathbb{I}_\mathrm{r}^{(\kappa)}$ ($\kappa \in \{0, ..., \psi-1\}$), where $\mathbb{I}_\mathrm{r}^{(0)} := \emptyset$. Finally, the cardinality of index set $\mathbb{I}_\mathrm{r}^{(\kappa)}$ is denoted as $|\mathbb{I}_\mathrm{r}^{(\kappa)}| =: M_\kappa$.

Let $\Pr\{\mathcal{E}^{(\alpha)} \,|\, P_\mathrm{S}, \sigma^2_{\mathrm{i,D}}\}$ and $\Pr\{\mathcal{E}^{(\beta_\kappa)} \,|\, P_\mathrm{S}\}$ denote the conditional probabilities associated with event $\mathcal{E}^{(\alpha)}$ and $\mathcal{E}^{(\beta_\kappa)}$, respectively, given a fixed source transmit power P_S and a fixed interference power $\sigma^2_{\mathrm{i,D}}$:

$$\Pr\{\mathcal{E}^{(\alpha)} \,|\, P_\mathrm{S}, \sigma^2_{\mathrm{i,D}}\} := \Pr\{\gamma_{\mathrm{S}\to\mathrm{D}} \geq \gamma_{\mathrm{D,target}} \,|\, P_\mathrm{S}, \sigma^2_{\mathrm{i,D}}\}, \quad (10)$$

$$\Pr\{\mathcal{E}^{(\beta_\kappa)} \,|\, P_\mathrm{S}\} := \prod_{i\in \mathbb{I}_\mathrm{r}^{(\kappa)}} \Pr\{\gamma_{\mathrm{S}\to\mathrm{R}_i} \geq \gamma_\mathrm{th} \,|\, P_\mathrm{S}\} \times \prod_{i\notin \mathbb{I}_\mathrm{r}^{(\kappa)}} \Pr\{\gamma_{\mathrm{S}\to\mathrm{R}_i} < \gamma_\mathrm{th} \,|\, P_\mathrm{S}\}. \quad (11)$$

Moreover, let $C^{(\alpha)}(\gamma_\mathrm{D} | P_\mathrm{S}, \sigma^2_{\mathrm{i,D}})$ and

$$C^{(\beta_\kappa)}(\gamma_\mathrm{D} | P_\mathrm{S}, \sigma^2_{\mathrm{i,D}}, P_{\mathrm{R}_{i_1}}, ..., P_{\mathrm{R}_{i_{M_\kappa}}})$$

denote the corresponding conditional CDFs of γ_D, given a fixed source transmit power P_S, a fixed interference power $\sigma^2_{\mathrm{i,D}}$, and fixed relay transmit powers $P_{\mathrm{R}_{i_1}}, ..., P_{\mathrm{R}_{i_{M_\kappa}}}$. Finally, let $p_1(P_\mathrm{S})$ and $p_2(\sigma^2_{\mathrm{i,D}})$ denote the probability density functions (PDFs) of P_S and $\sigma^2_{\mathrm{i,D}}$, respectively, and let $p_3(P_{\mathrm{R}_{i_1}}, ..., P_{\mathrm{R}_{i_{M_\kappa}}})$ denote the joint PDF of $P_{\mathrm{R}_{i_1}}, ..., P_{\mathrm{R}_{i_{M_\kappa}}}$. With these definitions, the average CDF of the overall MRC output SINR γ_D at the destination node can be written as:

$$\begin{aligned}
\bar{C}(\gamma_\mathrm{D}) = &\int_0^\infty \int_0^{P_{\mathrm{S,max}}} \Pr\{\mathcal{E}^{(\alpha)} \,|\, P_\mathrm{S}, \sigma^2_{\mathrm{i,D}}\} \\
&\times C^{(\alpha)}\left(\gamma_\mathrm{D} | P_\mathrm{S}, \sigma^2_{\mathrm{i,D}}\right) \cdot p_1(P_\mathrm{S})\, p_2(\sigma^2_{\mathrm{i,D}})\, \mathrm{d}P_\mathrm{S}\, \mathrm{d}\sigma^2_{\mathrm{i,D}} \\
&+ \int_0^\infty \int_0^{P_{\mathrm{S,max}}} \left(1 - \Pr\{\mathcal{E}^{(\alpha)} \,|\, P_\mathrm{S}, \sigma^2_{\mathrm{i,D}}\}\right) \\
&\times \Bigg(\sum_{\kappa=0}^{\psi-1} \int_0^{P_{\mathrm{R}_{i_{M_\kappa}},\mathrm{max}}} \cdots \int_0^{P_{\mathrm{R}_{i_1},\mathrm{max}}} \Pr\{\mathcal{E}^{(\beta_\kappa)} \,|\, P_\mathrm{S}\} \\
&\times C^{(\beta_\kappa)}\left(\gamma_\mathrm{D} | P_\mathrm{S}, \sigma^2_{\mathrm{i,D}}, P_{\mathrm{R}_{i_1}}, ..., P_{\mathrm{R}_{i_{M_\kappa}}}\right) \\
&\times p_3(P_{\mathrm{R}_{i_1}}, ..., P_{\mathrm{R}_{i_{M_\kappa}}})\, \mathrm{d}P_{\mathrm{R}_{i_1}} \cdots \mathrm{d}P_{\mathrm{R}_{i_{M_\kappa}}} \Bigg) \\
&\times p_1(P_\mathrm{S})\, p_2(\sigma^2_{\mathrm{i,D}})\, \mathrm{d}P_\mathrm{S}\, \mathrm{d}\sigma^2_{\mathrm{i,D}}. \quad (12)
\end{aligned}$$

In the following, we provide closed-form expressions for the conditional event probabilities (10) and (11), the conditional

CDFs $C^{(\alpha)}(\gamma_\mathrm{D}|\cdot)$ and $C^{(\beta_\kappa)}(\gamma_\mathrm{D}|\cdot)$, and the PDFs $p_i(\,.\,)$ $(i=1,2,3)$. Special notations introduced in the sequel are summarized in Table II.

A. Conditional Event Probabilities (10) and (11)

The conditional event probability (10) can be expressed as

$$\Pr\{\mathcal{E}^{(\alpha)}\,|\,P_\mathrm{S},\sigma^2_\mathrm{i,D}\} \;=\; 1-\Pr\left\{\alpha_\mathrm{S,D}<\frac{\gamma_\mathrm{D,target}(\sigma^2_\mathrm{i,D}+\sigma^2_\mathrm{n,D})}{P_\mathrm{S}}\;\middle|\;P_\mathrm{S},\sigma^2_\mathrm{i,D}\right\}. \tag{13}$$

According to the Rayleigh-fading assumption, the moment-generating function (MGF) of $\alpha_\mathrm{S,D}$ is given by $M_{\alpha_\mathrm{S,D}}(s)=\prod_{l=0}^{L_\mathrm{S,D}}(1-\sigma^2_{l,\mathrm{S,D}}\,s)^{-1}$. Invoking the residue calculus [29, Ch. 10], the PDF $p_{\alpha_\mathrm{S,D}}(\alpha_\mathrm{S,D})$ of $\alpha_\mathrm{S,D}$ can be determined as [19, Ch. 14.5]

$$p_{\alpha_\mathrm{S,D}}(\alpha_\mathrm{S,D})=\sum_{l=0}^{L_\mathrm{S,D}}\frac{w_{l,\mathrm{S,D}}}{\sigma^2_{l,\mathrm{S,D}}}\exp\left(-\frac{\alpha_\mathrm{S,D}}{\sigma^2_{l,\mathrm{S,D}}}\right), \tag{14}$$

where

$$w_{l,\mathrm{S,D}}:=\prod_{\substack{l'=0\\ l'\neq l}}^{L_\mathrm{S,D}}\frac{\sigma^2_{l,\mathrm{S,D}}}{\sigma^2_{l,\mathrm{S,D}}-\sigma^2_{l',\mathrm{S,D}}}.$$

Integration of (14) and evaluation of the resulting CDF at $\alpha_\mathrm{S,D}=\gamma_\mathrm{D,target}(\sigma^2_\mathrm{i,D}+\sigma^2_\mathrm{n,D})/P_\mathrm{S}$ yields

$$\Pr\{\mathcal{E}^{(\alpha)}\,|\,P_\mathrm{S},\sigma^2_\mathrm{i,D}\} \;=\; 1-\sum_{l=0}^{L_\mathrm{S,D}}w_{l,\mathrm{S,D}}\left(1-\exp\left(-\frac{\gamma_\mathrm{D,target}}{\bar\gamma_{l,\mathrm{S\to D}}}\right)\right), \tag{15}$$

where

$$\bar\gamma_{l,\mathrm{S\to D}}:=\frac{P_\mathrm{S}\,\sigma^2_{l,\mathrm{S,D}}}{\sigma^2_\mathrm{i,D}+\sigma^2_\mathrm{n,D}}.$$

The conditional event probability (11) can be expressed as

$$\Pr\{\mathcal{E}^{(\beta_\kappa)}\,|\,P_\mathrm{S}\} \;=\; \prod_{i\in\mathbb{I}_\mathrm{r}^{(\kappa)}}\left(1-\Pr\left\{\mu_{\mathrm{S,R}_i}<\frac{\gamma_\mathrm{th}}{P_\mathrm{S}}\;\middle|\;P_\mathrm{S}\right\}\right)\times\prod_{i\notin\mathbb{I}_\mathrm{r}^{(\kappa)}}\Pr\left\{\mu_{\mathrm{S,R}_i}<\frac{\gamma_\mathrm{th}}{P_\mathrm{S}}\;\middle|\;P_\mathrm{S}\right\}, \tag{16}$$

where $\mu_{\mathrm{S,R}_i}:=\alpha_{\mathrm{S,R}_i}/(\sigma^2_{\mathrm{i,R}_i}+\sigma^2_{\mathrm{n,R}_i})$. The PDF of $\alpha_{\mathrm{S,R}_i}$ is of the same form as (14). Moreover, we have

$$\sigma^2_{\mathrm{i,R}_i}=P_{\mathrm{U_{tx}}}\,\alpha_{\mathrm{U_{tx},R}_i}/N_\mathrm{sp}$$

with

$$p_{\alpha_{\mathrm{U_{tx},R}_i}}(\alpha_{\mathrm{U_{tx},R}_i})=1/\sigma^2_{0,\mathrm{U_{tx},R}_i}\exp(-\alpha_{\mathrm{U_{tx},R}_i}/\sigma^2_{0,\mathrm{U_{tx},R}_i}).$$

Based on this, the PDF of $\mu_{\mathrm{S,R}_i}$ is given by

$$p_{\mu_{\mathrm{S,R}_i}}(\mu_{\mathrm{S,R}_i}) \;=\; C_{1,\mathrm{R}_i}\sum_{l=0}^{L_{\mathrm{S,R}_i}}\frac{w_{l,\mathrm{S,R}_i}}{\sigma^2_{l,\mathrm{S,R}_i}}\,\frac{(C_{1,\mathrm{R}_i}+\mu_{\mathrm{S,R}_i}/\sigma^2_{l,\mathrm{S,R}_i})\,\sigma^2_{\mathrm{n,R}_i}+1}{(C_{1,\mathrm{R}_i}+\mu_{\mathrm{S,R}_i}/\sigma^2_{l,\mathrm{S,R}_i})^2}\times\exp\left(-\frac{\sigma^2_{\mathrm{n,R}_i}\,\mu_{\mathrm{S,R}_i}}{\sigma^2_{l,\mathrm{S,R}_i}}\right), \tag{17}$$

where $w_{l,\mathrm{S,R}_i}$ is of form (14) and

$$C_{1,\mathrm{R}_i}:=N_\mathrm{sp}/(P_\mathrm{U_{tx}}\,\sigma^2_{0,\mathrm{U_{tx},R}_i}).$$

Integration of (17) and evaluation of the resulting CDF at $\mu_{\mathrm{S,R}_i}=\gamma_\mathrm{th}/P_\mathrm{S}$ yields

$$\Pr\left\{\mu_{\mathrm{S,R}_i}<\frac{\gamma_\mathrm{th}}{P_\mathrm{S}}\;\middle|\;P_\mathrm{S}\right\}=\sum_{l=0}^{L_{\mathrm{S,R}_i}}w_{l,\mathrm{S,R}_i}\cdot\Big(1-K_{l,\mathrm{R}_i}\Big), \tag{18}$$

where

$$K_{l,\mathrm{R}_i}:=\mathrm{e}^{-C_{2,l,\mathrm{R}_i}\,\sigma^2_{\mathrm{n,R}_i}}/(1+C_{2,l,\mathrm{R}_i}/C_{1,\mathrm{R}_i})$$

and $C_{2,l,\mathrm{R}_i}:=\gamma_\mathrm{th}/(P_\mathrm{S}\,\sigma^2_{l,\mathrm{S,R}_i})$. Combining (18) with (16) yields the desired closed-form expression for $\Pr\{\mathcal{E}^{(\beta_\kappa)}\,|\,P_\mathrm{S}\}$.

B. Conditional CDFs $C^{(\alpha)}(\gamma_\mathrm{D}|\cdot)$ and $C^{(\beta_\kappa)}(\gamma_\mathrm{D}|\cdot)$

Consider first the case of non-cooperative transmission, i.e., no relays are available. In this case, the PDF of $\gamma_\mathrm{D}=\gamma_\mathrm{S\to D}$, given a fixed source transmit power P_S and a fixed interference power $\sigma^2_\mathrm{i,D}$ at the destination node, is of the same form as (14). The corresponding CDF can be calculated as

$$C_\mathrm{nc}(\gamma_\mathrm{D}|P_\mathrm{S},\sigma^2_\mathrm{i,D})=\sum_{l=0}^{L_\mathrm{S,D}}w_{l,\mathrm{S,D}}\left(1-\exp\left(-\frac{\gamma_\mathrm{D}}{\bar\gamma_{l,\mathrm{S\to D}}}\right)\right). \tag{19}$$

Next, consider the relaying case (α), where the source node is able to accomplish the desired target SINR $\gamma_\mathrm{D,target}$ on its own. By definition we have $\gamma_\mathrm{D}=\gamma_\mathrm{S\to D}\geq\gamma_\mathrm{D,target}$. Based on (19), we therefore obtain the following expression for the conditional CDF $C^{(\alpha)}(\gamma_\mathrm{D}|P_\mathrm{S},\sigma^2_\mathrm{i,D})$:

$$C^{(\alpha)}(\gamma_\mathrm{D}|P_\mathrm{S},\sigma^2_\mathrm{i,D}) \;=\; \begin{cases}0 & \text{for } \gamma_\mathrm{D}<\gamma_\mathrm{D,target}\\ 1+K_\alpha\cdot\Big(C_\mathrm{nc}(\gamma_\mathrm{D}|P_\mathrm{S},\sigma^2_\mathrm{i,D})-\varphi_\mathrm{S,D}\Big) & \text{for } \gamma_\mathrm{D}\geq\gamma_\mathrm{D,target}\end{cases}, \tag{20}$$

where $K_\alpha:=\left(\sum_{l=0}^{L_\mathrm{S,D}}w_{l,\mathrm{S,D}}\exp\left(-\frac{\gamma_\mathrm{D,target}}{\bar\gamma_{l,\mathrm{S\to D}}}\right)\right)^{-1}$ and $\varphi_\mathrm{S,D}:=\sum_{l=0}^{L_\mathrm{S,D}}w_{l,\mathrm{S,D}}$. Note that for $\gamma_\mathrm{D}<\gamma_\mathrm{D,target}$ the first integral in (12) becomes zero, as $C^{(\alpha)}(\gamma_\mathrm{D}|P_\mathrm{S},\sigma^2_\mathrm{i,D})$ is zero for $\gamma_\mathrm{D}<\gamma_\mathrm{D,target}$.

Next, we consider relaying case (β), where the source node is not able to accomplish the desired target SINR $\gamma_\mathrm{D,target}$ on its own. For the special case (β_0), where no relay is able to decode the message from the source node correctly ($\mathbb{I}_\mathrm{r}=\mathbb{I}_\mathrm{r}^{(0)}=\emptyset$), we have $\gamma_\mathrm{D}=\gamma_\mathrm{S\to D}<\gamma_\mathrm{D,target}$. We thus obtain the following expression for the conditional CDF $C^{(\beta_0)}(\gamma_\mathrm{D}|P_\mathrm{S},\sigma^2_\mathrm{i,D})$:

$$C^{(\beta_0)}(\gamma_\mathrm{D}|P_\mathrm{S},\sigma^2_\mathrm{i,D}) \;=\; \begin{cases}K_\beta\,C_\mathrm{nc}(\gamma_\mathrm{D}|P_\mathrm{S},\sigma^2_\mathrm{i,D}) & \text{for } \gamma_\mathrm{D}<\gamma_\mathrm{D,target}\\ 1 & \text{for } \gamma_\mathrm{D}\geq\gamma_\mathrm{D,target}\end{cases}, \tag{21}$$

where $K_\beta:=K_\alpha/(K_\alpha\varphi_\mathrm{S,D}-1)$. If a certain non-empty subset $\mathbb{I}_\mathrm{r}^{(\kappa)}\subseteq\{1,\ldots,N_\mathrm{r}\}$ of relays is able to decode the message from the source node correctly, γ_D is given by $\gamma_\mathrm{D}=\gamma_\mathrm{S\to D}+\gamma_\mathrm{R\to D}$, while $\gamma_\mathrm{S\to D}<\gamma_\mathrm{D,target}$. Therefore, for a fixed source transmit power P_S, a fixed interference power $\sigma^2_\mathrm{i,D}$ at the destination node, and fixed relay transmit powers P_{R_i} ($i \in \mathbb{I}_{\mathrm{r}}^{(\kappa)}$), the PDF of γ_{D} can be calculated by convolving the constrained PDF of $\gamma_{\mathrm{S}\to\mathrm{D}}$ with the PDF of $\gamma_{\mathrm{R}\to\mathrm{D}}$. The latter can be expressed as

$$p_{\gamma_{\mathrm{R}\to\mathrm{D}}}(\gamma_{\mathrm{R}\to\mathrm{D}}) = \sum_{i\in\mathbb{I}_{\mathrm{r}}^{(\kappa)}} \sum_{l=0}^{L_{\mathrm{R,D}}} \frac{\tilde{w}_{l,\mathrm{R}_i,\mathrm{D}}}{\bar{\gamma}_{l,\mathrm{R}_i\to\mathrm{D}}} \exp\left(-\frac{\gamma_{\mathrm{R}\to\mathrm{D}}}{\bar{\gamma}_{l,\mathrm{R}_i\to\mathrm{D}}}\right), \tag{22}$$

where

$$\tilde{w}_{l,\mathrm{R}_i,\mathrm{D}} = \prod_{\substack{i'\in\mathbb{I}_{\mathrm{r}}^{(\kappa)} \\ (i',l')\neq(i,l)}} \prod_{l'=0}^{L_{\mathrm{R,D}}} \frac{\bar{\gamma}_{l,\mathrm{R}_i\to\mathrm{D}}}{\bar{\gamma}_{l,\mathrm{R}_i\to\mathrm{D}} - \bar{\gamma}_{l',\mathrm{R}_{i'}\to\mathrm{D}}},$$

and $\bar{\gamma}_{l,\mathrm{R}_i\to\mathrm{D}} := P_{\mathrm{R}_i}\,\sigma^2_{l,\mathrm{R}_i,\mathrm{D}}/(\sigma^2_{\mathrm{i,D}}+\sigma^2_{\mathrm{n,D}})$. For simplicity, we have assumed in (22) that $L_{\mathrm{R}_i,\mathrm{D}} =: L_{\mathrm{R,D}}$ for all $i\in\mathbb{I}_{\mathrm{r}}^{(\kappa)}$. Integration of the resulting PDF finally yields the conditional CDF

$$C^{(\beta_\kappa)}(\gamma_{\mathrm{D}}|P_{\mathrm{S}},\sigma^2_{\mathrm{i,D}},P_{\mathrm{R}_{i_1}},\ldots,P_{\mathrm{R}_{i_{M_\kappa}}}) = \tag{23}$$
$$\begin{cases} K_\beta \sum\limits_{i\in\mathbb{I}_{\mathrm{r}}^{(\kappa)}} \sum\limits_{l=0}^{L_{\mathrm{S,D}}} \sum\limits_{\lambda=0}^{L_{\mathrm{R,D}}} v_{l,\lambda,i} \Bigg[\bar{\gamma}_{\lambda,\mathrm{R}_i\to\mathrm{D}}\left(\exp\left(-\frac{\gamma_{\mathrm{D}}}{\bar{\gamma}_{\lambda,\mathrm{R}_i\to\mathrm{D}}}\right)-1\right) \\ \qquad - \bar{\gamma}_{l,\mathrm{S}\to\mathrm{D}}\left(\exp\left(-\frac{\gamma_{\mathrm{D}}}{\bar{\gamma}_{l,\mathrm{S}\to\mathrm{D}}}\right)-1\right)\Bigg] \\ \qquad\qquad\qquad \text{for} \quad \gamma_{\mathrm{D}} < \gamma_{\mathrm{D,target}} \\ K_\beta \sum\limits_{i\in\mathbb{I}_{\mathrm{r}}^{(\kappa)}} \sum\limits_{l=0}^{L_{\mathrm{S,D}}} \sum\limits_{\lambda=0}^{L_{\mathrm{R,D}}} v_{l,\lambda,i} \\ \quad \times\Bigg[\left(\bar{\gamma}_{\lambda,\mathrm{R}_i\to\mathrm{D}} - \bar{\gamma}_{l,\mathrm{S}\to\mathrm{D}}\right)\left(\exp\left(-\frac{\gamma_{\mathrm{D,target}}}{\bar{\gamma}_{l,\mathrm{S}\to\mathrm{D}}}\right)-1\right) \\ \qquad - \bar{\gamma}_{\lambda,\mathrm{R}_i\to\mathrm{D}}\left(\exp\left(C_{3,l,\lambda,\mathrm{R}_i}\,\gamma_{\mathrm{D,target}}\right)-1\right) \\ \qquad\qquad \times \exp\left(-\frac{\gamma_{\mathrm{D}}}{\bar{\gamma}_{\lambda,\mathrm{R}_i\to\mathrm{D}}}\right)\Bigg] \\ \qquad\qquad\qquad \text{for} \quad \gamma_{\mathrm{D}} \geq \gamma_{\mathrm{D,target}}, \end{cases}$$

where $v_{l,\lambda,i} := w_{l,\mathrm{S,D}}\,\tilde{w}_{\lambda,\mathrm{R}_i,\mathrm{D}}/(C_{3,l,\lambda,\mathrm{R}_i}\,\bar{\gamma}_{l,\mathrm{S}\to\mathrm{D}}\,\bar{\gamma}_{\lambda,\mathrm{R}_i\to\mathrm{D}})$ and $C_{3,l,\lambda,\mathrm{R}_i} := 1/\bar{\gamma}_{\lambda,\mathrm{R}_i\to\mathrm{D}} - 1/\bar{\gamma}_{l,\mathrm{S}\to\mathrm{D}}$.

C. PDFs $p_1(\,.\,)$, $p_2(\,.\,)$, $p_3(\,.\,)$

The PDF $p_1(P_{\mathrm{S}})$ of the source transmit power P_{S} according to (1) can be evaluated as

$$p_1(P_{\mathrm{S}}) = \begin{cases} \frac{C_{4,\mathrm{S}}}{P_{\mathrm{S}}^2}\exp\left(-\frac{C_{4,\mathrm{S}}}{P_{\mathrm{S}}}\right) \\ \quad + \left(1-\exp\left(-\frac{C_{4,\mathrm{S}}}{P_{\mathrm{S,max}}}\right)\right)\delta_0\left(P_{\mathrm{S}}-P_{\mathrm{S,max}}\right) \\ \qquad\qquad\qquad \text{for} \quad P_{\mathrm{S}} \leq P_{\mathrm{S,max}} \\ \quad 0 \qquad\qquad\quad \text{for} \quad P_{\mathrm{S}} > P_{\mathrm{S,max}}, \end{cases} \tag{24}$$

where $C_{4,\mathrm{S}} = \xi_{\mathrm{rx}}/(\rho\,\sigma^2_{0,\mathrm{S,U_{rx}}})$ and $\delta_0(P_{\mathrm{S}}-P_{\mathrm{S,max}})$ denotes a Dirac impulse at $P_{\mathrm{S}}=P_{\mathrm{S,max}}$. In order to arrive at (24), we have used that the PDF of $\alpha_{\mathrm{S,U_{rx}}}$ is given by

$$p_{\alpha_{\mathrm{S,U_{rx}}}}(\alpha_{\mathrm{S,U_{rx}}}) = 1/\sigma^2_{0,\mathrm{S,U_{rx}}}\exp(-\alpha_{\mathrm{S,U_{rx}}}/\sigma^2_{0,\mathrm{S,U_{rx}}}).$$

If the interference constraint vanishes (i.e., $\rho\,\sigma^2_{0,\mathrm{S,U_{rx}}}\to 0$), we obtain $p_1(P_{\mathrm{S}}) = \delta_0(P_{\mathrm{S}}-P_{\mathrm{S,max}})$, as expected.

The PDF $p_2(\sigma^2_{\mathrm{i,D}})$ of the interference power

$$\sigma^2_{\mathrm{i,D}} = P_{\mathrm{U_{tx}}}\,\alpha_{\mathrm{U_{tx},D}}/N_{\mathrm{sp}}$$

at the destination node is given by

$$p_2(\sigma^2_{\mathrm{i,D}}) = C_{1,\mathrm{D}}\,\mathrm{e}^{-C_{1,\mathrm{D}}\,\sigma^2_{\mathrm{i,D}}}, \tag{25}$$

where $C_{1,\mathrm{D}} := N_{\mathrm{sp}}/(P_{\mathrm{U_{tx}}}\sigma^2_{0,\mathrm{U_{tx},D}})$. Here, we have used that

$$p_{\alpha_{\mathrm{U_{tx},D}}}(\alpha_{\mathrm{U_{tx},D}}) = 1/\sigma^2_{0,\mathrm{U_{tx},D}}\exp(-\alpha_{\mathrm{U_{tx},D}}/\sigma^2_{0,\mathrm{U_{tx},D}}).$$

For the FD-TPA scheme, the individual relay transmit powers $P_{\mathrm{R}_i} := \min\{P_{\mathrm{R}_i,\max}, \xi_{\mathrm{rx}}/(\rho\,N'_{\mathrm{r}}\,\alpha_{\mathrm{R}_i,\mathrm{U_{rx}}})\}$ are statistically independent. Correspondingly, the joint PDF of the relay transmit powers is given by $p_3(P_{\mathrm{R}_{i_1}},\ldots,P_{\mathrm{R}_{i_{M_\kappa}}}) = \prod_{i\in\mathbb{I}_{\mathrm{r}}^{(\kappa)}} p_{3,i}(P_{\mathrm{R}_i})$, where $M_\kappa = |\mathbb{I}_{\mathrm{r}}^{(\kappa)}| =: N'_{\mathrm{r}}$. Similar to (24), one obtains

$$p_{3,i}(P_{\mathrm{R}_i}) = \begin{cases} \frac{C_{4,\mathrm{R}_i}}{N'_{\mathrm{r}}\,P^2_{\mathrm{R}_i}}\exp\left(-\frac{C_{4,\mathrm{R}_i}}{N'_{\mathrm{r}}\,P_{\mathrm{R}_i}}\right) \\ \quad + \left(1-\exp\left(-\frac{C_{4,\mathrm{R}_i}}{N'_{\mathrm{r}}\,P_{\mathrm{R}_i,\max}}\right)\right) \\ \qquad\qquad \times\,\delta_0\left(P_{\mathrm{R}_i}-P_{\mathrm{R}_i,\max}\right) \\ \qquad\qquad\qquad \text{for } P_{\mathrm{R}_i} \leq P_{\mathrm{R}_i,\max} \\ \quad 0 \qquad\qquad\quad \text{for } P_{\mathrm{R}_i} > P_{\mathrm{R}_i,\max}, \end{cases} \tag{26}$$

where $C_{4,\mathrm{R}_i} := \xi_{\mathrm{rx}}/(\rho\,\sigma^2_{0,\mathrm{R}_i,\mathrm{U_{rx}}})$. Again, we have $p_{3,i}(P_{\mathrm{R}_i}) = \delta_0\left(P_{\mathrm{R}_i}-P_{\mathrm{R}_i,\max}\right)$ for $\rho\,\sigma^2_{0,\mathrm{R}_i,\mathrm{U_{rx}}}\to 0$.

D. Discussion

Based on the above expressions for the conditional event probabilities $\Pr\{\mathcal{E}^{(\alpha)}\,|\,\cdot\}$ and $\Pr\{\mathcal{E}^{(\beta_\kappa)}\,|\,\cdot\}$, the corresponding conditional CDFs $C^{(\alpha)}(\gamma_{\mathrm{D}}|\,\cdot)$ and $C^{(\beta_\kappa)}(\gamma_{\mathrm{D}}|\,\cdot)$, and the PDFs $p_1(P_{\mathrm{S}})$, $p_2(\sigma^2_{\mathrm{i,D}})$, and $p_{3,i}(P_{\mathrm{R}_i})$, a closed-form evaluation of the average CDF $\bar{C}(\gamma_{\mathrm{D}})$ of the overall MRC output SINR γ_{D} at the destination node, cf. (12), appears to be difficult. In the following section, we will therefore apply Monte-Carlo integration, in order to evaluate $\bar{C}(\gamma_{\mathrm{D}})$ numerically. We note that (12) is only valid if the random variables P_{S} and P_{R_i} ($i\in\mathbb{I}_{\mathrm{r}}^{(\kappa)}$) are statistically independent from the random variables $\alpha_{\mathrm{S,D}}$, $\alpha_{\mathrm{S,R}_i}$, $\alpha_{\mathrm{R}_i,\mathrm{D}}$, and $\alpha_{\mathrm{U_{tx},R}_i}$. This is the case for the FD-TPA scheme, but not for the DFA-TPA scheme and the OC solution. A corresponding extension of the above analysis therefore appears difficult. Moreover, for the DFA-TPA scheme and the OC solution there are no closed-form expressions for the relay transmit powers P_{R_i} as a function of the various system parameters (cf. Section II-C).

V. Numerical Performance Results

In the following, we evaluate the performance of the proposed TPA schemes and highlight their advantage over non-cooperative transmission (i.e., without relay assistance). We start with semi-analytical performance results for the FD-TPA scheme, given a single relaying phase ($N_{\max}=1$) and a single active primary link (Section V-A). We also include simulation-based performance results so as to corroborate our analysis in Section IV. Afterwards, we will present simulation-based performance results for the DFA-TPA scheme, as well as for the case of more than two hops and multiple primary transmitters and receivers (Section V-B and Section V-C).

As in the analysis in Section IV, quasi-static Rayleigh fading is assumed. All link lengths are normalized with respect to the distance between source and destination, i.e., $d_{\text{ref}} := d_{\text{S,D}}$. The average signal-to-noise ratio (SNR) of the source–destination link, denoted as $\bar{\gamma}_{0,\text{S}\to\text{D}}$, serves as a reference in the sequel. Throughout this section, transmit powers are normalized with respect to the average received power of the source–destination link. The locations of source and destination node are set to $(-0.5, 0)$ and $(+0.5, 0)$, respectively. The links within the CR network are assumed to have a channel memory length of $L_{\text{X,Y}} = 9$ and an exponentially decaying power delay profile, according to $\tilde{\sigma}^2_{l,\text{X,Y}} / \tilde{\sigma}^2_{0,\text{X,Y}} = \exp(-l/c_h)$, where we choose $c_h = 2$. Moreover, we choose a path-loss exponent of $p = 3$. For simplicity, all nodes within the CR network are assumed to have identical physical properties. To this end, we set $\sigma^2_{\text{n,D}} = \sigma^2_{\text{n,R}_i} =: \sigma^2_{\text{n}}$ for all indices $i \in \{1, ..., N_{\text{r}}\}$ and choose identical maximum transmit powers $P_{\text{S,max}} = P_{\text{R}_i,\text{max}} = 1$ for the source node and the relays. Finally, we set $\gamma_{\text{th}} = 10$ dB and $\gamma_{\text{D,target}} = 10$ dB. All simulation results presented in the following have been averaged over 10^5 to 10^6 statistically independent channel realizations.

A. Performance Results for a Two-hop FD-TPA Scheme and a Single Primary Link

As an example, we assume that $N_{\text{r}} = 5$ relays are available with positions $(0, 0)$, $(0, \pm 0.2)$, and $(0, \pm 0.4)$. As discussed earlier, we assume that the average transmit power $P_{\text{U}_{\text{tx}}}$ employed by the primary transmitter is much larger than the maximum transmit powers within the CR network. As an example we set $P_{\text{U}_{\text{tx}}} = 10,000$. Consequently, the primary transmitter needs to be located at some distance from the CR network, in order to allow for secondary spectrum usage. As an example, we consider different positions $(0, d{+}10)$ and $(0, d)$ of the primary transmitter and receiver, respectively, while we vary d between $1 \leq d \leq 20$. The maximum sum interference power tolerated by the primary receiver is expected to be rather small. As an example, we set $\xi_{\text{rx}} = 0.01$. Finally, the bandwidth ratio between the primary link and the CR network is set to $\rho = 0.1$.

Fig. 3 shows the average complementary CDF (CCDF) of the overall MRC output SINR γ_{D}, $1 - \bar{C}(\gamma_{\text{D}})$, resulting for non-cooperative transmission and the FD-TPA scheme, respectively, for four different cases specified in the figure caption. As can be seen, the (semi-)analytical results (dashed/solid lines) based on (12) are in good accordance with the simulation results (markers). Moreover, it can be seen that for all considered cases the FD-TPA scheme substantially outperforms non-cooperative transmission, as the associated average CCDFs are located significantly further to the right. The performance of both non-cooperative transmission and the FD-TPA scheme improves, if (i) the distance d between the primary system and the CR network is increased, (ii) the spreading length N_{sp} is increased, or (iii) the reference SNR $\bar{\gamma}_{0,\text{S}\to\text{D}}$ is increased. However, even for $\bar{\gamma}_{0,\text{S}\to\text{D}} = 10$ dB and $d = 12$ (Case 4), the probability that the target SINR $\gamma_{\text{D,target}} = 10$ dB is accomplished by means of non-cooperative transmission is only about 0.14, whereas the corresponding probability for the FD-TPA scheme is close to one.

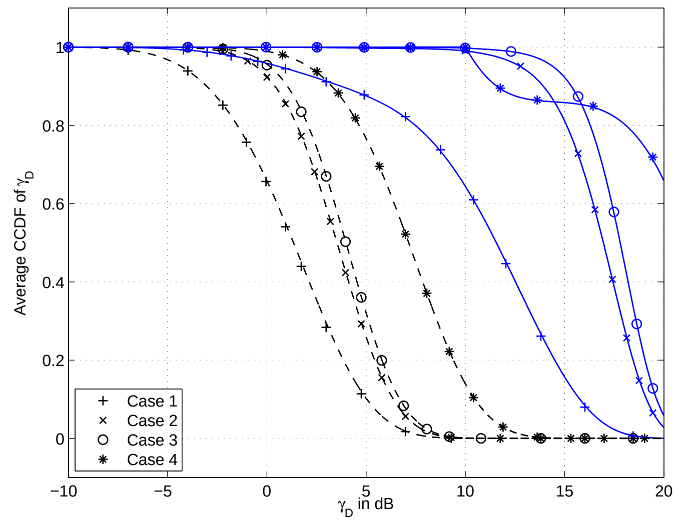

Fig. 3. Average CCDF $1 - \bar{C}(\gamma_{\text{D}})$ of the overall MRC output SINR γ_{D} at the destination node for different cases ($N_{\text{r}} = 5$ relays, one relaying phase, $N_{\text{p}} = 1$ primary link; primary transmitter and receiver located at $(0, d{+}10)$ and $(0, d)$, respectively). Case 1: $\bar{\gamma}_{0,\text{S}\to\text{D}} = 5$ dB, $d = 3$, $N_{\text{sp}} = 10$; Case 2: $\bar{\gamma}_{0,\text{S}\to\text{D}} = 5$ dB, $d = 12$, $N_{\text{sp}} = 10$; Case 3: $\bar{\gamma}_{0,\text{S}\to\text{D}} = 5$ dB, $d = 12$, $N_{\text{sp}} = 20$; Case 4: $\bar{\gamma}_{0,\text{S}\to\text{D}} = 10$ dB, $d = 12$, $N_{\text{sp}} = 10$. Lines represent (semi-)analytical results obtained by means of Monte-Carlo integration of (12), whereas markers represent simulation results. Dashed lines: non-cooperative transmission; solid lines: FD-TPA scheme.

In the case of the FD-TPA scheme, plateaus within $1 - \bar{C}(\gamma_{\text{D}})$ are due to the relaying process, since in those cases where the source node is not able to accomplish the target SINR $\gamma_{\text{D,target}}$ on its own, the participating relays maximize γ_{D} according to a best-effort strategy. As a consequence, the FD-TPA scheme exhibits a better CCDF curve for $\bar{\gamma}_{0,\text{S}\to\text{D}} = 5$ dB (Case 2) than for $\bar{\gamma}_{0,\text{S}\to\text{D}} = 10$ dB (Case 4), as long as $\gamma_{\text{D}} \leq 14$ dB.

The advantage of the FD-TPA scheme over non-cooperative transmission is even more apparent in terms of the average outage probability $\mathsf{E}\{\Pr\{\gamma_{\text{D}} < \gamma_{\text{D,target}}\}\} = \bar{C}(\gamma_{\text{D,target}})$ ($\mathsf{E}\{\cdot\}$ denotes statistical expectation). Fig. 4 shows $\bar{C}(\gamma_{\text{D,target}})$ as a function of the distance d between the primary system and the CR network (for $\bar{\gamma}_{0,\text{S}\to\text{D}} = 5$ dB, 10 dB and $N_{\text{sp}} = 10,\ 20$). As can be seen, the FD-TPA scheme substantially outperforms non-cooperative transmission, especially for $\bar{\gamma}_{0,\text{S}\to\text{D}} = 10$ dB. In particular, for the FD-TPA scheme $\bar{C}(\gamma_{\text{D,target}})$ decreases significantly with growing distance d, whereas the average outage probability for non-cooperative transmission remains relatively close to one for the considered range of d. As earlier, the (semi-)analytical results based on (12) are in good accordance with the simulation results. We have also included simulation results for the optimum centralized (OC) solution, which reveal that for the considered example the performance of the FD-TPA scheme is, in fact, very close to the optimum (for the entire range of d).

B. Performance Results for the DFA-TPA Scheme and more than Two Hops

For the time being, we again focus on the two-hop case and a single active primary link with $\xi_{\text{rx}} = 0.01$ and $\rho = 0.1$. As earlier, $N_{\text{r}} = 5$ relays with positions $(0, 0)$, $(0, \pm 0.2)$, and

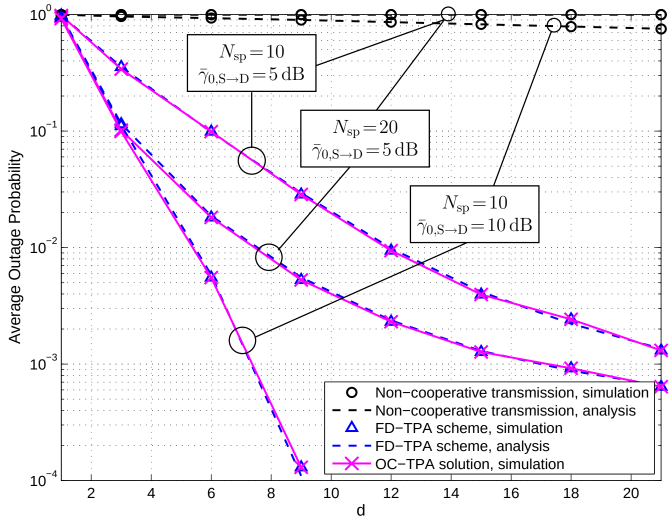

Fig. 4. Average outage probability $\bar{C}(\gamma_{\rm D,target})$ for non-cooperative transmission, the FD-TPA scheme, and the OC-TPA solution as a function of the distance between the primary system and the CR network (different SNR values $\bar{\gamma}_{0,{\rm S}\to{\rm D}}$ and spreading lengths $N_{\rm sp}$; $N_{\rm r}=5$ relays, one relaying phase, $N_{\rm P}=1$ primary link; primary transmitter and receiver located at $(0, d+10)$ and $(0, d)$, respectively). For non-cooperative transmission and the FD-TPA scheme, dashed lines represent (semi-)analytical results obtained by means of Monte-Carlo integration, whereas markers represent simulation results. The curves for the OC-TPA solution were obtained by means of simulations.

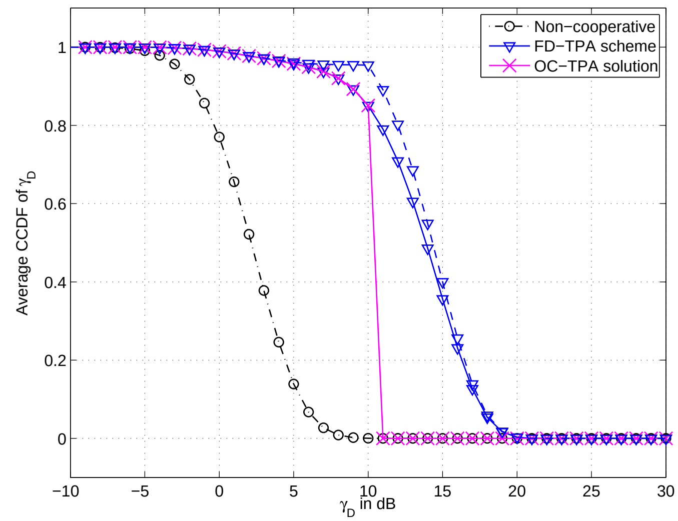

Fig. 5. Average CCDF $1-\bar{C}(\gamma_{\rm D})$ of the overall MRC output SINR $\gamma_{\rm D}$ at the destination node in the case of non-cooperative transmission, the FD-TPA scheme, and the OC-TPA solution ($N_{\rm r}=5$ relays, $N_{\rm P}=1$ primary link, $\bar{\gamma}_{0,{\rm S}\to{\rm D}}=5$ dB, $N_{\rm sp}=10$; primary transmitter and receiver located at $(0, 15)$ and $(0, 5)$, respectively). Solid lines: one relaying phase; dashed line: two relaying phases.

$(0, \pm 0.4)$ are assumed. Fig. 5 depicts simulation results for the average CCDF $1-\bar{C}(\gamma_{\rm D})$ in the case of non-cooperative transmission, the FD-TPA scheme, and the OC-TPA solution, respectively ($\bar{\gamma}_{0,{\rm S}\to{\rm D}}=5$ dB, $N_{\rm sp}=10$). The positions of the primary transmitter and receiver were set to $(0, 15)$ and $(0, 5)$, respectively. We note that the average CCDF of the OC-TPA solution exhibits a pronounced cliff around the target SINR $\gamma_{\rm D,target}$. This illustrates that the OC-TPA solution aims to approach $\gamma_{\rm D,target}$, whereas the FD-TPA scheme tends to exceed it. The average CCDF of the DFA-TPA scheme (not depicted) is virtually identical to that of the OC-TPA solution, despite the incomplete knowledge of the channel energies $\alpha_{{\rm R}_i,{\rm U}_{\rm rx}}$ at the destination node. For the FD-TPA scheme, we have also included the average CCDF resulting for $N_{\max}=2$ relaying phases (dashed line), which illustrates that multihop relaying indeed improves the outage performance in this case.

Fig. 6 shows the average outage probability $\bar{C}(\gamma_{\rm D,target})$ for non-cooperative transmission, the FD-TPA scheme, the DFA-TPA solution, and the OC-TPA solution as a function of the distance d between the primary system and the CR network ($\bar{\gamma}_{0,{\rm S}\to{\rm D}}=5$ dB, $N_{\rm sp}=10$). Here, we have assumed that altogether $N_{\rm r}=15$ relays are available with positions $(\pm\frac{m}{4}, \pm\frac{n}{5})$, where $m=0,1$ and $n=0,1,2$. Consider first the case labelled as 'Case I'. Given a single relaying phase (solid lines), we note that the presence of $N_{\rm r}=15$ instead of $N_{\rm r}=5$ relays leads to significant performance improvements (cf. Fig. 4). For $d \geq 5$, the performances of the FD-TPA scheme, the DFA-TPA solution, and the OC-TPA solution are virtually the same. Second, as expected additional relaying phases offer further substantial performance improvements, where the second relaying phase (dashed lines) yields the largest relative gain. In this example, it was found that the relative gains for $N_{\max}>3$ relaying phases are marginal. For the scenario with $N_{\rm r}=5$ relays (cf. Fig. 5), it was found that the relative gains diminish already for $N_{\max}>2$ relaying phases.

In practice, perfect knowledge of the channel energies $\alpha_{{\rm S},{\rm U}_j}$ and $\alpha_{{\rm R}_i,{\rm U}_j}$ ($j \in \mathbb{I}_{\rm P}^{(\rm rx)}$) in direction of the primary receiver might be difficult to obtain. This problem can, for example, be solved by multiplying the available estimates $\tilde{\alpha}_{{\rm S},{\rm U}_j}$ and $\tilde{\alpha}_{{\rm R}_i,{\rm U}_j}$ of the channel energies $\alpha_{{\rm S},{\rm U}_j}$ and $\alpha_{{\rm R}_i,{\rm U}_j}$ by an appropriate interference margin factor $C_{\rm margin}>1$. By this means, it can be ensured that the interference constraints at the primary receivers are met with a high probability, even if the available estimates $\tilde{\alpha}_{{\rm S},{\rm U}_j}$ and $\tilde{\alpha}_{{\rm R}_i,{\rm U}_j}$ are smaller than the actual channel energies. As an example, Fig. 6 shows performance results for the case, where all cognitive nodes employ an interference margin factor of $C_{\rm margin}=10$ ('Case II'). For simplicity, we have assumed that $\tilde{\alpha}_{{\rm S},{\rm U}_{\rm rx}}=\alpha_{{\rm S},{\rm U}_{\rm rx}}$ and $\tilde{\alpha}_{{\rm R}_i,{\rm U}_{\rm rx}}=\alpha_{{\rm R}_i,{\rm U}_{\rm rx}}$ for all indices $i \in \{1, \ldots, N_{\rm r}\}$. As can be seen, as long as the primary system is not too close to the CR network, the considered TPA schemes still achieve significant performance improvements over non-cooperative transmission (especially in the case of multiple hops). Given a single relaying phase, the FD-TPA scheme leaves a somewhat larger gap to the DFA-TPA and the OC-TPA solutions than in the scenario without interference margin ($C_{\rm margin}=1$), due to the more restrictive interference constraints.

In order to highlight the differences between the FD-TPA scheme and the DFA-TPA solution, Fig. 7 shows the average transmit power spent by the individual relays as a function of the relay position. As an example, we have focused on the case $C_{\rm margin}=1$, $d=5$, and a single relaying phase. We first note that relays which are far away from the source node are characterized by very small average transmit powers, since they are inactive with a high probability. In the case of the DFA-TPA

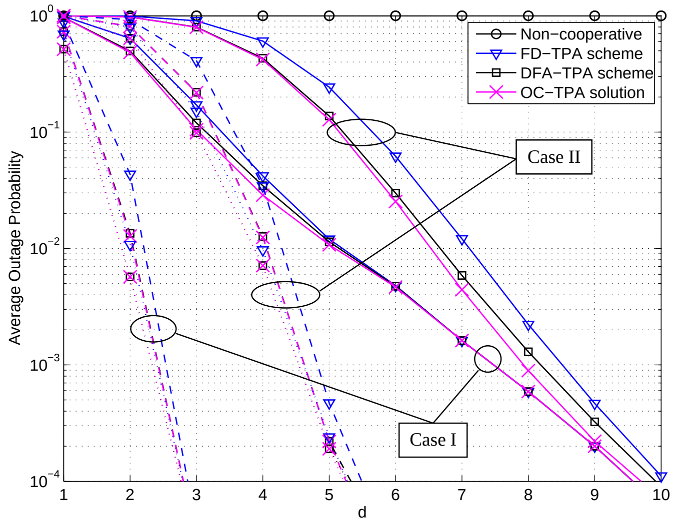

Fig. 6. Average outage probability $\bar{C}(\gamma_{\mathrm{D,target}})$ for non-cooperative transmission, the FD-TPA scheme, the DFA-TPA solution, and the OC-TPA solution as a function of the distance between the primary system and the CR network ($N_{\mathrm{r}}=15$ relays, $N_{\mathrm{P}}=1$ primary link, $\bar{\gamma}_{0,\mathrm{S}\rightarrow\mathrm{D}}=5$ dB, $N_{\mathrm{sp}}=10$; primary transmitter and receiver located at $(0,d+10)$ and $(0,d)$, respectively). Case I: no interference margin ($C_{\mathrm{margin}}=1$); Case II: interference margin $C_{\mathrm{margin}}=10$. Solid lines: one relaying phase; dashed lines: two relaying phases; dotted lines: three relaying phases.

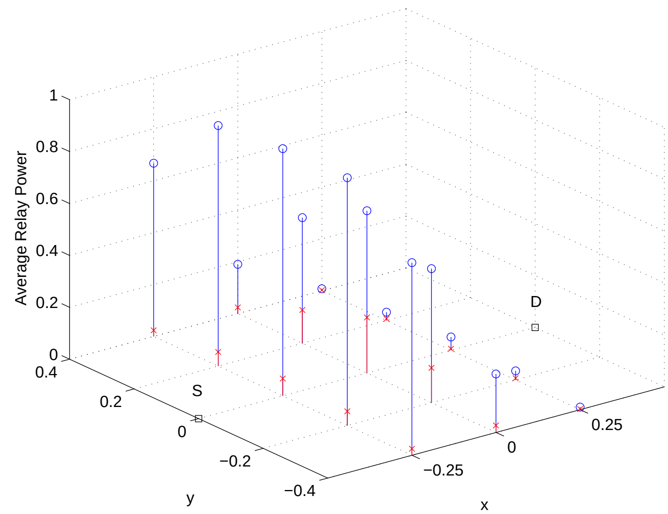

Fig. 7. Average transmit power spent by the individual relays ($N_{\mathrm{r}}=15$ relays, one relaying phase, $N_{\mathrm{P}}=1$ primary link, $\bar{\gamma}_{0,\mathrm{S}\rightarrow\mathrm{D}}=5$ dB, $N_{\mathrm{sp}}=10$, interference margin $C_{\mathrm{margin}}=1$), for the case of the FD-TPA scheme (marked by 'o') and the DFA-TPA solution (marked by 'x'). The position of the source node is marked by 'S', and the position of the destination node is marked by 'D'. The primary transmitter and receiver are located at $(0,15)$ and $(0,5)$, respectively (not depicted).

solution, it can be seen that relays which are far away from the destination node are on average assigned comparatively small transmit powers, as the DFA-TPA solution takes the quality of the $\mathrm{R}_i\rightarrow\mathrm{D}$ links into account (via the feedback information from the destination node). Consequently, the DFA-TPA solution mainly utilizes those relays, which have approximately the same distance from the source and from the destination node, whereas the FD-TPA scheme also utilizes those relays which are close to the source node. Finally, we note that on average the DFA-TPA solution requires substantially smaller relay transmit powers than the FD-TPA scheme (as expected). This is even more apparent in Fig. 8, where the overall average transmit power spent by the relays is depicted as a function of the distance d between the primary system and the CR network. As can be seen, if the primary receiver is located at some distance from the CR network, i.e., if the interference constraints are less restrictive, the FD-TPA scheme entails a relatively large energy consumption, whereas the energy consumptions in the case of the DFA-TPA and the OC-TPA solutions remain moderate.

C. Performance Results in the Presence of Multiple Active Primary Links

Finally, we consider the case where multiple active primary links are present in the vicinity of the CR network. As an example, we focus on a scenario with $N_{\mathrm{p}}=2$ primary systems with $\xi_{\mathrm{rx},1}=0.01$, $\xi_{\mathrm{rx},2}=0.02$, and $\rho_1=\rho_2=0.1$. As earlier, the positions of the first primary transmitter and receiver are set to $(0,d+10)$ and $(0,d)$, respectively. Similarly, the positions of the second primary transmitter and receiver are set to $(0,-(d+10))$ and $(0,-d)$, respectively. Similar to Fig. 6, we assume that all cognitive nodes employ an interference margin factor of $C_{\mathrm{margin}}=10$ for the channel energies $\alpha_{\mathrm{S,U_{rx,\mathit{j}}}}$ and $\alpha_{\mathrm{R}_i,\mathrm{U_{rx,\mathit{j}}}}$ in direction of the primary receivers ($j=1,2$, $i\in\{1,...,N_{\mathrm{r}}\}$). Fig. 9 depicts the average outage probability $\bar{C}(\gamma_{\mathrm{D,target}})$ for non-cooperative transmission, the FD-TPA scheme, the DFA-TPA solution, and the OC-TPA solution as a function of d, for $N_{\mathrm{r}}=5$ and $N_{\mathrm{r}}=15$ relays (same positions as earlier). Generally, the presence of a second primary system leads to significant performance degradations for all schemes, since (i) the interference stemming from the primary transmitters becomes more severe and (ii) the interference constraints posed by the primary receivers become more restrictive. As can be seen, in the case of $N_{\mathrm{r}}=5$ relays the performance of the FD-TPA scheme, the DFA-TPA solution, and the OC-TPA solution is very similar. Interestingly, in the case of $N_{\mathrm{r}}=15$ relays the DFA-TPA solution performs less close to the OC-TPA solution than in the case of a single primary system (cf. Fig. 6). This is partly due to the assumption made in the DFA-TPA solution that $\xi_{\mathrm{rx}}=\min_j\{\xi_{\mathrm{rx},j}\}$ for both primary users (cf. Section III-B). However, as can be seen in Fig. 9 this performance gap is reduced when a sufficient number of relaying phases is employed (e.g. $N_{\max}=3$ in this case).

VI. Conclusions

In this paper, two distributed transmit power allocation (TPA) schemes for relay-assisted cognitive-radio (CR) systems in the presence of a single or multiple active primary links have been developed, with the goal to optimize the performance of the CR system, while limiting the interference experienced by the primary receivers (cf. Table I for a summary of the proposed schemes). Analytical and simulation-based performance results have shown that both proposed schemes accomplish significant improvements over non-cooperative transmission, especially when more than two hops are employed. In particular, it was shown that both schemes usually perform close to

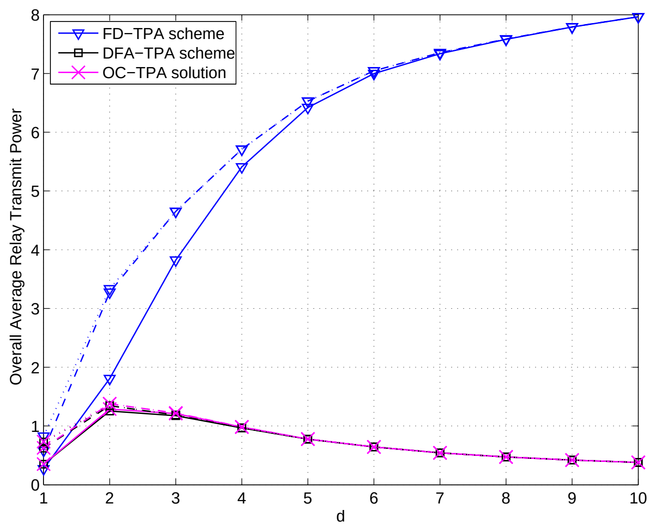

Fig. 8. Overall average transmit power spent by the relays in the case of the FD-TPA scheme, the DFA-TPA solution, and the OC-TPA solution as a function of the distance of the primary system from the CR network ($N_{\mathrm{r}}=15$ relays, one relaying phase, $N_{\mathrm{p}}=1$ primary link, $\bar{\gamma}_{0,\mathrm{S}\to\mathrm{D}}=5$ dB, $N_{\mathrm{sp}}=10$, interference margin $C_{\mathrm{margin}}=1$; primary transmitter and receiver located at $(0,d+10)$ and $(0,d)$, respectively). Solid lines: one relaying phase; dashed lines: two relaying phases; dotted lines: three relaying phases.

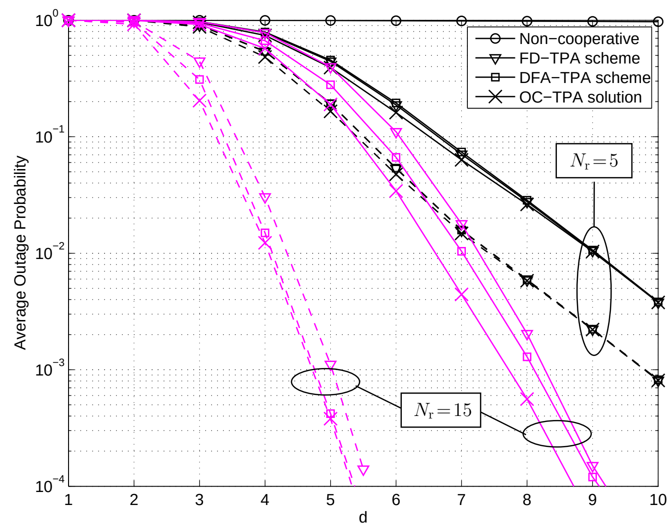

Fig. 9. Average outage probability $\bar{C}(\gamma_{\mathrm{D,target}})$ for non-cooperative transmission, the FD-TPA scheme, the DFA-TPA solution, and the OC-TPA solution as a function of the distance d of the primary systems from the CR network ($N_{\mathrm{p}}=2$ non-congenerous primary links, $\bar{\gamma}_{0,\mathrm{S}\to\mathrm{D}}=10$ dB, $N_{\mathrm{sp}}=10$, interference margin $C_{\mathrm{margin}}=10$). Dark color: $N_{\mathrm{r}}=5$ relays; light color: $N_{\mathrm{r}}=15$ relays. Solid lines: one relaying phase; dashed lines: three relaying phases.

the optimal centralized TPA solution. Moreover, the distributed feedback-assisted (DFA) TPA scheme was shown to effectively capitalize on the feedback from the destination node, so as to achieve a low average energy consumption at the relays.

Future work might yield more sophisticated distributed TPA schemes for the case of multiple (non-congenerous) primary systems and/or large numbers of available relays. Moreover, some work in the direction of a joint optimization of the individual relaying phases would be of interest. Finally, it would be interesting to study the impact of non-perfect channel knowledge and non-perfect measurements at the destination node on the performance of the proposed TPA schemes.

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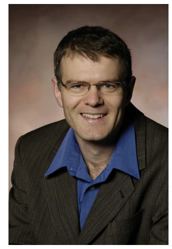

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