A Rigorous Analysis of the Statistical Properties of the Discrete-Time Triply-Selective MIMO Rayleigh Fading Channel Model

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***Abstract*— We analyze the correlation properties of the discrete-time triply-selective multiple-input multiple-output (MIMO) Rayleigh fading channel model, while triply-selective refers to selectivity in time, frequency, and space. A rigorous analysis of these correlation properties is indispensable to assess the performance of multiple-antenna techniques in realistic environments. To the best of the authors' knowledge, a similar analysis has not yet been presented in the literature.**

***Index Terms*— MIMO systems, Rayleigh channels, dispersive channels, time-varying channels, modeling.**

I. INTRODUCTION

THE USE of multiple antennas for wireless communication systems has gained much interest during the last decade, since multiple-antenna techniques offer capacity gains [1], diversity gains [2], and beamforming gains [3] over conventional single-antenna systems. Channel modeling for multiple-antenna systems, often called multiple-input multiple-output (MIMO) systems, is an important topic. On the one hand, accurate channel models are required in order to predict the theoretical limits of real-world MIMO systems. On the other hand, they are indispensable for designing novel transmitter- and receiver techniques and assess their efficiency in realistic environments. In fact, channel modeling for MIMO systems is still an active field of research [4].

In digital communications, transmitted signals carry discrete-time data symbols drawn from a finite alphabet. At the receiver, the received signals are filtered and sampled, and the transmitted data symbols are recovered by means of discrete-time signal processing. Correspondingly, it is useful to define a discrete-time channel model [5], especially with regard to computer simulations. The discrete-time channel model comprises the continuous-time physical channel, analog filters at transmitter and receiver, as well as the sampling rate and sampling phase. A statistical discrete-time model for single-input single-output Rayleigh fading channels with selectivity in time (due to motion of transmitter or receiver) and frequency (due to a non-negligible delay spread of the physical channel) was proposed in [6]. In [7], this concept was generalized to *triply-selective* MIMO fading channels with selectivity in time, frequency, and space (due to spatial diversity obtained by means of multiple antennas).

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In this letter, we present a rigorous analysis of the correlation properties of the discrete-time triply-selective MIMO Rayleigh fading channel model [7], which enables accurate computer simulations. It is claimed in [7] that the correlations between the coefficients of the discrete-time MIMO channel model, in the following called *channel coefficients*, can be written as the product of temporal correlation, intertap correlation, and spatial correlation. We show that this is, in fact, not true and may lead to significant modeling errors. Moreover, we show that the Kronecker model for the spatial correlations does, in general, not hold in the case of frequency-selective fading.

The remainder of this letter is organized as follows: The discrete-time MIMO Rayleigh fading channel model [7] under consideration is briefly recapitulated in Section II. The correlation properties of the channel model are analyzed in Section III. Finally, in order to illustrate our findings, some numerical examples are presented in Section IV.

II. DISCRETE-TIME MIMO RAYLEIGH FADING CHANNEL MODEL

Throughout this letter, we make use of the complex baseband notation. We assume a rich-scattering environment and a wide-sense stationary (WSS) scenario with uncorrelated scattering (US) [6]. Focus is on symbol-rate sampling, i.e., sampling is performed once per symbol duration T. The physical channel is assumed to be slowly time-varying.

Consider a MIMO system with M transmit and N receive antennas. The physical channel for the link from the μth transmit antenna ($1 \leq \mu \leq M$) to the νth receive antenna ($1 \leq \nu \leq N$) is modeled by an impulse response

$$f_{\nu,\mu}(\tau, t) \triangleq \sum_{n=0}^{N_\tau - 1} f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)\, \delta(\tau - \tau_{\mathrm{d},n}), \qquad (1)$$

where τ denotes the propagation delay with respect to the line-of-sight path, t the absolute time, N_τ the number of resolvable delays $\tau_{\mathrm{d},n}$, and $\delta(\tau - \tau_{\mathrm{d},n})$ a Dirac impulse at $\tau = \tau_{\mathrm{d},n}$. Due to rich scattering, the gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$ are assumed to be (circularly symmetric) complex Gaussian random variables [6], $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t) \sim \mathcal{CN}(\bar{f}_{\nu,\mu,n}, \sigma^2_{f_{\nu,\mu},n})$, where we focus on the case of Rayleigh fading here ($\bar{f}_{\nu,\mu,n} = 0$ for all indices μ, ν, and n). Moreover, for simplicity we use the same set of discrete delays $\tau_{\mathrm{d},n}$ for all links. (Possibly, some of the complex gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$ are zero for specific links.)

In the following, we assume that (i) a linear modulation scheme is used, while the same (analog) pulse-shaping filter $g_{\mathrm{Tx}}(t)$ is employed for all transmit antennas, (ii) coherent demodulation is performed at the receiver, while the same (analog) receiver filter $g_{\mathrm{Rx}}(t)$ is employed for all receive antennas, (iii) the same sampling phase[1] $\epsilon \in [0, T)$ is used for all receive antennas. The discrete-time channel model for the νth receive antenna is given by

$$y_\nu[k] = \sum_{\mu=1}^{M} \sum_{l=0}^{L} h_{\nu,\mu}[k,l]\, x_\mu[k-l] \,+\, n_\nu[k], \tag{2}$$

where k denotes the discrete time index, $y_\nu[k]$ the kth received sample of the νth receive antenna, L the effective memory length of the discrete-time channel model, $x_\mu[k]$ the kth transmitted data symbol of the μth transmit antenna, and $n_\nu[k]$ the kth additive Gaussian noise sample at the νth receive antenna. (Due to receive filtering, the noise samples $n_\nu[k]$ may be colored, i.e., correlated in time.) The complex channel coefficients $h_{\nu,\mu}[k,l]$ $(l=0,\dots,L)$ are defined as

$$h_{\nu,\mu}[k,l] \triangleq \sum_{n=0}^{N_\tau-1} f_{\nu,\mu}(\tau_{\mathrm{d},n}, kT)\, g(lT+\epsilon-\tau_{\mathrm{d},n}) \tag{3}$$

$(f_{\nu,\mu}(\tau_{\mathrm{d},n}, kT) = f_{\nu,\mu}(\tau_{\mathrm{d},n}, kT+\epsilon)$ due to slow time variance), where

$$g(t) \triangleq \int_{-\infty}^{+\infty} g_{\mathrm{Tx}}(t')\, g_{\mathrm{Rx}}(t-t')\, \mathrm{d}t' \tag{4}$$

denotes the overall impulse response of transmit and receive filtering[2].

III. Statistical Properties of the Channel Coefficients

According to (3) the channel coefficients $h_{\nu,\mu}[k,l]$ result from a weighted sum of statistically independent[3] complex Gaussian random variables and are thus also Gaussian distributed [6], [7], i.e., $h_{\nu,\mu}[k,l] \sim \mathcal{CN}(0, \sigma^2_{h_{\nu,\mu},l})$. Correspondingly, the (joint) statistical properties of the channel coefficients are fully captured by the variances $\sigma^2_{h_{\nu,\mu},l}$ and the complete set of correlations between any two channel coefficients $h_{\nu,\mu}[k,l]$ and $h_{\nu',\mu'}[k',l']$.

Utilizing the assumption of uncorrelated scattering, the variances $\sigma^2_{h_{\nu,\mu},l}$ of the channel coefficients can be written as

$$\sigma^2_{h_{\nu,\mu},l} = \sum_{n=0}^{N_\tau-1} \sigma^2_{f_{\nu,\mu},n}\, |g(lT+\epsilon-\tau_{\mathrm{d},n})|^2, \tag{5}$$

i.e., they depend solely on the variances of the complex gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$ and on the shape of $g(t)$.

In the following, we assume that the temporal and the spatial correlations of the complex gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$ can be modeled independently, which is a common assumption in the literature. Correspondingly,

$$\frac{\mathsf{E}\{f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)\, f^*_{\nu',\mu'}(\tau_{\mathrm{d},n}, t')\}}{\sqrt{\sigma^2_{f_{\nu,\mu},n}\sigma^2_{f_{\nu',\mu'},n}}} \triangleq \rho_{f_{\nu,\mu,\nu',\mu'},n} \cdot R_{f_{\nu,\mu},n}(t',t), \tag{6}$$

where

$$\rho_{f_{\nu,\mu,\nu',\mu'},n} \triangleq \frac{\mathsf{E}\{f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)\, f^*_{\nu',\mu'}(\tau_{\mathrm{d},n}, t)\}}{\sqrt{\sigma^2_{f_{\nu,\mu},n}\sigma^2_{f_{\nu',\mu'},n}}} \tag{7}$$

denotes the spatial correlation between the gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$ and $f_{\nu',\mu'}(\tau_{\mathrm{d},n}, t)$ and

$$R_{f_{\nu,\mu},n}(t',t) \triangleq \mathsf{E}\{f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)\, f^*_{\nu,\mu}(\tau_{\mathrm{d},n}, t')\}/\sigma^2_{f_{\nu,\mu},n} \tag{8}$$

denotes the temporal auto-correlation function of $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$.

Based on (3) and (6), the correlation between two channel coefficients $h_{\nu,\mu}[k,l]$ and $h_{\nu',\mu'}[k',l']$ results as

$$\begin{aligned} \rho_{h_{\nu,\mu,\nu',\mu',l,l',k,k'}} &\triangleq \\ &\frac{1}{\sqrt{\sigma^2_{h_{\nu,\mu},l}\sigma^2_{h_{\nu',\mu'},l'}}} \sum_{n=0}^{N_\tau-1} \sqrt{\sigma^2_{f_{\nu,\mu},n}\,\sigma^2_{f_{\nu',\mu'},n}} \cdot R_{f_{\nu,\mu},n}[k',k] \\ &\times \rho_{f_{\nu,\mu,\nu',\mu'},n} \cdot g(lT+\epsilon-\tau_{\mathrm{d},n})\, g^*(l'T+\epsilon-\tau_{\mathrm{d},n}), \end{aligned} \tag{9}$$

where $R_{f_{\nu,\mu},n}[k',k] \triangleq R_{f_{\nu,\mu},n}(t'=k'T, t=kT)$. Equation (9) subsumes several special cases, for example quasi-static fading with selectivity in frequency and space (i.e., $R_{f_{\nu,\mu},n}[k',k] \triangleq 1$ for all k') or spatially uncorrelated fading with selectivity in time and frequency (i.e., $\rho_{f_{\nu,\mu,\nu',\mu'},n} \triangleq 0$ for all $\mu' \neq \mu$ or $\nu' \neq \nu$). Specifically, for the temporal correlation between two channel coefficients $h_{\nu,\mu}[k,l]$ and $h_{\nu,\mu}[k',l]$ one obtains

$$\begin{aligned} \rho_{h_{\nu,\mu,l,k,k'}} &\triangleq \\ &\frac{1}{\sigma^2_{h_{\nu,\mu},l}} \sum_{n=0}^{N_\tau-1} \sigma^2_{f_{\nu,\mu},n}\, R_{f_{\nu,\mu},n}[k',k]\; |g(lT+\epsilon-\tau_{\mathrm{d},n})|^2. \end{aligned} \tag{10}$$

Similarly, the intertap correlation between two channel coefficients $h_{\nu,\mu}[k,l]$ and $h_{\nu,\mu}[k,l']$ results as

$$\begin{aligned} \rho_{h_{\nu,\mu,l,l'}} &\triangleq \frac{1}{\sqrt{\sigma^2_{h_{\nu,\mu},l}\sigma^2_{h_{\nu,\mu},l'}}} \\ &\times \sum_{n=0}^{N_\tau-1} \sigma^2_{f_{\nu,\mu},n} \cdot g(lT+\epsilon-\tau_{\mathrm{d},n})\, g^*(l'T+\epsilon-\tau_{\mathrm{d},n}). \end{aligned} \tag{11}$$

Finally, for the spatial correlation between two channel coefficients $h_{\nu,\mu}[k,l]$ and $h_{\nu',\mu'}[k,l]$ one finds

$$\begin{aligned} \rho_{h_{\nu,\mu,\nu',\mu',l}} &\triangleq \frac{1}{\sqrt{\sigma^2_{h_{\nu,\mu},l}\sigma^2_{h_{\nu',\mu'},l}}} \\ &\times \sum_{n=0}^{N_\tau-1} \sqrt{\sigma^2_{f_{\nu,\mu},n}\,\sigma^2_{f_{\nu',\mu'},n}} \cdot \rho_{f_{\nu,\mu,\nu',\mu'},n}\; |g(lT+\epsilon-\tau_{\mathrm{d},n})|^2. \end{aligned} \tag{12}$$

Due to the assumption of (wide-sense) stationarity, the correlations (9)-(12) do not depend on the time index k. (The correlation $\rho_{h_{\nu,\mu,l,k,k'}}$ depends only on the difference $k'-k$.) Although claimed in [7], the overall correlation (9) can *not*

[1] This assumption is not crucial, since different sampling phases may be represented by shifting the impulse responses of the physical channel accordingly.

[2] Theoretically the overall impulse response $g(t)$ is of infinite length (due to a limited bandwidth). However, practical impulse responses typically decay significantly for large absolute values of t. Correspondingly, one can find a certain window $l \in [-L_1, +L_2]$ such that channel coefficients $h_{\nu,\mu}[k,l]$ with $l < -L_1$ or $l > +L_2$ have very small average powers and can thus be neglected. For simplicity, we define the index l such that the window of interest is $l \in [0, L]$.

[3] Due to the assumption of uncorrelated scattering, two complex gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$ and $f_{\nu,\mu}(\tau_{\mathrm{d},n'}, t)$ are statistically independent for $n' \neq n$.

be written as the product of temporal correlation, intertap correlation, and spatial correlation:

$$\rho_{h_{\nu,\mu,\nu',\mu',l,l',k,k'}} \neq \rho_{h_{\nu,\mu,l,k,k'}} \cdot \rho_{h_{\nu,\mu,l,l'}} \cdot \rho_{h_{\nu,\mu,\nu',\mu',l}}. \quad (13)$$

As will be seen in Section IV, depending on the statistical properties of the complex gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$ and on the shape of $g(t)$, the result in [7] can lead to significant modeling errors.

A. Kronecker Correlation Model

It is often assumed in the literature that the spatial correlation $\rho_{f_{\nu,\mu,\nu',\mu'},n}$ between two gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$ and $f_{\nu',\mu'}(\tau_{\mathrm{d},n}, t)$ can be factorized according to

$$\rho_{f_{\nu,\mu,\nu',\mu'},n} \triangleq \rho_{f_{\mu,\mu'},n} \cdot \rho_{f_{\nu,\nu'},n} \quad (14)$$

(*Kronecker correlation*). This implies that the correlation between transmit antenna μ and μ' does not depend on the considered receive antenna, and the correlation between receive antenna ν and ν' does not depend on the considered transmit antenna. In this case, (12) becomes

$$\rho_{h_{\nu,\mu,\nu',\mu',l}} \triangleq \frac{1}{\sqrt{\sigma^2_{h_{\nu,\mu},l}\sigma^2_{h_{\nu',\mu'},l}}} \sum_{n=0}^{N_\tau-1} \sqrt{\sigma^2_{f_{\nu,\mu},n}\,\sigma^2_{f_{\nu',\mu'},n}} \times \rho_{f_{\mu,\mu'},n} \cdot \rho_{f_{\nu,\nu'},n}\, |g(lT+\epsilon-\tau_{\mathrm{d},n})|^2. \quad (15)$$

For the special cases $\nu' = \nu$ and $\mu' = \mu$, one obtains

$$\rho_{h_{\mu,\mu'},l} \triangleq \frac{1}{\sqrt{\sigma^2_{h_{\nu,\mu},l}\sigma^2_{h_{\nu,\mu'},l}}} \times \sum_{n=0}^{N_\tau-1} \sqrt{\sigma^2_{f_{\nu,\mu},n}\,\sigma^2_{f_{\nu,\mu'},n}} \cdot \rho_{f_{\mu,\mu'},n}\, |g(lT+\epsilon-\tau_{\mathrm{d},n})|^2 \quad (16)$$

and

$$\rho_{h_{\nu,\nu'},l} \triangleq \frac{1}{\sqrt{\sigma^2_{h_{\nu,\mu},l}\sigma^2_{h_{\nu',\mu},l}}} \times \sum_{n=0}^{N_\tau-1} \sqrt{\sigma^2_{f_{\nu,\mu},n}\,\sigma^2_{f_{\nu',\mu},n}} \cdot \rho_{f_{\nu,\nu'},n}\, |g(lT+\epsilon-\tau_{\mathrm{d},n})|^2, \quad (17)$$

respectively. Although claimed in [7], the Kronecker correlation model (14) for the complex gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$ does in general *not* transfer to the channel coefficients, i.e., in general we have

$$\rho_{h_{\nu,\mu,\nu',\mu',l}} \neq \rho_{h_{\mu,\mu'},l} \cdot \rho_{h_{\nu,\nu'},l}. \quad (18)$$

Exceptions are given by the case of frequency-flat fading ($N_\tau = 1$, $L = 0$), and by a MIMO channel model, where the variances $\sigma^2_{f_{\nu,\mu},n}$ do not depend on the antenna indices μ and ν and the spatial correlations $\rho_{f_{\mu,\mu'},n}$ and $\rho_{f_{\nu,\nu'},n}$ are identical for all indices n.

IV. Numerical Examples

In the sequel we illustrate, in which cases the MIMO channel model [7] employing the erroneous simplifications

$$\rho_{h_{\nu,\mu,\nu',\mu',l,l',k,k'}} = \rho_{h_{\nu,\mu,l,k,k'}} \cdot \rho_{h_{\nu,\mu,l,l'}} \cdot \rho_{h_{\nu,\mu,\nu',\mu',l}} \quad (19)$$

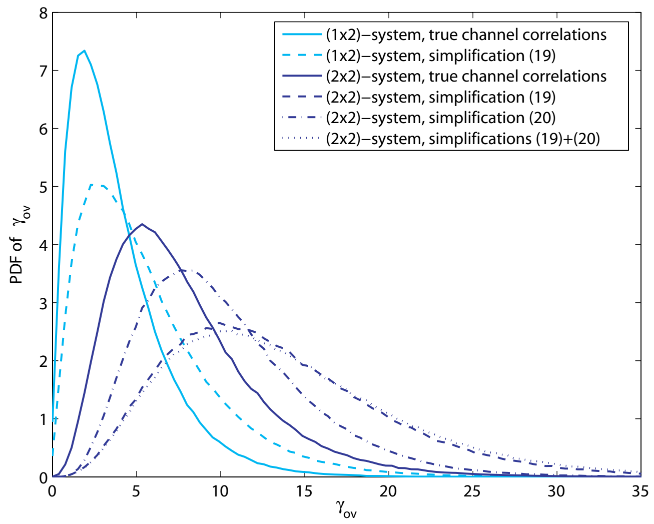

Fig. 1. PDF of the instantaneous SNR γ_{ov} for $M = 1, 2$ transmit and $N = 2$ receive antennas, $N_\tau = 2$ resolvable delays ($\tau_{\mathrm{d},0} = 0$, $\tau_{\mathrm{d},1} = T/2$), exponentially decaying variances $\sigma^2_{f,n}$ ($c_\tau = 5$), a roll-off factor $r = 0$, and spatial correlations $\rho_{f,0} = 0$, $\rho_{f,1} = 1$ ($\sigma^2_x/\sigma^2_{\mathrm{n}} = 1$).

and

$$\rho_{h_{\nu,\mu,\nu',\mu',l}} = \rho_{h_{\mu,\mu'},l} \cdot \rho_{h_{\nu,\nu'},l} \quad (20)$$

deviates significantly from a triply-selective MIMO channel model employing the true channel correlations $\rho_{h_{\nu,\mu,\nu',\mu',l,l',k,k'}}$ and $\rho_{h_{\nu,\mu,\nu',\mu',l}}$ according to (9) and (12). In order to compare the two channel models, we first consider the associated distributions of the instantaneous sum signal-to-noise ratio (SNR)

$$\gamma_{\mathrm{ov}} \triangleq \sum_{\mu=1}^{M}\sum_{\nu=1}^{N}\sum_{l=0}^{L} \frac{|h_{\nu,\mu}[k,l]|^2\sigma^2_x}{\sigma^2_{\mathrm{n}}} \quad (21)$$

resulting from maximum-ratio combining over all MN transmission links. Here, $\sigma^2_x \triangleq \mathsf{E}\{|x_\mu[k]|^2\}$ denotes the symbol variance and $\sigma^2_{\mathrm{n}} \triangleq \mathsf{E}\{|n_\nu[k]|^2\}$ the variance of the (complex) noise samples $n_\nu[k]$. Afterwards, we will briefly consider the impact on the corresponding *Matched Filter Bound (MFB)* [8, Ch. 14.5].[4]

For simplicity, we focus on the case of quasi-static fading, i.e, $\rho_{h_{\nu,\mu,l,k,k'}} = 1$ for all cases ($R_{f_{\nu,\mu},n}[k',k] \equiv 1$). Moreover, we focus on the case of $M = 1$ or $M = 2$ transmit antennas and $N = 2$ receive antennas. The variances $\sigma^2_{f_{\nu,\mu},n}$ ($n = 0, ..., N_\tau - 1$) of the complex gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n}, t)$ are assumed to decay exponentially with the delay $\tau_{\mathrm{d},n}$ (identically for all indices μ, ν), according to

$$\sigma^2_{f_{\nu,\mu},n} = \exp\left(-\frac{\tau_{\mathrm{d},n}}{c_\tau T}\right). \quad (22)$$

The spatial correlations $\rho_{f_{\nu,\mu,\nu',\mu'},n}$ are assumed to follow the Kronecker correlation model (14), where we assume for simplicity that $\rho_{f_{\mu,\mu'},n} = \rho_{f_{\nu,\nu'},n} \triangleq \rho_{f,n}$ for all indices $n = 0, ..., N_\tau - 1$ and $\mu \neq \mu'$, $\nu \neq \nu'$. For the impulse response

[4] The MFB (also called *Rake Receiver Bound*) constitutes an analytical lower bound on the symbol error rate performance of maximum-likelihood sequence estimation (MLSE). It is known to accurately predict the performance of MLSE in the presence of frequency-selective fading, e.g. [9].

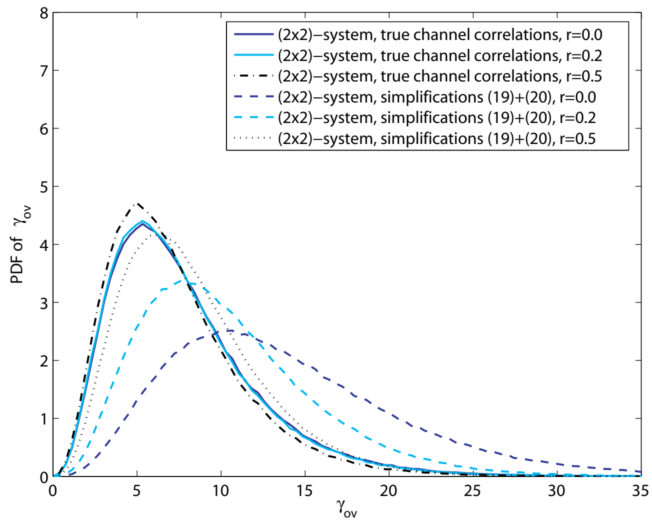

Fig. 2. PDF of the instantaneous SNR γ_{ov} for $M=2$ transmit and $N=2$ receive antennas, $N_\tau=2$ resolvable delays ($\tau_{\mathrm{d},0}=0$, $\tau_{\mathrm{d},1}=T/2$), a decay factor $c_\tau=5$, spatial correlations $\rho_{f,0}=0$, $\rho_{f,1}=1$, and different roll-off factors r ($\sigma_x^2/\sigma_{\mathrm{n}}^2=1$).

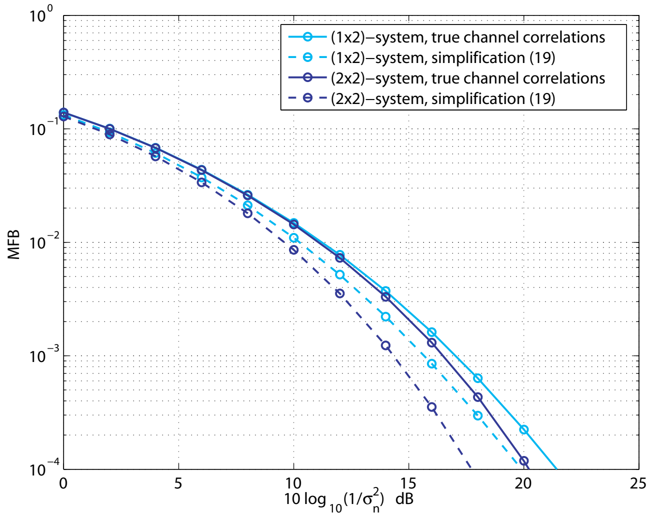

Fig. 4. MFB as a function of $1/\sigma_{\mathrm{n}}^2$ in dB, resulting for $M=1,2$ transmit and $N=2$ receive antennas, binary antipodal transmission, $N_\tau=2$ resolvable delays ($\tau_{\mathrm{d},0}=0$, $\tau_{\mathrm{d},1}=T/2$), a decay factor $c_\tau=5$, a roll-off factor $r=0$, and spatial correlations $\rho_{f,0}=0$ and $\rho_{f,1}=1$.

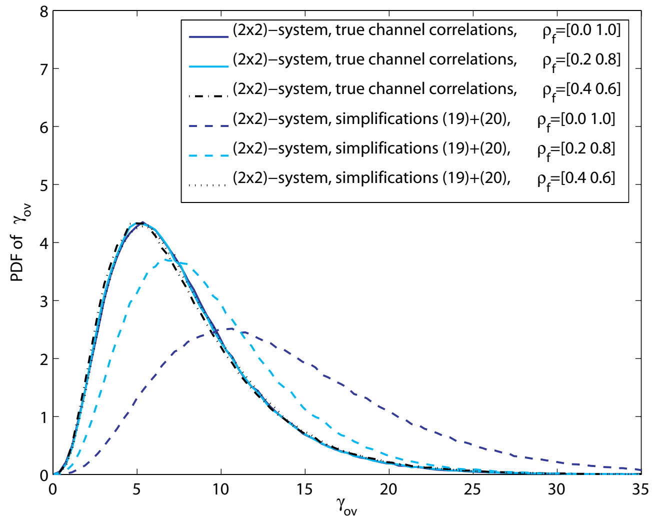

Fig. 3. PDF of the instantaneous SNR γ_{ov} for $M=2$ transmit and $N=2$ receive antennas, $N_\tau=2$ resolvable delays ($\tau_{\mathrm{d},0}=0$, $\tau_{\mathrm{d},1}=T/2$), a decay factor $c_\tau=5$, a roll-off factor $r=0$, and different spatial correlations $\rho_f=[\rho_{f,0},\,\rho_{f,1}]$ ($\sigma_x^2/\sigma_{\mathrm{n}}^2=1$).

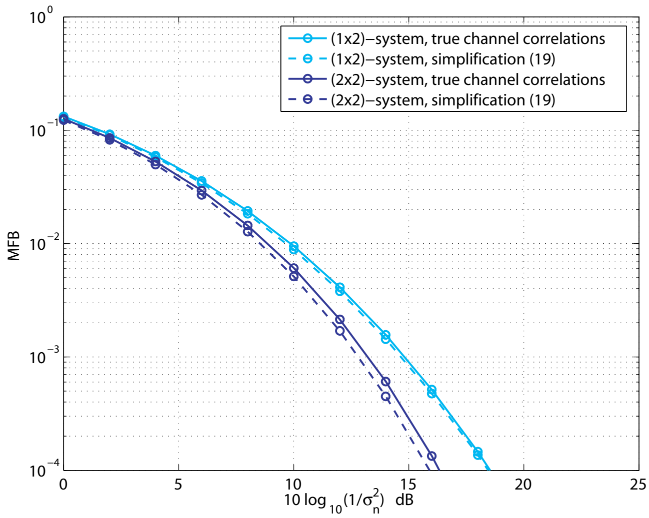

Fig. 5. MFB as a function of $1/\sigma_{\mathrm{n}}^2$ in dB, resulting for $M=1,2$ transmit and $N=2$ receive antennas, binary antipodal transmission, $N_\tau=2$ resolvable delays ($\tau_{\mathrm{d},0}=0$, $\tau_{\mathrm{d},1}=T/2$), a decay factor $c_\tau=5$, a roll-off factor $r=0$, and spatial correlations $\rho_{f,0}=0.1$ and $\rho_{f,1}=0.9$.

$g(t)$, a cosine roll-off impulse with roll-off factor r is assumed, while the sampling phase is adjusted with respect to delay $\tau_{\mathrm{d},0}$, i.e., $g(lT+\epsilon-\tau_{\mathrm{d},0})\stackrel{\triangle}{=}\delta[l]$. Throughout this section, we assume that the physical channel is characterized by $N_\tau=2$ complex gain factors $f_{\nu,\mu}(\tau_{\mathrm{d},n},t)$ with delays $\tau_{\mathrm{d},0}=0$ and $\tau_{\mathrm{d},1}=T/2$.

Fig. 1 displays the probability density function (PDF) of the instantaneous SNR γ_{ov} resulting for $M=1$ and $M=2$ transmit antennas, a decay parameter of $c_\tau=5$, a roll-off factor of $r=0$, and spatial correlations $\rho_{f,0}=0$ and $\rho_{f,1}=1$. The PDFs resulting for the true channel correlations according to (9) and (12) are marked by solid lines. As can be seen, when employing the simplifications (19) and/or (20), salient deviations from the true PDFs are observed. However, as illustrated by Fig. 2 and Fig. 3, the resulting deviations become less significant if

- the roll-off factor r is increased (Fig. 2) or
- the spatial correlations $\rho_{f,0}$ and $\rho_{f,1}$ become more balanced (Fig. 3).

Further numerical results not displayed here indicate that the resulting deviations also become less significant if

- the decay factor c_τ is reduced or
- the number N_τ of resolvable delays $\tau_{\mathrm{d},n}$ is increased.

In Fig. 4 and Fig. 5, the impact of the simplification (19) on the corresponding MFB is studied, for $M=1,2$ transmit antennas, binary antipodal transmission ($x_\mu[k]\in\{\pm 1\}$), a decay parameter of $c_\tau=5$, a roll-off factor of $r=0$, and two different sets of spatial correlations $\rho_{f,0}$ and $\rho_{f,1}$. As can be seen, for $\rho_{f,0}=0$ and $\rho_{f,1}=1$ the deviation from the true MFB is quite significant, both for $M=1$ and $M=2$ (Fig. 4). However, as soon as the the correlations $\rho_{f,0}$ and $\rho_{f,1}$ are slightly more balanced (e.g., $\rho_{f,0}=0.1$ and $\rho_{f,1}=0.9$, cf. Fig. 5), the resulting deviations are far less significant.

V. CONCLUSIONS

In this letter, a rigorous analysis of the correlation properties of the discrete-time triply-selective MIMO Rayleigh fading channel model has been presented, which enables accurate computer simulations for assessing the efficiency of multiple-antenna techniques in realistic environments. By means of numerical examples, the differences between our rigorous channel model and a known simplified channel model has been illustrated.

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