

A Statistical Transmit Power Allocation Scheme for Spatially Correlated MIMO Systems and its Robustness to Estimation Errors

Jan Mietzner¹, Peter A. Hoeher¹, Claudiu Krakowski², and Wen Xu²

¹ Information and Coding Theory Lab, Faculty of Engineering, University of Kiel, Kaiserstrasse 2, D-24143 Kiel, Germany
URL: www-ict.tf.uni-kiel.de E-mail: {jm,ph}@tf.uni-kiel.de

² BenQ Mobile GmbH & Co. OHG, Haidenauplatz 1, D-81667 Munich, Germany
URL: www.BenQmobile.com E-mail: {Claudiu.Krakowski,Wen.Xu}@BenQ.com

Abstract— We consider a simple statistical transmit power allocation scheme for spatially correlated multiple-input multiple-output (MIMO) systems, which is based on the Karhunen-Loève transform. The considered scheme is solely based on second-order channel statistics, i.e., no instantaneous channel knowledge is required at the transmitter. We consider different transmit power allocation strategies and propose a simple transmitter-sided maximum-ratio-combining scheme, which provides near-optimum performance over a wide signal-to-noise ratio range. Specifically, substantial gains over equal power allocation are obtained. Finally, the impact of estimation errors concerning the transmitter correlation matrix is studied, and it is shown that the considered transmit power allocation scheme provides a robust performance.

Index Terms— Wireless communications, MIMO systems, spatial correlation, space-time codes, transmit power allocation, Karhunen-Loève transform, estimation errors.

I. INTRODUCTION

SINCE THE END of the 1990's, it is well known that wireless communication systems with multiple antennas offer huge advantages over conventional single-antenna systems. On the one hand, it was shown in [1],[2] that the capacity of a multiple-input multiple-output (MIMO) system with M transmit (Tx) antennas and N receive (Rx) antennas grows *linearly* with $\min\{M, N\}$. Correspondingly, multiple antennas provide a promising means to increase the spectral efficiency of a system. On the other hand, it was shown in [3],[4] that multiple antennas can also be utilized, in order to provide a spatial diversity gain and thus to improve the error performance of a system.

The results in [1]-[4] are based on the assumption that the individual transmission links from the transmit antennas to the receive antennas are statistically *independent*. Spatial correlation, caused by insufficient antenna spacings or by a lack of scattering from the physical environment, can cause significant degradations in capacity and error performance [5],[6]. In cellular systems, spatial correlation is an issue both at the base station and at the mobile station: Though at the base station generous antenna spacings can be granted, there is comparably little scattering from the physical environment, because the transmitted/ received signals are typically concentrated within a small angular region. As opposed to this, the mobile station normally experiences rich scattering from many local scatterers, but the antenna spacings are often small due to a limited terminal size.

In several publications, it was shown that the performance of MIMO systems may be improved significantly by using some sort of channel knowledge at the transmitter, see e.g. [7]. The use of (full or partial) instantaneous channel knowledge at the

transmitter was, for example, considered in [2],[8]-[13]. However, accurate instantaneous channel knowledge at the transmitter is costly and may be difficult to acquire [14]. As an alternative, the use of statistical channel knowledge at the transmitter was studied in [7],[14]-[20]. Statistical channel knowledge can easily be gained in a practical system, for example off-line through field measurements, ray-tracing simulations or based on physical channel models, or on-line based on long-term averaging of the channel coefficients [14].

In this paper, we consider a simple statistical transmit power allocation scheme for spatially correlated MIMO systems, which is based on the Karhunen-Loève transform (KLT) [21, Ch. 8.5]. The scheme requires solely knowledge of the second-order channel statistics at the transmitter (in terms of the transmitter correlation matrix) and is therefore of high practical relevance. The general structure of the scheme was earlier considered in [14],[15],[17],[18]. Specifically, optimal transmit power allocation strategies were derived in [14],[15],[18] with regard to different optimization criteria: Minimum symbol error probability [14], minimum pair-wise error probability [15], and maximum ergodic capacity [18].

Within the scope of this paper, we combine the statistical transmit power allocation scheme with an outer space-time code. First, we discuss different transmit power allocation strategies and propose a simple transmitter-sided maximum-ratio-combining (MRC) scheme. By means of analytical performance results we show that the MRC scheme provides a near-optimum performance over a wide signal-to-noise ratio (SNR) range. Specifically, substantial gains over equal power allocation are obtained. Finally, we consider the impact of estimation errors concerning the transmitter correlation matrix and show that the considered statistical transmit power allocation scheme provides a robust performance.

In [22] it was shown that there is a certain duality between spatially correlated MIMO systems and MIMO system with distributed transmitters, where multiple cooperating transmitters (distributed on a larger scale) establish a *virtual* multiple-antenna system. Examples include simulcast systems [23] and relay-assisted systems, e.g. [24],[25]. Due to this duality, the statistical transmit power allocation scheme under consideration can also be applied in distributed MIMO systems, without any loss of optimality.

A. Paper Organization

The remainder of the paper is organized as follows: In Section II, the system and correlation model used throughout this paper is introduced. In Section III, the statistical transmit power

allocation scheme is discussed along with different power allocation strategies. Specifically, closed-form expressions and numerical results for the resulting bit error rates are presented. Finally, in Section IV the impact of estimation errors concerning the transmitter correlation matrix is analyzed, and conclusions are drawn in Section V.

B. Mathematical Notation

Matrices and vectors are written in upper case and lower case bold face, respectively. If not stated otherwise, all vectors are column vectors. The complex conjugate of a complex number a is marked as a^* , and the Hermitian transposed of a matrix \mathbf{A} as \mathbf{A}^H . The trace of an $(M \times M)$ -matrix, i.e., the sum over all diagonal elements, is denoted as $\text{tr}(\mathbf{A})$. The square-root $\mathbf{A}^{1/2}$ of a Hermitian matrix \mathbf{A} (i.e., $\mathbf{A} = \mathbf{A}^H$) is defined as $\mathbf{A}^{1/2H} \mathbf{A}^{1/2} = \mathbf{A}^{1/2} \mathbf{A}^{1/2H} = \mathbf{A}$. $\text{diag}(\mathbf{a})$ is a diagonal matrix with diagonal elements given by the vector \mathbf{a} , and $\text{vec}(\mathbf{A})$ is a vector which results from stacking the columns of an $(N \times M)$ -matrix \mathbf{A} in a joint vector. $\mathbb{E}\{\cdot\}$ denotes statistical expectation.

II. SYSTEM AND CORRELATION MODEL

Throughout this paper, the complex baseband notation is used. We consider a MIMO system with M transmit and N receive antennas. The corresponding discrete-time channel model for quasi-static frequency-flat fading is given by

$$\mathbf{y}[k] = \mathbf{H}\mathbf{x}[k] + \mathbf{n}[k], \quad (1)$$

where k denotes the discrete time index, $\mathbf{y}[k]$ the $(N \times 1)$ -received vector, \mathbf{H} the $(N \times M)$ -channel matrix, $\mathbf{x}[k]$ the $(M \times 1)$ -transmitted vector, and $\mathbf{n}[k]$ an $(N \times 1)$ -noise vector. It is assumed that \mathbf{H} , $\mathbf{x}[k]$ and $\mathbf{n}[k]$ are statistically independent. The channel matrix \mathbf{H} is assumed to be constant over an entire data block spanning K subsequent time indices, and changes randomly from one data block to the next. The entries h_{ji} of \mathbf{H} ($i = 1, \dots, M$, $j = 1, \dots, N$) are assumed to be zero-mean (circularly symmetric) complex Gaussian random variables with variance $\sigma_h^2/2$ per real dimension, i.e. $h_{ji} \sim \mathcal{CN}\{0, \sigma_h^2\}$ (Rayleigh fading).¹ The instantaneous realizations of the channel matrix \mathbf{H} are assumed to be perfectly known at the receiver. The entries $x_i[k]$ of $\mathbf{x}[k]$ are zero-mean random variables drawn from a finite symbol alphabet \mathcal{A} , and the entries of $\mathbf{n}[k]$ are zero-mean, spatially and temporally white complex Gaussian random variables with variance $\sigma_n^2/2$ per real dimension, i.e., $n_j[k] \sim \mathcal{CN}\{0, \sigma_n^2\}$ and $\mathbb{E}\{\mathbf{n}[k] \mathbf{n}^H[k']\} = \sigma_n^2 \cdot \delta[k-k'] \cdot \mathbf{I}_N$.

The spatial correlation between two channel coefficients h_{ji} and $h_{j'i'}$ is defined as

$$\rho_{ij,i'j'} := \mathbb{E}\{h_{ji} h_{j'i'}^*\} / \sigma_h^2 = \rho_{i'j',ij}^*. \quad (2)$$

(Note that the magnitude of $\rho_{ij,i'j'}$ is always between zero and one.) Moreover, we define

$$\mathbf{R}_{\text{Tx}} := \mathbb{E}\{\mathbf{H}^H \mathbf{H}\} / (N \sigma_h^2), \quad \mathbf{R}_{\text{Rx}} := \mathbb{E}\{\mathbf{H} \mathbf{H}^H\} / (M \sigma_h^2), \quad (3)$$

where \mathbf{R}_{Tx} denotes the transmitter correlation matrix and \mathbf{R}_{Rx} the receiver correlation matrix ($\text{tr}(\mathbf{R}_{\text{Tx}}) = M$, $\text{tr}(\mathbf{R}_{\text{Rx}}) = N$).

¹For simplicity, we assume equal variances for the individual channel coefficients h_{ji} . A generalization to unequal variances is, however, straight forward.

Within the scope of this paper, the Kronecker-correlation model [5] is used. This means that (i) the transmit antenna correlations $\rho_{ij,i'j'} =: \rho_{\text{Tx},ii'}$ ($i, i' = 1, \dots, M$) do not depend on the specific receive antenna j under consideration, (ii) the receive antenna correlations $\rho_{ij,i'j'} =: \rho_{\text{Rx},jj'}$ ($j, j' = 1, \dots, N$) do not depend on the specific transmit antenna i under consideration, and (iii) the spatial correlations $\rho_{ij,i'j'}$ can be written as the product $\rho_{ij,i'j'} := \rho_{\text{Tx},ii'} \cdot \rho_{\text{Rx},jj'}$. Altogether, the overall spatial correlation matrix $\mathbf{R} := \mathbb{E}\{\text{vec}(\mathbf{H}) \text{vec}(\mathbf{H})^H\} / \sigma_h^2$ of size $(MN \times MN)$ can be written as the Kronecker product

$$\mathbf{R} = \mathbf{R}_{\text{Tx}} \otimes \mathbf{R}_{\text{Rx}}, \quad (4)$$

$$\mathbf{R}_{\text{Tx}} := [\rho_{\text{Tx},ii'}]_{i,i'=1,\dots,M}, \quad \mathbf{R}_{\text{Rx}} := [\rho_{\text{Rx},jj'}]_{j,j'=1,\dots,N}. \quad (5)$$

Moreover, the channel matrix \mathbf{H} can be written as

$$\mathbf{H} := \mathbf{R}_{\text{Rx}}^{1/2} \mathbf{G} \mathbf{R}_{\text{Tx}}^{1/2}, \quad (6)$$

where \mathbf{G} denotes an $(N \times M)$ -matrix with spatially uncorrelated entries $g_{ji} \sim \mathcal{CN}\{0, \sigma_h^2\}$. The square-roots $\mathbf{R}_{\text{Tx}}^{1/2}$ and $\mathbf{R}_{\text{Rx}}^{1/2}$ can be obtained via the eigenvalue decompositions of \mathbf{R}_{Tx} and \mathbf{R}_{Rx} (e.g., by means of the Jacobian algorithm [26, Ch. 8.4]):

$$\mathbf{R}_{\text{Tx}}^{1/2} := \mathbf{U}_{\text{Tx}} \mathbf{\Lambda}_{\text{Tx}}^{1/2} \mathbf{U}_{\text{Tx}}^H, \quad \mathbf{R}_{\text{Rx}}^{1/2} := \mathbf{U}_{\text{Rx}} \mathbf{\Lambda}_{\text{Rx}}^{1/2} \mathbf{U}_{\text{Rx}}^H, \quad (7)$$

where $\mathbf{\Lambda}_{\text{Tx}}$, $\mathbf{\Lambda}_{\text{Rx}}$ are diagonal matrices containing the (real-valued) eigenvalues $\lambda_{\text{Tx},i}$ and $\lambda_{\text{Rx},j}$ of \mathbf{R}_{Tx} and \mathbf{R}_{Rx} , respectively, and \mathbf{U}_{Tx} , \mathbf{U}_{Rx} are unitary matrices containing the corresponding eigenvectors ($\mathbf{U}_{\text{Tx}} \mathbf{U}_{\text{Tx}}^H = \mathbf{I}_M$, $\mathbf{U}_{\text{Rx}} \mathbf{U}_{\text{Rx}}^H = \mathbf{I}_N$). Note that the eigenvalues $\lambda_{\text{Tx},i}$ and $\lambda_{\text{Rx},j}$ are always greater or equal to zero [27, Ch. 1.5]. Since $\mathbf{\Lambda}_{\text{Tx}}$ and $\mathbf{\Lambda}_{\text{Rx}}$ are diagonal, $\mathbf{\Lambda}_{\text{Tx}}^{1/2}$ and $\mathbf{\Lambda}_{\text{Rx}}^{1/2}$ are also diagonal and contain the (non-negative) square-roots of the eigenvalues $\lambda_{\text{Tx},i}$ and $\lambda_{\text{Rx},j}$, respectively.

III. STATISTICAL TRANSMIT POWER ALLOCATION FOR SPATIALLY CORRELATED MIMO SYSTEMS

The transmission model under consideration is depicted in Fig. 1. The information symbols $a[k]$ are space-time encoded, which yields an $(M \times 1)$ -vector $\mathbf{x}''[k]$. (The information symbols are assumed to be drawn (randomly) from a Q -ary symbol alphabet.) Within the scope of this paper, we focus on orthogonal space-time block codes (OSTBCs) [3],[4] such as the well-known Alamouti-STBC for $M=2$ transmit antennas. (However, the statistical transmit power allocation scheme considered here can be used in conjunction with any other space-time coding technique.) In the case of flat fading, the information symbols $a[k]$ can be recovered at the receiver by means of a simple linear detection operation based on an equivalent channel matrix \mathbf{H}_{eq} [3],[4].² For simplicity, the entries $x_i''[k]$ of the space-time encoded vector $\mathbf{x}''[k]$ are assumed to have zero means and equal variances. Moreover, we assume an overall power constraint of P/N , i.e., $\mathbb{E}\{|x_i''[k]|^2\} := P/(MN)$ for all $i = 1, \dots, M$. (Thus, a fair comparison is possible between systems with different numbers of antennas.) Typically, the entries

²The transmission system can easily be generalized to the case of frequency-selective fading, by using a bank of appropriate a-posteriori probability (APP) equalizers [28] at the receiver (one for each receive antenna). In a final step, the APP values provided by the individual equalizers have to be combined accordingly (e.g., in the case of log-likelihood ratios, by a summation). Alternatively, one might replace the OSTBC by a space-time coding scheme suitable for frequency-selective fading, such as (generalized) delay diversity [28]-[30] or the time-reversal STBC in [31].

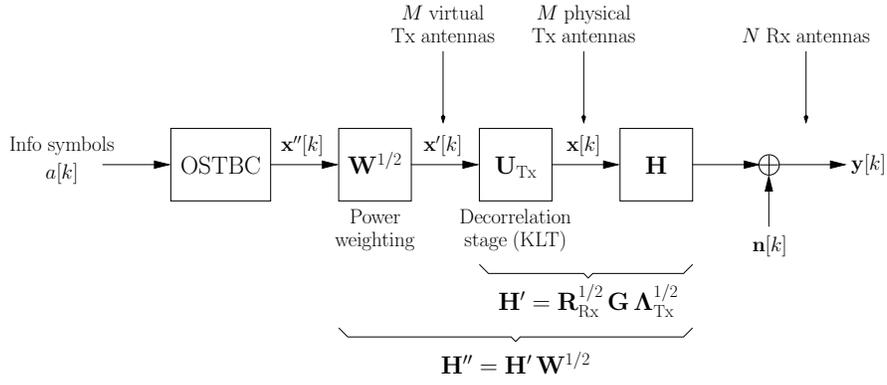


Fig. 1. Transmission model: OSTBC system with statistical transmit power allocation for spatially correlated MIMO channels.

of $\mathbf{x}''[k]$ are statistically independent random variables (only across the individual transmit antennas, not in time direction), i.e., $\mathbb{E}\{\mathbf{x}''[k] \mathbf{x}''^H[k]\} = P/(MN) \cdot \mathbf{I}_M$.

The statistical transmit power allocation scheme consists of an inner decorrelation stage based on the KLT and an outer power weighting stage with weighting matrix

$$\mathbf{W} := \text{diag}([w_1, \dots, w_M]), \quad \text{tr}(\mathbf{W}) = M. \quad (8)$$

The overall transmission model can be written as (cf. Fig. 1):

$$\begin{aligned} \mathbf{y}[k] &= \mathbf{H} \mathbf{x}[k] + \mathbf{n}[k] = \mathbf{H} \mathbf{U}_{\text{Tx}} \mathbf{W}^{1/2} \mathbf{x}''[k] + \mathbf{n}[k] \\ &=: \mathbf{H}' \mathbf{x}'[k] + \mathbf{n}[k] =: \mathbf{H}'' \mathbf{x}''[k] + \mathbf{n}[k], \end{aligned} \quad (9)$$

with $\mathbf{H}' := \mathbf{H} \mathbf{U}_{\text{Tx}}$, $\mathbf{x}'[k] := \mathbf{W}^{1/2} \mathbf{x}''[k]$, and $\mathbf{H}'' := \mathbf{H}' \mathbf{W}^{1/2}$. (Note that the detection of the space-time encoded information symbols has to be carried out based on the system model $\mathbf{y}[k] = \mathbf{H}'' \mathbf{x}''[k] + \mathbf{n}[k]$.) The decorrelation stage transforms the given channel matrix \mathbf{H} according to (6) into a semi-correlated channel matrix $\mathbf{H}' = \mathbf{R}_{\text{Rx}}^{1/2} \mathbf{G} \mathbf{\Lambda}_{\text{Tx}}^{1/2}$, by using the unitary matrix \mathbf{U}_{Tx} from the eigenvalue decomposition of \mathbf{R}_{Tx} as a precoding matrix:

$$\mathbb{E}\{\mathbf{H}'^H \mathbf{H}'\} = \mathbf{U}_{\text{Tx}}^H \mathbb{E}\{\mathbf{H}^H \mathbf{H}\} \mathbf{U}_{\text{Tx}} = N \sigma_h^2 \mathbf{\Lambda}_{\text{Tx}}. \quad (10)$$

The channel matrix \mathbf{H}' is often called *virtual* channel matrix in the literature, and the M inputs to the precoding matrix \mathbf{U}_{Tx} (vector $\mathbf{x}'[k]$) represent *virtual* transmit antennas. Finally, note that due to the trace constraint on \mathbf{W} , the transmitted vector $\mathbf{x}[k]$ will always meet the same overall power constraint as the vector $\mathbf{x}''[k]$, i.e., $\text{tr}(\mathbb{E}\{\mathbf{x}[k] \mathbf{x}^H[k]\}) = P/N$.

A. Signal-to-Noise Ratio Gain at the Receiver

If the transmitter correlation matrix \mathbf{R}_{Tx} is known at the transmitter (and thus the corresponding eigenvalue decomposition), the power weights w_1, \dots, w_M can be optimized with respect to the eigenvalues $\lambda_{\text{Tx},1}, \dots, \lambda_{\text{Tx},M}$ of \mathbf{R}_{Tx} . By this means the overall received SNR can be improved, which is seen by considering the covariance matrix of the received vector:

$$\begin{aligned} &\mathbb{E}\{\mathbf{y}[k] \mathbf{y}[k]^H\} \\ &= \mathbb{E}\left\{\mathbf{H}' \mathbf{W}^{1/2} \mathbb{E}\{\mathbf{x}''[k] \mathbf{x}''^H[k]\} \mathbf{W}^{1/2 H} \mathbf{H}'^H\right\} + \sigma_n^2 \mathbf{I}_N \\ &= \frac{P}{MN} \cdot \mathbb{E}\left\{\mathbf{H} \mathbf{U}_{\text{Tx}} \mathbf{W} \mathbf{U}_{\text{Tx}}^H \mathbf{H}^H\right\} + \sigma_n^2 \mathbf{I}_N \\ &= \frac{P}{MN} \cdot \mathbf{R}_{\text{Rx}}^{1/2} \mathbb{E}\left\{\mathbf{G} \mathbf{U}_{\text{Tx}} \mathbf{W} \mathbf{\Lambda}_{\text{Tx}} \mathbf{U}_{\text{Tx}}^H \mathbf{G}^H\right\} \mathbf{R}_{\text{Rx}}^{1/2 H} + \sigma_n^2 \mathbf{I}_N \end{aligned}$$

$$\begin{aligned} &= \underbrace{\frac{P \sigma_h^2}{MN} \sum_{i=1}^M w_i \lambda_{\text{Tx},i} \mathbf{R}_{\text{Rx}}}_{=: f_{\text{snr}}(\mathbf{W}, \mathbf{\Lambda}_{\text{Tx}})} + \sigma_n^2 \mathbf{I}_N. \end{aligned} \quad (11)$$

In the case of equal power allocation, i.e., $\mathbf{W} = \mathbf{I}_M$, one obtains $f_{\text{snr}}(\mathbf{W}, \mathbf{\Lambda}_{\text{Tx}}) = P \sigma_h^2 / N$ (since $\text{tr}(\mathbf{\Lambda}_{\text{Tx}}) = M$). However, using an appropriate transmit power allocation strategy, it is possible to achieve an overall SNR gain ($f_{\text{snr}}(\mathbf{W}, \mathbf{\Lambda}_{\text{Tx}}) > P \sigma_h^2 / N$).

Note that the transmitter requires solely knowledge about the second order statistics of the MIMO channel (in terms of the transmitter correlation matrix \mathbf{R}_{Tx}), in order to accomplish the SNR gain. No knowledge about specific channel realizations is required. Apart from the overall received SNR, the achieved diversity advantage is of major interest (especially for high SNR values), which is discussed next.

B. Diversity Advantage

In the sequel, the error performance of a spatially correlated OSTBC system with statistical transmit power allocation is evaluated, so as to study the diversity advantage for high SNRs.

To start with, we consider the case of equal power allocation and study the impact of the transmitter- and receiver-sided correlation on the error performance of the OSTBC system. The OSTBC (in conjunction with the appropriate linear detection step at the receiver) transforms the $(M \times N)$ -MIMO system (1) into an equivalent single-antenna system of form [32]

$$z[k] = \left(\sum_{i=1}^M \sum_{j=1}^N |h_{ji}|^2 \right) a[k] + w[k], \quad (12)$$

where $z[k]$ denotes the k th received symbol after the linear detection step, $a[k]$ the k th information symbol, and $w[k]$ an additive white Gaussian noise (AWGN) sample. Correspondingly, the $(M \times N)$ -OSTBC system is equivalent to an $(1 \times MN)$ maximum-ratio-combining (MRC) system [33], where we assume that (i) the OSTBC provides a temporal rate of 1 symbol/channel use ('full rate')³ and (ii) the underlying overall

³It should be noted that full-rate OSTBCs exist solely for two transmit antennas (Alamouti-STBC) [34]. However, since in this paper focus is on the statistical transmit power allocation stage, we will always assume a full-rate OSTBC for simplicity.

received energy per information symbol, E_s , after linear detection/ MRC is the same in both systems. Using the average power constraint $P/(MN)$ from Section II for the transmitted symbols, the overall received SNR after linear detection/ MRC results as $P\sigma_h^2/\sigma_n^2 =: E_s/N_0$, where N_0 denotes the single-sided noise power density.

The error performance of a spatially correlated $(1 \times MN)$ -MRC system (and thus of the corresponding $(M \times N)$ -OSTBC-system) can in turn be analyzed by means of the KLT. Consider the following $(1 \times MN)$ -system:

$$\mathbf{y}[k] = \mathbf{h} a[k] + \mathbf{n}[k]. \quad (13)$$

(For the $(1 \times MN)$ -channel vector \mathbf{h} and the $(1 \times MN)$ -noise vector $\mathbf{n}[k]$, the same statistical properties are assumed as in Section II.) Let $\mathbf{R} := E\{\mathbf{h}\mathbf{h}^H\}/\sigma_h^2$ denote the overall spatial correlation matrix, which corresponds to the Kronecker product of the transmitter and receiver correlation matrix in the associated $(M \times N)$ -OSTBC system, cf. (4). Based on the eigenvalue decomposition $\mathbf{R} := \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, the system (13) is decorrelated as

$$\mathbf{y}'[k] := \mathbf{U}^H \mathbf{y}[k] =: \mathbf{h}' a[k] + \mathbf{n}'[k], \quad (14)$$

where $E\{\mathbf{h}'\mathbf{h}'^H\} = \mathbf{\Lambda}$ and $E\{\mathbf{n}'[k]\mathbf{n}'^H[k]\} = \sigma_n^2 \mathbf{I}_N$. As can be seen, the decorrelated system is characterized by unequal average link SNRs determined by the eigenvalues $\lambda_1, \dots, \lambda_{MN}$ of \mathbf{R} . In [35] it was shown that the two systems (13) and (14) are equivalent in the sense that MRC provides the same average symbol error rate in both cases.

In the following, we focus on binary antipodal transmission⁴ (i.e., $a[k] \in \{\pm 1\}$). The average bit error rate (BER) of the (decorrelated) MRC-system – and thus of the associated OSTBC-system – can be calculated in closed form, according to [36, Ch. 14.5]

$$\bar{P}_b = \frac{1}{2} \sum_{j=1}^{MN} \left(\prod_{\substack{j'=1 \\ j' \neq j}}^{MN} \frac{\gamma_j}{\gamma_j - \gamma_{j'}} \right) \left(1 - \sqrt{\frac{\gamma_j}{1 + \gamma_j}} \right), \quad (15)$$

where $\gamma_j := P \sigma_h^2 \lambda_j / (MN \sigma_n^2)$, $j = 1, \dots, MN$, denotes the average SNR for the j th receive antenna. (The overall average SNR is given by $\gamma := \gamma_1 + \dots + \gamma_{MN} = E_s/N_0$.) A high-SNR approximation ($\sigma_n^2 \rightarrow 0$) of (15) yields [36, Ch. 14.5]

$$\bar{P}_b \approx \left(\frac{MN}{4\gamma} \right)^{MN} \binom{2MN-1}{MN} \prod_{j=1}^{MN} \frac{1}{\lambda_j}, \quad (16)$$

where it was assumed that all eigenvalues of the correlation matrix \mathbf{R} are greater than zero. Two important observations can be made in (16): (i) Asymptotically, \bar{P}_b is always proportional to γ^{-MN} , i.e., the diversity order of the system is not reduced as long as the correlation matrix \mathbf{R} has full rank; (ii) the product term in (16), which is solely determined by the eigenvalues of \mathbf{R} , causes an asymptotic up-shift of the BER curve (in a log-log plot): As shown in [22], the product term is always greater or equal to one (and it is only equal to one in the uncorrelated case, i.e., for $\mathbf{\Lambda} = \mathbf{I}_{MN}$).

⁴Channel coding is not taken into account. However, an outer channel coding scheme can be added to further improve performance.

Since \mathbf{R} is the Kronecker product of the transmitter correlation matrix \mathbf{R}_{Tx} and the receiver correlation matrix \mathbf{R}_{Rx} in the associated $(M \times N)$ -OSTBC system, the set of eigenvalues $\{\lambda_j | j = 1, \dots, MN\}$ of \mathbf{R} is given by all pairwise products $\{\lambda_{Tx,i} \cdot \lambda_{Rx,j} | i = 1, \dots, M, j = 1, \dots, N\}$ of the eigenvalues of \mathbf{R}_{Tx} and \mathbf{R}_{Rx} [37, Ch. 12.2]. Therefore, based on (15) the average BER of a spatially correlated $(M \times N)$ -OSTBC system with statistical transmit power allocation results as

$$\bar{P}_b = \frac{1}{2} \sum_{i=1}^M \sum_{j=1}^N \left(\prod_{\substack{i'=1 \\ (i',j') \neq (i,j)}}^M \prod_{j'=1}^N \frac{1}{1 - \frac{w_{i'} \lambda_{Tx,i'} \lambda_{Rx,j'}}{w_i \lambda_{Tx,i} \lambda_{Rx,j}}} \right) \times \left(1 - \sqrt{\frac{P \sigma_h^2 w_i \lambda_{Tx,i} \lambda_{Rx,j}}{MN \sigma_n^2 + P \sigma_h^2 w_i \lambda_{Tx,i} \lambda_{Rx,j}}} \right), \quad (17)$$

where it was assumed that $w_i > 0$ for all $i = 1, \dots, M$. The corresponding high-SNR approximation ($\sigma_n^2 \rightarrow 0$) is given by

$$\bar{P}_b \approx \left(\frac{MN}{4\gamma} \right)^{MN} \binom{2MN-1}{MN} \underbrace{\prod_{i=1}^M \prod_{j=1}^N \frac{1}{w_i \lambda_{Tx,i} \lambda_{Rx,j}}}_{=: f_{\text{div}}(\mathbf{W}, \mathbf{\Lambda}_{Tx}, \mathbf{\Lambda}_{Rx})}. \quad (18)$$

Correspondingly, in order to maximize the diversity advantage for high SNR values, the transmit power weights w_i should be chosen such that the term $f_{\text{div}}(\mathbf{W}, \mathbf{\Lambda}_{Tx}, \mathbf{\Lambda}_{Rx})$ is minimized.

C. Appropriate Transmit Power Allocation Strategies

In order to maximize the overall received SNR, i.e., to maximize the term $f_{\text{snr}}(\mathbf{W}, \mathbf{\Lambda}_{Tx})$ in (11), the optimal power allocation strategy is to concentrate the complete transmit power on the strongest eigenvalue $\lambda_{Tx, \max}$ of the transmitter correlation matrix \mathbf{R}_{Tx} , i.e., $\mathbf{W}_{\text{opt}} = \text{diag}([0, \dots, 0, M, 0, \dots, 0])$ [14],[15]. This power allocation strategy is in the sequel denoted as (one-dimensional) eigen-beamforming (EBF).

Similarly, in order to maximize the diversity advantage for high SNR values, i.e., to minimize the product term $f_{\text{div}}(\mathbf{W}, \mathbf{\Lambda}_{Tx}, \mathbf{\Lambda}_{Rx})$ in (18), the optimal power allocation strategy is to use equal power allocation (EPA), i.e., $\mathbf{W}_{\text{opt}} = \mathbf{I}_M$ [14],[15]. In other words, the asymptotic up-shift of the BER curve caused by the product term in (16) cannot be reduced by any statistical transmit power allocation strategy. Moreover, any statistical transmit power allocation scheme that yields an SNR gain with respect to the EPA scheme will at the same time lower the diversity advantage at high SNR values.

For arbitrary SNR values, the optimal statistical transmit power allocation strategy – in terms of a minimum symbol error probability – was derived in [14]. The result is a waterfilling solution with respect to the inverse eigenvalues $1/\lambda_{Tx,i}$ of the transmitter correlation matrix \mathbf{R}_{Tx} :

$$w_{i, \text{opt}} = M \left[\frac{1}{M'} - \frac{1}{\gamma} \left(\frac{1}{\lambda_{Tx,i}} - \frac{1}{M'} \sum_{i'=1}^{M'} \frac{1}{\lambda_{Tx,i'}} \right) \right]_+, \quad (19)$$

where $[x]_+ := \max\{0, x\}$ and M' denotes the number of virtual antennas actually used (i.e., the number of power weights

$w_{i,\text{opt}} > 0$). Note that the optimal waterfilling solution depends on the overall SNR $\gamma = E_s/N_0$. For high SNR values, the waterfilling solution tends to the EPA-solution, and for low SNR values one obtains the EBF-solution.

As an alternative to the optimal waterfilling solution (19), we propose to use a transmitter-sided MRC scheme, where the eigenvalues $\lambda_{T_x,i}$ themselves are used as weighting factors ($\mathbf{W} := \mathbf{\Lambda}_{T_x}$), i.e., strong eigenvalues are strongly weighted and weak eigenvalues are weakly weighted. As will be shown in the next section, the simple MRC scheme yields a near-optimum performance over a wide SNR range.

D. Numerical Results

As an example, we consider a (4×1)-OSTBC system with a transmitter correlation matrix $\mathbf{R}_{T_x} \neq \mathbf{I}_4$. Specifically, we use a single-parameter correlation matrix

$$\mathbf{R}_{M,\rho} := \begin{bmatrix} 1 & \rho & \rho^4 & \dots & \rho^{(M-1)^2} \\ \rho^* & 1 & \rho & \dots & \rho^{(M-2)^2} \\ \rho^{4*} & \rho^* & & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ \rho^{(M-1)^2*} & \rho^{(M-2)^2*} & \dots & \dots & 1 \end{bmatrix} \quad (20)$$

($\rho \in \mathbb{C}$) for \mathbf{R}_{T_x} , which was proposed in [38] for uniform linear antenna arrays with M antenna elements. In the sequel, we set $\rho := 0.8$ (real-valued). Fig. 2 displays the BER performance as a function of E_s/N_0 in dB, which results for the different transmit power allocation strategies discussed above (solid lines).

For low SNR values the EBF scheme is best (as expected), although the difference to the MRC scheme is barely visible.⁵ In fact, the MRC scheme provides a good performance over the complete SNR range under consideration and is quite close to the optimal waterfilling solution according to (19). Depending on the SNR value the MRC scheme provides a gain of up to 2 dB over the EBF scheme/ the EPA scheme. Interestingly, even for SNR values up to 15 dB the MRC scheme still outperforms the EPA scheme. However, for larger SNR values the EPA scheme becomes superior (not shown).

IV. IMPACT OF ESTIMATION ERRORS

So far, we have assumed that the correlation matrix \mathbf{R}_{T_x} is perfectly known at the transmitter. In the case of estimation errors, the statistical transmit power allocation scheme will be based on an erroneous transmitter correlation matrix $\hat{\mathbf{R}}_{T_x}$. In general, both the eigenvectors and the eigenvalues of $\hat{\mathbf{R}}_{T_x}$ will be different from those of the actual correlation matrix \mathbf{R}_{T_x} , i.e.,

$$\hat{\mathbf{R}}_{T_x} = \hat{\mathbf{U}}_{T_x} \hat{\mathbf{\Lambda}}_{T_x} \hat{\mathbf{U}}_{T_x}^H, \quad (21)$$

where $\hat{\mathbf{U}}_{T_x} \neq \mathbf{U}_{T_x}$ and $\hat{\mathbf{\Lambda}}_{T_x} \neq \mathbf{\Lambda}_{T_x}$. (We assume that $\hat{\mathbf{R}}_{T_x}$ is still a Hermitian matrix.) This has two effects: First, the transmitter will use a mismatched decorrelation stage $\hat{\mathbf{U}}_{T_x}$. If $\hat{\mathbf{U}}_{T_x} \neq \mathbf{U}_{T_x}$, the product $\hat{\mathbf{U}}_{T_x}^H \mathbf{U}_{T_x}$ does not yield the identity

⁵Note that the BER curve of the EBF scheme has the same asymptotic slope as the curve of the single-antenna system, because the EBF scheme reduces the diversity order from four to one. However, compared to the single-antenna system an SNR gain of $10 \log_{10}(2.72)$ dB ≈ 4.4 dB is achieved, since the maximum eigenvalue of \mathbf{R}_{T_x} is given by $\lambda_{T_x,\text{max}} = 2.72$.

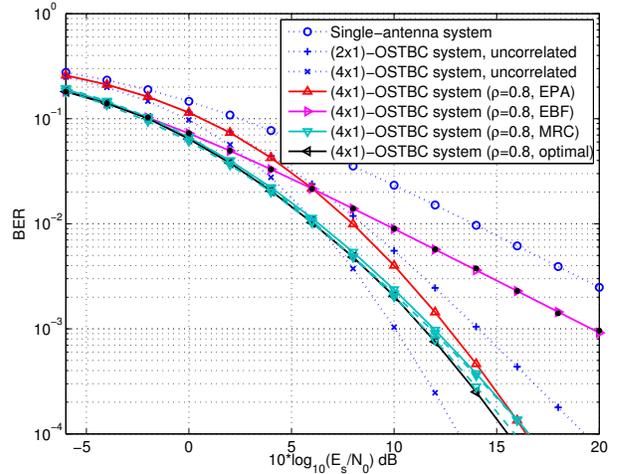


Fig. 2. BER performance of a (4×1)-OSTBC system with different statistical transmit power allocation strategies (analytical results according to (17)): Binary transmission, flat Rayleigh fading, transmitter correlation matrix $\mathbf{R}_{T_x} = \mathbf{R}_{M,\rho}$ with $M=4$ and $\rho=0.8$. All analytical results were validated by means of Monte Carlo simulations. As an example, simulation results are included for the EBF scheme (marked by black dots).

matrix, i.e., equation (10) does not hold anymore and generalizes to

$$\mathbb{E} \{ \mathbf{H}^H \mathbf{H} \} = N \sigma_h^2 \hat{\mathbf{U}}_{T_x}^H \mathbf{U}_{T_x} \mathbf{\Lambda}_{T_x} \mathbf{U}_{T_x}^H \hat{\mathbf{U}}_{T_x}. \quad (22)$$

Second, the transmit power allocation stage will be based on an erroneous eigenvalue matrix $\hat{\mathbf{\Lambda}}_{T_x}$. This means that a mismatched weighting matrix $\hat{\mathbf{W}}$ will be used, which might lower the obtained performance gains. Altogether, there will be an overall mismatch in the power weighting, which is captured by the diagonal elements ξ_{ii} ($i=1, \dots, M$) of the matrix

$$\Xi := \hat{\mathbf{U}}_{T_x}^H \mathbf{U}_{T_x} \hat{\mathbf{W}} \mathbf{\Lambda}_{T_x} \mathbf{U}_{T_x}^H \hat{\mathbf{U}}_{T_x}. \quad (23)$$

(If \mathbf{R}_{T_x} is perfectly known at the transmitter, one obtains $\Xi = \mathbf{W} \mathbf{\Lambda}_{T_x}$.)

In the following, the impact of estimation errors concerning the transmitter correlation matrix is illustrated by means of a simple example. For this purpose, we assume that the transmitter correlation matrix \mathbf{R}_{T_x} is of form (20), and that a direct estimate $\hat{\rho}$ of the correlation parameter $\rho := |\rho|e^{j\phi}$ is available at the transmitter. Correspondingly, the statistical transmit power allocation scheme will be based on an erroneous transmitter correlation matrix $\hat{\mathbf{R}}_{T_x} = \mathbf{R}_{M,\hat{\rho}}$.

In the case $M=2$, the diagonal entries of Ξ can be calculated in closed form: For $\mathbf{R}_{T_x} = \mathbf{R}_{2,\rho}$, one obtains

$$\mathbf{U}_{T_x} = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{j\phi} & -e^{j\phi} \\ 1 & 1 \end{bmatrix}, \quad \mathbf{\Lambda}_{T_x} = \begin{bmatrix} 1+|\rho| & 0 \\ 0 & 1-|\rho| \end{bmatrix}. \quad (24)$$

Thus, the diagonal entries of Ξ result as

$$\xi_{11} = \frac{1}{2} \left(\hat{w}_1 (1 + \cos \Delta\phi) + \hat{w}_2 (1 - \cos \Delta\phi) \right) \lambda_{T_x,1}, \quad (25)$$

$$\xi_{22} = \frac{1}{2} \left(\hat{w}_1 (1 - \cos \Delta\phi) + \hat{w}_2 (1 + \cos \Delta\phi) \right) \lambda_{T_x,2}, \quad (26)$$

where $\Delta\phi := \phi - \hat{\phi}$. Obviously, for perfect knowledge of \mathbf{R}_{T_x} we have $\hat{w}_{1,2} = w_{1,2}$ and $\Delta\phi = 0$, i.e., $\xi_{ii} = w_i \lambda_{T_x,i}$ ($i=1, 2$).

Next, we consider the case $M=4$. As an example, numerical results for the (4×1) -OSTBC system considered in Section III-D have been included in Fig. 2 ($\rho=0.8$) for the case of the MRC scheme (dashed lines). Specifically, for $|\hat{\rho}|$ values of 0.9ρ and 1.1ρ were assumed, and for $\Delta\phi$ a value of 0.1 rad. As can be seen, the BER performance of the MRC scheme is quite robust with regard to these estimation errors.⁶ (Moreover, since the MRC scheme is not optimal, estimation errors can even improve the performance slightly.)

V. CONCLUSIONS

In this paper, a simple statistical transmit power allocation scheme for spatially correlated MIMO systems has been considered, which consists of an inner decorrelation stage based on the Karhunen-Loève transform and an outer power weighting stage. The considered scheme requires solely statistical knowledge of the MIMO channel, which can easily be acquired in practical systems. It was shown that an appropriate choice of the power allocation strategy offers significant performance improvements compared to equal power allocation, especially in the case of low SNR values. Finally, the impact of estimation errors was investigated, and it was shown that the considered scheme yields quite a robust performance.

ACKNOWLEDGMENT

The authors would like to thank Ronald Böhnke (Dept. of Communications Engineering, University of Bremen) for fruitful discussions.

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⁶Note that the estimation errors considered here are already quite large. Usually, such an estimation accuracy can easily be achieved, for example, by averaging the channel coefficients over a reasonable number of channel realizations.

