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Equivalence of Spatially Correlated and Distributed MIMO Systems

Jan Mietzner and Peter A. Hoeher

Information and Coding Theory Lab, Faculty of Engineering, University of Kiel, Kaiserstrasse 2, D-24143 Kiel, Germany

URL: www-ict.tf.uni-kiel.de E-mail: {jm,ph}@tf.uni-kiel.de

***Abstract*— Wireless communication systems with multiple antennas, so-called multiple-input multiple-output (MIMO) systems, offer huge advantages over conventional single-antenna systems – both with regard to capacity and error performance. Typically, quite restrictive assumptions are made in the literature concerning the antenna spacings at transmitter and receiver: On the one hand, one normally assumes that the individual antenna elements are co-located, i.e., they are part of some antenna array. On the other hand, it is often assumed that antenna spacings are sufficiently large so as to justify the assumption of uncorrelated antennas. From numerous publications it is known that spatially correlated links lead to a loss in capacity and error performance. We show that this is also the case when the transmit and/or the receive antennas are spatially distributed on a larger scale. (Possible applications include simulcast networks, reach-back links for wireless sensors, as well as relay-assisted wireless networks.) Specifically, we show that any spatially correlated system that obeys the so-called Kronecker-correlation model can be transformed into an equivalent (with regard to the capacity distribution) spatially distributed system, and vice versa. Correspondingly, both types of MIMO system can be treated in a single unified framework. We also prove (asymptotic) equivalence with regard to the pairwise error probability of space-time codes. Moreover, we consider a simple performance measure originally proposed for spatially correlated systems and find the equivalent measure for distributed systems. Finally, we discuss appropriate transmit power allocation schemes that are based on second-order channel statistics.**

***Index Terms*— Wireless communications, MIMO systems, spatial correlation, distributed antennas, virtual antenna arrays.**

I. Introduction

Wireless communication systems with multiple antennas, so-called multiple-input multiple-output (MIMO) systems, have gained much attention during the last decade, because they offer huge advantages over conventional single-antenna systems. On the one hand, it was shown in [1]-[3] that the capacity of a MIMO system with M transmit (Tx) antennas and N receive (Rx) antennas grows linearly with $\min\{M, N\}$. Correspondingly, multiple antennas provide a promising means to increase the spectral efficiency of a system. On the other hand, it was shown in [4]-[6] that multiple antennas can also be utilized in order to provide a spatial diversity gain and thus to improve the error performance of a system.

The results in [1]-[6] are based on quite restrictive assumptions with regard to the antenna spacings at transmitter and receiver: On the one hand, it is assumed that the individual antenna elements are co-located, i.e., they are part of some antenna array (cf. Fig. 1 (a)). On the other hand, the antenna spacings are assumed to be sufficiently large so as to justify the assumption of independent fading on the individual transmission links. In [7]-[9], it was shown that spatial fading correlation, caused by insufficient antenna spacings (cf. Fig. 1 (b)), can lead to significant degradations in capacity and error performance.[1]

[1] Note that the notion of insufficient antenna spacings is relative, because spatial correlation effects are not only governed by the geometry of the antenna arrays, but also by the richness of scattering from the physical environment, the angular power distribution of the transmitted/ received signal, and the employed carrier frequency.

In this paper, we show that this is also the case when the individual transmit and/or receive antennas are distributed on a larger scale (cf. Fig. 1 (c)).

In such distributed MIMO systems, multiple distributed transmitting or receiving nodes cooperate in terms of a joint transmission/ reception strategy and thus establish a virtual antenna array. By this means, the cooperating nodes – possibly equipped with just a single antenna – can enjoy some of the benefits offered by conventional MIMO systems. Examples of distributed MIMO systems include (i) simulcast networks for broadcasting or paging applications [10], where distributed transmitting nodes (e.g., multiple base stations) perform a joint transmission scheme; (ii) reach-back links for wireless sensors, where measured data of wireless sensors are collected by multiple distributed receiving nodes (and are then processed in a joint fashion); (iii) relay-assisted wireless networks [11],[12] where multiple wireless relays forward messages of a certain source node to a certain destination node (in a cooperative fashion).

At the first glance, MIMO systems with co-located antennas and MIMO systems with distributed antennas have little in common. However, we show that these two types of system can, in fact, be treated in a single unified framework: For the case of flat Rayleigh fading, we prove that any MIMO system with co-located antennas, which follows the so-called Kronecker-correlation model [7], can be transformed into an equivalent (with regard to the capacity distribution) MIMO system with distributed antennas, and vice versa. Moreover, with regard to space-time coding we show that (asymptotically) both MIMO systems lead to identical pairwise error probabilities (PEPs). Finally, we discuss the use of transmit power allocation schemes that are based on statistical channel knowledge. Optimal transmit power allocation strategies for spatially correlated MIMO systems were, for example, proposed in [13]-[15]. Due to the above equivalence, these schemes can be reused for distributed MIMO systems, without any loss of optimality.

The equivalence proofs presented here are based on two unitary matrix transforms: The first transform is related to the well-known Karhunen-Loève transform (KLT) [16, Ch. 8.5], which is often used in the literature in order to analyze correlated systems. The second transform was earlier established in [17]. These transforms were already used in [17], in order to prove that any spatially correlated MIMO system employing an orthogonal space-time block code (OSTBC) can be transformed into an equivalent (with regard to the average symbol error probability) distributed OSTBC system, and vice versa. The present paper therefore constitutes an extension of [17] to arbitrary (coded) MIMO systems.

A. Paper Organization

The paper is organized as follows: In Section II, the system and correlation model used throughout this paper is introduced. In Section III, the capacity distribution of co-located and distributed MIMO systems is considered. The pairwise error prob-

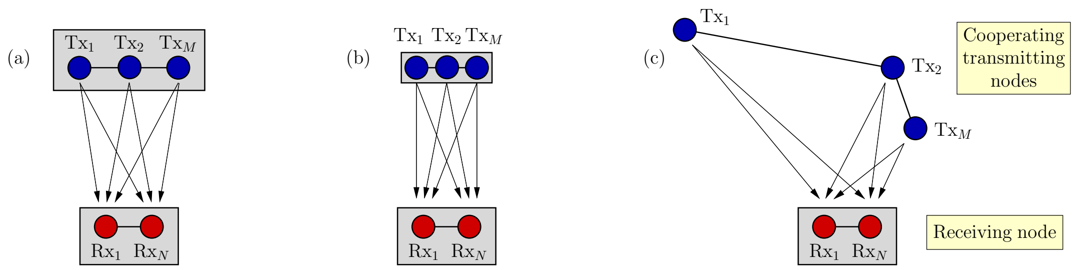

Fig. 1. MIMO systems with different antenna spacings. (a) Conventional MIMO system with co-located antennas and statistically independent links; (b) MIMO system with co-located antennas and correlated links (insufficient antenna spacing at the transmitter side); (c) MIMO system with distributed transmit antennas.

ability of space-time codes resulting for both types of system is considered in Section IV. In Section V, a simple performance measure originally proposed for spatially correlated MIMO systems is discussed [18], and the equivalent measure for distributed systems is derived. Finally, the use of statistical transmit power allocation schemes is discussed in Section VI. Conclusions are drawn in Section VII.

B. Mathematical Notation

Matrices and vectors are written in upper case and lower case bold face, respectively. If not stated otherwise, all vectors are column vectors. The complex conjugate of a complex number a is marked as a^*, and the Hermitian transposed of a matrix $\mathbf{A}$ as $\mathbf{A}^{\mathrm{H}}$. The (i,j)-th element of $\mathbf{A}$ is denoted as $[\mathbf{A}]_{i,j}$. The trace of an $(M{\times}M)$-matrix $\mathbf{A}$, i.e., the sum over all diagonal elements, is denoted as $\mathrm{tr}(\mathbf{A})$. The rank and the determinant of $\mathbf{A}$ is denoted as $\mathrm{rank}(\mathbf{A})$ and $\det(\mathbf{A})$, respectively. The square-root $\mathbf{A}^{1/2}$ of a Hermitian matrix $\mathbf{A}$ (i.e., $\mathbf{A}=\mathbf{A}^{\mathrm{H}}$) is defined as $\mathbf{A}^{1/2\,\mathrm{H}}\mathbf{A}^{1/2}=\mathbf{A}^{1/2}\mathbf{A}^{1/2\,\mathrm{H}}=\mathbf{A}$. Moreover, $||\mathbf{A}||_{\mathrm{F}}=\sqrt{\mathrm{tr}(\mathbf{A}\mathbf{A}^{\mathrm{H}})}$ denotes the Frobenius norm of $\mathbf{A}$. $\mathrm{diag}(\mathbf{a})$ is a diagonal matrix with diagonal elements given by the vector $\mathbf{a}$, and $\mathrm{vec}(\mathbf{A})$ is a vector which results from stacking the columns of an $(N{\times}M)$-matrix $\mathbf{A}$ in a joint vector. Finally, $\mathsf{E}\{.\}$ denotes statistical expectation.

II. System and Correlation Model

Throughout this paper, the complex baseband notation is used. We consider a point-to-point MIMO communication link with M transmit and N receive antennas. The transmit and receive antennas are either co-located or distributed and are assumed to have fixed positions. The discrete-time channel model for quasi-static frequency-flat fading is given by

$$\mathbf{y}[k]=\mathbf{H}\mathbf{x}[k]+\mathbf{n}[k], \tag{1}$$

where k denotes the discrete time index, $\mathbf{y}[k]$ the kth received vector of size $(N{\times}1)$, $\mathbf{H}$ the $(N{\times}M)$-channel matrix, $\mathbf{x}[k]$ the kth transmitted vector of size $(M{\times}1)$, and $\mathbf{n}[k]$ the kth additive noise vector. It is assumed that $\mathbf{H}$, $\mathbf{x}[k]$ and $\mathbf{n}[k]$ are statistically independent. The channel matrix $\mathbf{H}$ is assumed to be constant over an entire data block of length N_{b}, and changes randomly from one data block to the next. Correspondingly, we will sometimes use the following block transmission model:

$$\mathbf{Y}=\mathbf{H}\,\mathbf{X}+\mathbf{N}, \tag{2}$$

where

$$\mathbf{Y} := \big[\,\mathbf{y}[0],\,...,\,\mathbf{y}[k],\,...,\,\mathbf{y}[N_{\mathrm{b}}-1]\,\big], \tag{3}$$

$$\mathbf{X} := \big[\,\mathbf{x}[0],\,...,\,\mathbf{x}[k],\,...,\,\mathbf{x}[N_{\mathrm{b}}-1]\,\big], \tag{4}$$

$$\mathbf{N} := \big[\,\mathbf{n}[0],\,...,\,\mathbf{n}[k],\,...,\,\mathbf{n}[N_{\mathrm{b}}-1]\,\big]. \tag{5}$$

The entries h_{ji} of $\mathbf{H}$ $(i=1,...,M,\;\; j=1,...,N)$ are assumed to be zero-mean (circularly symmetric) complex Gaussian random variables with variance $\sigma_{ji}^2/2$ per real dimension, i.e. $h_{ji}\sim\mathcal{CN}\{0,\sigma_{ji}^2\}$ (Rayleigh fading). The instantaneous realizations of the channel matrix $\mathbf{H}$ are assumed to be perfectly known at the receiver. The covariance between two channel coefficients h_{ji} and $h_{j'i'}$ is denoted as

$$\sigma_{ij,i'j'} := \mathsf{E}\{h_{ji}\,h_{j'i'}^*\}=\sigma_{i'j',ij}^* \tag{6}$$

and the corresponding spatial correlation as

$$\rho_{ij,i'j'} := \sigma_{ij,i'j'}\Big/\sqrt{\sigma_{ji}^2\,\sigma_{j'i'}^2}. \tag{7}$$

(Note that $|\rho_{ij,i'j'}|$ is always between zero and one.)

The entries $x_i[k]$ of the transmitted vector $\mathbf{x}[k]$ are treated as zero-mean random variables with variance $\sigma_{x_i}^2$. (Possibly, they are correlated due to some underlying space-time code.) We assume an overall transmit power constraint of P, i.e., $\sum_i\sigma_{x_i}^2\le P$. For the time being, we consider the case of equal power allocation among the transmit antennas, i.e., $\sigma_{x_i}^2=P/M$ for all $i=1,...,M$. Finally, the entries of $\mathbf{n}[k]$ are assumed to be zero-mean, spatially and temporally white complex Gaussian random variables with variance $\sigma_{\mathrm{n}}^2/2$ per real dimension, i.e., $n_j[k]\sim\mathcal{CN}\{0,\sigma_{\mathrm{n}}^2\}$ and $\mathsf{E}\left\{\mathbf{n}[k]\,\mathbf{n}^{\mathrm{H}}[k']\right\}=\sigma_{\mathrm{n}}^2\cdot\delta[k-k']\cdot\mathbf{I}_N$.

A. MIMO System with Co-located Antennas

In the case of co-located antennas (both at the transmitter and the receiver side), all links experience – on average – similar propagation conditions. It is therefore reasonable to assume that the variance of the channel coefficients h_{ji} is the same for all transmission links. Correspondingly, we define $\sigma_{ji}^2:=\sigma^2$ for all $i=1,...,M$ and $j=1,...,N$. (A generalization to unequal variances is, however, straightforward.) Moreover, we define

$$\mathbf{R}_{\mathrm{Tx}}:=\mathsf{E}\{\mathbf{H}^{\mathrm{H}}\mathbf{H}\}/(N\sigma^2),\quad \mathbf{R}_{\mathrm{Rx}}:=\mathsf{E}\{\mathbf{H}\mathbf{H}^{\mathrm{H}}\}/(M\sigma^2), \tag{8}$$

where $\mathbf{R}_{\mathrm{Tx}}$ denotes the transmitter correlation matrix and $\mathbf{R}_{\mathrm{Rx}}$ the receiver correlation matrix $(\mathrm{tr}(\mathbf{R}_{\mathrm{Tx}})=M,\,\mathrm{tr}(\mathbf{R}_{\mathrm{Rx}})=N)$.

Throughout this paper, the so-called Kronecker-correlation model [7] is employed. This means that (i) the transmit antenna correlations $\rho_{ij,i'j}=:\rho_{\mathrm{Tx},ii'}$ $(i,i'=1,...,M)$ do not depend on the specific receive antenna j under consideration, (ii) the receive antenna correlations $\rho_{ij,ij'}=:\rho_{\mathrm{Rx},jj'}$ $(j,j'=1,...,N)$ do not depend on the specific transmit antenna i under consideration, and (iii) the spatial correlations $\rho_{ij,i'j'}$ can be written as the product $\rho_{ij,i'j'}:=\rho_{\mathrm{Tx},ii'}\cdot\rho_{\mathrm{Rx},jj'}$. Altogether, the overall spatial correlation matrix $\mathbf{R} := \mathsf{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\mathrm{H}}\}/\sigma^2$ of size $(MN\times MN)$ can be written as the Kronecker product

$$\mathbf{R} = \mathbf{R}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}}, \tag{9}$$

$$\mathbf{R}_{\mathrm{Tx}} := [\rho_{\mathrm{Tx},ii'}]_{i,i'=1,\ldots,M},\ \mathbf{R}_{\mathrm{Rx}} := [\rho_{\mathrm{Rx},jj'}]_{j,j'=1,\ldots,N}. \tag{10}$$

Moreover, the channel matrix $\mathbf{H}$ can be written as

$$\mathbf{H} := \mathbf{R}_{\mathrm{Rx}}^{1/2}\,\mathbf{G}\,\mathbf{R}_{\mathrm{Tx}}^{1/2}, \tag{11}$$

where $\mathbf{G}$ denotes an $(N\times M)$-matrix with independent and identically distributed (i.i.d.) entries $g_{ji} \sim \mathcal{CN}\{0,\sigma^2\}$, i.e., $\mathsf{E}\{\mathrm{vec}(\mathbf{G})\mathrm{vec}(\mathbf{G})^{\mathrm{H}}\} = \sigma^2\mathbf{I}_{MN}$. The square-roots of $\mathbf{R}_{\mathrm{Tx}}, \mathbf{R}_{\mathrm{Rx}}$ can be obtained via the corresponding eigenvalue decompositions (e.g., by means of the Jacobian algorithm [19, Ch. 8.4]):

$$\mathbf{R}_{\mathrm{Tx}}^{1/2} := \mathbf{U}_{\mathrm{Tx}}\,\mathbf{\Lambda}_{\mathrm{Tx}}^{1/2}\,\mathbf{U}_{\mathrm{Tx}}^{\mathrm{H}}, \quad \mathbf{R}_{\mathrm{Rx}}^{1/2} := \mathbf{U}_{\mathrm{Rx}}\,\mathbf{\Lambda}_{\mathrm{Rx}}^{1/2}\,\mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}, \tag{12}$$

where $\mathbf{\Lambda}_{\mathrm{Tx}}$, $\mathbf{\Lambda}_{\mathrm{Rx}}$ are diagonal matrices containing the (real-valued) eigenvalues $\lambda_{\mathrm{Tx},i}$ and $\lambda_{\mathrm{Rx},j}$ of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$, respectively, and $\mathbf{U}_{\mathrm{Tx}}$, $\mathbf{U}_{\mathrm{Rx}}$ are unitary matrices containing the corresponding eigenvectors ($\mathbf{U}_{\mathrm{Tx}}\mathbf{U}_{\mathrm{Tx}}^{\mathrm{H}} = \mathbf{I}_M$, $\mathbf{U}_{\mathrm{Rx}}\mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}} = \mathbf{I}_N$). Note that the eigenvalues $\lambda_{\mathrm{Tx},i}$ and $\lambda_{\mathrm{Rx},j}$ are always greater or equal to zero [20, Ch. 1.5]. Since $\mathbf{\Lambda}_{\mathrm{Tx}}$ and $\mathbf{\Lambda}_{\mathrm{Rx}}$ are diagonal, $\mathbf{\Lambda}_{\mathrm{Tx}}^{1/2}$ and $\mathbf{\Lambda}_{\mathrm{Rx}}^{1/2}$ are also diagonal and contain the (non-negative) square-roots of the eigenvalues $\lambda_{\mathrm{Tx},i}$ and $\lambda_{\mathrm{Rx},j}$, respectively.

B. MIMO System with Distributed Antennas

To start with, consider a MIMO system with distributed transmit antennas and co-located receive antennas, as depicted in Fig. 1 (c). The cooperating transmitting nodes may, for example, be part of a simulcast network [10] that serves a certain area around the receiving node. In this example, the receiving node would represent a single user with a fixed position or a subscriber home equipped with a fixed antenna array.[2] Alternatively, the cooperating transmitters may also be wireless relays [11],[12], forwarding messages of a certain source node to a certain destination node (in a cooperative fashion). In this example, the receiving node would represent the destination node.

As a generalization to Fig. 1 (c), the individual transmitting nodes may in the sequel be equipped with multiple antennas. To this end, let T denote the number of transmitting nodes, M_t the number of antennas employed at the tth transmitting node ($t = 1, ..., T$), and let M again denote the overall number of transmit antennas, i.e., $\sum_t M_t =: M$. As earlier, let N denote the number of receive antennas used. For simplicity, we assume that all transmit antennas are uncorrelated. (For antennas belonging to different transmitting nodes, this condition is surely met.) A generalization to the case of correlated transmit antennas is, however, straightforward.

Similar to Section II-A, it is again reasonable to assume that all channel coefficients associated with the same transmitting node t have the same variance σ_t^2. Correspondingly, assuming an appropriate ordering of the columns of $\mathbf{H}$, we obtain

$$\mathsf{E}\{\mathbf{H}^{\mathrm{H}}\mathbf{H}\}/N = \mathrm{diag}([\,\sigma_1^2, ..., \sigma_t^2, ..., \sigma_T^2\,]) =: \mathbf{\Sigma}_{\mathrm{Tx}}, \tag{13}$$

where each variance σ_t^2 occurs M_t times. Following the Kronecker-correlation model, we may thus write

$$\mathbf{H} := \mathbf{R}_{\mathrm{Rx}}^{1/2}\,\mathbf{G}\,\mathbf{\Sigma}_{\mathrm{Tx}}^{1/2}, \tag{14}$$

where $\mathbf{G}$ denotes an $(N\times M)$-matrix with i.i.d. entries $g_{ji} \sim \mathcal{CN}\{0,1\}$. Typically, the variances σ_t^2 (and thus the average link SNRs) vary significantly between the individual transmitting nodes, due to different link lengths and, possibly, due to shadowing effects.[3] (Note that the received power decays at least with the square of the link length.)

Similarly, in a MIMO system with co-located transmit antennas and distributed receive antennas, where R receiving nodes (possibly equipped with multiple antennas) cooperate, we have

$$\mathsf{E}\{\mathbf{H}\mathbf{H}^{\mathrm{H}}\}/M = \mathrm{diag}([\,\sigma_1^2, ..., \sigma_r^2, ..., \sigma_R^2\,]) =: \mathbf{\Sigma}_{\mathrm{Rx}} \tag{15}$$

and

$$\mathbf{H} := \mathbf{\Sigma}_{\mathrm{Rx}}^{1/2}\,\mathbf{G}\,\mathbf{R}_{\mathrm{Tx}}^{1/2}, \tag{16}$$

where $g_{ji} \sim \mathcal{CN}\{0,1\}$ with $\mathsf{E}\{\mathrm{vec}(\mathbf{G})\mathrm{vec}(\mathbf{G})^{\mathrm{H}}\} = \mathbf{I}_{MN}$. The cooperating receivers may, for example, be part of a reach-back network for wireless sensors, which serves a certain area around the transmitting node. (In this example, the transmitting node would represent a single wireless sensor broadcasting measurements to the nodes within the reach-back network.)

C. Normalization

In order to treat systems with co-located antennas and systems with distributed antennas in a single unified framework, we employ the following normalization in the sequel:

$$\mathrm{tr}\left(\mathsf{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\mathrm{H}}\}\right) := MN. \tag{17}$$

For MIMO systems with co-located antennas this means we set $\sigma^2 := 1$, i.e., $\mathsf{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\mathrm{H}}\} = \mathbf{R} = \mathbf{R}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}}$. For MIMO systems with distributed transmit or receive antennas, it means we set $\mathrm{tr}(\mathbf{\Sigma}_{\mathrm{Tx}}) := M$ or $\mathrm{tr}(\mathbf{\Sigma}_{\mathrm{Rx}}) := N$, respectively.

III. Equivalence of Spatially Correlated MIMO Systems and Distributed MIMO Systems

In the following, we will show that for any MIMO system with co-located antennas, which follows the Kronecker-correlation model (11), an equivalent MIMO system with distributed antennas can be found, and vice versa, in the sense that both systems are characterized by identical capacity distributions.

For the time being, we assume that no channel state information is available at the transmitter. In this case, the capacity of the MIMO system (1) is given by the well-known expression [2]

$$C(\mathbf{H}) = \log_2 \det\left(\mathbf{I}_N + \frac{P}{M\sigma_{\mathrm{n}}^2}\,\mathbf{H}\mathbf{H}^{\mathrm{H}}\right)\ \text{bit/channel use.} \tag{18}$$

$C(\mathbf{H})$ is in the following called instantaneous capacity, because it is associated with a single realization of the random channel matrix. Correspondingly, $C(\mathbf{H})$ itself is a random variable with probability density function (pdf) denoted as $p(C(\mathbf{H}))$.

A. Capacity Distribution in the Case of Co-located Antennas

To start with, consider a MIMO system with co-located transmit and receive antennas and an overall spatial covariance matrix $\mathsf{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\mathrm{H}}\} = \mathbf{R}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}}$.

[2] Simulcast networks are typically employed for broadcasting applications (where many users are served simultaneously) or for paging applications (where a single user with unknown position is served).

[3] As long as the transmitting nodes have fixed positions and a single receiving node is considered (also with a fixed position), no macroscopic diversity is available. Shadowing effects are solely captured by the variances σ_t^2. The benefits of macroscopic diversity would, for example, become apparent when averaging over many possible positions of the receiving node.

Let
$$\mathbf{R}_{\rm A},\, \mathbf{R}_{\rm B} := \begin{cases} \mathbf{R}_{\rm Tx},\, \mathbf{R}_{\rm Rx} & \text{if } M<N \\ \mathbf{R}_{\rm Rx},\, \mathbf{R}_{\rm Tx} & \text{else} \end{cases}, \tag{19}$$
i.e., the matrix $\mathbf{R}_{\rm A}$ is always related to the side with less antennas. Moreover, let
$$N_{\min} := \min\{M,N\} \;\text{ and }\; N_{\max} := \max\{M,N\}. \tag{20}$$
For simplicity, we assume that both matrices $\mathbf{R}_{\rm A}$ and $\mathbf{R}_{\rm B}$ have full rank and distinct eigenvalues
$$0<\lambda_{{\rm A},1}<\ldots<\lambda_{{\rm A},N_{\min}} \quad\text{and} \tag{21}$$
$$0<\lambda_{{\rm B},1}<\ldots<\lambda_{{\rm B},N_{\max}}, \tag{22}$$
respectively. Under these premises, the characteristic function (cf) of the instantaneous capacity $C(\mathbf{H})$,
$$\mathrm{cf}_C(\mathrm{j}\omega) := \mathsf{E}\{\mathrm{e}^{\mathrm{j}\omega C(\mathbf{H})}\} \tag{23}$$
($\mathrm{j}=\sqrt{-1},\ \omega\in\mathbb{R}$), was evaluated in [21]. The result is of form
$$\mathrm{cf}_C(\mathrm{j}\omega) = \frac{K\,\varphi(\mathrm{j}\omega)}{\psi(\mathbf{R}_{\rm A},\mathbf{R}_{\rm B})}\,\det\left(\begin{bmatrix} \mathbf{V}(\mathbf{R}_{\rm B}) \\ \mathbf{M}(\mathbf{R}_{\rm A},\mathbf{R}_{\rm B},\mathrm{j}\omega)\end{bmatrix}\right), \tag{24}$$
which depends on the number of transmit and receive antennas, the overall transmit power P, the noise variance $\sigma_{\rm n}^2$, and the matrices $\mathbf{R}_{\rm A}$ and $\mathbf{R}_{\rm B}$ (see [21],[22] for further details).[4] More specifically, the term $\psi(\mathbf{R}_{\rm A},\mathbf{R}_{\rm B})$ as well as the $((N_{\max}-N_{\min})\times N_{\max})$-Vandermonde matrix $\mathbf{V}(\mathbf{R}_{\rm B})$ and the $(N_{\min}\times N_{\max})$-matrix $\mathbf{M}(\mathbf{R}_{\rm A},\mathbf{R}_{\rm B},\mathrm{j}\omega)$ depend solely on the eigenvalues of $\mathbf{R}_{\rm A}$ and $\mathbf{R}_{\rm B}$, but not on specific entries of $\mathbf{R}_{\rm A}$ or $\mathbf{R}_{\rm B}$. Moreover, the constant K and the term $\varphi(\mathrm{j}\omega)$ are completely independent of $\mathbf{R}_{\rm A}$ and $\mathbf{R}_{\rm B}$. Correspondingly, any MIMO system having an overall spatial covariance matrix
$$\begin{aligned}\mathsf{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\rm H}\} &= (\mathbf{U}_M\mathbf{R}_{\rm Tx}\mathbf{U}_M^{\rm H})\otimes(\mathbf{U}_N\mathbf{R}_{\rm Rx}\mathbf{U}_N^{\rm H}) \\ &=: \mathbf{R}'_{\rm Tx}\otimes\mathbf{R}'_{\rm Rx},\end{aligned} \tag{25}$$
where $\mathbf{U}_M$ is an *arbitrary* unitary $(M{\times}M)$-matrix and $\mathbf{U}_N$ an *arbitrary* unitary $(N{\times}N)$-matrix, will exhibit exactly the same characteristic function (24) of the instantaneous capacity $C(\mathbf{H})$, because the eigenvalues of $\mathbf{R}'_{\rm Tx}$ and $\mathbf{R}_{\rm Tx}$ and of $\mathbf{R}'_{\rm Rx}$ and $\mathbf{R}_{\rm Rx}$ are identical. Specifically, we may choose $\mathbf{U}_M := \mathbf{U}_{\rm Tx}^{\rm H}$ and/or $\mathbf{U}_N := \mathbf{U}_{\rm Rx}^{\rm H}$, in order to find an equivalent (with respect to the characteristic function $\mathrm{cf}_C(\mathrm{j}\omega)$) MIMO system with distributed transmit and/or distributed receive antennas:
$$\mathbf{U}_{\rm Tx}^{\rm H}\mathbf{R}_{\rm Tx}\mathbf{U}_{\rm Tx} = \mathbf{\Lambda}_{\rm Tx} =: \mathbf{\Sigma}_{\rm Tx} \tag{26}$$
$$\mathbf{U}_{\rm Rx}^{\rm H}\mathbf{R}_{\rm Rx}\mathbf{U}_{\rm Rx} = \mathbf{\Lambda}_{\rm Rx} =: \mathbf{\Sigma}_{\rm Rx}. \tag{27}$$
The characteristic function $\mathrm{cf}_C(\mathrm{j}\omega)$ contains the complete information about the statistical properties of $C(\mathbf{H})$. Specifically, the pdf of $C(\mathbf{H})$ can be calculated as[5] [21]
$$p(C(\mathbf{H})) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\mathrm{cf}_C(\mathrm{j}\omega)\mathrm{e}^{-\mathrm{j}\omega C(\mathbf{H})}\mathrm{d}\omega. \tag{28}$$
Based on the pdf $p(C(\mathbf{H}))$, further statistical characteristics of $C(\mathbf{H})$ can be obtained, such as the cumulative distribution function (cdf) $\Pr\{C(\mathbf{H})<C_0\}$, the ergodic capacity $\bar{C}=\mathsf{E}\{C(\mathbf{H})\}$, or the $p\%$-outage capacity $C_{\rm out}^{p\%}$, i.e., the capacity value C_0 for which the cdf yields $p\%$ [2].

[4] If the eigenvalues of $\mathbf{R}_{\rm A}$ or $\mathbf{R}_{\rm B}$ are not distinct, the characteristic function of $C(\mathbf{H})$ can be obtained as a limiting case of (24).

[5] The characteristic function of a random variable can be interpreted as the Fourier transform of the corresponding pdf, evaluated at $-\mathrm{j}\omega$. Therefore, the pdf can be obtained from the characteristic function via the corresponding inverse transform.

B. Capacity Distribution in the Case of Distributed Antennas

Based on (26) and (27), for any spatially correlated MIMO system an equivalent distributed MIMO system can be found. Vice versa, given a MIMO system with distributed transmit and/or distributed receive antennas, the diagonal elements of the matrix $\mathbf{\Sigma}_{\rm Tx}$/ $\mathbf{\Sigma}_{\rm Rx}$ may be interpreted as the eigenvalues of a corresponding correlation matrix $\mathbf{R}_{\rm Tx}$/ $\mathbf{R}_{\rm Rx}$ (provided that the normalization according to Section II-C is applied).

In [17] it was shown that for any number of transmit/ receive antennas, a unitary matrix $\tilde{\mathbf{U}}_M$/ $\tilde{\mathbf{U}}_N$ can be found such that the transform
$$\tilde{\mathbf{U}}_M\mathbf{\Sigma}_{\rm Tx}\tilde{\mathbf{U}}_M^{\rm H} =: \mathbf{R}_{\rm Tx} \tag{29}$$
$$\tilde{\mathbf{U}}_N\mathbf{\Sigma}_{\rm Rx}\tilde{\mathbf{U}}_N^{\rm H} =: \mathbf{R}_{\rm Rx} \tag{30}$$
yields a correlation matrix $\mathbf{R}_{\rm Tx}$/ $\mathbf{R}_{\rm Rx}$ with diagonal entries equal to one and non-diagonal entries with magnitudes ≤ 1. Suitable unitary matrices are, for example, the $(n{\times}n)$-Fourier matrix with entries $\tilde{u}_{ij}=\mathrm{e}^{\mathrm{j}2\pi(i-1)(j-1)/n}/\sqrt{n}\ \ (i,j=1,\ldots,n)$, which exists for any number n, or the normalized $(n{\times}n)$-Hadamard matrix, which is known to exist for all $n=2^\nu$, where ν is an arbitrary positive integer number.

C. Two Simple Examples

Consider a system with two co-located transmit antennas and a single receive antenna, and let
$$\mathbf{R}_{\rm Tx} = \begin{bmatrix} 1 & \rho \\ \rho^* & 1\end{bmatrix}, \qquad \rho=|\rho|\mathrm{e}^{\mathrm{j}\phi}, \tag{31}$$
where $|\rho|\le 1$ and $\phi\in[0,2\pi)$. In this case, one obtains
$$\mathbf{U}_{\rm Tx} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathrm{e}^{\mathrm{j}\phi} & -\mathrm{e}^{\mathrm{j}\phi} \\ 1 & 1\end{bmatrix},\quad \mathbf{\Lambda}_{\rm Tx} = \begin{bmatrix} 1+|\rho| & 0 \\ 0 & 1-|\rho|\end{bmatrix}. \tag{32}$$
Thus, setting $\mathbf{\Lambda}_{\rm Tx}=:\mathbf{\Sigma}_{\rm Tx}$, we have found an equivalent system with two distributed transmit antennas and a single receive antenna: Let D denote the distance between transmitter and receiver in the co-located system. Assuming that the received power scales with D^{-p}, where p denotes the path-loss exponent (typically, $2\le p\le 4$), we obtain $D'_1=D(1+|\rho|)^{-1/p}$ and $D'_2=D(1-|\rho|)^{-1/p}$ for the distances in the equivalent distributed system. For example, for $\rho=0.8$, $D=100$ meters, and $p=2$ we get $D'_1\approx 74.5$ meters and $D'_2\approx 223.6$ meters.

Next, consider a system with two distributed transmit antennas and a single receive antenna, and let
$$\mathbf{\Sigma}_{\rm Tx} = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2\end{bmatrix}, \qquad \sigma_1^2,\sigma_2^2\in\mathbb{R}_{\ge 0}. \tag{33}$$
where $\sigma_1^2+\sigma_2^2=M=2$. Using the (2×2)-Hadamard matrix
$$\tilde{\mathbf{U}}_2 := \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1\end{bmatrix} \tag{34}$$
we find an equivalent system with two co-located transmit antennas, one receive antenna and a transmitter correlation matrix
$$\mathbf{R}_{\rm Tx} := \tilde{\mathbf{U}}_2\mathbf{\Sigma}_{\rm Tx}\tilde{\mathbf{U}}_2^{\rm H} = \begin{bmatrix} 1 & \frac{1}{2}(\sigma_1^2-\sigma_2^2) \\ \frac{1}{2}(\sigma_1^2-\sigma_2^2) & 1\end{bmatrix}. \tag{35}$$
Let D_1 and D_2 denote the distances between the distributed transmit antennas and the receive antenna. Then, the corresponding distance D' in the equivalent co-located system is given by $D'=D_1\cdot(\sigma_1^2)^{-1/p}=D_2\cdot(\sigma_2^2)^{-1/p}$.

D. Numerical Results

In order to illustrate the findings of Section III-A and III-B, some numerical results are presented in Fig. 2, for different MIMO systems with four transmit and three receive antennas. Displayed are the capacity distributions resulting for (i) a conventional MIMO system with uncorrelated links, (ii) a co-located MIMO system with correlated antennas, and (iii) the corresponding equivalent distributed MIMO system. (Moreover, the associated ergodic capacities are marked by dotted lines.) For the spatially correlated MIMO system, a single-parameter $(n\times n)$-correlation matrix

$$\mathbf{R}_{n,\rho} := \begin{bmatrix} 1 & \rho & \rho^4 & \cdots & \rho^{(n-1)^2} \\ \rho & 1 & \rho & \cdots & \rho^{(n-2)^2} \\ \rho^4 & \rho & & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ \rho^{(n-1)^2} & \rho^{(n-2)^2} & \cdots & \cdots & 1 \end{bmatrix} \qquad (36)$$

$(\rho\in\mathbb{R})$ was used for $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$, which was proposed in [23] for uniform linear antenna arrays with n antenna elements. For the transmitter side, the parameters $n=M$ and $\rho_{\mathrm{Tx}}=0.8$ were chosen, and for the receiver side the parameters $n=N$ and $\rho_{\mathrm{Rx}}=0.7$. The corresponding matrices $\mathbf{\Sigma}_{\mathrm{Tx}}$ and $\mathbf{\Sigma}_{\mathrm{Rx}}$ in the equivalent distributed MIMO system are given by

$$\mathbf{\Sigma}_{\mathrm{Tx}} = \mathrm{diag}([\,0.0198 \;\; 0.2125 \;\; 1.0459 \;\; 2.7217\,]), \qquad (37)$$
$$\mathbf{\Sigma}_{\mathrm{Rx}} = \mathrm{diag}([\,0.1228 \;\; 0.7599 \;\; 2.1173\,]). \qquad (38)$$

As can be seen in Fig. 2, the capacity distributions for the spatially correlated MIMO system and the equivalent distributed MIMO system are, in fact, identical. Compared to the conventional MIMO system with uncorrelated links, the ergodic capacity is significantly reduced (from 23.7 bit/channel use to 18.9 bit/channel use), and the width of the capacity distribution is (slightly) increased.

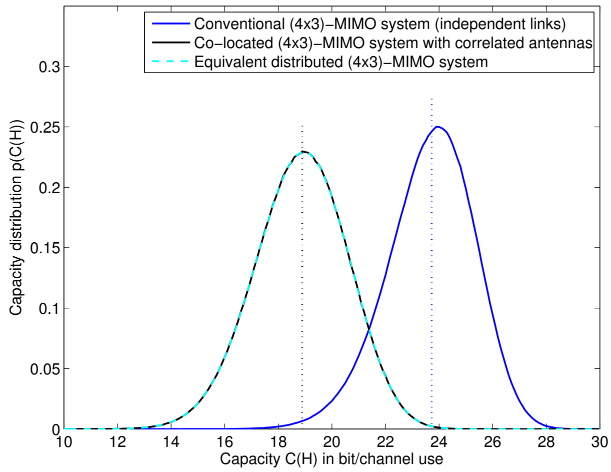

Fig. 2. Capacity distributions for different MIMO systems with four transmit and three receive antennas, at an SNR of $10\log_{10} P/(M\sigma_{\mathrm{n}}^2) = 20$ dB (analytical results according to (24) and (28); the results were validated by means of Monte-Carlo simulations over 10^7 independent channel realizations).

IV. Asymptotic Equivalence Concerning the Pairwise Error Probability of Space-Time Codes

The results in Section III are very general and provide theoretical limits for coded MIMO systems with co-located or distributed antennas. In the following, we focus on space-time coded MIMO systems with spatially correlated or distributed antennas. Specifically, we will show that (asymptotically) spatially correlated MIMO systems and distributed MIMO systems lead to identical pairwise error probabilities (PEPs).

Consider the block transmission model (2). We assume that a space-time encoder with memory length ν (e.g., a space-time trellis encoder) is used at the transmitter side – possibly in a distributed fashion. The space-time encoder maps a sequence of $(N_{\mathrm{b}}-\nu)$ information symbols (followed by ν known tailing symbols) onto an $(M\times N_{\mathrm{b}})$ space-time transmission matrix $\mathbf{X}$ $(N_{\mathrm{b}}>M)$, which is referred to as code matrix in the sequel. Assuming that the channel matrix $\mathbf{H}$ is perfectly known at the receiver, the metric for maximum-likelihood sequence estimation (MLSE) is given by [24]

$$\mu(\mathbf{Y},\tilde{\mathbf{X}}) := \big|\big|\mathbf{Y}-\mathbf{H}\,\tilde{\mathbf{X}}\big|\big|_{\mathrm{F}}^2, \qquad (39)$$

where $\tilde{\mathbf{X}}$ denotes a hypothesis for the code matrix $\mathbf{X}$. The PEP $P(\mathbf{X}\rightarrow\mathbf{E})$, i.e., the probability that the MLSE decoder decides in favor of an erroneous code matrix $\mathbf{E}\neq\mathbf{X}$, although the matrix $\mathbf{X}$ was transmitted, is given by [25]

$$\begin{aligned} P(\mathbf{X}\rightarrow\mathbf{E}) &= \Pr\{\,\mu(\mathbf{Y},\mathbf{E}) \le \mu(\mathbf{Y},\mathbf{X})\,\} \qquad (40)\\ &= \mathsf{E}\left\{\mathrm{Q}\left(\sqrt{\frac{P}{2M\sigma_{\mathrm{n}}^2}}\,\big|\big|\mathbf{H}\,(\mathbf{X}-\mathbf{E})\big|\big|_{\mathrm{F}}\right)\right\}, \end{aligned}$$

where $\mathrm{Q}(x)$ denotes the Gaussian Q-function. (The expectation is taken over the channel matrix $\mathbf{H}$.) The PEP can, for example, be used to approximate the resulting bit error rate (BER) [24].

A. PEP in the Case of Co-located Antennas

Consider again a system with co-located antennas and spatial covariance matrix $\mathsf{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\mathrm{H}}\} = \mathbf{R}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}}$. Let us denote

$$\mathbf{\Psi}_{\mathbf{X},\mathbf{E}} := (\mathbf{X}-\mathbf{E})(\mathbf{X}-\mathbf{E})^{\mathrm{H}}. \qquad (41)$$

In the sequel, we assume that the employed space-time code achieves a diversity order of MN (full spatial diversity). This implies that the matrix $\mathbf{\Psi}_{\mathbf{X},\mathbf{E}}$ has always full rank, i.e.,

$$\mathrm{rank}(\mathbf{\Psi}_{\mathbf{X},\mathbf{E}}) = M \qquad (42)$$

for any pair of code matrices $(\mathbf{X}\neq\mathbf{E})$. In [25], it was shown that the PEP (40) can be expressed in the form of a single finite-range integral, according to

$$P(\mathbf{X}\rightarrow\mathbf{E}) = \frac{1}{\pi}\int_0^{\pi/2}\prod_{i=1}^{M}\prod_{j=1}^{N}\left[1+\frac{P}{4M\sigma_{\mathrm{n}}^2}\,\frac{\xi_{\mathrm{Tx},i}\lambda_{\mathrm{Rx},j}}{\sin^2\theta}\right]^{-1}\mathrm{d}\theta, \qquad (43)$$

where $\xi_{\mathrm{Tx},1},\dots,\xi_{\mathrm{Tx},M}$ denote the eigenvalues of the matrix $\mathbf{\Psi}_{\mathbf{X},\mathbf{E}}\,\mathbf{R}_{\mathrm{Tx}}$ and $\lambda_{\mathrm{Rx},1},\dots,\lambda_{\mathrm{Rx},N}$ the eigenvalues of $\mathbf{R}_{\mathrm{Rx}}$, as earlier. Moreover, it was shown in [25] that the presence of receive antenna correlation always degrades the PEP (for any SNR value, particularly for high SNRs). As opposed to this, the impact of transmit antenna correlation depends on the employed space-time code: In the low SNR regime, the presence of transmit antenna correlation can improve the PEP, whereas for large SNR values the PEP is always degraded.

B. PEP in the Case of Distributed Antennas

Based on the same arguments as in Section III, we can always find a MIMO system with distributed *receive* antennas, which leads to exactly the same PEP as the above co-located system: Any MIMO system with overall spatial covariance matrix

$$\mathsf{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\mathrm{H}}\} = \mathbf{R}_{\mathrm{Tx}} \otimes (\mathbf{U}_N \mathbf{R}_{\mathrm{Rx}} \mathbf{U}_N^{\mathrm{H}}), \qquad (44)$$

with $\mathbf{U}_N$ being an arbitrary unitary ($N{\times}N$)-matrix, will lead to the same PEP expression (43). In particular, we may again choose $\mathbf{U}_N := \mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}$, in order to obtain $\mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}} \mathbf{R}_{\mathrm{Rx}} \mathbf{U}_{\mathrm{Rx}} =: \mathbf{\Sigma}_{\mathrm{Rx}}$.

As opposed to this, a MIMO system with distributed *transmit* antennas and overall spatial covariance matrix

$$\mathsf{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\mathrm{H}}\} = \mathbf{\Sigma}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}} \qquad (45)$$

($\mathbf{\Sigma}_{\mathrm{Tx}} := \mathbf{U}_{\mathrm{Tx}}^{\mathrm{H}} \mathbf{R}_{\mathrm{Tx}} \mathbf{U}_{\mathrm{Tx}}$) will normally *not* lead to the same PEP expression (43), because the eigenvalues of $\mathbf{\Psi}_{\mathbf{X},\mathbf{E}}\, \mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{\Psi}_{\mathbf{X},\mathbf{E}}\, \mathbf{\Sigma}_{\mathrm{Tx}}$ are, in general, different. Still it can be seen that asymptotically, i.e., for large SNR values, the PEP expressions again become the same: In [26] it was shown that for large SNR values (43) is well approximated by

$$\begin{aligned} P(\mathbf{X} \to \mathbf{E}) \;\leq\; & \left(\frac{P}{4M\sigma_{\mathrm{n}}^2} \right)^{-r_{\mathrm{Tx}}\, r_{\mathrm{Rx}}} \times \qquad (46) \\ & \times\; \det(\mathbf{\Psi}_{\mathbf{X},\mathbf{E}}\, \mathbf{R}_{\mathrm{Tx}})^{-r_{\mathrm{Rx}}} \det(\mathbf{R}_{\mathrm{Rx}})^{-r_{\mathrm{Tx}}}, \end{aligned}$$

where $r_{\mathrm{Tx}} := \mathrm{rank}(\mathbf{R}_{\mathrm{Tx}})$ and $r_{\mathrm{Rx}} := \mathrm{rank}(\mathbf{R}_{\mathrm{Rx}})$. Assuming that $\mathbf{R}_{\mathrm{Tx}}$ has full rank ($r_{\mathrm{Tx}} := M$), we have

$$\begin{aligned} \det(\mathbf{\Psi}_{\mathbf{X},\mathbf{E}}\, \mathbf{R}_{\mathrm{Tx}}) &= \det(\mathbf{\Psi}_{\mathbf{X},\mathbf{E}}) \det(\mathbf{R}_{\mathrm{Tx}}) \\ &= \det(\mathbf{\Psi}_{\mathbf{X},\mathbf{E}}) \det(\mathbf{\Sigma}_{\mathrm{Tx}}) = \det(\mathbf{\Psi}_{\mathbf{X},\mathbf{E}}\, \mathbf{\Sigma}_{\mathrm{Tx}}), \qquad (47) \end{aligned}$$

i.e., the expression (46) does not change if $\mathbf{R}_{\mathrm{Tx}}$ is replaced by $\mathbf{\Sigma}_{\mathrm{Tx}}$. Similarly, given a MIMO system with distributed transmit/ receive antennas, we can find an (asymptotically) equivalent co-located MIMO system by evaluating (29), (30).

V. A Simple Performance Measure

Section III and IV have shown that co-located and distributed MIMO systems are (asymptotically) equivalent with regard to many important performance measures, such as the capacity distribution (and thus the ergodic and outage capacity) and the PEP of space-time codes. Apart form this, it was shown in [17] that co-located and distributed MIMO systems are also equivalent with regard to the average symbol error rate of OSTBCs.

The antenna correlation matrices $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$ (or, equivalently, the matrices $\mathbf{\Sigma}_{\mathrm{Tx}}$ and $\mathbf{\Sigma}_{\mathrm{Rx}}$) do, however, not directly reflect the associated system performance in terms of capacity or error rate. Correspondingly, it is not immediately clear, how two systems with different correlation matrices compare. To this end, we consider a simple performance measure $\Delta(\mathbf{R})$ between zero and one, which allows for a classification of different systems. The performance measure was earlier proposed in [18] in order to categorize spatially correlated MIMO systems with regard to their ergodic capacity. Moreover, it was already successfully used in [17] for categorizing spatially correlated and distributed OSTBC-systems with regard to their average symbol error rate.

Consider again a MIMO system with co-located antennas and an overall spatial covariance matrix $\mathbf{R} = \mathbf{R}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}}$. The corresponding performance measure $\Delta(\mathbf{R})$ is defined as [18]

$$\Delta(\mathbf{R}) \;=\; \frac{1}{\sqrt{MN(MN-1)}} \sqrt{\sum_{i=1}^{MN} \sum_{\substack{j=1 \\ j\neq i}}^{MN} |\, [\mathbf{R}]_{i,j}\, |^2} \,. \qquad (48)$$

Note that $\Delta(\mathbf{R})$ is always between zero and one, where zero corresponds to the uncorrelated case ($\Delta(\mathbf{I}_{MN}) = 0$) and one to the fully correlated case, where $|\, [\mathbf{R}]_{i,j}\, | = 1$ for all i, j. We may reformulate (48) according to

$$\Delta(\mathbf{R}) \;=\; \frac{||\mathbf{R} - \mathbf{I}_{MN}||_{\mathrm{F}}}{\sqrt{MN(MN-1)}} \,. \qquad (49)$$

Moreover, let the eigenvalue decomposition of $\mathbf{R}$ be given by

$$\mathbf{R} := \mathbf{U}_{MN}\, \mathbf{\Lambda}\, \mathbf{U}_{MN}^{\mathrm{H}}, \qquad (50)$$

where $\mathbf{U}_{MN} \mathbf{U}_{MN}^{\mathrm{H}} = \mathbf{I}_{MN}$. Since $\mathbf{R}$ is the Kronecker product of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$, the set $\{\lambda_i |\, i = 1, ..., MN\}$ of the eigenvalues of $\mathbf{R}$ is given by all pairwise products $\lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j}$ of the eigenvalues of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$ [27, Ch. 12.2]. Utilizing the fact that the Frobenius norm is invariant under a unitary matrix transform, we have

$$||\mathbf{R} - \mathbf{I}_{MN}||_{\mathrm{F}} = ||\mathbf{\Lambda} - \mathbf{I}_{MN}||_{\mathrm{F}}, \qquad (51)$$

i.e., (49) may be reformulated as

$$\Delta(\mathbf{R}) \;=\; \sqrt{\frac{\sum_{i=1}^{M} \sum_{j=1}^{N} (\lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j} - 1)^2}{MN(MN-1)}} \,. \qquad (52)$$

Thus we have found a new expression for $\Delta(\mathbf{R})$ as a function of the eigenvalues of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$, which can directly be used for MIMO systems with distributed transmit and/or receive antennas (by replacing the eigenvalues $\lambda_{\mathrm{Tx},i}$ by the variances σ_t^2 and/or the eigenvalues $\lambda_{\mathrm{Rx},j}$ by the variances σ_r^2).

VI. Transmit Power Allocation Schemes based on Statistical Channel Knowledge

In several publications, it was shown that the performance of MIMO systems may be improved significantly by using some sort of channel knowledge at the transmitter, see e.g. [28]. The use of (full or partial) instantaneous channel knowledge at the transmitter was, for example, considered in [3],[29],[30]. However, accurate instantaneous channel knowledge at the transmitter is costly and may be difficult to acquire [13].

As an alternative, the use of statistical channel knowledge at the transmitter was studied in [13]-[15],[28]. Statistical channel knowledge can easily be gained in practical systems, for example off-line through field measurements, ray-tracing simulations or based on physical channel models, or on-line based on long-term averaging of the channel coefficients [13]. Optimal statistical transmit power allocation schemes for spatially correlated MIMO systems were, for example, derived in [13]-[15] with regard to different optimization criteria: Minimum symbol error probability [13], minimum PEP of space-time codes [14], and maximum ergodic capacity [15]. Due to the (asymptotic) equivalence of co-located and distributed MIMO systems, these power allocation strategies can also be used in distributed systems, without any loss of optimality.

As an example, we will consider the ergodic capacity as the performance measure of interest.

A. Maximizing Ergodic Capacity

Consider again a MIMO system with co-located antennas and an overall spatial covariance matrix $\mathbf{R} = \mathbf{R}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}}$. In order to maximize the ergodic capacity, it was shown in [15] that the optimal strategy is to transmit in the directions of the eigenvectors of the transmitter correlation matrix $\mathbf{R}_{\mathrm{Tx}}$. To this end, the transmitted vector in (1) is pre-multiplied with the unitary matrix $\mathbf{U}_{\mathrm{Tx}}$ from the eigenvalue decomposition of $\mathbf{R}_{\mathrm{Tx}}$. Moreover, a diagonal weighting matrix

$$\mathbf{W}^{1/2} := \mathrm{diag}([\sqrt{w_1}, ..., \sqrt{w_M}]), \qquad \mathrm{tr}(\mathbf{W}) := M, \qquad (53)$$

is used in order to perform the transmit power weighting along the eigenvectors of $\mathbf{R}_{\mathrm{Tx}}$ (see, for example, [31] for further details). Altogether, the transmitted vector can be expressed as

$$\mathbf{x}[k] := \mathbf{U}_{\mathrm{Tx}} \mathbf{W}^{1/2} \mathbf{x}'[k], \qquad (54)$$

where $\sigma^2_{x'_i} := \mathsf{E}\{x'_i[k] x'^*_i[k]\} = P/M$ for all $i = 1, ..., M$. Under these premises, the instantaneous capacity (18) becomes

$$C(\mathbf{H}, \mathbf{Q}_{\mathbf{x}}) = \log_2 \det \left(\mathbf{I}_N + \frac{1}{\sigma_{\mathrm{n}}^2} \mathbf{H} \mathbf{Q}_{\mathbf{x}} \mathbf{H}^{\mathrm{H}} \right) \text{ bit/channel use}, \qquad (55)$$

where $\mathbf{Q}_{\mathbf{x}} := \mathsf{E}\{\mathbf{x}[k]\mathbf{x}^{\mathrm{H}}[k]\} = P/M \cdot \mathbf{U}_{\mathrm{Tx}} \mathbf{W} \mathbf{U}_{\mathrm{Tx}}^{\mathrm{H}}$ denotes the covariance matrix of $\mathbf{x}[k]$.

Unfortunately, a closed-form solution for the optimal weighting matrix $\mathbf{W}_{\mathrm{opt}}$, which maximizes the ergodic capacity $\bar{C}(\mathbf{Q}_{\mathbf{x}}) := \mathsf{E}\{C(\mathbf{H}, \mathbf{Q}_{\mathbf{x}})\}$, is not known. The optimal power weighting results from solving the optimization problem [15]

$$\text{maximize} \quad g(\mathbf{W}) := \mathsf{E}\left\{ \log_2 \det \left(\mathbf{I}_N + \sum_{i=1}^{M} \frac{w_i \lambda_{\mathrm{Tx},i}\, \mathbf{z}_i \mathbf{z}_i^{\mathrm{H}}}{\sigma_{\mathrm{n}}^2} \right) \right\} \qquad (56)$$

$$\text{subject to} \quad \mathrm{tr}(\mathbf{W}) := M \quad \text{and} \quad w_i \geq 0 \quad \text{for all} \quad i,$$

where the vectors $\mathbf{z}_i$ are i.i.d. complex Gaussian random vectors with zero mean and covariance matrix $\mathbf{\Lambda}_{\mathrm{Rx}}$, i.e., $z_{i,j} \sim \mathcal{CN}\{0, \lambda_{\mathrm{Rx},j}\}$ for all $i = 1, ..., M$. (Note that the optimum power weighting depends both on the eigenvalues of $\mathbf{R}_{\mathrm{Tx}}$ and on the eigenvalues of $\mathbf{R}_{\mathrm{Rx}}$.) Based on the same arguments as in Section III, the resulting transmit power weighting will also be optimal for a distributed MIMO system with overall covariance matrix $\mathbf{R} = \mathbf{\Sigma}_{\mathrm{Tx}} \otimes \mathbf{R}_{\mathrm{Rx}}$ or $\mathbf{R} = \mathbf{R}_{\mathrm{Tx}} \otimes \mathbf{\Sigma}_{\mathrm{Rx}}$ with $\mathbf{\Sigma}_{\mathrm{Tx}}$, $\mathbf{\Sigma}_{\mathrm{Rx}}$ given by (26) and (27).

The expression (56) is, in general, difficult to evaluate. In the following, we will therefore consider a tight upper bound on $\bar{C}(\mathbf{Q}_{\mathbf{x}})$, which greatly simplifies the optimization of $\mathbf{W}$. Since the log-function is convex-$\cap$, we may apply Jensen's inequality to the expression for $\bar{C}(\mathbf{Q}_{\mathbf{x}})$ [32], which yields

$$\bar{C}(\mathbf{Q}_{\mathbf{x}}) \leq \log_2 \mathsf{E}\left\{ \det \left(\mathbf{I}_N + \frac{1}{\sigma_{\mathrm{n}}^2} \mathbf{H} \mathbf{Q}_{\mathbf{x}} \mathbf{H}^{\mathrm{H}} \right) \right\}. \qquad (57)$$

For the case $\mathbf{Q}_{\mathbf{x}} = P/M \cdot \mathbf{I}_M$, the right-hand side of (57) was further evaluated in [32], based on the principal minor determinants of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$. In this context, the only constraint on $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$ is that they have to be positive definite Hermitian matrices. Now, from Section III it is known that the pdf of $C(\mathbf{H})$ does not change, if $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$ are replaced by the corresponding eigenvalue matrices $\mathbf{\Lambda}_{\mathrm{Tx}}$ and $\mathbf{\Lambda}_{\mathrm{Rx}}$ (which are, of course also positive definite Hermitian matrices). Moreover, since the log-function is bijective, this will also hold for the pdf of $\det(\mathbf{I}_N + P/(M\sigma_{\mathrm{n}}^2)\, \mathbf{H}\mathbf{H}^{\mathrm{H}})$, and in particular for the expected value. Correspondingly, the principal minor determinants of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$ occurring in [32] may be replaced by appropriate products of eigenvalues. Finally, moving on to the case of unequal power allocation, i.e., $\mathbf{Q}_{\mathbf{x}} = P/M \cdot \mathbf{U}_{\mathrm{Tx}} \mathbf{W} \mathbf{U}_{\mathrm{Tx}}^{\mathrm{H}}$, we may simply replace $\mathbf{R}_{\mathrm{Tx}}$ by $\mathbf{U}_{\mathrm{Tx}} \mathbf{W} \mathbf{\Lambda}_{\mathrm{Tx}} \mathbf{U}_{\mathrm{Tx}}^{\mathrm{H}}$ (or, equivalently, $\mathbf{\Lambda}_{\mathrm{Tx}}$ by $\mathbf{W}\mathbf{\Lambda}_{\mathrm{Tx}}$), since the latter is still a positive definite Hermitian matrix. Altogether, one thus obtains

$$\bar{C}(\mathbf{Q}_{\mathbf{x}}) \;\leq\; \log_2 \left(1 + \sum_{m=1}^{N_{\min}} \left(\frac{P}{M\sigma_{\mathrm{n}}^2} \right)^m m! \;\times \right. \qquad (58)$$

$$\left. \times \sum_{\mathbf{i} \in \mathcal{I}_m} w_{i_1} \lambda_{\mathrm{Tx},i_1} \cdots w_{i_m} \lambda_{\mathrm{Tx},i_m} \sum_{\mathbf{j} \in \mathcal{J}_m} \lambda_{\mathrm{Rx},j_1} \cdots \lambda_{\mathrm{Rx},j_m} \right),$$

where $\mathcal{I}_m$ and $\mathcal{J}_m$ denote index sets defined as

$$\mathcal{I}_m \;:=\; \{\mathbf{i} := [i_1, ..., i_m] \,|\, 1 \leq i_1 < i_2 < \cdots < i_m \leq M\} \qquad (59)$$

$$\mathcal{J}_m \;:=\; \{\mathbf{j} := [j_1, ..., j_m] \,|\, 1 \leq j_1 < j_2 < \cdots < j_m \leq N\} \qquad (60)$$

($m \in \mathbb{Z}$, $1 \leq m \leq N_{\min}$). For a fixed SNR value $P/(M\sigma_{\mathrm{n}}^2)$, the right-hand side of (58) may now be maximized numerically in order to find the optimum power weighting matrix $\mathbf{W}_{\mathrm{opt}}$.

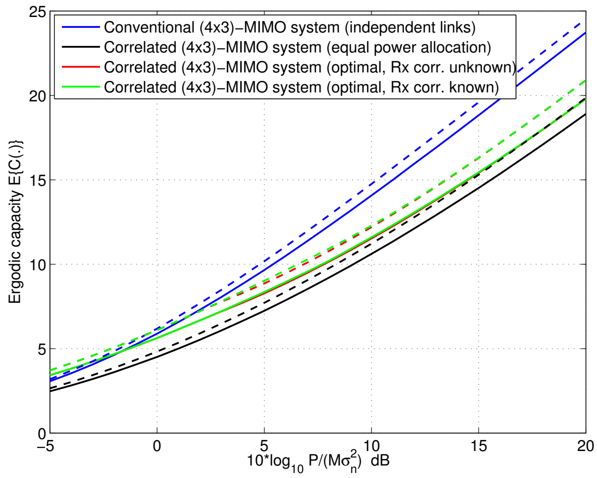

Fig. 3. Ergodic capacity as a function of the SNR $P/(M\sigma_{\mathrm{n}}^2)$ in dB, for uncorrelated and correlated MIMO systems with four transmit and three receive antennas. Solid lines: Simulative results obtained by means of Monte-Carlo simulations over 10^5 independent channel realizations. Dashed lines: Corresponding analytical upper bounds based on (58). The transmit power weights for the red and the green curve were optimized numerically.

B. Numerical Results

As an example, we consider a co-located MIMO system with four transmit and three receive antennas. As in Section III-D, the correlation matrices $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$ are assumed to be of form (36) with correlation parameters $\rho_{\mathrm{Tx}} := 0.8$ and $\rho_{\mathrm{Rx}} := 0.7$. Fig. 3 displays the ergodic capacity as a function of the SNR $P/(M\sigma_{\mathrm{n}}^2)$ in dB that results for different MIMO systems. (Simulative results are represented by solid lines; the corresponding analytical upper bounds are represented by dashed lines.) As can be seen, compared to the uncorrelated system (blue curve) the ergodic capacity of the correlated system (equal power allocation, black curve) is reduced significantly, especially for large SNR values. For the red and the green curve, the transmit power weights $w_1, ..., w_M$ were optimized numerically, based on (58). The green curve represents the case, where both $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$ is known at the transmitter side. For the red curve, however, it was assumed that the transmitter knows only $\mathbf{R}_{\mathrm{Tx}}$ (i.e., the transmitter presumes $\mathbf{R}_{\mathrm{Rx}} = \mathbf{I}_N$.) As can be seen, in both cases the ergodic capacity of the correlated system is improved significantly, especially in the low SNR regime. (For SNR values smaller than -2 dB, the achieved capacity is even larger than in the uncorrelated case.) Interestingly, although the upper bound (58) depends both on the eigenvalues of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$, the difference between the red and the green curve is negligible in the considered example, i.e., the knowledge of $\mathbf{R}_{\mathrm{Rx}}$ at the transmitter is of little benefit. Finally, it should be noted that for $P/(M\sigma_{\mathrm{n}}^2) \rightarrow 0$ the optimal power weighting tends to the (one-dimensional) beamforming solution, where the complete transmit power is focussed on the largest eigenvalue of $\mathbf{R}_{\mathrm{Tx}}$ (see also [15]). For $P/(M\sigma_{\mathrm{n}}^2) \rightarrow \infty$, however, one obtains (in the considered example) an equal power allocation solution over the $N_{\mathrm{min}} = 3$ largest eigenvalues of $\mathbf{R}_{\mathrm{Tx}}$.

VII. Conclusions

This paper has shown that MIMO systems with co-located antennas and MIMO-systems with distributed antennas can be treated in a single unified framework. Specifically, it was shown that for any MIMO system with correlated antennas an equivalent MIMO system with distributed antennas can be found, and vice versa, in the sense that both systems are characterized by identical capacity distributions. Moreover, with regard to space-time coding is was shown that both systems offer the same (asymptotic) pairwise error probability. Finally, the benefits of statistical transmit power allocation schemes was demonstrated. Due to the above equivalence, these schemes can be used both for spatially correlated and for distributed MIMO systems, without any loss of optimality.

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