On the Soft Outputs Provided by the Probabilistic Data Association MIMO Decoder

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Probabilistic data association (PDA), originally developed for target tracking, has been applied in many different areas, including code-division multiple access (CDMA) systems [1] and recently uncoded multiple-input multiple-output (MIMO) systems [2]. Since the complexity of the optimal MIMO decoder, the *a posteriori* probability (APP) decoder, grows exponentially with the data rate, the use of sub-optimal decoders with reduced complexity, such as the PDA decoder, is inevitable.

We consider a MIMO system with M transmit and N receive antennas. A single data stream is encoded by a channel encoder, interleaved and multiplexed onto M complex modulation symbols, which are transmitted simultaneously by the corresponding antennas. Assuming flat fading, the equivalent discrete-time channel model can be written in complex baseband notation as $\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{v}$, where baud-rate sampling, block fading and perfect knowledge of the channel matrix $\mathbf{H} \in \mathbb{C}^{N\times M}$ at the receiver is assumed. The channel matrix coefficients $h_{n,m}$ represent the gain between transmit antenna m $(1 \leq m \leq M)$ and receive antenna n $(1 \leq n \leq N)$. The vector $\mathbf{x} \in \mathbb{Q}^{M\times 1}$ consists of the complex-valued transmitted modulation symbols taken from a symbol alphabet $\mathbb{Q}$ with cardinality Q, while the vector $\mathbf{r} \in \mathbb{C}^{N\times 1}$ contains the received samples. The vector $\mathbf{v} \in \mathbb{C}^{N\times 1}$ represents additive noise and its elements are independent and identically distributed white Gaussian noise samples with zero mean and variance $\sigma_v^2 = \mathrm{E}\left\{|v_n|^2\right\}$. At the receiver, the MIMO decoding operation is performed by the PDA deccoder providing soft outputs to a subsequent deinterleaver and a soft input channel decoder.

The conventional PDA decoder uses two approximations. Firstly, the PDA decoder looks only at one transmitted symbol at a time, treating the received symbols as statistically independent. The second approximation is the Gaussian approximation ("Gaussian forcing") of the PDF of the interference and noise. The PDA decoder approximates *a posteriori* probabilities $\Pr(x_m|\mathbf{r})$ for every element x_m of $\mathbf{x}$. All symbols interfering with x_m and the noise are modeled as a single vector $\mathbf{w} = \sum_{k\neq m} x_k \mathbf{h}_k + \mathbf{v}$, where $\mathbf{h}_k$ denotes the k^{th} column of $\mathbf{H}$. The interference and noise term $\mathbf{w}$ is assumed to be an n-variate Gaussian distributed random variable with mean

$$\boldsymbol{\mu}_w = \mathrm{E}\left\{\sum_{k\neq m} x_k \mathbf{h}_k + \mathbf{v}\right\} = \sum_{k\neq m} \mathrm{E}\left\{x_k\right\} \mathbf{h}_k \qquad (1)$$

and variance

$$\mathbf{R}_{ww} = \mathrm{E}\left\{(\mathbf{w} - \boldsymbol{\mu}_w)(\mathbf{w} - \boldsymbol{\mu}_w)^H\right\} = \sum_{k\neq m} \mathrm{Var}\{x_k\} \mathbf{h}_k \mathbf{h}_k^H \ . \qquad (2)$$

If no *a priori* information is available, the PDA decoder initializes the symbol probabilities as a uniform distribution and computes the *a posteriori* probabilities for the symbol x_m as

$$\Pr(x_m|\mathbf{r}) = c e^{\left(-(\mathbf{r} - x_m \mathbf{h}_m - \boldsymbol{\mu}_w)^H \mathbf{R}_{ww}^{-1} (\mathbf{r} - x_m \mathbf{h}_m - \boldsymbol{\mu}_w)\right)} \ . \qquad (3)$$

For an estimate of the symbol x_m no information on symbols x_k, $k \geq m$, is available. In order to provide information on these symbols, the PDA decoder may use multiple iterations. The mean $\boldsymbol{\mu}_w$ (1) and the variance $\mathbf{R}_{ww}$ (2) are updated for every symbol x_m, incorporating the new information gained from symbol probabilities already computed in the current or previous iterations.

Let $\mathbb{X}^s$ be the set of all possible symbol vector combinations causing interference for a fixed x_m. The actual PDF of $\mathbf{w}$ is a summation of $Q^{M-1} = |\mathbb{X}^s|$ single Gaussian distributions, each of them caused by one possible interfering symbol constellation as a convolution of the discrete symbol probabilities and the PDF of the Gaussian noise vector $\mathbf{v}$. Therefore each of the Gaussian distributions, which sum up to

$$p_{\mathbf{W}}(\mathbf{w}(\mathbf{v})) = \sum_{\mathbf{x}\in\mathbb{X}^s} \Pr(\mathbf{x}) \left(\frac{1}{\pi\sigma_v^2}\right)^N e^{\left(\frac{-1}{\sigma_v^2}\left\|\mathbf{v} - \sum_{k\neq m} x_k \mathbf{h}_k\right\|^2\right)} \ , \qquad (4)$$

has a mean depending on the channel as well as the interfering modulation symbols. According to the central limit theorem, the quality of the Gaussian approximation used in the PDA decoder improves by increasing the number of transmit antennas. On the other hand, the approximation becomes worse when modulation schemes with more constellation points are used. Moreover, a smaller noise variance σ_v^2 is more likely to make the sum in (4) non-Gaussian.

Soft input channel decoders use reliability information based on the symbol probabilities which are calculated from the approximated PDF of the interference and noise term. In order to illustrate the influence of the Gaussian assumption on the performance of a PDA decoder, simulation results for an $M \times N$ MIMO system in conjunction with a rate 1/2 turbo code with polynomials (5,7) are shown in Fig. 1. The elements $h_{n,m}$ of $\mathbf{H}$ are statistically independent random variables (each component being complex Gaussian distributed with zero mean and variance $\mathrm{E}\left\{|h_{n,m}|^2\right\} = 1$. As a benchmark, the BER performance for an APP decoder and a PDA decoder using the actual PDF of the interference and noise have been simulated as well.

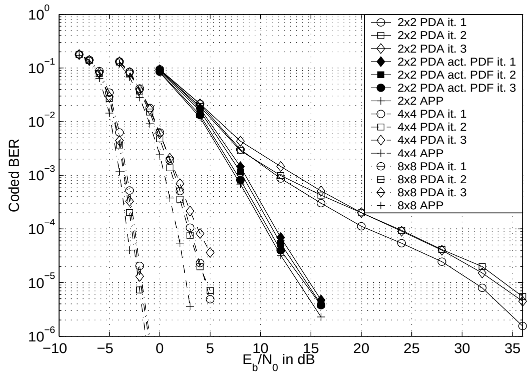

Fig. 1. BER performance of a turbo-coded $M \times N$ MIMO system using QPSK modulation with different decoders.

It can be seen that the difference between APP and PDA decoding improves with an increasing number of antennas as the Gaussian approximation is improving. Especially for the 2×2 system the gap between APP and PDA is becoming larger with a decrease of the noise variance σ_v^2. Furthermore, the third iteration is not the best one, as the quality of the soft output generated by the PDA degrades with every iteration. Since the PDA using the actual PDF for the interference and noise term achieves near-optimal results, it can be concluded that the problems when using a channel code arise from the Gaussian approximation and not the symbol-by-symbol decoding done by the PDA. Similar results have also been obtained for convolutional codes and different code rates.

References

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[2] D. Pham, K. Pattipati, P. K. Willet, and J. Luo, "A generalized probabilistic data association detector for multiple antenna systems," *IEEE Communications Letters*, vol. 8, pp. 205–207, April 2004.