

On the Performance of Non-Coherent Transmission Schemes with Equal-Gain Combining in Correlated Generalized K -Fading[†]

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Abstract—The generalized K -fading model, characterized by two parameters, k and m , was recently shown to accurately capture the effects of composite shadowing and multipath fading in wireless communication systems. In this paper, we derive closed-form expressions for the bit error probability of two non-coherent transmission schemes over L diversity branches being subject to generalized K -fading. Specifically, focus is on binary differential phase-shift keying (DPSK) and binary non-coherent frequency-shift keying (FSK) modulation with equal-gain combining at the receiver. We also derive expressions for the asymptotic diversity order, which reveal an interesting interplay between the two fading parameters k and m . Moreover, we show that the diversity order of the considered non-coherent transmission schemes is the same as in the case of coherent transmission. Finally, numerical performance results are presented, and our analytical results are corroborated by means of Monte-Carlo simulations.

I. INTRODUCTION

THE performance of wireless communication systems is largely governed by shadowing and multipath fading effects [1, Ch. 2]. While major obstacles between transmitter and receiver cause macroscopic fading effects, i.e., fluctuations in the average received signal-to-noise ratio (SNR), scatterers in the vicinity of transmitter and receiver entail microscopic fading effects, i.e., fluctuations in the instantaneous received SNR. Recently, the generalized K -fading model, which is characterized by two parameters, $k > 0$ and $m > 0$, was shown to accurately capture the effects of composite shadowing and multipath fading [2]. In particular, it comprises a large variety of channel conditions, ranging from severe shadowing (small values of k) to mild shadowing (large values of k) and from severe multipath fading (small values of m) to mild multipath fading (large values of m).

A favorable property of the generalized K -fading model is that it allows for a closed-form expression for the probability density function (PDF) of the instantaneous received SNR, which is in contrast to, e.g., competing composite shadowing/multipath fading models that are based on the lognormal PDF [2]. As a result, several analytical performance results for generalized K -fading and ‘ordinary’ K -fading channels ($m=1$) have been reported in the literature [3]–[8].

Most of the papers mentioned above have focussed on coherent transmission schemes, which rely on the availability of accurate channel knowledge at the receiver side. In contrast to this, non-coherent transmission schemes eliminate the need for channel estimation at the receiver and are thus attractive for high-mobility and low-SNR scenarios as well

as for low-cost receiver implementations. In this paper, we derive closed-form expressions for the bit error probability (BEP) of two non-coherent transmission schemes over L generalized K -fading branches with (post-detection) equal-gain combining (EGC) at the receiver. Specifically, focus is on binary differential phase-shift keying (DPSK) modulation with conventional differential detection at the receiver (i.e., based on two subsequent received symbols) and orthogonal binary frequency-shift keying (FSK) modulation with non-coherent detection at the receiver [9, Ch. 9.4]. The L diversity branches could, for example, be created by multiple receive antennas. We also present a high-SNR analysis and provide expressions for the resulting asymptotic diversity order, which reveal an interesting interplay between the two fading parameters k and m . Concerning the K -fading model, we consider the scenario where the shadowing part is fully correlated across links, whereas the multipath fading is independent and identically distributed (i.i.d.) across the L branches. Since shadowing represents a large-scale fading effect, it can be expected to affect all diversity branches simultaneously, while in a rich-scattering environment the multipath fading part can typically be considered independent across links, e.g., if the antenna spacings are chosen sufficiently large.

It is worth noting that the existing papers on non-coherent transmission schemes over (generalized) K -fading links [3], [4], [6] are all restricted to a single branch ($L=1$). For $L > 1$, to the best of the authors’ knowledge no closed-form expressions for the BEP and the asymptotic diversity order of the considered non-coherent transmission schemes in generalized K -fading (or for alternative, e.g., coherent, transmission schemes) have yet been presented in the literature. Also, there are no similar analyses for the competing composite lognormal shadowing/multipath fading models.

The remainder of this paper is organized as follows. In Section II, the generalized K -fading model is briefly recapitulated. In Section III, the closed-form BEP expressions for binary DPSK/non-coherent FSK modulation over L generalized K -fading branches are presented. In Section IV, asymptotic performance results are reported and the diversity order of the non-coherent transmission schemes is determined. Moreover, it is shown that the diversity order of the considered non-coherent transmission schemes is, in fact, the same as in the case of coherent transmission. Finally, numerical performance results are presented in Section V, and conclusions are offered in Section VI.

II. THE GENERALIZED K -FADING MODEL

The generalized K -fading model describes a composite Gamma-shadowing/Nakagami- m fading process. The PDF of

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the instantaneous SNR γ , conditioned on the average SNR $\bar{\gamma}$, is given by

$$p_{\gamma|\bar{\gamma}}(\gamma|\bar{\gamma}) = \frac{m^m \gamma^{m-1}}{\Gamma(m) \bar{\gamma}^m} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right), \quad m > 0, \quad \gamma \geq 0, \quad (1)$$

where $\Gamma(x)$ denotes the Gamma function. The average SNR $\bar{\gamma}$ itself is a random variable with PDF given by

$$p_{\bar{\gamma}}(\bar{\gamma}) = \frac{\bar{\gamma}^{k-1}}{\Gamma(k) \bar{\gamma}^k} \exp\left(-\frac{\bar{\gamma}}{\bar{\gamma}}\right), \quad k > 0, \quad \bar{\gamma} \geq 0, \quad (2)$$

where $\bar{\gamma} \triangleq E\{\bar{\gamma}\}$ and $E\{\cdot\}$ denotes statistical expectation. Combining (1) and (2), the PDF of the instantaneous SNR γ results as [2]

$$p_{\gamma}(\gamma) = \frac{a^{\beta+1}}{\Gamma(k)\Gamma(m)2^{\beta}} \gamma^{\frac{\beta-1}{2}} K_{\alpha}(a\sqrt{\gamma}), \quad (3)$$

where $a \triangleq 2\sqrt{m/\bar{\gamma}}$, $\alpha \triangleq k-m$, $\beta \triangleq k+m-1$, and $K_{\nu}(x)$ denotes the modified Bessel function of the second kind and order ν .

III. PERFORMANCE ANALYSIS FOR NON-COHERENT TRANSMISSION SCHEMES

In this section, we derive closed-form BEP expressions for binary DPSK/non-coherent FSK modulation over L generalized K -fading branches with EGC at the receiver. As explained above, we assume that the shadowing part is fully correlated, whereas the multipath fading is i.i.d. across the L branches. Correspondingly, all branches are characterized by identical fading parameters, k and m , and by the same average SNR, $\bar{\gamma}$, which itself is a random variable with PDF given by (2).

Considering binary DPSK/non-coherent FSK modulation over L branches with EGC at the receiver, the instantaneous EGC output SNR is given by [9, Ch. 9.4]

$$\gamma_t \triangleq \sum_{l=1}^L \gamma_l, \quad (4)$$

where γ_l denotes the instantaneous SNR associated with the l th branch. For a fixed value of γ_t , the BEP of the considered non-coherent transmission schemes is given by [10, Ch. 14.4]

$$P_b(\gamma_t) = \frac{1}{2^{2L-1}} e^{-g\gamma_t} \sum_{l=0}^{L-1} c_l (g\gamma_t)^l, \quad (5)$$

where

$$c_l \triangleq \frac{1}{l!} \sum_{\kappa=0}^{L-1-l} \binom{2L-1}{\kappa}, \quad (6)$$

and $g \triangleq 1$ for binary DPSK and $g \triangleq 1/2$ for binary non-coherent FSK modulation. In order to arrive at a closed-form expression for the average BEP $\bar{P}_b(\bar{\gamma})$, we first average (5) over the instantaneous branch SNRs γ_l , while conditioning on $\bar{\gamma}$. In the final step, the resulting conditional BEP, denoted as $\bar{P}_b(\bar{\gamma})$, is then averaged over $\bar{\gamma}$.

We first note that – due to the assumption of independent multipath fading across the L branches – the joint PDF of the instantaneous branch SNRs γ_l ($l \in \{1, \dots, L\}$), conditioned on the average SNR $\bar{\gamma}$, is given by

$$p_{\gamma_1, \dots, \gamma_L|\bar{\gamma}}(\gamma_1, \dots, \gamma_L|\bar{\gamma}) = \prod_{l=1}^L p_{\gamma_l|\bar{\gamma}}(\gamma_l|\bar{\gamma}). \quad (7)$$

Second, we define the index vector $\kappa \triangleq [\kappa_1, \dots, \kappa_L] \in \mathbb{N}_0^L$ and the index set

$$\mathbb{K}_l \triangleq \{\kappa \in \mathbb{N}_0^L \mid \kappa_1 + \dots + \kappa_L = l\}, \quad (8)$$

where $\mathbb{N}_0$ denotes the set of all integers greater than or equal to zero, and we note that the term $\gamma_t^l = (\gamma_1 + \dots + \gamma_L)^l$ can be expressed as [11, Ch. 24]

$$\gamma_t^l = (\gamma_1 + \dots + \gamma_L)^l = \sum_{\kappa \in \mathbb{K}_l} \binom{l}{\kappa} \gamma_1^{\kappa_1} \dots \gamma_L^{\kappa_L}, \quad (9)$$

where $\binom{l}{\kappa} \triangleq l! / (\kappa_1! \dots \kappa_L!)$. Based on the above findings, the conditional BEP $\bar{P}_b(\bar{\gamma})$ can be written as

$$\begin{aligned} \bar{P}_b(\bar{\gamma}) &= \frac{1}{2^{2L-1}} \sum_{l=0}^{L-1} c_l g^l \sum_{\kappa \in \mathbb{K}_l} \binom{l}{\kappa} \\ &\times \left(\prod_{\lambda=1}^L \int_0^\infty e^{-g\gamma_\lambda} \gamma_\lambda^{\kappa_\lambda} p_{\gamma_\lambda|\bar{\gamma}}(\gamma_\lambda|\bar{\gamma}) d\gamma_\lambda \right). \end{aligned} \quad (10)$$

Plugging in (1) for the conditional PDFs $p_{\gamma_l|\bar{\gamma}}(\gamma_l|\bar{\gamma})$ and employing [§3.381, no. 4] from [12], we find the following expression for $\bar{P}_b(\bar{\gamma})$:

$$\begin{aligned} \bar{P}_b(\bar{\gamma}) &= \frac{1}{2^{2L-1}} \left(\frac{m^m}{\Gamma(m)} \right)^L \sum_{l=0}^{L-1} c_l g^l \sum_{\kappa \in \mathbb{K}_l} \binom{l}{\kappa} \\ &\times \left(\prod_{\lambda=1}^L \Gamma(m + \kappa_\lambda) \frac{\bar{\gamma}^{\kappa_\lambda}}{(g\bar{\gamma} + m)^{m + \kappa_\lambda}} \right). \end{aligned} \quad (11)$$

Based on the PDF (2) of the average SNR $\bar{\gamma}$, the average BEP $\bar{P}_b(\bar{\gamma}) \triangleq E_{\bar{\gamma}}\{\bar{P}_b(\bar{\gamma})\}$ can be written as

$$\begin{aligned} \bar{P}_b(\bar{\gamma}) &= \frac{1}{2^{2L-1}} \frac{1}{\Gamma(k) (\Gamma(m))^L \bar{\gamma}^k} \sum_{l=0}^{L-1} c_l g^l \sum_{\kappa \in \mathbb{K}_l} \binom{l}{\kappa} \\ &\times \frac{\prod_{\lambda=1}^L \Gamma(m + \kappa_\lambda)}{m^l} \int_0^\infty \frac{\bar{\gamma}^{k+l-1} \cdot e^{-\bar{\gamma}/\bar{\gamma}}}{\left(\frac{g}{m}\bar{\gamma} + 1\right)^{mL+l}} d\bar{\gamma}. \end{aligned} \quad (12)$$

Employing [§3.383, no. 5] from [12] and assuming that (i) m is a finite non-integer value¹ and (ii) $k \neq mL$, we find the following closed-form expression for the average BEP $\bar{P}_b(\bar{\gamma})$:

$$\begin{aligned} \bar{P}_b(\bar{\gamma}) &= \frac{1}{2^{2L-1}} \frac{1}{\Gamma(k)} \frac{\pi}{\sin(\pi \Delta_{k,m})} \\ &\times \sum_{l=0}^{L-1} c_l \left[\sum_{\kappa \in \mathbb{K}_l} \binom{l}{\kappa} \left(\prod_{\lambda=1}^L \Gamma(m + \kappa_\lambda) \right) \right] \\ &\times \left[\left(\frac{m}{g\bar{\gamma}} \right)^{mL} \frac{\Gamma(1 - \varphi_{m,l})}{\Gamma(1 - \psi_{k,l})} L_{-\varphi_{m,l}}^{\Delta_{k,m}} \left(\frac{m}{g\bar{\gamma}} \right) \right. \\ &\quad \left. - \left(\frac{m}{g\bar{\gamma}} \right)^k \frac{\sin(\pi \varphi_{m,l})}{\sin(\pi \psi_{k,l})} L_{-\psi_{k,l}}^{\Delta_{k,m}} \left(\frac{m}{g\bar{\gamma}} \right) \right], \end{aligned} \quad (13)$$

where $(x)_\nu \triangleq \Gamma(x + \nu) / \Gamma(x)$ denotes the Pochhammer symbol and $L_a^\nu(x)$ the generalized Laguerre function. Moreover, we have used the identity $\Gamma(x)\Gamma(1-x) = \pi / \sin(\pi x)$ for the Gamma function and introduced the short-hand notations $\Delta_{k,m} \triangleq k - mL$, $\psi_{k,l} \triangleq k + l$, and $\varphi_{m,l} \triangleq mL + l$.

¹As will be seen in Section V, error probabilities for values $m \in \mathbb{N}$, where $\mathbb{N}$ denotes the set of all integers greater than zero, can typically be evaluated with a high accuracy by replacing m with a slightly different value $m \pm \epsilon \notin \mathbb{N}$, where $\epsilon > 0$ is a small perturbation value.

IV. ASYMPTOTIC ANALYSIS AND DIVERSITY ORDER

The closed-form BEP expression (13) is relatively easy to evaluate, but involves the non-standard generalized Laguerre function. Correspondingly, the primary behavior of the resulting BEP curve is not obvious. In the following, we will therefore study the behavior of (13) for high SNR values ($\bar{\gamma} \rightarrow \infty$). In particular, we derive an expression for the resulting (asymptotic) diversity order²

$$d \triangleq \lim_{\bar{\gamma} \rightarrow \infty} -\frac{\partial \log(\bar{P}_b(\bar{\gamma}))}{\partial \log(\bar{\gamma})}. \quad (14)$$

Subsequently, we present a corresponding analysis for the case of a coherent transmission scheme and show that the resulting diversity order is, in fact, the same as that of the considered non-coherent transmission schemes.

A. Non-Coherent Transmission Schemes

For $x \rightarrow 0$, the generalized Laguerre function $L_a^b(x)$ can be approximated as [13, Ch. 13.2]

$$L_a^b(x) \doteq \frac{(b+1)_a}{\Gamma(a+1)}, \quad (15)$$

where $\doteq$ denotes asymptotic equality. For $\bar{\gamma} \rightarrow \infty$, the average BEP (13) can thus be approximated as

$$\begin{aligned} \bar{P}_b(\bar{\gamma}) &\doteq \frac{1}{2^{2L-1}} \frac{\text{sign}(\Delta_{k,m})}{\Gamma(k)} \frac{\pi}{\sin(\pi \Delta_{k,m})} \left(\frac{m}{g \bar{\gamma}} \right)^{\zeta_1} \\ &\quad \times \sum_{l=0}^{L-1} c_l \Phi_l \Xi_l \frac{(1 - |\Delta_{k,m}|)_{-\zeta_1-l}}{\Gamma(1 - \psi_{k,l})}, \end{aligned} \quad (16)$$

where $\zeta_1 \triangleq \min\{k, mL\}$, $\text{sign}(x)$ denotes the sign function (i.e., $\text{sign}(x) = +1$ for all $x \geq 0$ and $\text{sign}(x) = -1$ otherwise),

$$\Phi_l \triangleq \sum_{\kappa \in \mathbb{K}_l} \binom{l}{\kappa} \left(\prod_{\lambda=1}^L (m)_{\kappa_\lambda} \right), \quad (17)$$

and³

$$\Xi_l \triangleq \begin{cases} \sin(\pi \varphi_{m,l}) / \sin(\pi \psi_{k,l}) & \text{for } k < mL \\ 1 & \text{for } k > mL \end{cases}. \quad (18)$$

Correspondingly, the asymptotic diversity order is obtained as

$$d = \zeta_1 = \min\{k, mL\}. \quad (19)$$

This result reveals an interesting interplay between macroscopic diversity due to shadowing effects and microscopic diversity due to multipath fading: the asymptotic diversity order is always limited by either the shadowing effect ($k \leq mL$) or the multipath fading ($mL < k$), depending on which one of the two fading effects is more severe.

In order to arrive at (16), we have utilized that for $\bar{\gamma} \rightarrow \infty$ only one of the two $L_a^b(x)$ -terms in (13) dominates, namely the one which is associated with the term $(\frac{m}{g \bar{\gamma}})^{\zeta_1}$. Correspondingly, if $k \approx mL$ the convergence of the asymptotic solution

²The (asymptotic) diversity order is the negative slope of the BEP curve for high SNR values on a log-log scale. It has been shown to be a useful measure for characterizing the principal behavior of digital transmission schemes over various fading channels [10, Ch. 14.4].

³As earlier, we assume that $k \neq mL$, since otherwise (13) is not valid. However, it turns out that (16) yields nearly identical results for $k = mL + \epsilon$ and $k = mL - \epsilon$, if ϵ is chosen sufficiently small.

(16) to the exact expression (13) can be expected to be rather slow, since the dominant term will only emerge for very large values of $\bar{\gamma}$. However, if k and mL are sufficiently different, the convergence of (16) is typically quite fast, as will be seen from the numerical performance results presented in Section V.

B. Coherent Transmission Scheme

Next, we compare the above result for binary DPSK/non-coherent FSK modulation with the asymptotic diversity order obtained in the case of a coherent transmission scheme.

As an example, we consider a binary PSK scheme with maximum-ratio combining (MRC) at the receiver. The corresponding average BEP can be determined via the following finite-range integral [14]:

$$\bar{P}_b(\bar{\gamma}) = \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_t} \left(-\frac{1}{\sin^2(\phi)} \right) d\phi, \quad (20)$$

where $M_{\gamma_t}(x) \triangleq E\{e^{x\gamma_t}\}$ denotes the moment-generating function (MGF) of the instantaneous MRC output SNR $\gamma_t = \sum_{l=1}^L \gamma_l$. Note that (20) is valid for arbitrary fading correlations, provided that an expression for the MGF $M_{\gamma_t}(x)$ is available.

In order to derive an expression for $M_{\gamma_t}(x)$, recall that the joint PDF $p_{\gamma_1, \dots, \gamma_L | \bar{\gamma}}(\gamma_1, \dots, \gamma_L | \bar{\gamma})$, conditioned on the average SNR $\bar{\gamma}$, can be written as the product of the conditional PDFs $p_{\gamma_l | \bar{\gamma}}(\gamma_l | \bar{\gamma})$ of the instantaneous branch SNRs γ_l ($l \in \{1, \dots, L\}$), cf. (7). Correspondingly, the conditional MGF of the instantaneous MRC output SNR γ_t , $M_{\gamma_t | \bar{\gamma}}(x) \triangleq \int_0^\infty e^{x\gamma_t} p_{\gamma_t | \bar{\gamma}}(\gamma_t | \bar{\gamma}) d\gamma_t$, is given by

$$M_{\gamma_t | \bar{\gamma}}(x) = \prod_{l=1}^L M_{\gamma_l | \bar{\gamma}}(x). \quad (21)$$

Based on (1) and [§3.381, no. 4] from [12], the conditional MGF of the instantaneous branch SNR γ_l , $M_{\gamma_l | \bar{\gamma}}(x)$, can be calculated as

$$M_{\gamma_l | \bar{\gamma}}(x) = \left(\frac{m}{m - x \bar{\gamma}} \right)^m, \quad \text{Re}\{x\} < 0, \quad (22)$$

which is the well-known MGF for Nakagami- m fading [9, Ch. 2.2]. Based on (2), (21) and (22), the (unconditional) MGF of γ_t can be written as

$$M_{\gamma_t}(x) = \frac{1}{\Gamma(k) \bar{\gamma}^k} \int_0^\infty \frac{\bar{\gamma}^{k-1}}{\left(1 - \frac{x}{m} \bar{\gamma}\right)^{mL}} \cdot e^{-\bar{\gamma}/\bar{\gamma}} d\bar{\gamma}. \quad (23)$$

Assuming that m is a finite non-integer value and employing [§3.383, no. 5] from [12], we find the following closed-form expression for the MGF of γ_t :

$$\begin{aligned} M_{\gamma_t}(x) &= (k)_{-mL} \cdot (mL)_{1-k} \\ &\quad \times \left[\left(\frac{-m}{x \bar{\gamma}} \right)^{mL} \frac{\Gamma(mL) \Gamma(1-mL)}{\Gamma(1-k)} \cdot L_{-mL}^{-\Delta_{k,m}} \left(\frac{-m}{x \bar{\gamma}} \right) \right. \\ &\quad \left. - \left(\frac{-m}{x \bar{\gamma}} \right)^k \frac{\Gamma(k) \Gamma(1-k)}{\Gamma(1-mL)} \cdot L_{-k}^{\Delta_{k,m}} \left(\frac{-m}{x \bar{\gamma}} \right) \right], \\ &\quad \bar{\gamma} < \infty, \quad \text{Re}\{x\} < 0. \end{aligned} \quad (24)$$

As earlier, we have used that $\Gamma(x)\Gamma(1-x) = \pi / \sin(\pi x)$. We note that the derived MGF expression (24) could also be

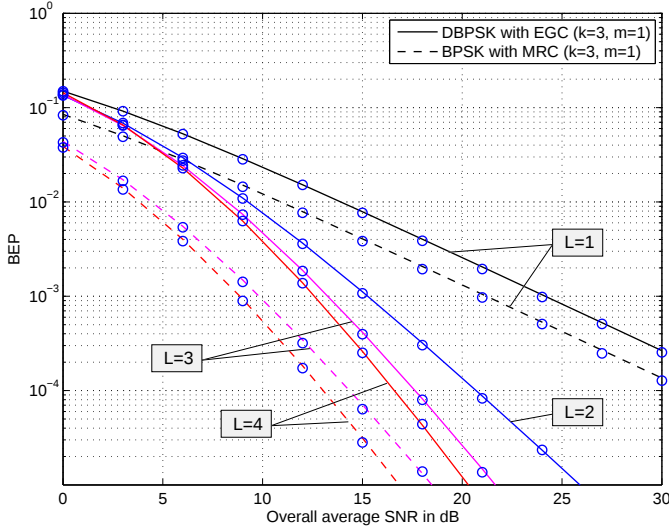


Fig. 1. Average BEP $\bar{P}_b(\bar{\gamma})$ versus overall average SNR $L\bar{\gamma}$ in dB for the case $k=3$ and $m=1$ (mild shadowing). Solid lines represent analytical results for binary DPSK (DBPSK) modulation with EGC at the receiver evaluated based on (13) using the values $k=3.01$ and $m=0.99$. Dashed lines represent corresponding analytical results for coherent BPSK modulation with MRC at the receiver evaluated based on (20), (24) using numerical integration. Corresponding simulation results for $k=3$ and $m=1$ are indicated by markers 'o' (both for DPSK and PSK modulation).

useful for other performance analyses (e.g., outage analysis) and is therefore of general interest. Also, as shown in [15], the MGF expression (24) can be employed to extend the above results to the case of non-binary modulation schemes. A further evaluation of (20) based on (24) appears to be difficult, however.

Now, based on (15) and employing [§3.621, no. 1] from [12], the average BEP (20) for $\bar{\gamma} \rightarrow \infty$ can be approximated as

$$\bar{P}_b(\bar{\gamma}) \doteq \frac{\text{sign}(\Delta_{k,m}) (k)_{-mL} \cdot (mL)_{1-k}}{2\pi} \times \frac{\Gamma(\zeta_1) (1 - |\Delta_{k,m}|)_{-\zeta_1}}{\Gamma(1 - \zeta_2)} \times \left(\frac{4m}{\bar{\gamma}}\right)^{\zeta_1} B(\zeta_1 + 1/2, \zeta_1 + 1/2), \quad (25)$$

where $\zeta_2 \triangleq \max\{k, mL\}$ and $B(x, y)$ denotes the Beta function. Correspondingly, the diversity order of binary PSK modulation with MRC at the receiver is given by

$$d = \zeta_1 = \min\{k, mL\}, \quad (26)$$

just as in the case of the considered non-coherent transmission schemes, cf. (19).

V. NUMERICAL PERFORMANCE RESULTS

In the following, numerical performance results are presented which illustrate our findings in Section III and Section IV. In particular, we will present Monte-Carlo simulation results, so as to corroborate our analytical performance results. As an example, we focus on the BEP performance of binary DPSK modulation with EGC at the receiver.

Fig. 1 presents numerical results for the average BEP $\bar{P}_b(\bar{\gamma})$ as a function of the overall average received SNR $L\bar{\gamma}$ in dB for

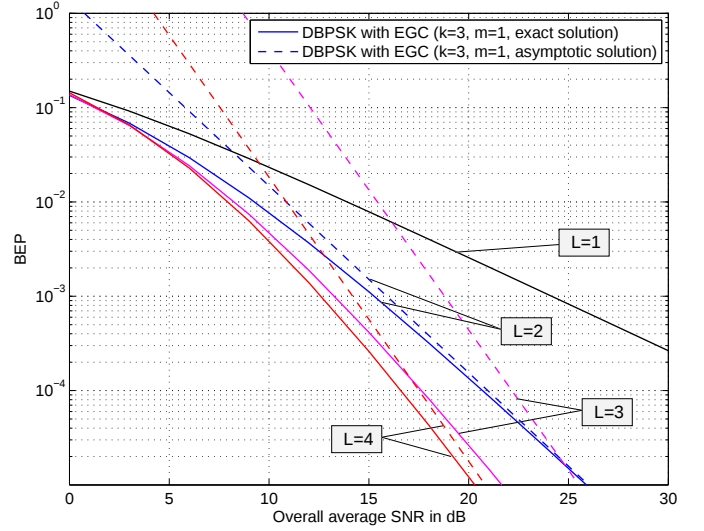


Fig. 2. Average BEP $\bar{P}_b(\bar{\gamma})$ versus overall average SNR $L\bar{\gamma}$ in dB for the case $k=3$ and $m=1$ (mild shadowing). Solid lines represent analytical results for DBPSK modulation with EGC at the receiver, evaluated based on (13) using the values $k=3.01$ and $m=0.99$. Dashed lines represent corresponding asymptotic results evaluated based on (16).

the case $k=3$ and $m=1$ (mild shadowing) and $L \in \{1, \dots, 4\}$. Solid lines represent analytical results evaluated based on (13), using the values $k=3.01$ and $m=0.99$. Dashed lines represent analytical results for coherent binary PSK modulation with MRC at the receiver (for the cases $L \in \{1, 3, 4\}$), evaluated based on (20) and (24) using the same values $k=3.01$ and $m=0.99$. Corresponding simulation results for $k=3$ and $m=1$, obtained by Monte-Carlo simulations over a large number of independent channel realizations, are indicated by markers 'o' (both for DPSK and PSK modulation). As can be seen, the analytical results and the simulation results are in good agreement, which corroborates our analysis in Section III. Note that significant diversity gains are accomplished for $L > 1$, both in the case of DPSK and PSK modulation. As can be seen, the general behavior of the BEP curves is the same for coherent and non-coherent transmission. The asymptotic advantage of binary PSK over binary DPSK modulation is about 3 dB, similar to the case of pure Rayleigh fading.

In Fig. 2, we compare the exact analytical BEPs for DPSK modulation according to (13) with the asymptotic BEPs according to (16).⁴ As earlier, the values $k=3.01$ and $m=0.99$ were employed for evaluating the expressions (13) and (16). It can be seen that convergence is comparatively fast for the cases $L=2$ and $L=4$. In particular, the BEP curves exhibit the predicted diversity orders of $d=2m=2$ and $d=k=3$, respectively. However, as discussed in Section IV, in the case $L=3$ convergence is very slow, since $k \approx mL$. In this example, SNR values on the order of 100 dB are required, until the exact analytical BEP (13) approaches the asymptotic BEP (16) and assumes the predicted asymptotic diversity order of $d=3m \approx k=3$. Note that since the maximum diversity order is accomplished for $L=3$, the relative performance advantage for $L > 3$ branches is comparatively small in this example.

⁴For binary PSK modulation with MRC at the receiver we have obtained very similar results (not depicted).

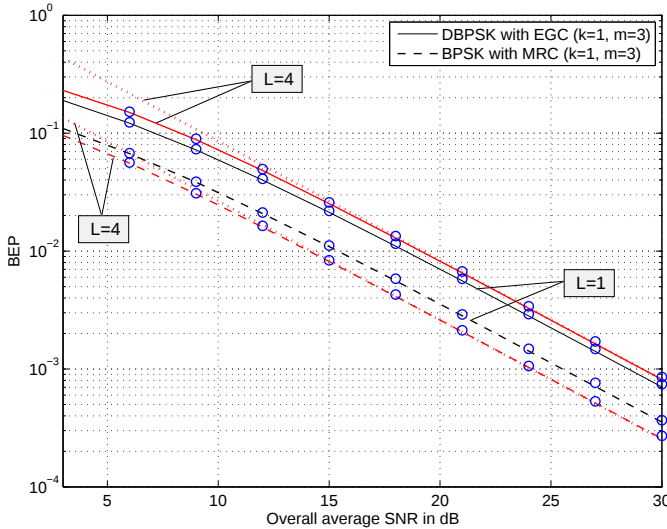


Fig. 3. Average BEP $\bar{P}_b(\bar{\gamma})$ versus overall average SNR $L\bar{\gamma}$ in dB for the case $k=1$ and $m=3$ (severe shadowing). Solid lines represent analytical results for DBPSK modulation with EGC at the receiver evaluated based on (13) using the values $k=1.01$ and $m=2.99$. Dashed lines represent corresponding analytical results for coherent BPSK modulation with MRC at the receiver evaluated based on (20), (24) using numerical integration. Corresponding simulation results for $k=1$ and $m=3$ are indicated by markers ‘o’ (both for DPSK and PSK modulation). The dotted lines represent asymptotic BEP curves for the case $L=4$ evaluated based on (16) for DPSK modulation and based on (25) for PSK modulation.

Finally, in Fig. 3 numerical performance results for the case $k=1$ and $m=3$ (severe shadowing) and $L \in \{1, 4\}$ are presented. Again it can be seen that the analytical results (solid lines for binary DPSK and dashed lines for binary PSK modulation) and the simulation results (markers ‘o’) are in good agreement. The analytical results for binary DPSK and binary PSK modulation were again evaluated based on (13) and (20), (24), respectively, using the values $k=1.01$ and $m=2.99$. Interestingly, in contrast to the case of mild shadowing, $L > 1$ branches offer no diversity benefit at all. As can be seen, in the case of binary DPSK modulation the BEP curve for $L=4$ is even slightly worse than the BEP curve for $L=1$ (due to the SNR normalization). The BEP curves for $L=2$ and $L=3$ (not depicted) lie in between the curves for $L=1$ and $L=4$. As predicted by the asymptotic BEP (16), included here for the case $L=4$ (dotted line), the BEP curves of binary DPSK for $L \geq 1$ are all characterized by the same asymptotic diversity order of $d=k=1$. Also note that the convergence of the asymptotic BEP (16) to the exact BEP (13) is comparatively fast in this example. Finally, we note that while in the case of binary PSK modulation the asymptotic diversity order is the same as for binary DPSK modulation, the order of the curves is swapped here, i.e., $L=4$ offers a slight performance advantage over $L=1$ (the BEP curves for $L=2$ and $L=3$ were again found in between the curves for $L=1$ and $L=4$).

VI. CONCLUSIONS

The generalized K -fading model, which is characterized by two fading parameters, $k > 0$ and $m > 0$, has recently been recognized as an accurate model for wireless scenarios with composite shadowing and multipath fading. In this paper, we

have derived closed-form expressions for the BEP of binary DPSK modulation and binary non-coherent FSK modulation over L generalized K -fading links with EGC at the receiver. Moreover, we have conducted an asymptotic performance analysis for high SNR values and have studied the resulting diversity order for various cases. Our results have shown that there is an interesting interplay between the two fading parameters k and m : the asymptotic diversity order is always limited by either the shadowing effect or the multipath fading, depending on which one of the two fading effects is more severe. Moreover, we have shown that the diversity order of the considered non-coherent transmission schemes is the same as in the case of coherent transmission. Finally, numerical performance results were presented, in order to illustrate the above findings, and our analytical performance results were corroborated by means of Monte-Carlo simulations.

An extension of the presented results to the case of non-binary coherent and non-coherent transmission schemes can be found in [15]. Moreover, it is worth noting that the generalized K -fading model is also useful to model cascade fading, which occurs, e.g., in mobile-to-mobile communication scenarios [16], [17]. A corresponding analysis can also be found in [15].

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