A Robust Receive Diversity Scheme for Spatially Correlated Multiple-Antenna Systems Using Second-Order Channel Statistics

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***Abstract*— We consider a simple reduced-complexity receive diversity scheme for spatially correlated multiple-antenna systems, which consists of an inner decorrelation stage based on the Karhunen-Loève transform and an outer selection stage. The task of the selection stage is to provide an optimal trade-off between complexity and performance, by selecting an appropriate subset of the eigenvalues/ eigenvectors of the receiver correlation matrix for further processing. The considered receive diversity scheme is solely based on second-order channel statistics. The main focus of the paper is to analyze the complexity-performance trade-off offered by the reduced-dimension receiver and to study the impact of estimation errors concerning the receiver correlation matrix. By means of numerical results, it is shown that the considered receive diversity scheme provides quite a robust performance. We also consider a statistical transmit power allocation scheme, which might be employed in conjunction with the reduced-dimension receiver so as to improve performance.**

I. Introduction

DURING THE LAST decade, the use of multiple antennas for wireless communication systems has attracted considerable interest, because multiple-antenna systems offer huge advantages over conventional single-antenna systems. On the one hand, it was shown in [1],[2] that the capacity of a multiple-input multiple-output (MIMO) system with M transmit (Tx) antennas and N receive (Rx) antennas grows *linearly* with $\min\{M, N\}$. Correspondingly, multiple antennas provide a promising means to increase the spectral efficiency of a system. On the other hand, it was shown in [3],[4] that multiple antennas can also be utilized, in order to provide a spatial diversity gain and thus to improve the error performance of a system.

The results in [1]-[4] are based on the assumption that the individual transmission links from the transmit antennas to the receive antennas are statistically *independent*. Spatial correlation, caused by insufficient antenna spacings or a lack of scattering from the physical environment, can cause significant degradations in capacity and error performance [5],[6]. In cellular systems, spatial correlation is an issue both at the base station and at the mobile station: Though at the base station generous antenna spacings can be granted, there is comparably little scattering from the physical environment, because the transmitted/ received signals are typically concentrated within a small angular region. As opposed to this, the mobile station normally experiences rich scattering from many local scatterers, but the antenna spacings are often small due to a limited terminal size.

In this paper, we consider a simple reduced-complexity receive diversity scheme for spatially correlated multiple-antenna systems, which solely requires knowledge of the second-order statistics of the MIMO channel. Such statistical channel knowledge can easily be acquired, for example off-line through field measurements, ray-tracing simulations or based on physical channel models, or on-line based on long-term averaging of the channel coefficients [7]. The receive diversity scheme under consideration consists of an inner decorrelation stage based on the Karhunen-Loève transform (KLT) [8, Ch. 8.5] and an outer selection stage. (The general structure of this reduced-dimension receiver was earlier considered in [9]. Here, we combine it with subsequent space-time decoding/ equalization.)

The benefit of such a receiver structure is that it provides a flexible trade-off between complexity and performance: Using all spatial dimensions offered by the individual receive antennas is of course optimal, but it leads to a high complexity for subsequent receiver stages. In the case of correlated receive antennas, however, an appropriate subset of spatial dimensions is usually sufficient in order to achieve a performance close to the optimum. In other words, the complexity of subsequent receiver stages can be reduced significantly at the expense of only a small performance loss. This complexity reduction is carried out by the selection stage, by selecting an appropriate subset of the eigenvalues (and the associated eigenvectors) of the receiver correlation matrix for further processing.

The main focus of this paper is to analyze the complexity-performance trade-off offered by the reduced-dimension receiver and to study the influence of estimation errors concerning the receiver correlation matrix. As will be see, a mismatched decorrelation and selection stage due to estimation errors can have some impact on the complexity-performance trade-off, since a wrong number of eigenvalues/ eigenvectors might be retained for further processing. Still, it is shown that the considered receive diversity scheme is quite robust against these effects. Specifically, the robustness of the scheme can be improved by modifying the selection rule originally proposed in [9] accordingly. We also consider a statistical transmit power allocation scheme, which might be employed in conjunction with the reduced-dimension receiver so as to improve performance.

A. Paper Organization

The paper is organized as follows: In Section II, the system and correlation model used throughout this paper is introduced. In Section III, the receive diversity scheme under consideration is discussed. Specifically, closed-form expressions and numerical results for the resulting bit error rates are presented. The impact of estimation errors is analyzed in Section IV. Finally, the statistical transmit power allocation scheme is considered in Section V, and conclusions are drawn in Section VI.

B. Mathematical Notation

Matrices and vectors are written in upper case and lower case bold face, respectively. If not stated otherwise, all vectors are column vectors. The complex conjugate of a complex number a is marked as a^*, and the Hermitian transposed of a matrix $\mathbf{A}$ as $\mathbf{A}^{\mathrm{H}}$. The trace of an $(M{\times}M)$-matrix, i.e., the sum over all diagonal elements, is denoted as $\mathrm{tr}(\mathbf{A})$. The square-root $\mathbf{A}^{1/2}$ of a Hermitian matrix $\mathbf{A}$ (i.e., $\mathbf{A}=\mathbf{A}^{\mathrm{H}}$) is defined as $\mathbf{A}^{1/2\,\mathrm{H}}\mathbf{A}^{1/2}=\mathbf{A}^{1/2}\mathbf{A}^{1/2\,\mathrm{H}}=\mathbf{A}$. $\mathrm{diag}(\mathbf{a})$ is a diagonal matrix with diagonal elements given by the vector $\mathbf{a}$, and $\mathrm{vec}(\mathbf{A})$ is a vector which results from stacking the columns of an $(N{\times}M)$-matrix $\mathbf{A}$ in a joint vector. $\mathsf{E}\{.\}$ denotes statistical expectation.

II. System and Correlation Model

Throughout this paper, the complex baseband notation is used. We consider a MIMO system with M transmit and N receive antennas. The corresponding discrete-time channel model for quasi-static frequency-flat fading is given by

$$\mathbf{y}[k]=\mathbf{H}\mathbf{x}[k]+\mathbf{n}[k], \tag{1}$$

where k denotes the discrete time index, $\mathbf{y}[k]$ the $(N{\times}1)$-received vector, $\mathbf{H}$ the $(N{\times}M)$-channel matrix, $\mathbf{x}[k]$ the $(M{\times}1)$-transmitted vector, and $\mathbf{n}[k]$ an $(N{\times}1)$-noise vector. It is assumed that $\mathbf{H}$, $\mathbf{x}[k]$ and $\mathbf{n}[k]$ are statistically independent.

The channel matrix $\mathbf{H}$ is assumed to be constant over an entire data block, spanning K subsequent time indices, and changes randomly from one data block to the next. Specifically, the entries h_{ji} of $\mathbf{H}$ ($i=1,...,M$, $j=1,...,N$) are assumed to be zero-mean (circularly symmetric) complex Gaussian random variables with variance $\sigma_h^2/2$ per real dimension, i.e. $h_{ji}\sim\mathcal{CN}\{0,\sigma_h^2\}$ (Rayleigh fading).[1]

The vector $\mathbf{x}[k]$ is assumed to contain space-time encoded data symbols $a[k]$ (randomly) drawn from a Q-ary symbol alphabet $\mathcal{A}$. Within the scope of this paper, we focus on orthogonal space-time block codes (OSTBCs) [3],[4] such as the well-known Alamouti-STBC for $M=2$ transmit antennas. However, the reduced-dimension receiver considered here (as well as the statistical transmit power allocation scheme) can be used in conjunction with any other space-time coding technique. The entries $x_i[k]$ of the vector $\mathbf{x}[k]$ are assumed to have zero means and equal variances.[2] Moreover, we assume an overall power constraint of P/N, i.e., $\mathsf{E}\{|x_i[k]|^2\}:=P/(MN)$, $i=1,...,M$. (Thus, a fair comparison is possible between systems with different numbers of antennas.) Typically, the entries of $\mathbf{x}[k]$ are statistically independent random variables (only across the individual transmit antennas, not in time direction), i.e., $\mathsf{E}\{\mathbf{x}[k]\,\mathbf{x}^{\mathrm{H}}[k]\}=P/(MN)\cdot\mathbf{I}_M$.

Finally, the entries of $\mathbf{n}[k]$ are zero-mean, spatially and temporally white (circularly symmetric) complex Gaussian random variables with variance $\sigma_n^2/2$ per real dimension, i.e., $n_j[k]\sim\mathcal{CN}\{0,\sigma_n^2\}$ and $\mathsf{E}\left\{\mathbf{n}[k]\,\mathbf{n}^{\mathrm{H}}[k']\right\}=\sigma_n^2\cdot\delta[k-k']\cdot\mathbf{I}_N$. (The noise variance σ_n^2 is assumed to be known at the receiver.)

The spatial correlation between two channel coefficients h_{ji} and $h_{j'i'}$ is defined as

$$\rho_{ij,i'j'}:=\mathsf{E}\{h_{ji}\,h^*_{j'i'}\}/\sigma_h^2=\rho^*_{i'j',ij}. \tag{2}$$

(Note that the magnitude of $\rho_{ij,i'j'}$ is always between zero and one.) Moreover, we define

$$\mathbf{R}_{\mathrm{Tx}}:=\mathsf{E}\{\mathbf{H}^{\mathrm{H}}\mathbf{H}\}/(N\sigma_h^2),\;\;\mathbf{R}_{\mathrm{Rx}}:=\mathsf{E}\{\mathbf{H}\mathbf{H}^{\mathrm{H}}\}/(M\sigma_h^2), \tag{3}$$

where $\mathbf{R}_{\mathrm{Tx}}$ denotes the transmitter correlation matrix and $\mathbf{R}_{\mathrm{Rx}}$ the receiver correlation matrix ($\mathrm{tr}(\mathbf{R}_{\mathrm{Tx}})=M$, $\mathrm{tr}(\mathbf{R}_{\mathrm{Rx}})=N$).

Within the scope of this paper, the Kronecker-correlation model [5] is used. This means that (i) the transmit antenna correlations $\rho_{ij,i'j}=:\rho_{\mathrm{Tx},ii'}$ ($i,i'=1,...,M$) do not depend on the specific receive antenna j under consideration, (ii) the receive antenna correlations $\rho_{ij,ij'}=:\rho_{\mathrm{Rx},jj'}$ ($j,j'=1,...,N$) do not depend on the specific transmit antenna i under consideration, and (iii) the spatial correlations $\rho_{ij,i'j'}$ can be written as $\rho_{ij,i'j'}:=\rho_{\mathrm{Tx},ii'}\cdot\rho_{\mathrm{Rx},jj'}$. Altogether, the overall spatial correlation matrix $\mathbf{R}:=\mathsf{E}\{\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^{\mathrm{H}}\}/\sigma_h^2$ of size $(MN{\times}MN)$ can be written as the Kronecker product

$$\mathbf{R}=\mathbf{R}_{\mathrm{Tx}}\otimes\mathbf{R}_{\mathrm{Rx}}, \tag{4}$$

$$\mathbf{R}_{\mathrm{Tx}}:=[\rho_{\mathrm{Tx},ii'}]_{i,i'=1,\dots,M},\;\;\mathbf{R}_{\mathrm{Rx}}:=[\rho_{\mathrm{Rx},jj'}]_{j,j'=1,\dots,N}. \tag{5}$$

Moreover, the channel matrix $\mathbf{H}$ can be written as

$$\mathbf{H}:=\mathbf{R}_{\mathrm{Rx}}^{1/2}\,\mathbf{G}\,\mathbf{R}_{\mathrm{Tx}}^{1/2}, \tag{6}$$

where $\mathbf{G}$ denotes an $(N{\times}M)$-matrix with spatially uncorrelated entries $g_{ji}\sim\mathcal{CN}\{0,\sigma_h^2\}$. The square-roots $\mathbf{R}_{\mathrm{Tx}}^{1/2}$ and $\mathbf{R}_{\mathrm{Rx}}^{1/2}$ can be obtained via the eigenvalue decompositions of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$ (e.g., by means of the Jacobian algorithm [10, Ch. 8.4]):

$$\mathbf{R}_{\mathrm{Tx}}^{1/2}:=\mathbf{U}_{\mathrm{Tx}}\,\mathbf{\Lambda}_{\mathrm{Tx}}^{1/2}\,\mathbf{U}_{\mathrm{Tx}}^{\mathrm{H}},\qquad\mathbf{R}_{\mathrm{Rx}}^{1/2}:=\mathbf{U}_{\mathrm{Rx}}\,\mathbf{\Lambda}_{\mathrm{Rx}}^{1/2}\,\mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}, \tag{7}$$

where $\mathbf{\Lambda}_{\mathrm{Tx}}$, $\mathbf{\Lambda}_{\mathrm{Rx}}$ are diagonal matrices containing the (real-valued) eigenvalues $\lambda_{\mathrm{Tx},i}$ and $\lambda_{\mathrm{Rx},j}$ of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$, respectively, and $\mathbf{U}_{\mathrm{Tx}}$, $\mathbf{U}_{\mathrm{Rx}}$ are unitary matrices containing the corresponding eigenvectors ($\mathbf{U}_{\mathrm{Tx}}\mathbf{U}_{\mathrm{Tx}}^{\mathrm{H}}=\mathbf{I}_M$, $\mathbf{U}_{\mathrm{Rx}}\mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}=\mathbf{I}_N$). Note that the eigenvalues $\lambda_{\mathrm{Tx},i}$ and $\lambda_{\mathrm{Rx},j}$ are always greater or equal to zero [11, Ch. 1.5]. Since $\mathbf{\Lambda}_{\mathrm{Tx}}$ and $\mathbf{\Lambda}_{\mathrm{Rx}}$ are diagonal, $\mathbf{\Lambda}_{\mathrm{Tx}}^{1/2}$ and $\mathbf{\Lambda}_{\mathrm{Rx}}^{1/2}$ are also diagonal and contain the (non-negative) square-roots of the eigenvalues $\lambda_{\mathrm{Tx},i}$ and $\lambda_{\mathrm{Rx},j}$, respectively.

With the above assumptions, the following covariance matrix results for the received vector $\mathbf{y}[k]$:

$$\begin{aligned}\mathsf{E}\left\{\mathbf{y}[k]\,\mathbf{y}[k]^{\mathrm{H}}\right\}&=\mathsf{E}\left\{\mathbf{H}\mathbf{x}[k]\,\mathbf{x}^{\mathrm{H}}[k]\mathbf{H}^{\mathrm{H}}\right\}+\mathsf{E}\left\{\mathbf{n}[k]\,\mathbf{n}^{\mathrm{H}}[k]\right\}\\&=\frac{P}{MN}\cdot\mathsf{E}\left\{\mathbf{H}\mathbf{H}^{\mathrm{H}}\right\}+\sigma_n^2\,\mathbf{I}_N\\&=\frac{P}{N}\cdot\sigma_h^2\,\mathbf{R}_{\mathrm{Rx}}+\sigma_n^2\,\mathbf{I}_N\,.\end{aligned} \tag{8}$$

In the case of frequency-selective fading, (1) generalizes to

$$\mathbf{y}[k]=\sum_{l=0}^{L}\mathbf{H}^{(l)}\mathbf{x}[k-l]+\mathbf{n}[k], \tag{9}$$

where L denotes the channel memory length (assumed identical for all transmission links). The variance of the entries $h_{ji}^{(l)}$ of $\mathbf{H}^{(l)}$ is in the sequel denoted by $\sigma_{h,l}^2$. The spatial correlation between two channels coefficients $h_{ji}^{(l)}$ and $h_{j'i'}^{(l)}$ is defined as

$$\rho_{ij,i'j'}^{(l)}:=\mathsf{E}\{h_{ji}^{(l)}\,h_{j'i'}^{(l)*}\}/\sigma_{h,l}^2. \tag{10}$$

In the case of frequency-selective fading, (8) generalizes to

$$\begin{aligned}&\mathsf{E}\left\{\mathbf{y}[k]\,\mathbf{y}[k]^{\mathrm{H}}\right\}\\&=\underbrace{\sum_{l=0}^{L}\sum_{l'=0}^{L}\mathsf{E}\left\{\mathbf{H}^{(l)}\,\mathbf{x}[k-l]\,\mathbf{x}^{\mathrm{H}}[k-l']\,\mathbf{H}^{(l')\mathrm{H}}\right\}}_{=:\,\mathbf{\Theta}_{\mathrm{Rx}}}+\sigma_n^2\,\mathbf{I}_N.\end{aligned} \tag{11}$$

In accordance with the flat-fading case, we therefore define

$$\mathbf{R}_{\mathrm{Rx}}:=N\,\mathbf{\Theta}_{\mathrm{Rx}}/(P\tilde{\sigma}_h^2), \tag{12}$$

[1] For simplicity, we assume equal variances for the individual channel coefficients h_{ji}. A generalization to unequal variances is, however, straight forward.

[2] For the time being, we consider equal power allocation at the transmitter.

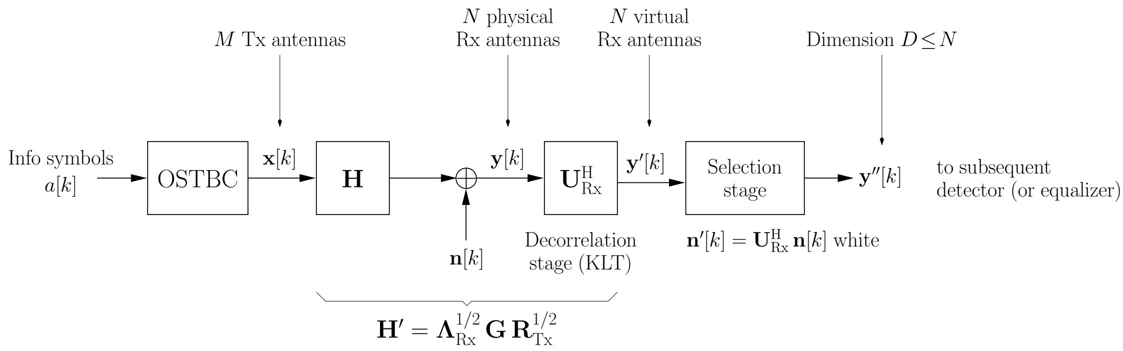

Fig. 1. Transmission model with receive diversity scheme for spatially correlated MIMO systems (frequency-flat fading).

where $\tilde{\sigma}_h^2$ is chosen such that the trace of the resulting receiver correlation matrix $\mathbf{R}_{\mathrm{Rx}}$ is equal to N. By this means, equation (8) can be used both for flat-fading and frequency-selective fading (with $\tilde{\sigma}_h^2$ instead of σ_h^2). Note that in the frequency-selective case $\mathbf{R}_{\mathrm{Rx}}$ represents an *effective* spatial correlation matrix seen at the receiver, which includes the spatial correlations of the MIMO channel, the intertap correlations, as well as the temporal correlations of the space-time encoded vector $\mathbf{x}[k]$. (In the case of temporally uncorrelated vectors $\mathbf{x}[k]$, we have $\tilde{\sigma}_h^2 = \sigma_{h,0}^2 + \ldots + \sigma_{h,L}^2$.) Unfortunately, the definition of a similar effective spatial correlation matrix for the transmitter side is less straight forward, unless $\mathsf{E}\{\mathbf{H}^{(l)\mathrm{H}}\mathbf{H}^{(l)}\}/(N\sigma_{h,l}^2) =: \mathbf{R}_{\mathrm{Tx}}$ is the same for all indices l.

III. Receive Diversity Scheme for Spatially Correlated MIMO Systems

In the following, the basic principle of the receive diversity scheme under consideration is discussed. For the sake of simplicity, we restrict the discussion to the case of frequency-flat fading. However, based on (8) and (12) a generalization to frequency-selective fading is straight forward.

A. Decorrelation and Selection Stage

The transmission model under consideration is depicted in Fig. 1. The information symbols $a[k]$ are space-time encoded using an OSTBC, and the resulting $(M{\times}1)$-vector $\mathbf{x}[k]$ is transmitted over the MIMO channel with channel matrix $\mathbf{H}$. The receiver consists of an inner decorrelation stage (based on the KLT) and a subsequent selection stage. The overall transmission model (without selection stage) can be written as follows:

$$\mathbf{y}'[k] = \mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}\mathbf{H}\,\mathbf{x}[k] + \mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}\mathbf{n}[k] =: \mathbf{H}'\,\mathbf{x}[k] + \mathbf{n}'[k], \quad (13)$$

where $\mathbf{H}' := \mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}\mathbf{H}$ and $\mathbf{n}'[k] := \mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}\mathbf{n}[k]$. The decorrelation stage transforms the given channel matrix $\mathbf{H}$ according to (6) into a semi-correlated channel matrix $\mathbf{H}' = \mathbf{\Lambda}_{\mathrm{Rx}}^{1/2}\,\mathbf{G}\,\mathbf{R}_{\mathrm{Tx}}^{1/2}$, by using the unitary matrix $\mathbf{U}_{\mathrm{Rx}}$ from the eigenvalue decomposition of $\mathbf{R}_{\mathrm{Rx}}$:

$$\mathsf{E}\left\{\mathbf{H}'\mathbf{H}'^{\mathrm{H}}\right\} = \mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}\,\mathsf{E}\left\{\mathbf{H}\mathbf{H}^{\mathrm{H}}\right\}\,\mathbf{U}_{\mathrm{Rx}} = M\sigma_h^2\,\mathbf{\Lambda}_{\mathrm{Rx}}. \quad (14)$$

Note that the resulting noise vector $\mathbf{n}'[k]$ is still spatially white (with unaltered variance):

$$\mathsf{E}\left\{\mathbf{n}'[k]\mathbf{n}'^{\mathrm{H}}[k]\right\} = \mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}\mathsf{E}\left\{\mathbf{n}[k]\mathbf{n}^{\mathrm{H}}[k]\right\}\mathbf{U}_{\mathrm{Rx}} = \sigma_n^2 \cdot \mathbf{I}_N. \quad (15)$$

The channel matrix $\mathbf{H}'$ is often called *virtual* channel matrix in the literature, and the N outputs of the decorrelation stage $\mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}}$ (vector $\mathbf{y}'[k]$) represent *virtual* receive antennas.

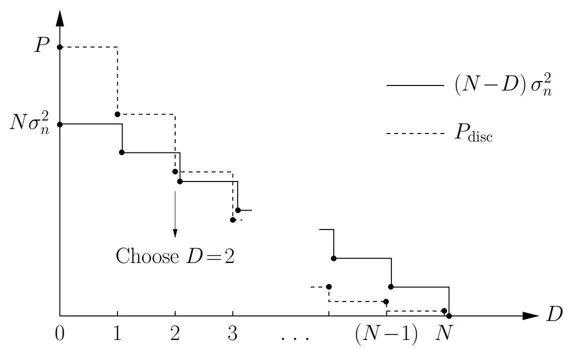

Fig. 2. Choice of the parameter D in the selection stage (example, $\sigma_h^2 = 1$).

The task of the selection stage is to provide an optimal trade-off between complexity and performance, by selecting an appropriate subset of the N virtual receive antennas for further processing [9]. More specifically, the selection stage selects those $D \leq N$ components of the vector $\mathbf{y}'[k]$ that correspond to the strongest eigenvalues $\lambda_{\mathrm{Rx},j}$ (and the associated eigenvectors) of the receiver correlation matrix $\mathbf{R}_{\mathrm{Rx}}$. This yields a new vector $\mathbf{y}''[k]$ of size $(D \times 1)$. On the one hand, D should be chosen as small as possible, in order to keep the resulting complexity of subsequent receiver stages small. On the other hand, if the chosen number of discarded signal dimensions is too large, a significant performance loss will occur. In [9], the following criterion was proposed for the selection of the parameter D:

$$P_{\mathrm{disc}} := \frac{P}{N} \cdot \sigma_h^2 \sum_{j \in \mathcal{J}_{\mathrm{disc}}} \lambda_{\mathrm{Rx},j} \;\overset{!}{\leq}\; (N-D) \cdot \sigma_n^2\,, \quad (16)$$

where $\mathcal{J}_{\mathrm{disc}}$ denotes the index set for the discarded eigenvalues $\lambda_{\mathrm{Rx},j}$ ($|\mathcal{J}_{\mathrm{disc}}| = N-D$). In other words, the parameter D should be chosen as small as possible, but such that the average discarded sum power of the desired signal, P_{disc}, is smaller or equal to the average sum power of the discarded noise. The optimization criterion (16) is illustrated in Fig. 2.[3]

B. Equalization and Detection Stage

Obviously, the receive diversity scheme under consideration is solely based on second-order channel statistics (in terms of the receiver correlation matrix, the channel variance, and the noise

[3] Note that both the decorrelation stage and the selection stage is completely independent of the transmitter correlation matrix $\mathbf{R}_{\mathrm{Tx}}$.

variance). For the subsequent equalization/ detection step, however, we assume perfect knowledge of the instantaneous channel matrix $\mathbf{H}'$. In the case of flat fading, the information symbols $a[k]$ can be recovered at the receiver by means of a simple linear detection operation based on an equivalent channel matrix $\mathbf{H}_{\mathrm{eq}}$ [3],[4]. In the case of frequency-selective fading, a bank of appropriate a-posteriori probability (APP) equalizers [12] can be used at the receiver, in order to recover the information symbols $a[k]$ (one APP equalizer for each receive antenna). In a final step, the APP values provided by the individual equalizers have to be combined accordingly (e.g., in the case of log-likelihood ratios, by a summation).[4]

C. Complexity-Performance Trade-off

In the following, the error performance of a spatially correlated OSTBC system with reduced-dimension receiver is evaluated, so as to illustrate the offered complexity-performance trade-off. To start with, we consider a full-dimension receiver ($D=N$). Moreover, for the time being we focus on the case of flat fading.

The OSTBC (in conjunction with the appropriate linear detection step at the receiver) transforms the $(M\times N)$-MIMO system (1) into an equivalent single-antenna system of form [16]

$$z[k] = \left(\sum_{i=1}^{M}\sum_{j=1}^{N} |h_{ji}|^2\right) a[k] + w[k], \tag{17}$$

where $z[k]$ denotes the kth received symbol after the linear detection step, $a[k]$ the kth information symbol, and $w[k]$ an additive white Gaussian noise (AWGN) sample. Correspondingly, the $(M\times N)$-OSTBC system is equivalent to an $(1\times MN)$ maximum-ratio-combining (MRC) system [17], where we assume that (i) the OSTBC provides a temporal rate of 1 symbol/ channel use ('full rate')[5] and (ii) the underlying overall received energy per information symbol, E_{s}, after linear detection/ MRC is the same in both systems. Using the average power constraint $P/(MN)$ from Section II for the transmitted symbols, the overall received signal-to-noise ratio (SNR) after linear detection/ MRC results as $P\sigma_h^2/\sigma_n^2 =: E_{\mathrm{s}}/N_0$, where N_0 denotes the single-sided noise power density.

The error performance of a spatially correlated $(1\times MN)$-MRC system (and thus of the corresponding $(M\times N)$-OSTBC-system) can in turn be analyzed by means of the KLT. Consider the following $(1\times MN)$-system:

$$\mathbf{y}[k] = \mathbf{h}\, a[k] + \mathbf{n}[k]. \tag{18}$$

(For the $(1\times MN)$-channel vector $\mathbf{h}$ and the $(1\times MN)$-noise vector $\mathbf{n}[k]$, the same statistical properties are assumed as in Section II.) Let $\mathbf{R} := \mathsf{E}\{\mathbf{h}\mathbf{h}^{\mathrm{H}}\}/\sigma_h^2$ denote the overall spatial correlation matrix, which corresponds to the Kronecker product of the transmitter and receiver correlation matrix in the equivalent $(M\times N)$-OSTBC system, cf. (4). Based on the eigenvalue decomposition $\mathbf{R} := \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathrm{H}}$, the system (18) is decorrelated as

$$\mathbf{y}'[k] := \mathbf{U}^{\mathrm{H}}\,\mathbf{y}[k] =: \mathbf{h}'\, a[k] + \mathbf{n}'[k], \tag{19}$$

[4] Alternatively, one might replace the OSTBC by a space-time coding scheme suitable for frequency-selective fading, such as (generalized) delay diversity [12]-[14] or the time-reversal STBC in [15].

[5] It should be noted that full-rate OSTBCs exist solely for two transmit antennas (Alamouti-STBC) [18]. However, since in this paper focus is on the complexity-performance trade-off offered by the reduced-dimension receiver, we will always assume a full-rate OSTBC for simplicity.

where $\mathsf{E}\{\mathbf{h}'\mathbf{h}'^{\mathrm{H}}\} = \mathbf{\Lambda}$ and $\mathsf{E}\{\mathbf{n}'[k]\mathbf{n}'^{\mathrm{H}}[k]\} = \sigma_n^2\mathbf{I}_N$. As can be seen, the decorrelated system is characterized by unequal average link SNRs determined by the eigenvalues $\lambda_1, ..., \lambda_{MN}$ of $\mathbf{R}$. In [19] it was shown that the two systems (18) and (19) are equivalent in the sense that MRC provides the same average symbol error rate in both cases.

In the following, we focus on binary antipodal transmission[6] (i.e., $a[k] \in \{\pm 1\}$). The average bit error rate (BER) of the (decorrelated) MRC-system – and thus of the equivalent OSTBC-system – can be calculated in closed form, according to [20, Ch. 14.5]

$$\bar{P}_{\mathrm{b}} = \frac{1}{2}\sum_{j=1}^{MN}\left(\prod_{\substack{j'=1\\ j'\neq j}}^{MN}\frac{\gamma_j}{\gamma_j-\gamma_{j'}}\right)\left(1-\sqrt{\frac{\gamma_j}{1+\gamma_j}}\right), \tag{20}$$

where $\gamma_j := P\,\sigma_h^2\,\lambda_j/(MN\,\sigma_n^2)$, $j=1, ..., MN$, denotes the average SNR for the jth receive antenna. (The overall average SNR is given by $\gamma := \gamma_1 + ... + \gamma_{MN} = E_{\mathrm{s}}/N_0$.) A high-SNR approximation ($\sigma_n^2 \to 0$) of (20) yields [20, Ch. 14.5]

$$\bar{P}_{\mathrm{b}} \approx \left(\frac{MN}{4\gamma}\right)^{MN}\binom{2MN-1}{MN}\prod_{j=1}^{MN}\frac{1}{\lambda_j}, \tag{21}$$

where it was assumed that all eigenvalues of the correlation matrix $\mathbf{R}$ are greater than zero. Two important observations can be made in (21): (i) Asymptotically, $\bar{P}_{\mathrm{b}}$ is always proportional to γ^{-MN}, i.e., the diversity order of the system is not reduced as long as the correlation matrix $\mathbf{R}$ has full rank; (ii) the product term in (21), which is solely determined by the eigenvalues of $\mathbf{R}$, causes an asymptotic up-shift of the BER curve (in a log-log plot): As shown in [21], the product term is always greater or equal to one (and it is only equal to one in the uncorrelated case, i.e., for $\mathbf{\Lambda} = \mathbf{I}_{MN}$).

Next, we consider a reduced-dimension receiver ($D<N$). Since $\mathbf{R}$ is the Kronecker product of the transmitter correlation matrix $\mathbf{R}_{\mathrm{Tx}}$ and the receiver correlation matrix $\mathbf{R}_{\mathrm{Rx}}$ in the equivalent $(M\times N)$-OSTBC system, the set of eigenvalues $\{\lambda_j \,|\, j=1, ..., MN\}$ of $\mathbf{R}$ is given by all pairwise products $\{\lambda_{\mathrm{Tx},i}\cdot\lambda_{\mathrm{Rx},j} \,|\, i=1, ..., M,\ j=1, ..., N\}$ of the eigenvalues of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$ [22, Ch. 12.2]. Therefore, based on (20) the average BER of a spatially correlated $(M\times N)$-OSTBC system with reduced-dimension receiver results as

$$\bar{P}_{\mathrm{b}} = \frac{1}{2}\sum_{i=1}^{M}\sum_{j\in\mathcal{J}_{\mathrm{ret}}}\left(\prod_{\substack{i'=1\\(i',j')\neq(i,j)}}^{M}\prod_{j'\in\mathcal{J}_{\mathrm{ret}}}\frac{1}{1-\dfrac{\lambda_{\mathrm{Tx},i'}\lambda_{\mathrm{Rx},j'}}{\lambda_{\mathrm{Tx},i}\lambda_{\mathrm{Rx},j}}}\right)\times$$
$$\times\left(1-\sqrt{\frac{P\,\sigma_h^2\,\lambda_{\mathrm{Tx},i}\lambda_{\mathrm{Rx},j}}{MN\,\sigma_n^2+P\,\sigma_h^2\,\lambda_{\mathrm{Tx},i}\lambda_{\mathrm{Rx},j}}}\right), \tag{22}$$

where $\mathcal{J}_{\mathrm{ret}}$ denotes the index set for the retained eigenvalues $\lambda_{\mathrm{Rx},j}$, i.e., $\mathcal{J}_{\mathrm{ret}} \cup \mathcal{J}_{\mathrm{disc}} = \{1, ..., N\}$. The corresponding high-SNR approximation ($\sigma_n^2 \to 0$) is given by

$$\bar{P}_{\mathrm{b}} \approx \left(\frac{MN}{4\gamma}\right)^{MD}\binom{2MD-1}{MD}\prod_{i=1}^{M}\prod_{j\in\mathcal{J}_{\mathrm{ret}}}\frac{1}{\lambda_{\mathrm{Tx},i}\lambda_{\mathrm{Rx},j}}. \tag{23}$$

[6] Channel coding is not taken into account. However, an outer channel coding scheme can be added to further improve performance.

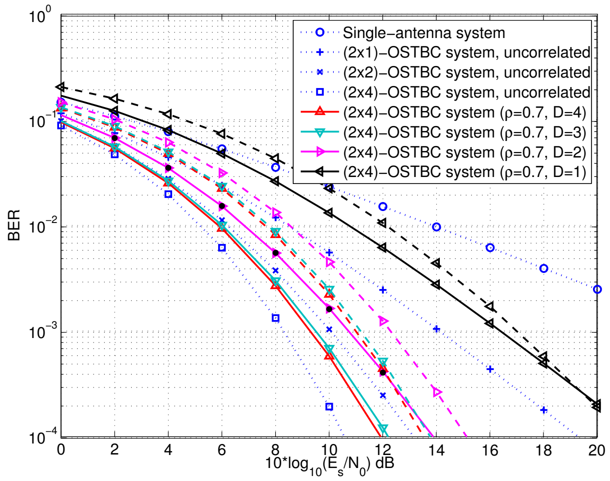

Fig. 3. BER performance of a (2×4)-OSTBC system with reduced-dimension receiver: Binary transmission, flat Rayleigh fading (analytical results, solid curves) and frequency-selective Rayleigh fading (simulative results for the GSM Typical Urban scenario [24], dashed curves), uncorrelated transmit antennas, receiver correlation matrix $\mathbf{R}_{\mathrm{Rx}} = \mathbf{R}_{M,\rho}$ with $M = 4$ and $\rho = 0.7$. The analytical results for flat fading were validated by means of Monte Carlo simulations. As an example, simulation results are included for the case $D = 2$ (marked by black dots). The simulative results for the frequency-selective case were obtained by means of Monte-Carlo simulations over 10,000 independent data blocks. A root-raised cosine receive filter with roll-off factor $r = 0.5$ was used, leading to a channel memory length of $L \approx 3$. For simplicity, it was assumed that the antenna correlations according to (10) are identical for all indices $l = 0, ..., L$ and comply with the Kronecker correlation model (4).

As can be seen, due to the selection stage the diversity order is reduced from MN to MD. Additionally, the overall received SNR is reduced from $\gamma = P\sigma_h^2/\sigma_n^2$ to $\gamma' = (P\sigma_h^2 - P_{\mathrm{disc}})/\sigma_n^2$. Due to these two effects, the BER performance of the system will deteriorate if a reduced dimension $D < N$ is selected. However, if the discarded eigenvalues of the receiver correlation matrix are small, the performance loss will be negligible (at least for low SNR values), i.e., the complexity of subsequent receiver stages can be reduced significantly while retaining (virtually) the same performance. This principle is illustrated in the following section by means of numerical results.

D. Numerical Results

As an example, we consider a (2×4)-OSTBC system with uncorrelated transmit antennas and a receiver correlation matrix $\mathbf{R}_{\mathrm{Rx}} \neq \mathbf{I}_4$. Specifically, we use a single-parameter correlation matrix

$$\mathbf{R}_{M,\rho} := \begin{bmatrix} 1 & \rho & \rho^4 & \cdots & \rho^{(M-1)^2} \\ \rho^* & 1 & \rho & \cdots & \rho^{(M-2)^2} \\ \rho^{4*} & \rho^* & & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ \rho^{(M-1)^2 *} & \rho^{(M-2)^2 *} & \cdots & \cdots & 1 \end{bmatrix} \quad (24)$$

($\rho \in \mathbb{C}$) for $\mathbf{R}_{\mathrm{Rx}}$, which was proposed in [23] for uniform linear antenna arrays with M antenna elements. In the sequel, we set $\rho := 0.7$ (real-valued). Fig. 3 displays the BER performance as a function of E_{s}/N_0 in dB, which results for different receiver dimensions $D = 1, ..., 4$. Both flat fading (solid curves) and frequency-selective fading (dashed curves) is considered.

The basic behavior of the BER curves is the same for flat fading and frequency-selective fading. (Note, however, that the the BER curves for frequency-selective fading exhibit a steeper asymptotic slope, due to a larger inherent diversity order.) The curves for $D = 4$ represent the respective optimum BER performance and the highest receiver complexity. As can be seen, when discarding the weakest eigenvalue of $\mathbf{R}_{\mathrm{Rx}}$ (which saves one fourth of the complexity for subsequent equalization/ detection), the associated performance loss is very small throughout the complete SNR range under consideration ($D = 3$). When discarding also the second smallest eigenvalue of $\mathbf{R}_{\mathrm{Rx}}$ ($D = 2$), a significant performance loss occurs for large SNR values. Note, however, that the selection criterion (16) suggests this choice of D only if E_{s}/N_0 is smaller or equal to 4 dB. Obviously, in this SNR region the associated performance loss is rather small, i.e., for $E_{\mathrm{s}}/N_0 \leq 4$ dB one half of the receiver complexity can be saved at the expense of only a small performance loss. Finally, when reducing the number of dimensions to $D = 1$, a significant performance loss occurs for the complete SNR range under consideration. (In fact, the selection criterion suggests this choice only if E_{s}/N_0 is smaller or equal to 2 dB.)[7]

IV. Impact of Estimation Errors

So far, we have assumed that the correlation matrix $\mathbf{R}_{\mathrm{Rx}}$ is perfectly known at the receiver. In a practical system, however, $\mathbf{R}_{\mathrm{Rx}}$ needs to be estimated. In the case of estimation errors, the reduced-dimension receiver will thus be based on an erroneous receiver correlation matrix $\hat{\mathbf{R}}_{\mathrm{Rx}}$. In general, both the eigenvectors and the eigenvalues of $\hat{\mathbf{R}}_{\mathrm{Rx}}$ will be different from those of the actual receiver correlation matrix $\mathbf{R}_{\mathrm{Rx}}$, i.e.,

$$\hat{\mathbf{R}}_{\mathrm{Rx}} = \hat{\mathbf{U}}_{\mathrm{Rx}}\, \hat{\mathbf{\Lambda}}_{\mathrm{Rx}}\, \hat{\mathbf{U}}_{\mathrm{Rx}}^{\mathrm{H}}, \quad (25)$$

where $\hat{\mathbf{U}}_{\mathrm{Rx}} \neq \mathbf{U}_{\mathrm{Rx}}$ and $\hat{\mathbf{\Lambda}}_{\mathrm{Rx}} \neq \mathbf{\Lambda}_{\mathrm{Rx}}$. (We assume that $\hat{\mathbf{R}}_{\mathrm{Rx}}$ is still a Hermitian matrix.) Correspondingly, Eq. (14) does not hold anymore and generalizes to

$$\mathsf{E}\left\{\mathbf{H}'\mathbf{H}'^{\mathrm{H}}\right\} = M\sigma_h^2\, \underbrace{\hat{\mathbf{U}}_{\mathrm{Rx}}^{\mathrm{H}} \mathbf{U}_{\mathrm{Rx}} \mathbf{\Lambda}_{\mathrm{Rx}} \mathbf{U}_{\mathrm{Rx}}^{\mathrm{H}} \hat{\mathbf{U}}_{\mathrm{Rx}}}_{=:\,\mathbf{\Xi}_{\mathrm{Rx}}}. \quad (26)$$

(If $\hat{\mathbf{U}}_{\mathrm{Rx}} \neq \mathbf{U}_{\mathrm{Rx}}$, the product $\hat{\mathbf{U}}_{\mathrm{Rx}}^{\mathrm{H}} \mathbf{U}_{\mathrm{Rx}}$ does not yield the identity matrix.) The mismatched decorrelation stage $\hat{\mathbf{U}}_{\mathrm{Rx}}$ will cause a certain shift between the received powers (desired signal) of the individual virtual receive antennas. This power shift is captured by the diagonal elements $\xi_{\mathrm{Rx},jj}$ of the matrix $\mathbf{\Xi}_{\mathrm{Rx}}$ ($\xi_{\mathrm{Rx},jj} \neq \lambda_{\mathrm{Rx},j}$). Moreover, the selection stage will be based on erroneous eigenvalues $\hat{\lambda}_{\mathrm{Rx},j}$, i.e., the criterion (16) changes to

$$\hat{P}_{\mathrm{disc}} := \frac{P}{N} \cdot \sigma_h^2 \sum_{j \in \mathcal{J}_{\mathrm{disc}}} \hat{\lambda}_{\mathrm{Rx},j} \overset{!}{\leq} (N - D) \cdot \sigma_n^2 \,. \quad (27)$$

This might lead to a wrong number of spatial dimensions retained for further processing. Furthermore, due to the mismatched decorrelation stage $\hat{P}_{\mathrm{disc}}$ does not exactly represent the discarded sum power of the desired signal: The sum power that is actually discarded is given by

$$P_{\mathrm{disc}} = \frac{P}{N} \cdot \sigma_h^2 \sum_{j \in \mathcal{J}_{\mathrm{disc}}} \xi_{\mathrm{Rx},jj} \,. \quad (28)$$

[7]Note that the asymptotic slope of the curve for $D = 2$ is the same as for the (2×2)-OSTBC system: Since two of the four eigenvalues $\lambda_{j,\mathrm{Rx}}$ are discarded, the effective diversity order is reduced from eight to four. The sum of the discarded eigenvalues is equal to 0.43, which leads to an SNR loss of $10\log_{10}(4/(4-0.43))$ dB ≈ 0.5 dB with respect to the uncorrelated (2×2)-OSTBC system. In the case $D = 1$, the asymptotic slope of the curve is the same as for the (2×1)-OSTBC system, because the diversity order is reduced to two. Moreover, an SNR loss of about 2.3 dB results with respect to the uncorrelated (2×1)-OSTBC system.

TABLE I

MINIMUM NUMBER D OF SPATIAL DIMENSIONS TO BE RETAINED IN ORDER TO MEET THE SELECTION CRITERION (16), FOR PERFECT KNOWLEDGE OF THE CORRELATION PARAMETER ρ. CHANGES RESULTING FOR ESTIMATES $|\hat{\rho}| = 0.9\,\rho$ AND $|\hat{\rho}| = 1.1\,\rho$ ARE MARKED WITH † AND ‡, RESPECTIVELY.

Correlation value ρ (and $\hat{\rho}$)	$10\,\log_{10}(E_\mathrm{s}/N_0)$ dB										
	0	1	2	3	4	5	6	7	8	9	10
0.0 $(0.00^\dagger,\ 0.00^\ddagger)$	1	4	4	4	4	4	4	4	4	4	4
0.1 $(0.09^\dagger,\ 0.11^\ddagger)$	1	4	4	4	4	4	4	4	4	4	4
0.2 $(0.18^\dagger,\ 0.22^\ddagger)$	1	2	4	4	4	4	4	4	4	4	4
0.3 $(0.27^\dagger,\ 0.33^\ddagger)$	1	2	3	$4\,(3^\ddagger)$	4	4	4	4	4	4	4
0.4 $(0.36^\dagger,\ 0.44^\ddagger)$	1	1	2	3	$3\,(4^\dagger)$	4	4	4	4	4	4
0.5 $(0.45^\dagger,\ 0.55^\ddagger)$	1	1	2	$2\,(3^\dagger)$	$3\,(2^\ddagger)$	$3\,(4^\dagger)$	$3\,(4^\dagger)$	$4\,(3^\ddagger)$	4	4	4
0.6 $(0.54^\dagger,\ 0.66^\ddagger)$	1	1	$2\,(1^\ddagger)$	$2\,(3^\dagger)$	$2\,(3^\dagger)$	$3\,(2^\ddagger)$	$3\,(4^\dagger)$	$3\,(4^\dagger)$	$3\,(4^\dagger)$	$4\,(3^\ddagger)$	$4\,(3^\ddagger)$
0.7 $(0.63^\dagger,\ 0.77^\ddagger)$	1	1	$1\ (2^\dagger)$	$2\,(1^\ddagger)$	$2\ (3^\dagger)$	$3\,(2^\ddagger)$	$3\,(2^\ddagger)$	$3\,(2^\ddagger)$	$3\,(4^\dagger, 2^\ddagger)$	$3\,(4^\dagger)$	$3\,(4^\dagger)$
0.8 $(0.72^\dagger,\ 0.88^\ddagger)$	1	1	1	$1\ (2^\dagger)$	$2\ (3^\dagger, 1^\ddagger)$	$3\,(1^\ddagger)$	$3\,(2^\ddagger)$	$3\,(2^\ddagger)$	$3\,(2^\ddagger)$	$3\,(2^\ddagger)$	$3\,(2^\ddagger)$
0.9 $(0.81^\dagger,\ 0.99^\ddagger)$	1	1	1	1	$1\ (2^\dagger)$	$1\ (2^\dagger)$	$3\,(1^\ddagger)$	$3\,(1^\ddagger)$	$3\,(1^\ddagger)$	$3\,(1^\ddagger)$	$3\,(1^\ddagger)$
1.0 $(0.90^\dagger)$	1	1	1	1	1	1	$1\ (3^\dagger)$	$1\,(3^\dagger)$	$1\,(3^\dagger)$	$1\,(3^\dagger)$	$1\ (3^\dagger)$

This will lead to a change in the overall received SNR γ' after the selection stage, compared to the case of perfect knowledge of $\mathbf{R}_\mathrm{Rx}$ (given the same number of retained spatial dimensions). However, this effect is usually quite small, as long as the estimate for $\mathbf{R}_\mathrm{Rx}$ is not too bad. (This has, for example, been demonstrated in [7] by means of simulation results.)

In the following, the impact of estimation errors is illustrated by means of a simple numerical example. For this purpose, we assume that the receiver correlation matrix $\mathbf{R}_\mathrm{Rx}$ is of form (24), and that a direct estimate $\hat{\rho}$ of the correlation parameter $\rho := |\rho| \mathrm{e}^{\mathrm{j}\phi}$ is available at the receiver. Correspondingly, the reduced-dimension receiver is based on an erroneous receiver correlation matrix $\hat{\mathbf{R}}_\mathrm{Rx} = \mathbf{R}_{M,\hat{\rho}}$. We consider again the (2×4)-OSTBC system from Section III-D ($\rho = 0.7$). Moreover, for $|\hat{\rho}|$ we assume a value of $(1 \pm 0.1)\,\rho$, and for $\Delta\phi := |\phi - \hat{\phi}|$ a value of 0.1 rad. (Note that an estimation error of 10% for $|\rho|$ and an estimation error of 0.1 rad for ϕ is already quite large. Usually, such an estimation accuracy can easily be achieved, for example, by averaging the channel coefficients over a reasonable number of channel realizations.)

To start with, we consider the impact of the mismatched decorrelation stage. The eigenvalues of the receiver correlation matrix $\mathbf{R}_\mathrm{Rx}$ are given by $\lambda_{\mathrm{Rx},1} = 2.367$, $\lambda_{\mathrm{Rx},2} = 1.196$, $\lambda_{\mathrm{Rx},3} = 0.374$ and $\lambda_{\mathrm{Rx},4} = 0.064$. For comparison, the diagonal elements of $\mathbf{\Xi}_\mathrm{Rx}$ result as $\xi_{\mathrm{Rx},11} = 2.335$, $\xi_{\mathrm{Rx},22} = 1.222$, $\xi_{\mathrm{Rx},33} = 0.374$, $\xi_{\mathrm{Rx},44} = 0.069$ for $|\hat{\rho}| = 1.1\rho$. (For $|\hat{\rho}| = 0.9\rho$ one obtains $\xi_{\mathrm{Rx},11} = 2.345$, $\xi_{\mathrm{Rx},22} = 1.210$, $\xi_{\mathrm{Rx},33} = 0.381$, $\xi_{\mathrm{Rx},44} = 0.065$.) As can be seen, the resulting power shift between the individual virtual receive antennas is indeed quite small. We will therefore neglect this effect in the following.

Next, we study the impact of the eigenvalue mismatch. To start with, consider the case where the correlation parameter ρ of the receiver correlation matrix is perfectly known. Table I displays the minimum number D of spatial dimensions (eigenvalues/ eigenvectors of $\mathbf{R}_\mathrm{Rx}$) that have to be retained, in order to meet the selection criterion (16), given different values of E_s/N_0 and different (real-valued) correlation parameters ρ between zero and one. For example, consider the case $\rho = 0.7$: Given an SNR value of 8 dB, the selection criterion suggests to retain three of the four eigenvalues $\lambda_{\mathrm{Rx},j}$ for further processing. Going back to Fig. 3, it can be seen that this choice is indeed reasonable: On the one hand, reducing the number of retained eigenvalues to two causes a significant performance loss. On the other hand, increasing the number of retained eigenvalues to four gives virtually no performance improvement and solely increases the receiver complexity.

If the correlation parameter ρ is not perfectly known, the selection stage will consider erroneous eigenvalues $\hat{\lambda}_{\mathrm{Rx},j}$, cf. (27). Correspondingly, for some correlation values and SNR values the number of retained eigenvalues might change, compared to the case of perfect knowledge of ρ. This is also shown in Table I for the examples $|\hat{\rho}| = 0.9\,\rho$ and $|\hat{\rho}| = 1.1\,\rho$ (the phase offset $\Delta\phi$ does not have any impact on the estimates of the eigenvalues $\lambda_{\mathrm{Rx},j}$): Changes compared to the case of perfect knowledge of ρ are marked with † ($|\hat{\rho}| = 0.9\,\rho$) and with ‡ ($|\hat{\rho}| = 1.1\,\rho$). Consider again the example $\rho = 0.7$ and $E_\mathrm{s}/N_0 = 8$ dB: In the case $|\hat{\rho}| = 0.63$, the selection stage retains all four eigenvalues for further processing, which leads to an unnecessarily high receiver complexity. In the case $|\hat{\rho}| = 0.77$, however, the selection stage retains only two of the four eigenvalues, which leads to a notable performance loss, cf. Fig. 3.

Altogether, it can be said that (for the considered example) the performance of the reduced-dimension receiver is quite robust with regard to estimation errors, because the performance loss or the complexity overhead due to possible wrong decisions made by the selection stage is limited. In order to further improve the robustness of the reduced-dimension receiver, the selection criterion (27) can be modified according to

$$\hat{P}_\mathrm{disc} \overset{!}{\leq} (N - D) \cdot \sigma_n^2 \; + \; \psi \,, \tag{29}$$

where the parameter ψ has to be adjusted accordingly: If it is rather affordable to accept some performance loss for certain correlation values and SNR values, one should choose a value $\psi > 0$. However, if it is rather affordable to accept an unnecessarily high receiver complexity in some cases, one should choose a value $\psi < 0$.

V. STATISTICAL TRANSMIT POWER ALLOCATION

In several publications, it was shown that the performance of MIMO systems may be improved significantly by using some sort of channel knowledge at the transmitter, see e.g. [25]. Since accurate instantaneous channel knowledge at the transmitter is costly and may be difficult to acquire in a practical system [7], we study the use of statistical channel knowledge at the

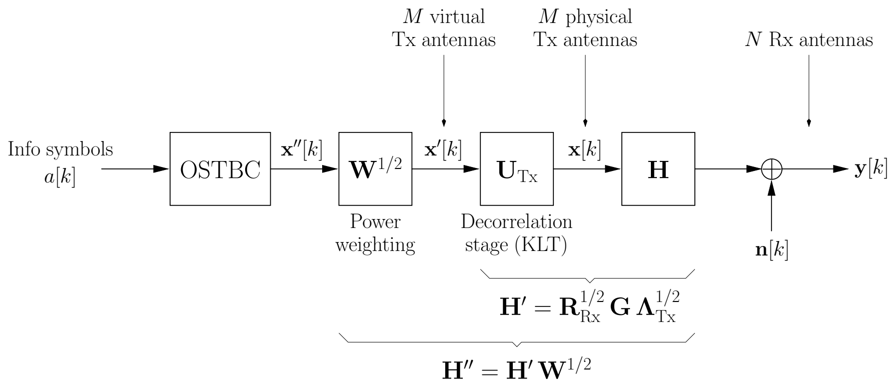

Fig. 4. Transmission model with transmit diversity scheme for spatially correlated MIMO systems (frequency-flat fading).

transmitter here. Specifically, we consider a statistical transmit power allocation scheme, which requires solely the knowledge of the transmitter correlation matrix $\mathbf{R}_{\text{Tx}}$. The scheme might, for example, be combined with the reduced-dimension receiver so as to improve performance.[8]

The general structure of the statistical transmit power allocation scheme under consideration is depicted in Fig. 4 (for flat fading). It was earlier studied in [7],[25] and consists of an inner decorrelation stage $\mathbf{U}_{\text{Tx}}$ (similarly to the reduced-dimension receiver, cf. Fig. 1) and an outer transmit power allocation stage, represented by a diagonal weighting matrix

$$\mathbf{W} := \text{diag}([w_1, ..., w_M]), \qquad \text{tr}(\mathbf{W}) = M. \tag{30}$$

The decorrelation stage transforms the given channel matrix $\mathbf{H}$ according to (6) into a semi-correlated channel matrix $\mathbf{H}' = \mathbf{R}_{\text{Rx}}^{1/2}\,\mathbf{G}\,\mathbf{\Lambda}_{\text{Tx}}^{1/2}$, by using the unitary matrix $\mathbf{U}_{\text{Tx}}$ from the eigenvalue decomposition of $\mathbf{R}_{\text{Tx}}$ as a precoding matrix:

$$\mathsf{E}\left\{\mathbf{H}'^{\text{H}}\mathbf{H}'\right\} = \mathbf{U}_{\text{Tx}}^{\text{H}}\,\mathsf{E}\left\{\mathbf{H}^{\text{H}}\mathbf{H}\right\}\,\mathbf{U}_{\text{Tx}} = N\sigma_h^2\,\mathbf{\Lambda}_{\text{Tx}} \tag{31}$$

(cf. (14)). The overall transmission model can be written as:

$$\begin{aligned} \mathbf{y}[k] \;&=\; \mathbf{H}\,\mathbf{x}[k] + \mathbf{n}[k] \;=\; \mathbf{H}\,\mathbf{U}_{\text{Tx}}\mathbf{W}^{1/2}\mathbf{x}''[k] + \mathbf{n}[k] \\ &=: \;\mathbf{H}'\,\mathbf{x}'[k] + \mathbf{n}[k] \;=:\; \mathbf{H}''\,\mathbf{x}''[k] + \mathbf{n}[k]\,, \end{aligned} \tag{32}$$

with $\mathbf{H}' {:=} \mathbf{H}\,\mathbf{U}_{\text{Tx}}$, $\mathbf{x}'[k] {:=} \mathbf{W}^{1/2}\mathbf{x}''[k]$, and $\mathbf{H}'' {:=} \mathbf{H}'\,\mathbf{W}^{1/2}$, where $\mathbf{x}''[k]$ denotes the space-time encoded vector without statistical transmit power allocation. (Note that the detection of the space-time encoded information symbols $a[k]$ has to be carried out based on the system model $\mathbf{y}[k] = \mathbf{H}''\mathbf{x}''[k] + \mathbf{n}[k]$.) As earlier, we assume that the entries of $\mathbf{x}''[k]$ are statistically independent random variables with variance $P/(MN)$. Due to the power constraint on $\mathbf{W}$, the transmitted vector $\mathbf{x}[k]$ will always meet the same overall power constraint as the vector $\mathbf{x}''[k]$.

A. Error Performance and Optimal Transmit Power Allocation

In the following, we again focus on binary antipodal transmission (i.e., $a[k] \in \{\pm 1\}$) and frequency-flat fading. Moreover, for simplicity we assume that a full-dimension receiver is employed ($D = N$). Under the assumptions made in Section III-C, the average BER of a spatially correlated $(M{\times}N)$-OSTBC system with statistical transmit power allocation stage is given by[9]

$$\begin{aligned} \bar{P}_{\text{b}} \;&=\; \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{N}\left(\prod_{\substack{i'=1 \\ (i',j')\neq(i,j)}}^{M}\prod_{j'=1}^{N}\frac{1}{1-\dfrac{w_{i'}\lambda_{\text{Tx},i'}\lambda_{\text{Rx},j'}}{w_i\lambda_{\text{Tx},i}\lambda_{\text{Rx},j}}}\right)\times \\ &\quad\times\left(1-\sqrt{\frac{P\,\sigma_h^2\,w_i\lambda_{\text{Tx},i}\lambda_{\text{Rx},j}}{MN\,\sigma_n^2 + P\,\sigma_h^2\,w_i\lambda_{\text{Tx},i}\lambda_{\text{Rx},j}}}\right). \end{aligned} \tag{33}$$

The optimal transmit power allocation strategy in terms of a minimum symbol error probability was derived in [7] as a function of the overall SNR $\gamma = E_{\text{s}}/N_0$. The result is a waterfilling solution with respect to the inverse eigenvalues $1/\lambda_{\text{Tx},i}$ of the transmitter correlation matrix $\mathbf{R}_{\text{Tx}}$:

$$w_{i,\text{opt}} = M\left[\frac{1}{M'} - \frac{1}{\gamma}\left(\frac{1}{\lambda_{\text{Tx},i}} - \frac{1}{M'}\sum_{i'=1}^{M'}\frac{1}{\lambda_{\text{Tx},i'}}\right)\right]_+, \tag{34}$$

where $[\,x\,]_+ {:=} \max\{0, x\}$ and M' denotes the number of spatial dimensions actually used (i.e., the number of power weights $w_{i,\text{opt}} > 0$). For high SNR values, this solution tends to the equal-power-allocation (EPA) solution, i.e., $w_{i,\text{opt}} = 1$ for all i ($M' = M$). For low SNR values, one obtains the eigen-beamforming (EBF) solution, where the complete transmit power is concentrated on the strongest eigenvalue of $\mathbf{R}_{\text{Tx}}$ ($M' = 1$). A particularly simple power allocation strategy is to employ a transmitter-sided MRC scheme, where the eigenvalues $\lambda_{\text{Tx},i}$ themselves are used as weighting factors ($w_i := \lambda_{\text{Tx},i}$, $i = 1, ..., M$), i.e, strong eigenvalues are strongly weighted and weak eigenvalues are weakly weighted. As will be shown in the next section, the MRC scheme yields a near-optimum performance over a wide SNR range.

B. Numerical Results and Impact of Estimation Errors

As an example, we consider a (4×1)-OSTBC system with a transmitter correlation matrix $\mathbf{R}_{\text{Tx}} = \mathbf{R}_{M,\rho}$ according to (24), where $M = 4$ and $\rho := 0.8$ (real-valued). Fig. 5 displays the BER performance as a function of E_{s}/N_0 in dB, which results for the different transmit power allocation strategies discussed above (solid lines). For low SNR values the EBF scheme is

[8] It is important to note that the statistical transmit power allocation scheme can be employed completely independently from the reduced-dimension receiver. Specifically, the decorrelation stage and the selection stage in the reduced-dimension receiver remain unchanged, when statistical transmit power allocation is performed.

[9] If the statistical transmit power allocation scheme is employed in conjunction with the reduced-dimension receiver, the average BER can be determined based on (22), by replacing the eigenvalues $\lambda_{\text{Tx},i}$ by $w_i\lambda_{\text{Tx},i}$, respectively.

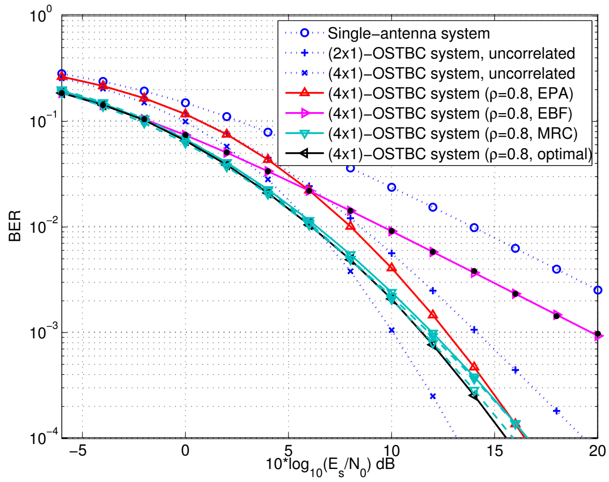

Fig. 5. BER performance of a (4×1)-OSTBC system with different statistical transmit power allocation strategies (analytical results): Binary transmission, flat Rayleigh fading, transmitter correlation matrix $\mathbf{R}_{\mathrm{Tx}} = \mathbf{R}_{M,\rho}$ with $M = 4$ and $\rho = 0.8$. All analytical results were validated by means of Monte Carlo simulations. As an example, simulation results are included for the EBF scheme (marked by black dots).

best (as expected), although the difference to the MRC scheme is barely visible.[10] In fact, the MRC scheme provides a good performance over the complete SNR range under consideration and is quite close to the optimal waterfilling solution (34). Depending on the SNR value the MRC scheme provides a gain of up to 2 dB over the EBF scheme/ the EPA scheme. Interestingly, even for SNR values up to 15 dB the MRC scheme still outperforms the EPA scheme. However, for larger SNR values the EPA scheme becomes superior (not shown).

If the transmitter correlation matrix is not perfectly known, the statistical transmit power allocation scheme will use a mismatched decorrelation stage $\hat{\mathbf{U}}_{\mathrm{Tx}} \neq \mathbf{U}_{\mathrm{Tx}}$ and a mismatched power allocation stage $\hat{\mathbf{W}}$ (due to an erroneous eigenvalue matrix $\hat{\mathbf{\Lambda}}_{\mathrm{Tx}} \neq \mathbf{\Lambda}_{\mathrm{Tx}}$). In effect, this will cause an overall mismatch in the power weighting, which is captured by the diagonal elements of the matrix

$$\mathbf{\Xi}_{\mathrm{Tx}} := \hat{\mathbf{U}}_{\mathrm{Tx}}^{\mathrm{H}} \mathbf{U}_{\mathrm{Tx}} \, \hat{\mathbf{W}} \mathbf{\Lambda}_{\mathrm{Tx}} \, \mathbf{U}_{\mathrm{Tx}}^{\mathrm{H}} \hat{\mathbf{U}}_{\mathrm{Tx}}. \tag{35}$$

As an example, numerical results have been included in Fig. 3 for the case of the MRC scheme (dashed lines). As earlier, it was assumed that a direct estimate $\hat{\rho}$ of the correlation parameter ρ is available, where for $|\hat{\rho}|$ values of $0.9\,\rho$ and $1.1\,\rho$ were considered and for $\Delta\phi$ a value of 0.1 rad. As can be seen, the BER performance of the MRC scheme is quite robust with regard to these estimation errors. (Moreover, since the MRC scheme is not optimal, estimation errors can even improve the performance slightly.)

VI. Conclusions

In this paper, a simple reduced-complexity receive diversity scheme for spatially correlated MIMO systems has been considered, which consists of an inner decorrelation stage and an outer selection stage. The considered reduced-dimension receiver requires solely statistical knowledge of the MIMO channel, which can easily be acquired in practical systems. Using an appropriate selection criterion, it was shown that the scheme offers a good trade-off between complexity and performance and is quite robust with regard to estimation errors. Finally, a statistical transmit power allocation scheme was considered, which might be combined with the reduced-dimension receiver so as to improve performance.

[10] Note that the BER curve of the EBF scheme has the same asymptotic slope as the curve of the single-antenna system, because the EBF scheme reduces the diversity order from four to one. However, compared to the single-antenna system an SNR gain of $10\log_{10}(2.72)\,\mathrm{dB} \approx 4.4\,\mathrm{dB}$ is achieved, since the maximum eigenvalue of $\mathbf{R}_{\mathrm{Tx}}$ is given by $\lambda_{\mathrm{Tx,max}} = 2.72$.

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