Prefiltering and Trellis-Based Equalization for Sparse ISI Channels

Jan Mietzner, Sabah Badri-Hoeher, and Peter A. Hoeher

Information and Coding Theory Lab, Faculty of Engineering, University of Kiel, Kaiserstrasse 2, D-24143 Kiel, Germany.
WWW: `www-ict.tf.uni-kiel.de` E-mail: `{jm,sbh,ph}@tf.uni-kiel.de`

***Abstract*— Sparse intersymbol-interference (ISI) channels are encountered in a variety of high-data-rate communication systems. Such channels have a large channel memory length, but only a small number of significant channel coefficients. In this paper, trellis-based equalization of sparse ISI channels is revisited. Due to the large channel memory length, the complexity of maximum-likelihood detection, e.g., by means of the Viterbi algorithm, is normally prohibitive, and efficient equalization with an acceptable complexity-performance trade-off is a demanding task. We investigate a unified approach to tackle general sparse ISI channels: It is shown that the use of a linear filter at the receiver renders the application of standard reduced-state trellis-based equalizer algorithms feasible, without significant loss of optimality. Numerical results verify the efficiency of the proposed receiver structure.**

***Index Terms*— Trellis-based equalization, sparse ISI channels, complexity reduction, prefiltering.**

I. Introduction

Sparse intersymbol-interference (ISI) channels are encountered in a wide range of communication systems, such as high-data-rate mobile radio systems (especially in hilly terrain), wireline systems, or aeronautical/satellite communication systems. For mobile radio applications, fading channels are of particular interest. The equivalent discrete-time channel impulse response (CIR) of a sparse ISI channel has a large channel memory length L, but only a small number of significant channel coefficients.

Due to the large memory length, equalization of sparse ISI channels is a demanding task. The topics of linear and decision-feedback equalization for sparse ISI channels are, e.g., addressed in [1], where the sparse structure of the channel is explicitly utilized for the design of the corresponding finite-impulse-response (FIR) filter(s). Decision-feedback equalization for sparse channels is also considered in [2]-[5].

Trellis-based equalization for sparse channels is addressed in [6]-[8]. The complexity in terms of trellis states of an optimal trellis-based equalizer, based on the Viterbi algorithm (VA) [9] or the Bahl-Cocke-Jelinek-Raviv algorithm (BCJRA) [10], is normally prohibitive for sparse ISI channels, because it grows exponentially with the channel memory length L[1]. However, reduced-complexity algorithms can be derived by exploiting the sparseness of the channel. In [6], it is observed that given a sparse channel, there is only a comparably small number of possible branch metrics within each trellis segment. By avoiding to compute the same branch metric several times, the computational complexity is reduced significantly without loss of optimality. However, the complexity in terms of trellis states remains the same and thus the storage expense. As an alternative, another equalizer concept coined multi-trellis Viterbi algorithm (M-VA) is proposed in [6] that is based on multiple parallel *irregular* trellises (i.e., time-variant trellises). However, it can be shown that the M-VA does, in fact, *not* lead to a reduction of computational complexity, compared to the conventional VA [11] (if optimality is supposed to be retained).

A particularly simple solution to reduce the complexity of the conventional VA without loss of optimality can be found in [7]: The parallel-trellis Viterbi algorithm (P-VA) is based on multiple parallel *regular* trellises. However, an application of the P-VA is only possible for a certain class of sparse channels having a so-called *zero-pad* structure. In order to tackle more general sparse channels with a CIR close to a zero-pad channel, it is proposed in [7] to exchange tentative decisions between the parallel trellises and thus cancel residual ISI. This modified version of the P-VA is, however, suboptimal and is denoted sub-P-VA in the sequel. A generalization of the P-VA and the sub-P-VA can be found in [8], where corresponding algorithms based on the BCJRA are presented. These are in the sequel denoted as parallel-trellis BCJR algorithms (P-BCJRA and sub-P-BCJRA, respectively). Some interesting enhancements of the (sub-)P-BCJRA are also discussed in [8]. Specifically, it is shown that the performance of the sub-P-BCJRA can be improved by means of minimum-phase prefiltering [12]-[14] at the receiver. A specific FIR approximation of the infinite-length linear minimum-phase filter is used, which preserves the sparse structure of the channel. This guarantees that the sub-P-BCJRA can still be applied after the prefiltering.

Alternatives to trellis-based equalization are the tree-based LISS algorithm [15] and the Joint Gaussian (JG) approach in [16]. Turbo equalization for sparse ISI channels is addressed in [17]. A non-trellis based equalizer algorithm for fast-fading sparse ISI channels, based on the symbol-by-symbol MAP criterion, is presented in [18].

In this paper, trellis-based equalization for sparse channels is revisited. In order to equalize general sparse ISI channels, a simple alternative to the sub-P-VA/ sub-P-BCJRA is investigated. For this purpose, the idea in [8] to employ prefiltering at the receiver is picked up. It is demonstrated that the use of a linear minimum-phase filter renders the application of reduced-state equalizers such as [19],[20] feasible, without significant loss of optimality. As an alternative receiver structure, the use of a linear channel shortening filter [21] is investigated, in conjunction with a conventional Viterbi equalizer operating on a shortened memory length. The proposed receiver structures are notably simple: The employed equalizer algorithms are standard (i.e., not specifically designed for sparse channels), because the sparse channel structure is normally lost after prefiltering. Solely the linear filters are adjusted to the current CIR (which is particularly favorable with regard to fading channels). The filter coefficients can be computed according to standard techniques available in the literature.

In Section II, the system model considered throughout this paper is introduced. The two prefiltering approaches are briefly recapitulated in Section III, and the overall complexity of the receiver structures under consideration is discussed. Afterwards, the structure of the filtered CIR is studied. In order to illustrate the efficiency of the proposed receiver structures, numer-

[1] The VA is optimal in the sense of maximum-likelihood sequence estimation (MLSE) and the BCJRA in the sense of maximum a-posteriori (MAP) symbol-by-symbol estimation. Both algorithms operate on the same trellis diagram. Correspondingly, statements concerning complexity hold for both the VA and the BCJRA.

ical results are presented in Section IV for various types of sparse ISI channels. Using minimum-phase prefiltering in conjunction with a delayed decision-feedback sequence estimation (DDFSE) equalizer [20], bit error rates are achieved that deviate only 1-2 dB from the matched filter bound (at a bit error rate of 10^{-3}). To the authors' best knowledge, similar performance studies for prefiltering in the case of sparse ISI channels have not yet been presented in the literature.

II. System Model

A *general* sparse ISI channel has a comparably large channel memory length L, but only a small number of significant channel coefficients $h_g,\ g = 0, ..., G \ll L$, according to

$$\mathbf{h} \;:=\; [\,h_0\ \underbrace{0\ldots0}_{f_0\ \text{zeros}}\ h_1\ \underbrace{0\ldots0}_{f_1\ \text{zeros}}\ \cdots\ \underbrace{0\ldots0}_{f_{G-1}\ \text{zeros}}\ h_G\,]^{\mathrm{T}}, \qquad (1)$$

where the f_i are non-negative integers and $L = \sum_{i=0}^{G-1}(f_i + 1)$. A sparse ISI channel, for which $f_0 = ... = f_{G-1} =: f \geq 1$ holds, is called *zero-pad* channel [7].

In the sequel, the channel vector $\mathbf{h}$ is assumed to be known at the receiver. Moreover, an M-ary alphabet for the data symbols is assumed. Throughout this paper, complex baseband notation is used. The k-th transmitted M-ary data symbol is denoted as $x[k]$, where k is the time index. A hard decision of $x[k]$ is denoted by $\hat{x}[k]$. For simplicity, the channel coefficients are assumed to be constant over an entire block of data symbols (block length $N > L$). The equivalent discrete-time channel model is given by

$$y[k] \;=\; h_0\, x[k] \;+\; \sum_{g=1}^{G} h_g\, x[k - d_g] \;+\; n[k]\,, \qquad (2)$$

where $y[k]$ denotes the k-th received sample and $n[k]$ the k-th sample of a complex additive white Gaussian noise (AWGN) process with zero mean and variance σ_{n}^2. Moreover,

$$d_g \;:=\; \sum_{i=1}^{g}(f_{i-1} + 1), \qquad 1 \leq g \leq G \qquad (3)$$

denotes the position of h_g within the channel vector $\mathbf{h}$.

The complexity in terms of trellis states of the conventional Viterbi/BCJR algorithm is given by $\mathcal{O}(M^L)$ and is thus normally prohibitive. Given a zero-pad channel, the conventional trellis diagram with $M^L = M^{(f+1)G}$ states can without loss of optimality be decomposed into $(f + 1)$ parallel regular trellises, each having only M^G states [7]. Such a decomposition is *not* possible in the case of a more general sparse ISI channel. In this case, one possibility is to resort to the suboptimal sub-P-VA/ sub-P-BCJRA with residual ISI cancellation. However, for a good performance the CIR should at least be close to a zero-pad structure.

In order to tackle general sparse ISI channels, a simple alternative to the sub-P-VA/ sub-P-BCJRA is proposed in the sequel: We investigate the use of prefiltering at the receiver, in conjunction with a standard (reduced-state) trellis-based equalizer algorithm. The receiver structure under consideration is illustrated in Fig. 1, where $z[k]$ denotes the k-th received sample after prefiltering and $\mathbf{h}_{\mathrm{f}}$ the filtered CIR. Two types of linear filters are considered in the following, namely a minimum-phase filter [12]-[14] and a channel shortening filter [21]. In

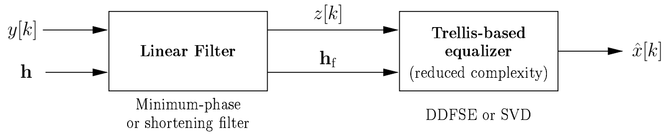

Fig. 1. Receiver structure under consideration.

the case of minimum-phase prefiltering, a reduced-state trellis-based equalizer is employed. Within the scope of this paper, focus is on delayed decision-feedback sequence estimation (DDFSE)[2] [20]. As an alternative receiver structure, the channel shortening filter is used in conjunction with a conventional Viterbi equalizer. The Viterbi equalizer operates on a shortened CIR with memory length $L_{\mathrm{s}} < L$, which is indicated by the term shortened Viterbi detector (SVD) in the sequel. The SVD equalizer is no longer optimal in the sense of MLSE.

III. Prefiltering for Sparse Channels

To start with, the two prefiltering approaches are briefly recapitulated. Then, the overall complexity of the receiver structures under consideration is discussed, and it is shown that the sparse channel structure is normally lost after prefiltering.

A. Minimum-Phase Filter

Consider a static ISI channel with CIR $\mathbf{h} := [h_0, h_1, ..., h_L]^{\mathrm{T}}$ and let $H(z)$ denote the z-transform of $\mathbf{h}$. Furthermore, let $\mathbf{h}_{\min} := [h_{\min,0}, h_{\min,1}, ..., h_{\min,L}]^{\mathrm{T}}$ denote the equivalent minimum-phase CIR of $\mathbf{h}$ and $H_{\min}(z)$ the corresponding z-transform. In the z-domain, all zeros of $H_{\min}(z)$ are either inside or on the unit circle [22, Ch. 3.4]. In the time domain, $\mathbf{h}_{\min}$ is characterized by an energy concentration in the first channel coefficients [12],[13] (especially if the zeros of $H(z)$ are not too close to the unit circle):

$$\sum_{l=0}^{\lambda} |h_{\min,l}|^2 \;\geq\; \sum_{l=0}^{\lambda} |h_l|^2\,, \qquad (4)$$

for all $\lambda \leq L$.

The z-transform of the ideal linear filter, which transforms $\mathbf{h}$ into the minimum-phase CIR $\mathbf{h}_{\mathrm{f}} = \mathbf{h}_{\min}$, is given by [13]

$$A(z) \;=\; \frac{H_{\min}(z)}{H(z)} \;=\; \prod_{\mu=1}^{m} (-z_{0,\mu}^*)\, \frac{z - 1/z_{0,\mu}^*}{z - z_{0,\mu}}\,, \qquad (5)$$

where $(.)^*$ denotes complex conjugation and $z_{0,\mu}$, $\mu = 1, ..., m$, denote the zeros of $H(z)$ that are outside the unit circle ($0 \leq m \leq L$). The z-transform $H_{\min}(z)$ is obtained by reflecting the zeros $z_{0,\mu}$ into the unit circle, whereas those zeros of $H(z)$ inside or on the unit circle are retained for $H_{\min}(z)$. If all zeros of $H(z)$ are close to the unit circle, minimum-phase prefiltering does not have a significant effect in the time domain. The ideal filter $A(z)$ has allpass characteristic [13], i.e., it does not color the noise. A direct realization of $A(z)$ would, however, result in a non-stable recursive filter. One alternative is to use a stable IIR (infinite impulse response) allpass, which generates a maximum-phase overall impulse response,

[2] A DDFSE equalizer is obtained from the conventional VA by applying the principle of parallel decision feedback [20]. By this means, the effective memory length of the equalizer can be reduced to $K < L$, i.e., the number of trellis states is reduced by a factor of M^{L-K}.

TABLE I

COMPUTATIONAL COMPLEXITY OF THE DIFFERENT RECEIVERS.

Conventional VA (memory length L)	**DDFSE + WMF** ($K<L$)	**SVD + CSF** ($L_\mathrm{s}<L$)
$\mathcal{O}(M^{L+1})$	$\mathcal{O}(M^{K+1})$	$\mathcal{O}(M^{L_\mathrm{s}+1})$
–	$\mathcal{O}(L_\mathrm{F}\,L^2)$	$\mathcal{O}(L_\mathrm{F}^3)$

and to employ a time-reversed equalizer [23]. Another alternative, among others, is to approximate $A(z)$ by an FIR filter of length $L_\mathrm{F}<\infty$.

In this paper, the approach in [12],[14] is used to calculate the filter coefficients. For reasons of conciseness, the derivation of the filter coefficients is not repeated here. The interested reader is referred to [14]. The resulting FIR filter approximates a discrete-time whitened matched filter (WMF). The computational complexity of calculating the filter coefficients is $\mathcal{O}(L_\mathrm{F}\,L^2)$, i.e., only linear with respect to the filter length (see [12],[14] for further details). Correspondingly, comparably large filter lengths are feasible.

B. Channel Shortening Filter

In this approach, a linear filter is used to transform a given CIR $\mathbf{h}:=[h_0,h_1,...,h_L]^\mathrm{T}$ into a shortened CIR $\mathbf{h}_\mathrm{f}=\mathbf{h}_\mathrm{s}:=[h_{\mathrm{s},0},h_{\mathrm{s},1},...,h_{\mathrm{s},L_\mathrm{s}}]^\mathrm{T}$, where $L_\mathrm{s}<L$ denotes the desired channel memory length, which is assumed in the subsequent equalizer. Several methods to design a linear channel shortening filter (CSF) can be found in the literature, e.g. [21],[24]-[27]. In this paper, the method described in [21] is used. This CSF is based on the feed-forward filter (FFF) of a minimum mean-squared error decision-feedback equalizer (MMSE-DFE). The filter design is as follows: For the feed-back filter (FBF) of the MMSE-DFE, a fixed filter length of $(L_\mathrm{s}+1)$ is chosen. Under this constraint, the FFF and the FBF of the DFE are then optimized with respect to the MMSE criterion, where the length L_F of the FFF can be chosen irrespective of L_s. The optimized FFF finally constitutes a linear finite-length CSF: The mean-squared error between the CIR $\mathbf{h}_\mathrm{s}$ after the FFF and the coefficients of the FBF is minimized, i.e., the channel coefficients $h_{\mathrm{s},l}$ with $l<0$ and $l>L_\mathrm{s}$ are optimally suppressed in the MMSE sense. As opposed to the minimum-phase filter, an arbitrary power distribution among the desired coefficients $h_{\mathrm{s},l}$, $0\le l\le L_\mathrm{s}$, results. Moreover, the CSF does not approximate an allpass filter, i.e., depending on the given CIR $\mathbf{h}$ the CSF can color the noise. The computational complexity of calculating the filter coefficients is $\mathcal{O}(L_\mathrm{F}^3)$ (see [21] for further details).

C. Computational Complexity of the Proposed Receivers

Altogether, three different receiver structures are considered in the sequel (cf. Fig. 1):

- A full-state Viterbi equalizer (MLSE, memory length L, no prefiltering[3])
- A DDFSE equalizer with effective memory length $K<L$ in conjunction with a minimum-phase filter (WMF)
- An SVD equalizer with effective memory length $L_\mathrm{s}<L$ in conjunction with a channel shortening filter (CSF).

The computational complexity of these three receiver structures is summarized in Table I. For the equalizer algorithms, the overall number of branch metrics is stated that is computed for each symbol decision $\hat{x}[k]$. For the linear filters the approximate computational complexity of calculating the filter coefficients is stated. The parameters K,L_s are design parameters. In order to obtain a complexity that is similar to the P-VA/P-BCJRA equalizer, K and L_s should be chosen such that[4]

$$K,L_\mathrm{s}\;\le\;\log_M(f+1)\;+\;G\,. \tag{6}$$

D. Channel Structure After Prefiltering

The sparse structure of a given CIR $\mathbf{h}$ is normally lost after prefiltering. This is obvious in the case of the shortening filter, since an arbitrary power distribution results among the desired $(L_\mathrm{s}+1)$ channel coefficients. However, the sparse structure is – in general – also lost when applying a minimum-phase filter. An exception is the zero-pad channel, where the sparse CIR structure is always preserved after minimum-phase prefiltering: Let $\mathbf{h}:=[\,h_0\;h_1\;\ldots\;h_G\,]^\mathrm{T}$ denote a (non-sparse) CIR with z-transform $H(z)$, and let $\mathbf{h}_\mathrm{ZP}$ denote the corresponding zero-pad CIR with memory length $(f{+}1)G$ and z-transform $H_\mathrm{ZP}(z)$, which results from inserting f zeros in between the individual coefficients of $\mathbf{h}$. Furthermore, let $z_{0,\,1},...,z_{0,\,G}$ denote the zeros of $H(z)$. An insertion of f zeros in the time domain corresponds to a transform $z\;\mapsto\;z^{1/(f+1)}$ in the z-domain, i.e., $H_\mathrm{ZP}(z)=H(z^{f+1})$. This means, the $(f{+}1)G$ zeros of $H_\mathrm{ZP}(z)$ are given by the $(f{+}1)$ complex roots of $z_{0,\,1},...,z_{0,\,G}$, respectively. Consider a certain zero $z_{0,\,g}:=r_{0,\,g}\exp(\mathrm{j}\,\varphi_{0,\,g})$ of $H(z)$ that is outside the unit circle ($r_{0,\,g}>1$). This zero will lead to $(f{+}1)$ zeros

$$z_{0,\,g}^{(\lambda)}\;:=\;r_{0,\,g}^{1/(f+1)}\exp\left(\mathrm{j}\,\frac{2\pi\lambda+\varphi_{0,\,g}}{f+1}\right) \tag{7}$$

of $H_\mathrm{ZP}(z)$ ($\lambda\;=\;0,...,f$) that are located on a circle of radius $r_{0,\,g}^{1/(f+1)}>1$, i.e., also outside the unit circle. By means of minimum-phase prefiltering, these zeros are reflected into the unit circle, i.e., the corresponding zeros of $H_{\mathrm{ZP,min}}(z)$ are given by $1/z_{0,\,g}^{(\lambda)*}$. Therefore, the sparse CIR structure is retained after minimum-phase prefiltering (with the same zero-pad grid), since the zeros of $H_{\mathrm{ZP,min}}(z)$ are the $(f+1)$ roots of the zeros of $H_\mathrm{min}(z)$. Specifically, the non-zero coefficients of $\mathbf{h}_{\mathrm{ZP,min}}$ are given by the CIR $\mathbf{h}_\mathrm{min}$. If the zeros of $H(z)$ (or equivalently of $H_{\mathrm{ZP,min}}(z)$) are not too close to the unit circle, $\mathbf{h}_\mathrm{min}$ is characterized by a significant energy concentration in the first channel coefficients. In this case, the effective channel memory length of $\mathbf{h}_\mathrm{ZP}$ is significantly reduced by minimum-phase prefiltering, namely by some multiples of $(f{+}1)$, cf. (1).

IV. NUMERICAL RESULTS

In the sequel, numerical results obtained by means of Monte-Carlo simulations are presented, in order to illustrate the efficiency of the proposed receiver structures. In all cases, perfect channel knowledge at the receiver was assumed. To start with, a *static* sparse ISI channel is considered, and the bit-error-rate (BER) performance of the proposed receiver structure is compared with that of the sub-P-BCJRA equalizer [8]. As an example, we consider the

[3]The bit-error-rate performance of the full-state VA is (virtually) not influenced by prefiltering [14].

[4]In order to obtain an appropriate value for K or L_s in the case of a general sparse ISI channel, one first has to find an underlying zero-pad channel with a structure as close as possible to the CIR under consideration. This yields the parameters f and G for the right hand side of (6). Then, choosing K,L_s according to (6) gives an overall receiver complexity similar to that of the corresponding sub-P-VA/ sub-P-BCJRA equalizer. It should be noted that due to the parallel decision feedback, the complexity of the DDFSE equalizer is slightly larger than that of the SVD equalizer (given the same value for K and L_s).

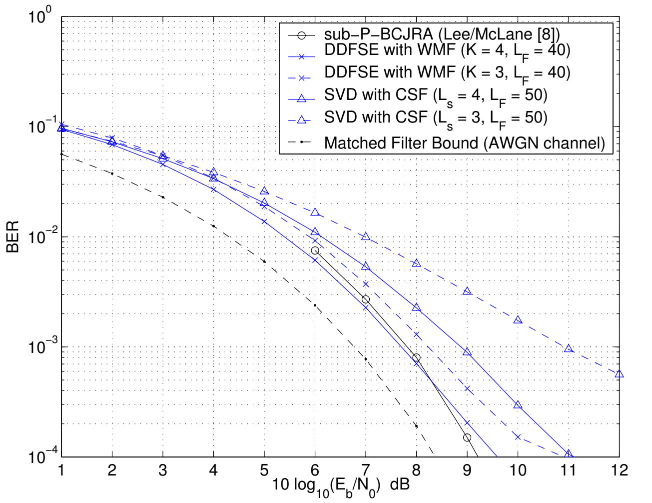

Fig. 2. BER performance of the proposed receiver structures in the case of a static sparse ISI channel.

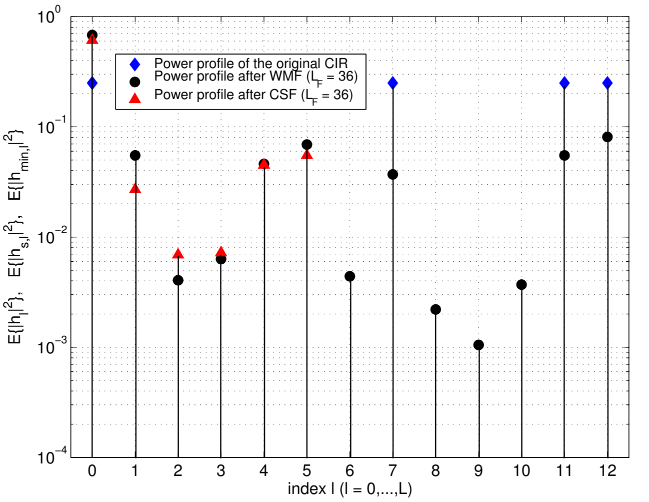

Fig. 3. Power profiles after prefiltering with the WMF/CSF, resulting for large values of E_b/N_0.

CIR $\mathbf{h} = [\,h_0\; 0\ldots 0\; h_4\; 0\ldots 0\; h_7\; 0\ldots 0\; h_{15}\,]^\mathrm{T}$ with $h_0 = 0.87$ and $h_4 = h_7 = h_{15} = 0.29$ from [8], which has a general sparse structure (i.e., no zero-pad structure). The BER performance (binary antipodal transmission, $M = 2$) of the sub-P-BCJRA equalizer and the DDFSE equalizer with WMF as well as the SVD equalizer with CSF is displayed in Fig. 2, as a function of E_b/N_0 in dB, where E_b denotes the average energy per bit and N_0 the single-sided noise power density ($E_\mathrm{b}/N_0 := 1/\sigma_\mathrm{n}^2$). Due to the given channel memory length, the complexity of MLSE detection is prohibitive. As a reference curve, however, the matched filter bound (MFB) is included, which constitutes a lower bound on the BER of MLSE detection [28, Ch. 14.5]. The filter lengths for the WMF and the CSF were chosen sufficiently large ($L_\mathrm{F} = 40$ for the WMF and $L_\mathrm{F} = 50$ for the CSF), i.e., a further increase of the filter lengths gives only marginal performance improvements[5]. Since the channel is static, the filters have to be computed only once. When the parameters K and L_s for the DDFSE and the SVD equalizer, respectively, are chosen as $K, L_\mathrm{s} = 4$, the overall receiver complexity is approximately the same as for the sub-P-BCJRA equalizer. In this case, the DDFSE equalizer in conjunction with the WMF achieves a similar BER performance as the sub-P-BCJRA equalizer, as can be seen in Fig. 2. At a BER of 10^{-3}, the loss with respect to the MFB is only about 1 dB. At the expense of a small loss due to residual ISI (0.5 dB at the same BER), the complexity of the DDFSE equalizer can be further reduced to $K = 3$. The BER performance of the SVD equalizer in conjunction with the CSF is worse than that of the DDFSE equalizer with WMF: At a BER of 10^{-3}, the gap to the MFB is about 2.1 dB for $L_\mathrm{s} = 4$ and 4.2 dB for $L_\mathrm{s} = 3$.

Next, we consider the case of a sparse Rayleigh fading channel model, i.e., the channel coefficients h_g ($g = 0, \ldots, G$) in (1) are now zero-mean complex Gaussian random variables with variance $\mathrm{E}\{|h_g|^2\} =: \sigma_{\mathrm{h},g}^2$. It is assumed in the sequel that the individual channel coefficients are statistically independent. Moreover, block fading is considered for simplicity. As an example, we assume a CIR with $G = 3$ and a power profile

$$\mathbf{p} \;:=\; [\,\sigma_{\mathrm{h},0}^2\; \underbrace{0\ldots0}_{f \text{ zeros}}\; \sigma_{\mathrm{h},1}^2\;\; 0\;\; 0\;\; 0\;\;\; \sigma_{\mathrm{h},2}^2\; \sigma_{\mathrm{h},3}^2\,]^\mathrm{T}. \qquad (8)$$

[5] According to a rule-of-thumb, the filter length for the WMF should be chosen as $L_\mathrm{F} \geq 2.5\,(L+1)$ [14].

Note that this CIR again does not have a zero-pad structure. By choosing different values for the parameter f, different channel memory lengths $L = f + 6$ can be studied. To start with, consider a power profile with equal variances $\sigma_{\mathrm{h},0}^2 = \ldots = \sigma_{\mathrm{h},3}^2 = 0.25$ and a memory length of $L = 12$. Fig. 3 shows the power profiles that result after prefiltering with the WMF and the CSF, respectively, for large values of E_b/N_0. The filter length was $L_\mathrm{F} = 36$ in both cases. As can be seen, after prefiltering with the WMF the sparse structure of the power profile is lost (cf. Section III-D): Significant variances $\mathrm{E}\{|h_{\mathrm{min},l}|^2\}$ occur, for example, at $l = 1$, $l = 4$, and $l = 5$. The power profile after the WMF exhibits a considerable energy concentration in the first channel coefficient, whereas the variances $\mathrm{E}\{|h_{\mathrm{min},l}|^2\}$ for $l = 7$, $l = 11$, and $l = 12$ are smaller than for the original CIR. As will be seen, this significantly improves the performance of the subsequent DDFSE equalizer. For the CSF a desired channel memory length of $L_\mathrm{s} = 5$ was chosen. After prefiltering with the CSF, the variances $\mathrm{E}\{|h_{\mathrm{s},l}|^2\}$ for $l < 0$ and $l > L_\mathrm{s}$ are virtually zero[6]. Correspondingly, a subsequent SVD equalizer with memory length $L_\mathrm{s} = 5$ will not (much) suffer from residual ISI.

Fig. 4 shows the BER performance of the proposed receiver structures for binary transmission and three different channel memory lengths L (solid lines: $L = 6$, dashed lines: $L = 12$, dotted lines: $L = 20$). The filter lengths have been chosen as $L_\mathrm{F} = 20$ ($L = 6$), $L_\mathrm{F} = 36$ ($L = 12$), and $L_\mathrm{F} = 60$ ($L = 20$), for both the WMF and the CSF. As reference curves, the BER for flat Rayleigh fading ($L = 1$) is included as well as the MFB[7]. In the case $L = 6$, MLSE detection is still feasible. As can be seen in Fig. 4, its performance is very close to the MFB. The DDFSE equalizer with $K = 5$ in conjunction with the WMF achieves a BER performance very close to MLSE detection (the loss at a BER of 10^{-3} is only about 0.6 dB). Even when the channel memory length is increased to $L = 20$, the BER curve of the DDFSE equalizer with WMF deviates only 2 dB from the MFB. When the DDFSE equalizer is used without WMF, a significant performance loss occurs already for $L = 6$. Considering the case $L = 12$, it can be seen that the influence of the WMF

[6] As discussed in Section III-B, the CSF is designed such that a given CIR is optimally shortened in the sense of the MMSE criterion. In the case of large E_b/N_0 values, the MMSE solution and the zero-forcing (ZF) solution become equivalent, i.e., the channel coefficients with $l < 0$ and $l > L_\mathrm{s}$ are nulled.

[7] The MFB does not depend on the channel memory length L, as long as the variances $\sigma_{\mathrm{h},g}^2$ remain unchanged.

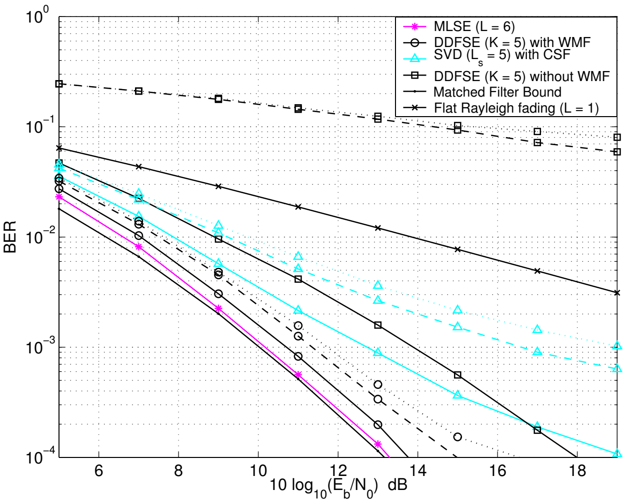

Fig. 4. BER performance of the proposed receiver structures in the case of a sparse Rayleigh fading channel (equal variances $\sigma^2_{\mathrm{h},g}$ of the non-zero channel coefficients; solid lines: $L = 6$, dashed: $L = 12$, dotted: $L = 20$).

(cf. Fig. 3) makes a huge difference: The BER increases by several decades when the WMF is not used. Similar to the case of the static sparse ISI channel (cf. Fig. 2), the performance of the SVD equalizer with $L_\mathrm{s} = 5$ in conjunction with the CSF is worse than that of the DDFSE equalizer, especially for large channel memory lengths L. Still, a significant gain compared to flat Rayleigh fading is achieved, i.e., a good portion of the inherent diversity (due to the independently fading channel coefficients) is captured.

Similar performance results were also obtained for unequal variances $\sigma^2_{\mathrm{h},g}$. Specifically, the performance of the SVD equalizer in conjunction with the CSF was always inferior to that of the DDFSE equalizer with WMF. For the latter, the following observation was made: When the power profile $\mathbf{p}$ of the original CIR does already exhibit an energy concentration in the first channel coefficients, the benefit of the WMF is smaller, but still significant.

V. Conclusions

In this paper, trellis-based equalization of sparse intersymbol-interference channels has been revisited. Due to the large memory length of sparse channels, efficient equalization with an acceptable complexity-performance trade-off is a demanding task. In order to tackle general sparse channels, receiver structures with a linear filter and a reduced-complexity equalizer have been studied. The employed equalizer algorithms are standard (i.e., not specifically designed for sparse channels), because the sparse channel structure is normally lost after prefiltering. Moreover, the coefficients of the linear filters can be computed using standard techniques from the literature. Using a minimum-phase filter in conjunction with a delayed decision-feedback sequence estimation equalizer, bit error rates are achieved that deviate only 1-2 dB from the matched filter bound (at a bit error rate of 10^{-3}).

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References

[1] F. K. H. Lee and P. J. McLane, "Design of nonuniformly spaced tapped-delay-line equalizers for sparse multipath channels," *IEEE Trans. Commun.*, vol. 52, no. 4, pp. 530-535, April 2004.

[2] I. J. Fevrier, S. B. Gelfand, and M. P. Fitz, "Reduced complexity decision feedback equalization for multipath channels with large delay spreads," *IEEE Trans. Commun.*, vol. 47, no. 6, pp. 927-937, June 1999.

[3] S. F. Cotter and B. D. Rao, "The adaptive matching pursuit algorithm for estimation and equalization of sparse time-varying channels," in *Proc. 34th Asilomar Conf. on Signals, Systems, and Computers*, 2000, pp. 1772-1776.

[4] E. F. Haratsch, A. J. Blanksby, and K. Azadet, "Reduced-state sequence estimation with tap-selectable decision-feedback," in *Proc. IEEE Int. Conf. Commun. (ICC'00)*, 2000, pp. 372-376.

[5] A. A. Rontogiannis and K. Berberidis, "Efficient decision feedback equalization for sparse wireless channels," *IEEE Trans. Wireless Commun.*, vol. 2, no. 3, pp. 570-581, May 2003.

[6] N. Benvenuto and R. Marchesani, "The Viterbi algorithm for sparse channels," *IEEE Trans. Commun.*, vol. 44, no. 3, pp. 287-289, Mar. 1996.

[7] N. C. McGinty, R. A. Kennedy, and P. Hoeher, "Parallel trellis Viterbi algorithm for sparse channels," *IEEE Commun. Lett.*, vol. 2, no. 5, pp. 143-145, May 1998. See also: ———, "Equalization of sparse ISI channels using parallel trellises," in *Proc. 7th Commun. Theory Mini-Conf.* in conjunction with *IEEE Globecom'98*, 1998, pp. 65-70.

[8] F. K. H. Lee and P. J. McLane, "Iterative parallel-trellis MAP equalizers with nonuniformly-spaced prefilters for sparse multipath channels," in *Proc. IEEE Veh. Technol. Conf. (VTC'02-Fall)*, 2002, pp. 2201-2205.

[9] G. D. Forney, Jr., "Maximum likelihood sequence estimation of digital sequences in the presence of intersymbol interference," *IEEE Trans. Inform. Theory*, vol. IT-18, pp. 363-378, May 1972.

[10] L. R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate," *IEEE Trans. Inform. Theory*, vol. IT-20, pp. 284-287, Mar. 1974.

[11] J. Mietzner, S. Badri-Hoeher, I. Land, and P. A. Hoeher, "Equalization of sparse intersymbol-interference channels revisited," in preparation.

[12] S. Badri-Hoeher, *Digitale Empfängeralgorithmen für TDMA-Mobilfunksysteme mit besonderer Berücksichtigung des EDGE-Systems.* Ph.D. dissertation, Erlangen, Germany, 2001.

[13] W. H. Gerstacker, F. Obernosterer, R. Meyer, and J. B. Huber, "On prefilter computation for reduced-state equalization," *IEEE Trans. Wireless Commun.*, vol. 1, no. 4, pp. 793-800, Oct. 2002.

[14] S. Badri-Hoeher and P. A. Hoeher, "Fast computation of a discrete-time whitened matched filter based on Kalman filtering," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 2417-2424, Nov. 2004.

[15] J. Hagenauer, "A soft-in/soft-out list sequential (LISS) decoder for turbo schemes," in *Proc. IEEE Int. Symp. Inform. Theory (ISIT'03)*, 2003, p. 382.

[16] L. Liu, W. K. Leung, and Li Ping, "Simple iterative chip-by-chip multiuser detection for CDMA systems," in *Proc. IEEE Veh. Technol. Conf. (VTC'03-Spring)*, 2003, pp. 2157-2161.

[17] J. Park and S. B. Gelfand, "Turbo equalizations for sparse channels," in *Proc. IEEE Wireless Commun. and Networking Conf. (WCNC'04)*, 2004, pp. 2301-2306.

[18] R. Cusani and J. Mattila, "Equalization of digital radio channels with large multipath delay for cellular land mobile applications," *IEEE Trans. Commun.*, vol. 47, no. 3, pp. 348-351, Mar. 1999.

[19] M. V. Eyuboğlu and S. U. Qureshi, "Reduced-state sequence estimation with set partitioning and decision feedback," *IEEE Trans. Commun.*, vol. 36, no. 1, pp. 13-20, Jan. 1988.

[20] A. Duel-Hallen and C. Heegard, "Delayed decision-feedback sequence estimation," *IEEE Trans. Commun.*, vol. 37, no. 5, pp. 428-436, May 1989.

[21] K. D. Kammeyer, "Time truncation of channel impulse responses by linear filtering: A method to reduce the complexity of Viterbi equalization," *AEÜ Int. J. Electron. Commun.*, vol. 48, no. 5, pp. 237-243, 1994.

[22] S. Haykin, *Adaptive Filter Theory.* 4th ed., Upper Saddle River (NJ): Prentice Hall, 2002.

[23] M. Schnell and A. J. Han Vinck, "A low complexity equalization concept for frequency-selective mobile radio channels", *AEÜ Int. J. Electron. Commun.*, vol. 50, no. 6, pp. 343-352, Nov. 1996.

[24] S. U. H. Qureshi and E. E. Newhall, "An adaptive receiver for data transmission over time-dispersive channels," *IEEE Trans. Inform. Theory*, vol. IT-19, no. 4, pp. 448-457, Jul. 1973.

[25] D. D. Falconer and F. R. Magee, "Adaptive channel memory truncation for maximum-likelihood sequence estimation," *Bell Syst. Tech. J.*, vol. 52, pp. 1541-1562, Nov. 1973.

[26] C. Beare, "The choice of the desired impulse response in combined linear-Viterbi algorithm equalizers," *IEEE Trans. Commun.*, vol. 26, no. 8, pp. 1301-1307, Aug. 1978.

[27] N. Al-Dhahir and J. M. Cioffi, "Efficiently computed reduced-parameter input-aided MMSE equalizers for ML detection: A unified approach," *IEEE Trans. Inform. Theory*, vol. 42, no. 3, pp. 903-915, May 1996.

[28] J. G. Proakis, *Digital Communications.* 4th ed., New York: McGraw-Hill, 2001.