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Compatible Improvement of the GSM/GPRS System by Means of Delay Diversity

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***Abstract*— On basis of the GSM/GPRS system, we investigate the application of the delay diversity scheme. The delay diversity scheme is a simple special case of a Space-Time Trellis Code (STTC). Delay diversity may be offered by network operators even in existing systems, since the standard is not affected at all. The performance improvements obtainable by means of this technique are demonstrated both on basis of analytical and simulation results. A lower bound on the bit error probability is derived and an optimization of the intrinsic delay parameter is considered.**

I. Introduction

The application of multiple transmit antenna techniques in wireless communications systems has recently gained much interest. It introduces an additional *spatial* component to the signal processing carried out in the transmitter and offers many possibilities of performance improvements upon systems employing only a single transmit antenna. In this context, *Space-Time Trellis Codes (STTC)* [1]-[4] and *Space-Time Block Codes (STBC)* [5],[6] are subject to current research activities. They exploit spatial diversity yielding an additional diversity and/or coding gain and thus an improved bit error performance compared to a single transmit antenna system. Multiple antennas at the receiver are optional. Spatial diversity results from the fact that the individual transmission paths from the transmit antennas to the receive antenna(s) are likely to fade *independently*.

The application of STTC and STBC in future wireless communications systems promises reliable transmission of high data rates, e.g., required for *3rd generation (3G)* bearer services. Such services are usually characterized by asymmetric data traffic, where the predominant part of data transfer occurs in the downlink (DL) direction. Therefore, in order to enhance the crucial DL direction, the application of STTC and STBC is very attractive because solely the base station needs to be equipped with additional antennas.

This paper considers the application of *delay diversity* [7],[8] in a GSM/GPRS[1] system [9], often referred to as a '2.5G system'. Delay diversity is the simplest special case of a STTC.

Original aspects of this paper include a transmitter structure designed with respect to compatibility aspects (Section II). New analytical results concerning the performance of delay diversity over a wireless channel are given in Section III. First, an improved version of the so-called RAKE receiver bound (RRB) [10] is derived. By means of this lower bound on the bit error probability it is then possible to optimize the intrinsic delay parameter of the delay diversity scheme. For related work see [11]-[13]. Simulation results for the GSM/GPRS system enhanced by delay diversity are presented in Section IV, demonstrating the bit error performance improvements for the example of a typical urban wireless channel. Moreover, the influence of non-perfect knowledge of the channel coefficients at the receiver is pointed out. Finally, a summary and concluding remarks are given in Section V.

[1] GPRS ('General Packet Radio Service') is part of the GSM specifications ('Phase 2+') and is used for the transfer of packet-switched data.

II. Structure of the Enhanced System

The compatible enhancement of the GSM/GPRS system by means of delay diversity shall be carried out in a way that no changes have to be applied to current specifications [9]. In this context, the GSM/GPRS binary Gaussian Minimum Shift Keying (GMSK) modulation scheme and the burst structure are retained in the extended system. Throughout this paper, the equivalent complex baseband notation is used.

In the delay diversity scheme, the same signal is transmitted over n_T antennas applying different delays δ_i at each antenna $1 \leq i \leq n_T$. If these delays are chosen as $\delta_i = (i-1)T$ [8], where T denotes the symbol interval, delay diversity can be regarded as the simplest special case of a STTC.

The transmitter structure of the GSM/GPRS system enhanced by delay diversity is depicted in Fig. 1, for $n_T = 2$. First of all, channel coding and interleaving is performed according to one of the GPRS coding schemes 'CS 1-4', which yields $2 \cdot 58$ data symbols $x(k) \in \{\pm 1\}$ per burst. Together with one of the eight GSM training sequences, the $x(k)$ are then mapped on a GSM/GPRS burst. Finally, GMSK pulse shaping is done (including a symbol-wise phase rotation of $\pi/2$) and the modulated signal is transmitted over the antennas, where at the second transmit antenna a delay δ is applied. Note that the single transmit antenna case ((1x1)-system) is included in the enhanced structure as the special case, when the second transmit antenna is switched off. The enhanced system is therefore compatible with the (1x1)-system and yields the same data rate.

Both for the (1x1)-system and for the enhanced system the same receiver structure can be applied. The received signal is first filtered, then sampled at time instants kT and derotated yielding received symbols $y(k)$. A channel estimator provides estimates $\hat{\mathbf{h}} = [\hat{h}^{(0)}, \hat{h}^{(1)}, ..., \hat{h}^{(L)}]$ of the coefficients of the equivalent discrete-time channel model, here referred to as channel coefficients (L denotes the effective channel memory length). The channel estimates are typically obtained by means of the *correlation method* [14], on basis of the training sequence used and the corresponding received symbols. Eventually, equalization and detection is performed utilizing the channel estimates. Throughout this paper we consider a 'Max-Log-MAP'

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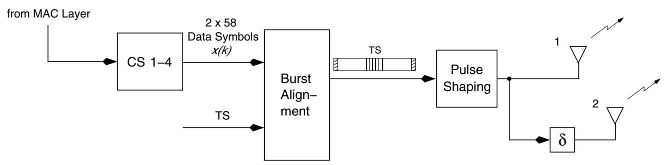

Fig. 1. Transmitter structure of the GSM/GPRS system enhanced by delay diversity ($n_T = 2$).

equalizer/detector [15], which is a soft-output algorithm approximating the log-likelihood ratios (LLRs) $L(x(k))$ of the data symbols $x(k)$. It is important to note that the same equalizer/detector algorithm can be used in the enhanced system as in the (1x1)-system, provided that it is capable of handling the increased channel memory length (see Section III-A).

The system described above applies, in principle, as well for EDGE/EGPRS[2], when the GPRS coding schemes 'CS 1-4' are replaced by the EGPRS modulation and coding schemes 'MCS 1-9'. The symbols $x(k)$ may then either be binary or 8-ary.

III. Improved RAKE Receiver Bound

The RAKE receiver bound (RRB) [10] is a lower bound on the bit error probability of a slowly time-varying frequency-selective channel, where perfect knowledge of the channel coefficients at the receiver is assumed. Performance loss due to inter-symbol interference (ISI) is not taken into account [16]. The channel coefficients are assumed to fade *independently*.

In order to compute the RRB the mean power of the channel coefficients $\rho_l \doteq E\{|h^{(l)}(k)|^2\}$, $0 \le l \le L$, is required ($E\{.\}$ denotes the expected value and k the time index). In case of the delay diversity scheme with $n_T = 2$ transmit antennas, ρ_l is a function of the delay δ applied to the signal at the second transmit antenna. In the following, an analytical expression is derived for ρ_l, which will later be utilized in order to find an optimal delay δ minimizing the RRB.

A. Mean Power of the Channel Coefficients

As in the delay diversity scheme the same signal is transmitted over each antenna, the transmission paths from either transmit antenna $1 \le i \le n_T$ to a certain receive antenna $1 \le j \le n_R$ can be combined in a *joint* channel model [17], where the delays δ_i applied at the different transmit antennas need to be taken into account. In case of $n_T = 2$ and $n_R = 1$ the overall channel model can be described by a vector $\mathbf{h}(k,\delta) = [h^{(0)}(k,\delta), h^{(1)}(k,\delta), ..., h^{(L)}(k,\delta)]$ of channel coefficients. Therefore, the received symbols $y(k)$ are given by:

$$y(k) = \sum_{l=0}^{L} h^{(l)}(k,\delta)\, x(k-l) + n(k)\,, \qquad (1)$$

where $x(k)$ denotes the k-th data symbol and $n(k)$ an additive white Gaussian noise sample. Furthermore, for the special case that $\delta = mT$, with m being an integer number, it is $h^{(l)}(k,\delta) = h_1^{(l)}(k) + h_2^{(l-m)}(k)$, where the coefficients $h_1^{(l)}(k)$ and $h_2^{(l)}(k)$ correspond to transmit antenna 1 and 2, respectively.

[2] EDGE ('Enhanced Data Rates for GSM Evolution') is a further development of GSM towards 3G data rates. The EDGE specifications comprise both binary GMSK and linearized GMSK with an 8-PSK mapping. The GSM burst structure as well as the eight training sequences have been retained for EDGE. EGPRS stands for 'Enhanced GPRS'.

The derivation of ρ_l is done corresponding to the way described in [16] (Appendix II). One obtains

$$\begin{aligned}\rho_l(\delta,\epsilon) \;&=\; \int_0^{\tau_{max}} p(\tau)\Big(\left|g_{TxRx}(lT-\tau-\epsilon)\right|^2 + \\ &\qquad + \left|g_{TxRx}(lT-\delta-\tau-\epsilon)\right|^2\Big)\,d\tau \\ &\doteq\; \rho_{l,1}(\epsilon) + \rho_{l,2}(\delta,\epsilon)\,, \qquad 0 \le l \le L\,, \end{aligned} \qquad (2)$$

where $g_{TxRx}(\xi) \doteq g_{Tx}(\xi) * g_{Rx}(\xi)$ denotes the overall impulse response of transmitter and receiver comprising a pulse shaping filter $g_{Tx}(\xi)$ and a receiver filter $g_{Rx}(\xi)$ (the asterisk means convolution). The power density function (pdf) $p(\tau)$ is proportional to the delay power density profile, where $0 \le \tau \le \tau_{max}$. An example for a delay power density profile are the GSM 05.05 propagation profiles [9]. The pdf $p(\tau)$ is assumed to apply for both transmission paths. As in [16] an additional sampling phase $\epsilon \in \mathbb{R}$ $(0 \le \epsilon < T)$ is taken into account. The values $\rho_l(\delta,\epsilon)$ are normalized such that for any δ, ϵ

$$\sum_{l=0}^{L} \rho_l(\delta,\epsilon) \;=\; 1\,. \qquad (3)$$

Fig. 2 illustrates ρ_l resulting for different delays δ and a fixed sampling phase ϵ_0. Note that the channel memory length L resulting for the joint channel model is actually a function of δ. The diversity gain accomplished by means of delay diversity $(\delta > 0)$ is therefore due to an increased degree of frequency-selectivity.

In the following, a trellis-based equalizer of length L_{eq} is assumed to be employed at the receiver, i.e., in the branch metric computation the equalizer takes into account the first $(L_{eq}+1)$ channel coefficients of the vector $\mathbf{h}(k,\delta)$ $(L_{eq} \le L)$.

B. RRB Resulting for an Optimal Trellis-Based Equalizer

In case of a binary modulation scheme and an equalizer of length $L_{eq} = L$, the RRB is given by

$$\begin{aligned} P_b^{(RRB)} \;&=\; \frac{1}{2}\sum_{\lambda=0}^{L}\left(\prod_{\substack{\nu=0\\ \rho_\nu \neq \rho_\lambda}}^{L} \frac{\rho_\lambda(\delta,\epsilon)}{\rho_\lambda(\delta,\epsilon)-\rho_\nu(\delta,\epsilon)}\right)\cdot \\ &\qquad \cdot\left(1 - \frac{1}{\sqrt{1+\frac{N_0}{E_s}\,\frac{1}{\rho_\lambda(\delta,\epsilon)}}}\right), \end{aligned} \qquad (4)$$

where E_s denotes the mean energy per data symbol and N_0 the single-sided noise power density. For the derivation of (4) refer

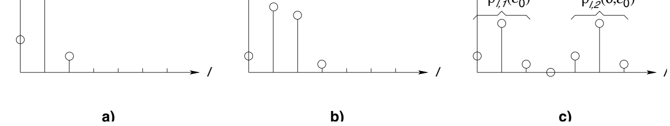

Fig. 2. Mean power $\rho_l(\delta,\epsilon)$ of the channel coefficients ($n_T = 2$ transmit antennas, sampling phase $\epsilon = \epsilon_0$), a) delay $\delta = 0$, b) delay $\delta = T$, and c) delay $\delta = 4T$.

to [10]. Note that the RRB is a function of δ, ϵ, and the signal-to-noise ratio (SNR) E_s/N_0.

In Fig. 3 the RRB is plotted as a function of δ (dashed line), resulting for the GSM 05.05 'typical urban (TU)' channel model [9], GMSK modulation, a sampling phase $\epsilon = 0$, and an SNR of 10 dB. Throughout the paper a non-adaptive root-raised-cosine receiver filter is considered, wth a roll-off factor $r = 0.5$ and a 3dB-bandwidth $f_{3\text{dB}} = 180\text{kHz}$. Fig. 3 shows that within the interval $0 \leq \delta \leq 3T$ the RRB decreases with growing delay δ, i.e., with an increased degree of frequency-selectivity. Note that currently $\delta = 0$ is applied in GSM/GPRS base stations, which corresponds to the case that both signals are transmitted simultaneously. Further note that, given a dispersive channel, the choice $\delta = T$ typically made in a delay diversity scheme with two transmit antennas is not optimal either. For $\delta = 3T$ the two terms $\rho_{l,1}(\epsilon)$ and $\rho_{l,2}(\delta, \epsilon)$ in ρ_l according to (2) are virtually disjunctive in the time domain (cf. Fig. 2 c)), which leads to the maximum possible diversity gain. Therefore, delays $\delta > 3T$ solely increase the resulting channel memory length L but do not accomplish further diversity gain. Accordingly, the RRB does not decrease further for $\delta > 3T$. Corresponding simulation results also included in Fig. 3 (dotted line) are in accordance with the general shape of the RRB curve.

As Fig. 3 illustrates, the conventional RRB leads to bit error probabilities significantly smaller than the simulated bit error rates. This is due to the fact that the RRB presumes *independent* fading of the individual channel coefficients corresponding to ρ_l. However, the channel coefficients are characterized by both *dynamic* ISI, which is due to the frequency-selective fading channel, and *static* ISI, which is due to the overall impulse response of transmitter and receiver $g_{TxRx}(\xi)$, i.e., the assumption of independent fading is not valid. This means, the RRB overestimates the degree of diversity utilized and therefore yields bit error probabilities that are too optimistic.

For the purpose of analysis, the static ISI can be eliminated from ρ_l by means of a linear zero-forcing (ZF) equalizer. The equalizer can be characterized by a finite-impulse-response (FIR) filter structure and its coefficients $e_{ZF}^{(\epsilon)}(l)$ are matched to the samples $g_{TxRx}(lT - \epsilon) \doteq g^{(\epsilon)}(l)$ of $g_{TxRx}(\xi)$. The convolution of the original channel coefficients and the ZF equalizer yields a set of modified channel coefficients $\tilde{h}^{(l)}(k, \delta)$ $(0 \leq l \leq \tilde{L})$, which are solely characterized by the dynamic ISI. The mean power $\tilde{\rho}_l$ of the modified channel coefficients is given by

$$\begin{aligned} \tilde{\rho}_l(\delta, \epsilon) \;&=\; \int_0^{\tau_{max}} p(\tau) \left(\left| g_{TxRx}(lT - \tau - \epsilon) * e_{ZF}^{(\epsilon)}(l) \right|^2 \right. \\ &\qquad \left. + \left| g_{TxRx}(lT - \delta - \tau - \epsilon) * e_{ZF}^{(\epsilon)}(l) \right|^2 \right) d\tau \\ &\doteq\; \tilde{\rho}_{l,1}(\epsilon) + \tilde{\rho}_{l,2}(\delta, \epsilon) \, , \qquad 0 \leq l \leq \tilde{L}. \end{aligned} \tag{5}$$

The derivation of (5) is given in the Appendix A. In turn, the $\tilde{\rho}_l$ are normalized such that their sum according to (3) is one.

The RRB curve computed on basis of $\tilde{\rho}_l$ is shown in Fig. 3 as well (solid line). The general shape of the curve corresponds to that obtained for ρ_l. However, due to the fact that the static ISI has been removed by means of the ZF equalizer, the bit error probabilities of the new RRB curve are less optimistic and therefore closer to the simulation results. Yet it is important to note that the channel coefficients $\tilde{h}^{(l)}(k, \delta)$ still do *not* fade independently, as filtering with the ZF equalizer leads to residual correlation between the $\tilde{h}^{(l)}(k, \delta)$.

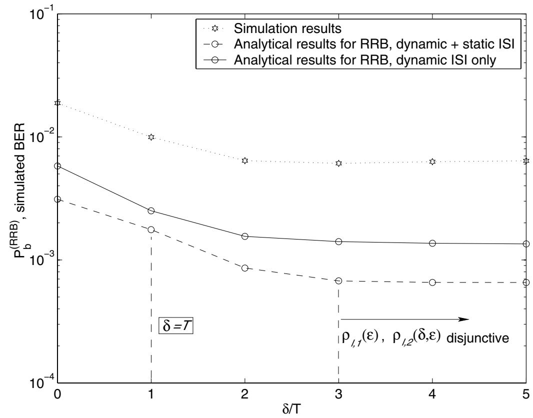

Fig. 3. RRB as a function of δ (equalizer length $L_{eq} = L$, 'TU'-profile, $E_s/N_0 = 10$ dB, sampling phase $\epsilon = 0$).

The influence of the SNR (E_s/N_0) on the improved RRB is such that with growing SNR the RRB tends to diminish. For example, for large δ and an SNR of 6 dB and 16 dB, bit error probabilities of $1.3 \cdot 10^{-2}$ and $7 \cdot 10^{-6}$ result, respectively.

C. RRB Resulting for a Truncated Trellis-Based Equalizer

In the following, we focus on the improved RRB curve. The case of a trellis-based equalizer of length $L_{eq} < \tilde{L}$ corresponds to a modified channel model, which is characterized by only $(L_{eq} + 1)$ channel coefficients and a transformed SNR denoted as $(E_s/N_0)'$:

$$\begin{aligned} P_b^{(RRB)} \;&=\; \frac{1}{2} \sum_{\lambda=0}^{L_{eq}} \left(\prod_{\substack{\nu=0 \\ \tilde{\rho}_\nu \neq \tilde{\rho}_\lambda}}^{L_{eq}} \frac{\tilde{\rho}_\lambda(\delta, \epsilon)}{\tilde{\rho}_\lambda(\delta, \epsilon) - \tilde{\rho}_\nu(\delta, \epsilon)} \right) \cdot \\ &\qquad \cdot \left(1 - \frac{1}{\sqrt{1 + \left(\frac{N_0}{E_s} \right)' \frac{1}{\tilde{\rho}_\lambda(\delta, \epsilon)}}} \right). \end{aligned} \tag{6}$$

$(N_0/E_s)'$ comprises the mean energy of the residual ISI term, which results from the fact that the channel coefficients $[\tilde{h}^{(L_{eq}+1)}(k, \delta), ..., \tilde{h}^{(\tilde{L})}(k, \delta)]$ are neglected in the modified channel model. One obtains

$$\left(\frac{N_0}{E_s} \right)' = \frac{N_0}{E_s} + \sum_{l=L_{eq}+1}^{\tilde{L}} \tilde{\rho}_l(\delta, \epsilon) \, . \tag{7}$$

For the derivation of (7) refer to Appendix B.

Fig. 4 shows the RRB as a function of δ resulting for different equalizer lengths L_{eq} (GSM 05.05 'TU' profile, SNR $E_s/N_0 = 10$ dB, sampling phase $\epsilon = 0$). With growing δ, the RRB curve for a given equalizer length complies with the ideal curve ($L_{eq} = \tilde{L}$), as long as the equalizer metric spans the predominant fraction of the sum in (3), i.e., $(\tilde{L} - L_{eq})$ is sufficiently small. Greater values of δ lead to degradation of the RRB due to residual ISI. At a certain delay δ_s the equalizer

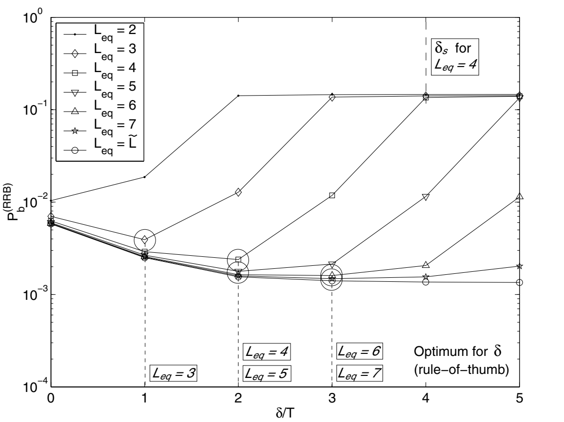

Fig. 4. RRB as a function of δ (different equalizer lengths L_{eq}, 'TU'-profile, $E_s/N_0 = 10$ dB, sampling phase $\epsilon = 0$ (analytical results)). Note that conventional GSM/GPRS receivers use $L_{eq} = 4 \dots 5$.

metric spans solely that fraction of (3) which corresponds to the term $\tilde{\rho}_{l,1}(\epsilon)$ in $\tilde{\rho}_l$ (cf. (5)). Therefore delays $\delta > \delta_s$ do not lead to further degradation of the RRB.

D. Optimization of the Delay Parameter

Given an optimal equalizer of length $L_{eq} = \tilde{L}$, the delay δ applied to the signal at the second transmit antenna should be $\delta \geq 3T$ in the 'typical urban' case, so as to minimize the RRB (cf. Fig. 3). In case of an equalizer of a fixed length L_{eq}, a rule-of-thumb can be derived from Fig. 4 concerning the optimal choice of δ (sampling phase $\epsilon = 0$, $L_{eq} \geq 3$):

$$\delta_{opt} \;\approx\; \lfloor L_{eq}/2 \rfloor \cdot T, \tag{8}$$

where $\lfloor x \rfloor$ is the greatest integer value i satisfying $i \leq x$. The values resulting for δ_{opt} given different equalizer lengths are included in Fig. 4. The rule-of-thumb (8) still holds, if ϵ is varied between $\pm T/4$.

IV. Simulation Results

The simulation results presented here demonstrate the performance of the GSM/GPRS system enhanced by delay diversity for the example of a GSM 05.05 'typical urban (TU)' channel model [9], which is a frequency-selective channel model with a memory length of about $L_{TU} = 3$. The simulations are restricted to the transmission of *uncoded* data, i.e., channel coding is not performed. Two transmit antennas and up to two receive antennas are considered. In this context, the overall transmitter power is the same as the transmitter power for a single transmit antenna system, i.e., the transmission power is normalized by a factor 1/2 at each antenna. Normalization with respect to the number n_R of receive antennas has not been performed. The receiver filter used is the non-adaptive root-raised-cosine filter introduced in Section III-B. The employed 'Max-Log-MAP' equalizer has $2^{L_{eq}} = 2^{L_{TU}} = 8$ states in case of the (1x1)-system. In the enhanced system utilizing a delay δ, $2^{L_{TU}+\vartheta}$ equalizer states are required for optimal equalization, where $\vartheta = \lfloor \delta/T \rfloor$.
The bit error performance of the GSM/GPRS system enhanced by delay diversity on the time-invariant 'typical urban' channel model ('TU0') is shown in Fig. 5, where the channel coefficients are perfectly known at the receiver. All simulation results were obtained by means of Monte-Carlo simulations over 10.000 bursts. The bit error rate (BER) curves are plotted as a function of the average SNR (E_s/N_0). As a reference, the BER curve resulting for the (1x1)-system is included. Moreover, analytical curves for *diversity reception* of uncoded BPSK are included [10], where data symbols $x(k) \in \{\pm 1\}$ are transmitted over ν individual paths subject to *independent* Rayleigh fading and where the ν paths are characterized by an identical average SNR of $(E_s/N_0)/\nu$.
As shown in Fig. 5, the delay diversity scheme yields significant performance improvements upon the (1x1)-system. Deploying a delay $\delta = T$, the gain accomplished at a BER of 10^{-3} is about 3 dB when a single receive antenna is employed ('(2x1)-Delay Diversity') and about 10 dB in case of a second receive antenna ('(2x2)-Delay Diversity'). A delay $\delta = 3T$ even leads to a gain of 4.7 dB for $n_R = 1$, which marks the maximum gain attainable by means of a single receive antenna (cf. Fig. 3). This gain comes, however, at the expense of an increased equalizer complexity.
In order to investigate the influence of channel estimation, in further investigations we also considered a time-varying 'typical urban' channel model. As a result, we observed that the performance loss with respect to perfect channel knowledge at the receiver was similar for all systems under consideration.

V. Summary and Conclusion

The application of delay diversity in a GSM/GPRS system has been investigated. First of all, the structure of the enhanced GSM/GPRS system was presented, which is compatible with current specifications and, in principle, also applies for an EDGE system[3]. An important advantage of delay diversity is the fact that virtually the same equalizer/detector algorithm can be used as in the (1x1)-system.
For the (2x1)-delay diversity scheme, an improved RAKE receiver bound (RRB) was derived as a function of the delay δ applied to the signal at the second transmit antenna. Both the case of an optimal and the case of a sub-optimal trellis-based equalizer were considered and a rule-of-thumb was deduced from the RRB concerning the optimal choice of δ. It was shown that the conventional choices $\delta = 0$ and $\delta = T$ are not necessarily the best ones.

[3] In case of 8-PSK, however, the equalizer complexity becomes impractical, and reduced-complexity techniques are required.

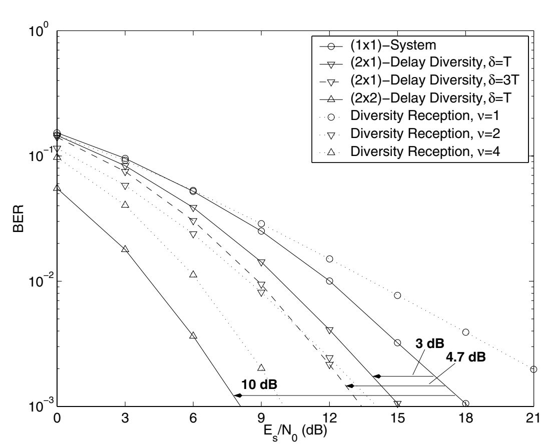

Fig. 5. Performance of the delay diversity scheme on the 'TU0' channel, perfect knowledge of channel coefficients at the receiver.

For the example of the GSM 05.05 'typical urban' channel model, simulation results demonstrated that delay diversity yields significant performance improvements upon a single transmit antenna system. Similar results hold for other channel models. Delay diversity enables higher bit rates and is therefore an attractive technique for an extension of the current GSM/GPRS specifications. With a single receive antenna performance improvements up to 4.7 dB were accomplished at a BER of 10^{-3}, with respect to E_s/N_0. Utilization of a second receive antenna lead to performance improvements even up to 10 dB ($\delta = T$). Concerning non-perfect knowledge of the channel coefficients at the receiver, the performance loss occurring in the enhanced systems turned out to be similar to that occurring in the conventional (1x1)-system.

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APPENDIX

A. Elimination of the Static ISI for the Purpose of Analysis

A convolution of the received symbols $y(k)$ according to (1) with the ZF equalizer yields

$$\begin{aligned} y(k) * e_{ZF}^{(\epsilon)}(k) &= \\ &= \left(\sum_{l=0}^{L} h^{(l)}(k,\delta)\, x(k-l)\right) * e_{ZF}^{(\epsilon)}(k) + \overbrace{n(k) * e_{ZF}^{(\epsilon)}(k)}^{\tilde{n}(k)} \\ &= \sum_{l=0}^{L} \underbrace{\left(\sum_{l'=0}^{L} h^{(l')}(k,\delta) e_{ZF}^{(\epsilon)}(l-l')\right)}_{h^{(l)}(k,\delta) * e_{ZF}^{(\epsilon)}(l) \doteq \tilde{h}^{(l)}(k,\delta)} x(k-l) + \tilde{n}(k) \\ &= \sum_{l=0}^{L} \tilde{h}^{(l)}(k,\delta)\, x(k-l) + \tilde{n}(k)\,. \end{aligned} \tag{9}$$

In turn, the derivation of the values $\tilde{\rho}_l \doteq E\{|\tilde{h}^{(l)}(k,\delta)|^2\} = E\{|\, h^{(l)}(k,\delta) * e_{ZF}^{(\epsilon)}(l)\,|^2\}$ is essentially done as in [16] (Appendix II), leading to (5).

B. Transformed SNR Value for the Modified Channel Model

Consider a channel model characterized by a channel memory length L, which in the following shall be transformed into a modified channel model of memory length $K < L$. The received signal $y(k)$ can be split into two terms:

$$y(k) = \sum_{l=0}^{L} h^{(l)}(k)\, x(k-l) \,+\, n(k) \tag{10}$$

$$= \sum_{l=0}^{K} h^{(l)}(k)\, x(k-l) + \underbrace{\overbrace{\sum_{l=K+1}^{L} h^{(l)}(k)\, x(k-l)}^{\doteq\, y_{\mathrm{ISI}}(k)} + n(k)}_{\doteq\, n'(k)}.$$

The noise term $n'(k)$ comprises an ISI expression $y_{\mathrm{ISI}}(k)$, which is due to the neglect of the channel coefficients $[h^{(K+1)}(k), ..., h^{(L)}(k)]$ in the modified channel model. For the corresponding mean normalized noise power $(N_0/E_s)'$ one obtains:

$$\begin{aligned} \left(\frac{N_0}{E_s}\right)' &= E\{|n'(k)|^2\} = E\{|y_{\mathrm{ISI}}(k) + n(k)|^2\} \\ &\overset{(*)}{=} E\{|y_{\mathrm{ISI}}(k)|^2\} + E\{|n(k)|^2\} \\ &= \sum_{l=K+1}^{L} \underbrace{E\{|h^{(l)}(k)|^2\}}_{=\rho_l} + \frac{N_0}{E_s}\,, \end{aligned} \tag{11}$$

where a binary modulation scheme is assumed in conjunction with independent and identically distributed (i.i.d.) data symbols $x(k)$, i.e., $E\{|x(k)|^2\} = 1$. Step $(*)$ is permitted under the assumptions that $y_{\mathrm{ISI}}(k)$ and $n(k)$ are statistically independent and that $E\{n(k)\}$ and $E\{y_{\mathrm{ISI}}(k)\}$ are zero. The latter assumption is granted due to the i.i.d. data symbols.

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