Distributed Transmit Power Allocation for Relay-Assisted Cognitive-Radio Systems†

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***Abstract*—We address the issue of optimal transmit power allocation in relay-assisted cognitive-radio (CR) systems. In particular, we assume that the frequency band chosen for unlicensed spectrum usage is not completely unoccupied, but contains one or more licensed narrowband users. For such a setting, we develop distributed transmit power allocation schemes, which optimize the performance of the CR system, while at the same time the interference experienced by the licensed users is limited. Numerical performance results illustrate that notable improvements compared to non-cooperative transmission are achieved by our proposed schemes.**

I. Introduction

RECENTLY, the concept of cognitive-radio (CR) systems has attracted considerable interest in the wireless communications community [1]. While traditionally spectrum usage has been organized according to fixed frequency plans defined through government licenses, CR systems are envisioned to take advantage of unused or partially occupied bands in an adaptive and unlicensed fashion, thus allowing for a more efficient spectrum utilization. To this end, CR systems will require spectrum-sensing capabilities, based on which they will adjust key transmission parameters such as operating frequency and radiated transmit power. In particular, CR capabilities will be relevant for ultra wideband (UWB) radio systems [2], which have been approved by several regulatory bodies around the world for unlicensed spectrum usage in (parts of) the 3.1-10.6 GHz band.

In order to limit the interference experienced by licensed ('primary') users, CR systems will naturally operate at comparatively low transmit powers. For UWB devices, for example, the Federal Communications Commission (FCC) has defined a spectral mask which explicitly limits the permitted transmit power level [3]. Correspondingly, in order to guarantee a certain quality of service, (cooperative) relaying techniques appear to be very attractive [4]-[7], since by this means relatively large distances between transmitter and receiver can be covered.

In this paper, we address the problem of cooperative relaying in CR systems for the case that the frequency band chosen for unlicensed usage is not completely unoccupied, but contains one or more primary users. In particular, we establish distributed transmit power allocation schemes, which optimize the performance of the CR system, while guaranteeing that a certain pre-defined maximum interference level experienced by the primary users is not exceeded. Numerical results presented confirm that notable performance improvements in comparison to non-cooperative transmission are achieved by our proposed schemes.

The remainder of this paper is organized as follows: In Section II, the system model and the optimization problem under consideration are introduced. Starting from the optimal centralized solution, several (suboptimal) distributed transmit power allocation schemes are developed in Section III, and their performance is assessed in Section IV. Finally, conclusions are drawn in Section V.

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II. System Model and Problem Formulation

We consider a relay-assisted wideband or UWB CR system that is based on code-division multiple access (CDMA). The source-destination node pair (S-D) is assisted by N_{r} perfectly synchronized (in time and frequency) relay nodes R_i ($i \in \{1, ..., N_{\mathrm{r}}\}$), which are equipped with mutually orthogonal spreading codes. For simplicity, all nodes within the CR system are assumed to employ a single omni-directional antenna.

The number of primary users residing within the frequency band of the CR system is in the sequel denoted by N_{p}. The bandwidth B_{U_j} occupied by primary user U_j ($j \in \{1, ..., N_{\mathrm{p}}\}$) is assumed to be small compared to the bandwidth B_{CR} of the CR system. The bandwidth ratio $B_{\mathrm{U}_j}/B_{\mathrm{CR}}$ is in the sequel denoted by ρ_j. The maximum sum interference power tolerated by primary user U_j is in the following denoted by ξ_j.

The channel impulse response (CIR) associated with a certain link $\mathrm{X} \rightarrow \mathrm{Y}$ from one node X to another node Y, where $\mathrm{X}, \mathrm{Y} \in \{\mathrm{S}, \mathrm{D}, \mathrm{R}_1, ..., \mathrm{R}_{N_{\mathrm{r}}}, \mathrm{U}_1, ..., \mathrm{U}_{N_{\mathrm{p}}}\}$, is in the following denoted as

$$\mathbf{h}_{\mathrm{X,Y}} := [\, h_{\mathrm{X,Y}}^{(0)}, \ \ldots, \ h_{\mathrm{X,Y}}^{(L_{\mathrm{X,Y}})} \,]^{\mathrm{T}}, \tag{1}$$

where $L_{\mathrm{X,Y}}$ denotes the corresponding channel memory length. Moreover, we define the CIR energy

$$\alpha_{\mathrm{X,Y}} := \sum_{l=0}^{L_{\mathrm{X,Y}}} |h_{\mathrm{X,Y}}^{(l)}|^2. \tag{2}$$

Since the bandwidths B_{U_j} have been assumed to be comparatively small, all links associated with the primary users are in the sequel modeled with a channel memory length of zero (frequency-flat fading). The system model under consideration is illustrated in Fig. 1, for the example $N_{\mathrm{r}} = 2$ and $N_{\mathrm{p}} = 1$.

Throughout this paper, a quasi-static scenario is considered. The destination node D is assumed to have perfect knowledge of the CIRs $\mathbf{h}_{\mathrm{S,D}}$ and $\mathbf{h}_{\mathrm{R}_i,\mathrm{D}}$ associated with the source-destination link $\mathrm{S} \rightarrow \mathrm{D}$ and the relay-destination links $\mathrm{R}_i \rightarrow \mathrm{D}$ ($i \in \{1, ..., N_{\mathrm{r}}\}$), respectively. Similarly, each relay node R_i is assumed to have perfect knowledge of the CIR $\mathbf{h}_{\mathrm{S,R}_i}$. Furthermore, it is assumed that the source node and each relay node is aware of the channel power gains

$$\alpha_{\mathrm{S,U}_j} = |h_{\mathrm{S,U}_j}^{(0)}|^2 \quad \text{or} \quad \alpha_{\mathrm{R}_i,\mathrm{U}_j} = |h_{\mathrm{R}_i,\mathrm{U}_j}^{(0)}|^2 \tag{3}$$

associated with its own links in direction of the primary users U_j ($j \in \{1, ..., N_{\mathrm{p}}\}$). This requires some network acquisition phase while the primary users are sensed to be transmitting.[1]

A. Transmission Protocol

The transmission protocol under consideration consists of two orthogonal time slots. Within the first time slot, the source node

[1]We assume that the primary users operate in a time-division duplex (TDD) mode. In order to estimate the channel power gains from the signal strengths received from the primary users, the (average) transmit powers of the primary users must be known. This appears to be a reasonable assumption, since due to the fixed frequency plans within the licensed spectrum it is known which primary systems will operate in the frequency band under consideration.

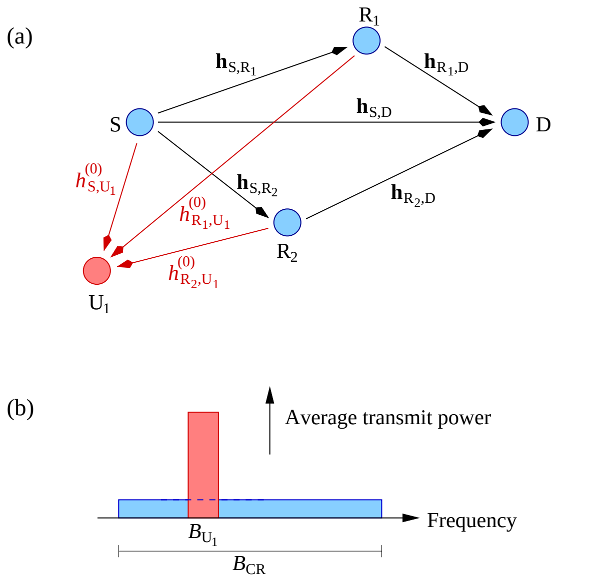

Fig. 1. (a) System model under consideration ($N_\mathrm{r}=2$ relays and $N_\mathrm{p}=1$ primary user), (b) corresponding interference scenario in the frequency domain.

S broadcasts a coded message to the relay nodes $\mathrm{R}_1,\ldots,\mathrm{R}_{N_\mathrm{r}}$ and the destination node D, while the transmit power P_S is adjusted such that all interference constraints are met, i.e.,

$$\rho_j \, P_\mathrm{S} \, \alpha_{\mathrm{S},\mathrm{U}_j} \leq \xi_j \tag{4}$$

for all $j \in \{1,\ldots,N_\mathrm{p}\}$. Furthermore, P_S is limited by some maximum available transmit power $P_{\mathrm{S},\max}$. Each relay node is assumed to employ a Rake receiver which performs optimal maximum-ratio combining (MRC) of the signal received from the source node. All relays that receive the coded message with an MRC output signal-to-noise ratio (SNR) of

$$\gamma_{\mathrm{R}_i} := \frac{P_\mathrm{S}\,\alpha_{\mathrm{S},\mathrm{R}_i}}{\sigma^2_{\mathrm{n},\mathrm{R}_i}} \geq \gamma_\mathrm{th}, \tag{5}$$

where $\sigma^2_{\mathrm{n},\mathrm{R}_i}$ denotes the noise variance at relay node R_i and γ_th denotes some threshold value, are assumed to decode the message without any errors. These $N'_\mathrm{r} \leq N_\mathrm{r}$ relays then broadcast a short beacon signal, so as to inform the other relays and the destination node that they will participate in the upcoming relaying process.[2] Within the second time slot, the N'_r relays that have decoded successfully re-encode the message and simultaneously retransmit it using the orthogonal spreading codes. The destination node finally performs optimal MRC of the signals received from the source node and the N'_r relay nodes, respectively.

B. Optimization Problem

Our design goal is to improve the overall MRC output SNR γ_D at the destination node according to a best-effort strategy, in order to establish a quick connection between source and destination node. Correspondingly, within the second time slot the transmit powers P_{R_i} of the participating relays shall be adjusted such that γ_D is maximized, under the constraint that the sum

[2]Since the relays are equipped with orthogonal spreading codes, one-bit beacons are sufficient in order to be able to identify the participating relays. Throughout this paper, we assume that the beacons are sufficiently protected using some low-rate channel code, so that they can be received reliably throughout the entire CR network.

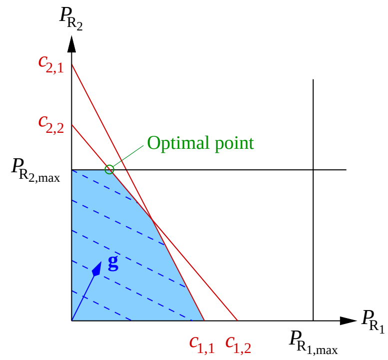

Fig. 2. Graphical illustration of the optimization problem (6), for the example $N'_\mathrm{r}=N_\mathrm{p}=2$. The feasible region for P_{R_1} and P_{R_2} is shaded. The level curves of γ_D are marked by dashed lines. The corresponding gradient vector is given by $\mathbf{g}=[\alpha_{\mathrm{R}_1,\mathrm{D}},\ \alpha_{\mathrm{R}_2,\mathrm{D}}]^\mathrm{T}/\sigma^2_{\mathrm{n},\mathrm{D}}$. Moreover, the parameters $c_{i,j}$ are given by $c_{i,j}=\xi_j/(\rho_j\,\alpha_{\mathrm{R}_i,\mathrm{U}_j})$.

interference power experienced by each primary user remains smaller than the pre-defined maximum interference power ξ_j. The optimal (centralized) transmit power allocation strategy thus results from the following linear optimization problem, which can be solved using standard linear programming methods, such as the well-known simplex algorithm [8, Ch. 4]:

$$\begin{aligned}
\text{maximize} \quad & \gamma_\mathrm{D} = \frac{1}{\sigma^2_{\mathrm{n},\mathrm{D}}}\left(\sum_{i=1}^{N'_\mathrm{r}} P_{\mathrm{R}_i}\,\alpha_{\mathrm{R}_i,\mathrm{D}} + P_\mathrm{S}\,\alpha_{\mathrm{S},\mathrm{D}}\right) \qquad (6)\\
\text{subject to} \quad & \rho_j \sum_{i=1}^{N'_\mathrm{r}} P_{\mathrm{R}_i}\,\alpha_{\mathrm{R}_i,\mathrm{U}_j} \leq \xi_j \quad \text{for all } j\in\{1,\ldots,N_\mathrm{p}\}\\
& P_{\mathrm{R}_i} \leq P_{\mathrm{R}_i,\max} \qquad\qquad \text{for all } i\in\{1,\ldots,N'_\mathrm{r}\},
\end{aligned}$$

where $\sigma^2_{\mathrm{n},\mathrm{D}}$ denotes the noise variance at the destination node and $P_{\mathrm{R}_i,\max}$ the maximum available transmit power of relay R_i.[3] The above optimization problem is illustrated in Fig. 2.

III. Distributed Transmit Power Allocation

In order to solve (6), a central network node is required (e.g., the destination node or one of the relays) which needs to be aware of *all* (equivalent) channel power gains $\alpha_{\mathrm{R}_i,\mathrm{D}}$ and $\alpha_{\mathrm{R}_i,\mathrm{U}_j}$ ($i\in\{1,\ldots,N'_\mathrm{r}\},\ j\in\{1,\ldots,N_\mathrm{p}\}$). After computing the optimal solution, the central node would then forward the resulting transmit power levels to the participating relay nodes. Obviously, this requires a lot of overhead, since each relay node R_i needs to communicate its own channel power gains $\alpha_{\mathrm{R}_i,\mathrm{D}}$ and $\alpha_{\mathrm{R}_i,\mathrm{U}_j}$ to the central node. In the following, we therefore develop several (suboptimal) distributed transmit power allocation schemes, which do not require any further exchange of channel information.

A. Fully Decentralized (FD) Transmit Power Allocation

We start with a fully decentralized (FD) transmit power allocation scheme, which is performed solely by the relays. Based on the beacon signals, the number N'_r of the participating relays is

[3]The maximum transmit power levels $P_{\mathrm{R}_i,\max}$ ($i\in\{1,\ldots,N'_\mathrm{r}\}$), the maximum interference powers ξ_j, and the parameters ρ_j ($j\in\{1,\ldots,N_\mathrm{p}\}$) are assumed to be known throughout the CR network. In contrast to this, the term $P_\mathrm{S}\,\alpha_{\mathrm{S},\mathrm{D}}$ in the objective function of (6) is irrelevant for the resulting solution and does not need to be known throughout the CR network.

known throughout the CR network. Moreover, each relay node R_i was assumed to be aware of the channel power gains $\alpha_{\mathrm{R}_i,\mathrm{U}_j}$ associated with its links in direction of the primary users U_j. Correspondingly, each relay can adjust its transmit power level according to

$$P_{\mathrm{R}_i} := \min\left\{P_{\mathrm{R}_i,\max},\ \min_{j\in\{1,\dots,N_\mathrm{p}\}}\left\{\frac{\xi_j}{\rho_j\, N_\mathrm{r}'\, \alpha_{\mathrm{R}_i,\mathrm{U}_j}}\right\}\right\}. \quad (7)$$

By this means, it can be guaranteed that each primary user experiences a sum interference power of at most ξ_j, without any further interaction between the relays. (In the special case $N_\mathrm{p}=1$ and $\xi_1/(\rho_1 N_\mathrm{r}' \alpha_{\mathrm{R}_i,\mathrm{U}_1}) \leq P_{\mathrm{R}_i,\max}$ for all $i \in \{1,\dots,N_\mathrm{r}'\}$, each relay will cause an interference power of exactly ξ_1/N_r'.) Moreover, it is guaranteed that the maximum transmit power available at each relay is not exceeded.

B. Distributed Scheme with Little Feedback (LF)

The performance of the above FD transmit power allocation scheme can be improved by allowing for some feedback from the destination node to the relay nodes. Based on this feedback, the transmit powers P_{R_i} can then be adjusted during an additional phase within the above transmission protocol. In the sequel, we propose a distributed transmit power allocation scheme, which requires particularly little feedback (LF) from the destination node.

The relay nodes start with the FD transmit power allocation (7), and the destination node measures the resulting overall MRC output SNR γ_D. (Within the scope of this paper, we assume that γ_D is measured with arbitrary accuracy.) Having perfect knowledge of the CIRs $\mathbf{h}_{\mathrm{R}_i,\mathrm{D}}$, the destination node then determines that relay R_k, which is associated with the largest equivalent channel power gain $\alpha_{\mathrm{R}_i,\mathrm{D}}$, i.e., with the largest component within the gradient vector $\mathbf{g}$ (cf. Fig. 2):

$$k := \arg\max_{i\in\{1,\dots,N_\mathrm{r}'\}}\{\alpha_{\mathrm{R}_i,\mathrm{D}}\}. \quad (8)$$

The destination node then computes the MRC output SNR

$$\gamma_\mathrm{D}' := \left(P_{\mathrm{R}_k,\max}\,\alpha_{\mathrm{R}_k,\mathrm{D}} + P_\mathrm{S}\,\alpha_{\mathrm{S},\mathrm{D}}\right)/\sigma^2_{\mathrm{n},\mathrm{D}} \quad (9)$$

which would result, if relay R_k transmitted at the maximum possible power level $P_{\mathrm{R}_k,\max}$ (disregarding the interference constraints), while all other relays remain silent. If $\gamma_\mathrm{D}' > \gamma_\mathrm{D}$ results, the destination node sends a corresponding one-bit beacon to each of the participating relays (using the corresponding spreading codes in conjunction with a low-rate channel code), signalizing which relay has been chosen to transmit at maximum power. Upon reception of the beacons, the participating relays then change their transmit power levels to

$$P_{\mathrm{R}_i} := \begin{cases} \min\left\{P_{\mathrm{R}_k,\max},\ \min_{j\in\{1,\dots,N_\mathrm{p}\}}\left\{\frac{\xi_j}{\rho_j\,\alpha_{\mathrm{R}_k,\mathrm{U}_j}}\right\}\right\}, & i=k \\ 0 & \text{else.} \end{cases} \quad (10)$$

Thus it is still guaranteed that each primary user experiences an interference power of at most ξ_j. Note, however, that the resulting MRC output SNR might in general be smaller than γ_D' (and, possibly, even smaller than γ_D), since due to the interference constraints, relay R_k might not be allowed to transmit at the maximum possible power level $P_{\mathrm{R}_k,\max}$. Finally, note that feedback from the destination node is only required, if the new LF transmit power allocation (10) promises to be superior to the FD power allocation (7). If the relay nodes do not receive any beacon signal from the destination node (i.e., $\gamma_\mathrm{D}' \leq \gamma_\mathrm{D}$), they simply retain the FD power allocation (7).

In the sequel, we focus on the case of congenerous primary users, i.e., $\xi_1/\rho_1 = \dots = \xi_{N_\mathrm{p}}/\rho_{N_\mathrm{p}} =: \theta$.

C. Distributed Quasi-Optimal (QO) Power Allocation

Next, we propose a quasi-optimal (QO) distributed transmit power allocation scheme for the case of congenerous primary users, which also utilizes feedback from the destination node (similar to the above LF scheme).

The relay nodes again start with the FD transmit power allocation (7), and the destination node measures the corresponding MRC output SNRs

$$\gamma_{\mathrm{D},i} := \frac{P_{\mathrm{R}_i}\,\alpha_{\mathrm{R}_i,\mathrm{D}}}{\sigma^2_{\mathrm{n},\mathrm{D}}}, \quad (11)$$

$i\in\{1,\dots,N_\mathrm{r}'\}$. (Again, we assume that these measurements are conducted with arbitrary accuracy.) Having perfect knowledge of the CIRs $\mathbf{h}_{\mathrm{R}_i,\mathrm{D}}$ (and the noise variance $\sigma^2_{\mathrm{n},\mathrm{D}}$), the destination node can determine the transmit power level P_{R_i} of each participating relay. Based on this, the destination node can now retrieve some information about the values $\alpha_{\mathrm{R}_i,\mathrm{U}_j}$.

Let R_m ($m \in \{1,\dots,N_\mathrm{r}'\}$) denote a relay node which has chosen a transmit power level of $P_{\mathrm{R}_m} < P_{\mathrm{R}_m,\max}$. In this case, it is known that

$$P_{\mathrm{R}_m} = \frac{\theta}{N_\mathrm{r}'\,\alpha_{\mathrm{R}_m,\mathrm{U},\max}}, \quad (12)$$

where $\alpha_{\mathrm{R}_m,\mathrm{U},\max} := \max_{j\in\{1,\dots,N_\mathrm{p}\}}\{\alpha_{\mathrm{R}_m,\mathrm{U}_j}\}$. Since it is known that $P_{\mathrm{R}_m} \leq \theta/(N_\mathrm{r}'\,\alpha_{\mathrm{R}_m,\mathrm{U}_j})$ for all $j \in \{1,\dots,N_\mathrm{p}\}$, the destination node can obtain the following worst-case estimate for the values $\alpha_{\mathrm{R}_m,\mathrm{U}_j}$ ($j\in\{1,\dots,N_\mathrm{p}\}$):

$$\tilde{\alpha}_{\mathrm{R}_m,\mathrm{U}_j} := \frac{\theta}{N_\mathrm{r}'\,P_{\mathrm{R}_m}} \geq \alpha_{\mathrm{R}_m,\mathrm{U}_j}. \quad (13)$$

Note that (13) holds with equality for $j=\arg\max_{j'}\{\alpha_{\mathrm{R}_m,\mathrm{U}_{j'}}\}$. In particular, in the special case of a single primary user ($N_\mathrm{p}=1$), the destination node is always able to retrieve the true value of $\alpha_{\mathrm{R}_m,\mathrm{U}_1}$.

Now let $\mathrm{R}_{m'}$ denote a relay node which operates at the maximum available transmit power $P_{\mathrm{R}_{m'},\max}$. In this case, the destination node simply adopts

$$\tilde{\alpha}_{\mathrm{R}_{m'},\mathrm{U}_j} := \frac{\theta}{N_\mathrm{r}'\,P_{\mathrm{R}_{m'},\max}} \geq \alpha_{\mathrm{R}_{m'},\mathrm{U}_j}, \quad (14)$$

($j\in\{1,\dots,N_\mathrm{p}\}$).

Based on the above results, the destination node can now solve the optimization problem (6), while replacing (some of) the parameters $\alpha_{\mathrm{R}_i,\mathrm{U}_j}$ by the corresponding estimates $\tilde{\alpha}_{\mathrm{R}_i,\mathrm{U}_j}$, and forward the resulting transmit power levels to the participating relay nodes. Since we always have $\tilde{\alpha}_{\mathrm{R}_i,\mathrm{U}_j} \geq \alpha_{\mathrm{R}_i,\mathrm{U}_j}$, the resulting quasi-optimal solution will always meet the interference constraint posed within the original optimization problem.

D. Enhanced Distributed Scheme with Little Feedback (ELF)

For the special case of a single or multiple congenerous primary users, the LF transmit power allocation scheme introduced in Section III-B can be further improved, using similar ideas as presented in Section III-C. To this end, the relay nodes again start with the FD transmit power allocation (7), and the destination node measures the MRC output SNRs $\gamma_{\mathrm{D},i}$ associated with the individual relays, cf. (11), as well as the overall MRC output SNR γ_D. Based on the values $\gamma_{\mathrm{D},i}$, it then determines the applied transmit power levels P_{R_i} ($i\in\{1,\dots,N_\mathrm{r}'\}$). For each relay R_m that has chosen a transmit power level of $P_{\mathrm{R}_m} < P_{\mathrm{R}_m,\max}$, it then computes the corresponding value $\alpha_{\mathrm{R}_m,\mathrm{U,max}}$, according to $\alpha_{\mathrm{R}_m,\mathrm{U,max}} = \theta/(N_\mathrm{r}' P_{\mathrm{R}_m})$, cf. (12). Finally, from these relays the destination node determines that relay R_k, which is associated with the largest MRC output SNR γ_D' which would result, if relay R_k transmitted at the maximum possible power level

$$P_{\mathrm{R}_k} = \min\left\{P_{\mathrm{R}_k,\mathrm{max}}, \frac{\theta}{\alpha_{\mathrm{R}_k,\mathrm{U,max}}}\right\}, \quad (15)$$

while all other relays remain silent. If $\gamma_\mathrm{D}' > \gamma_\mathrm{D}$ results, the destination node again sends a corresponding one-bit beacon to each of the participating relays, signalizing which relay has been chosen to transmit at the maximum possible power. Compared to the LF transmit power allocation scheme presented in Section III-B, the enhanced scheme takes the interference constraint into account when determining the 'best' relay. Moreover, it avoids to choose a relay that already transmits at the maximum available transmit power $P_{\mathrm{R}_k,\mathrm{max}}$, which can only deteriorate the overall MRC output SNR ($\gamma_\mathrm{D}' \leq \gamma_\mathrm{D}$).

IV. Numerical Performance Results

In the following, the performance of the above distributed transmit power allocation schemes will be assessed for various scenarios. In particular, we will compare it to the performance of direct transmission ($P_{\mathrm{R}_i} = 0$ for all $i \in \{1, ..., N_\mathrm{r}\}$) and the performance of the optimal centralized transmit power allocation.

In the sequel, all link lengths are normalized with respect to the distance between the source node and the destination node (cf. Fig. 1 (a)). Correspondingly, the locations of the source node and the destination node are set to $(-0.5, 0)$ and $(+0.5, 0)$, respectively. The relay nodes and the primary users are assumed to have random positions within square-shaped areas of side length 0.8, according to a uniform distribution. The center points of the two areas are given by $(0, 0)$ and $(x_\mathrm{p}, 0)$, respectively. Throughout this section, we assume a fixed (overall) number of $N_\mathrm{r} = 20$ relay nodes.

For simplicity, all nodes within the CR network are assumed to have identical physical properties. To this end, we set

$$P_{\mathrm{S,max}} = P_{\mathrm{R}_1,\mathrm{max}} = ... = P_{\mathrm{R}_{N_\mathrm{r}},\mathrm{max}} =: P_\mathrm{max} \quad (16)$$

and

$$\sigma^2_{\mathrm{n,D}} = \sigma^2_{\mathrm{n,R}_1} = ... = \sigma^2_{\mathrm{n,R}_{N_\mathrm{r}}} = \sigma^2_\mathrm{n}. \quad (17)$$

All transmission links are assumed to be subject to quasi-static Rayleigh fading. While the links associated with the primary users are modeled by frequency-flat fading, the links within the CR network are modeled by frequency-selective fading with a channel memory length of $L_{\mathrm{X,Y}} = 9$ and an exponentially decaying power profile, according to

$$\frac{\mathsf{E}\{|h^{(l)}_{\mathrm{X,Y}}|^2\}}{\mathsf{E}\{|h^{(0)}_{\mathrm{X,Y}}|^2\}} := \exp\left(-\frac{l}{c_h}\right), \quad (18)$$

$l \in \{0, ..., L_{\mathrm{X,Y}}\}$. Throughout this section, we choose $c_h := 2$. Additionally, we choose a path-loss exponent of two, so as to account for different link lengths. All simulation results presented in the following have been averaged over 1,000 random locations of the relay nodes and the primary users, while 100 statistically independent channel realizations per spatial constellation have been generated for each transmission link. Normalization is done such that $\mathsf{E}\{\alpha_{\mathrm{S,D}}\} = 1$.

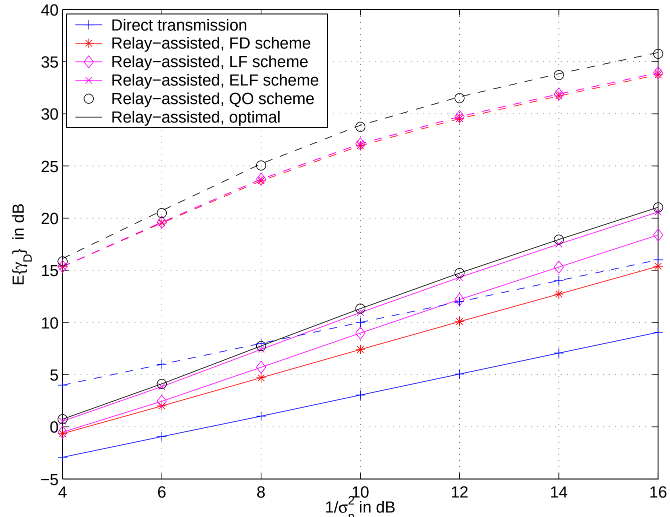

Fig. 3. Performance of different distributed transmit power allocation schemes. Depicted is the overall average MRC output SNR $\mathsf{E}\{\gamma_\mathrm{D}\}$ in dB versus $1/\sigma^2_\mathrm{n}$ in dB ($N_\mathrm{r} = 20$, $P_\mathrm{max} = 1$, $\gamma_\mathrm{th} = 10$ dB, $N_\mathrm{p} = 1$, $\xi_1 = 0.1$, $\rho_1 = 0.5$). Solid lines: $x_\mathrm{p} = 0$; dashed lines: $x_\mathrm{p} = 5$.

A. Single Primary User

To start with, we consider the case of a single primary user ($N_\mathrm{p} = 1$). Fig. 3 depicts the performance of the different distributed transmit power allocation schemes in terms of the overall average MRC output SNR $\mathsf{E}\{\gamma_\mathrm{D}\}$ as a function of $1/\sigma^2_\mathrm{n}$. Two different scenarios are considered:

(i) The relays and the primary user are found within the same square-shaped area ($x_\mathrm{p} = 0$, solid lines)
(ii) The area with the primary user is located at some distance ($x_\mathrm{p} = 5$, dashed lines).

As can be seen, the proposed distributed transmit power allocation schemes yield significant performance improvements over direct transmission, especially if the primary user is located at some distance from the CR network. Interestingly, already the fully decentralized (FD) scheme significantly outperforms direct transmission, even for the worst case where the relays and the primary user are found within the same area ($x_\mathrm{p} = 0$). Moreover, for $x_\mathrm{p} = 0$ the distributed schemes using little feedback from the destination node (LF scheme/ ELF scheme) are able to accomplish further performance improvements. In particular, the performance of the ELF scheme is already very close to the optimum. If the primary user is located at some distance from the CR network ($x_\mathrm{p} = 5$), the performance of the FD scheme, the LF scheme, and the ELF scheme are virtually the same. Finally, the performance of the distributed quasi-optimal (QO) scheme is virtually the same as that of the optimal centralized transmit power allocation scheme, both for $x_\mathrm{p} = 0$ and $x_\mathrm{p} = 5$ (over the entire range of $1/\sigma^2_\mathrm{n}$).

Fig. 4 depicts the performance of the distributed transmit power allocation schemes resulting for different maximum transmit power levels of the CR network ($x_\mathrm{p} = 1$). As can be seen, the relative performance of the different transmit power allocation schemes remains more or less unchanged. In particular, significant performance gains over direct transmission are retained for all power levels P_max under consideration. For low power levels, e.g. $P_\mathrm{max} = 0.1$, the performance of the FD scheme is already close to the optimum. For large power levels, the slope of the different performance curves becomes rather flat, due to the imposed interference constraint.

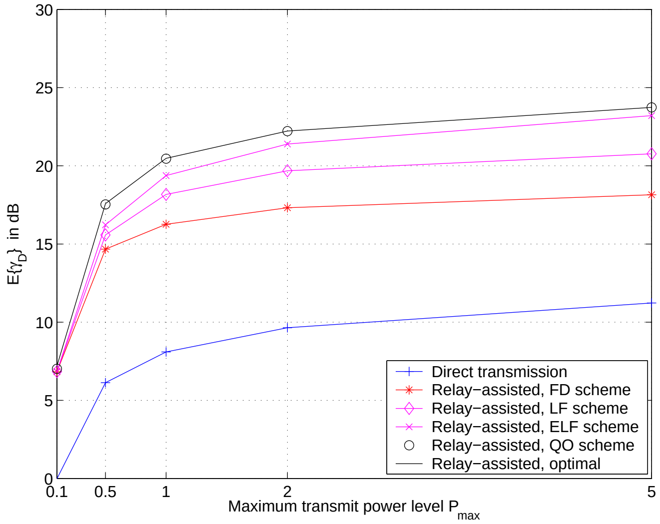

Fig. 4. Performance of the distributed transmit power allocation schemes as a function of the maximum transmit power level $P_{\max}$, resulting for $1/\sigma_{\mathrm{n}}^2 = 10$ dB ($N_{\mathrm{r}} = 20$, $\gamma_{\mathrm{th}} = 10$ dB, $N_{\mathrm{p}} = 1$, $\xi_1 = 0.1$, $\rho_1 = 0.5$, $x_{\mathrm{p}} = 1$).

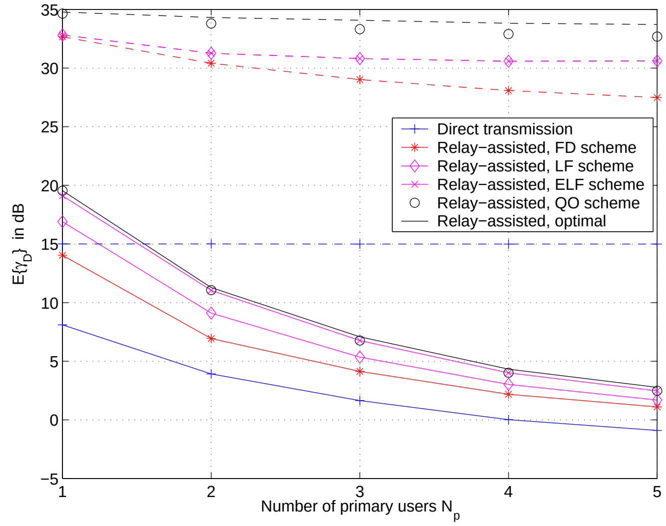

Fig. 5. Performance of the distributed transmit power allocation schemes as a function of the number N_{p} of primary users, resulting for $1/\sigma_{\mathrm{n}}^2 = 15$ dB ($N_{\mathrm{r}} = 20$, $P_{\max} = 1$, $\gamma_{\mathrm{th}} = 10$ dB, $\xi_j = 0.1$ and $\rho_j = 0.5$ for all $j \in \{1, ..., N_{\mathrm{p}}\}$). Solid lines: $x_{\mathrm{p}} = 0$; dashed lines: $x_{\mathrm{p}} = 5$.

Further simulation results not presented here indicate that the performance gains over direct transmission become even more significant when the parameter ρ_1 is reduced and/or when the maximum interference power ξ_1 tolerated by the primary user is increased. Moreover, it was found that the performance improvements over direct transmission tend to be notably larger when the path-loss exponent is increased. Furthermore, the performance improvements over direct transmission tend to degrade gracefully, when the threshold SNR γ_{th} is increased. Moreover, for larger values of γ_{th}, the performance of the FD scheme is already fairly close to that of the optimal centralized transmit power allocation.

B. *Multiple Primary Users*

For simplicity, we focus on the case of congenerous primary users here, i.e., $\xi_1/\rho_1 = ... = \xi_{N_{\mathrm{p}}}/\rho_{N_{\mathrm{p}}} =: \theta$. As can be seen in Fig. 5, if the primary users are found within the same area as the relays ($x_{\mathrm{p}} = 0$, solid lines), the performance of the above schemes degrades significantly when N_{p} increases. Still, notable performance improvements over direct transmission are achieved already by the FD scheme. Moreover, the performance of the ELF scheme and the QO scheme is very close to that of the optimal centralized transmit power allocation scheme, similar to the case of a single primary user (cf. Fig. 3). If the primary users are located at some distance from the CR network ($x_{\mathrm{p}} = 5$, dashed lines), the performance of the above transmit power allocation schemes degrades rather gracefully, when the number of primary users is increased. (In comparison, the performance degradation observed for the FD scheme is somewhat more significant than for the other schemes.) Interestingly, the ELF scheme yields virtually no improvements over the LF scheme in this case, which can be explained by the fact that the interference constraints are less critical than for $x_{\mathrm{p}} = 0$. Moreover, the QO scheme performs slightly worse than the optimal centralized transmit power allocation scheme.

Further simulation results not presented here indicate that if the primary users are characterized by different parameters ξ_j and ρ_j, the FD scheme and the LF scheme provide similar performance gains as in the case of congenerous primary users.

V. Conclusions

In this paper, several suboptimal distributed transmit power allocation schemes for relay-assisted cognitive-radio networks in the presence of a single or multiple primary users have been developed. Numerical results have shown that all proposed schemes accomplish significant performance improvements over direct transmission. In particular, for the case of congenerous primary users a distributed quasi-optimal transmit power allocation scheme has been developed, which offers a performance that is very close to that of the optimal centralized solution. Moreover, our proposed (enhanced) LF scheme utilizing little feedback from the destination node achieves a performance which is still fairly close to the optimal one. Future work might yield more sophisticated solutions for the case of multiple non-congenerous primary users, so as to further approach the performance of the optimal centralized power allocation.

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