A Robust Receive Diversity Scheme for Spatially Correlated Multiple-Antenna Systems Using Second-Order Channel Statistics

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... which can be readily combined with IDMA ;-)

Introduction

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Mobile station: Insufficient antenna spacings
Base station: Lack of scattering from physical environment

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Mobile station: Insufficient antenna spacings
Base station: Lack of scattering from physical environment

Here: Transmit and **receive diversity scheme** for correlated MIMO systems based on **statistical** channel knowledge

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Transmit and receive diversity scheme under consideration:

- **only** knowledge of second-order channel statistics required (can **easily** be acquired in practical systems)
- can be employed **independently** of each other

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- **Basic structure:** Outer **power weighting** stage followed by inner **decorrelation** stage

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Reduced-Dimension Receiver

- **Basic structure:** Inner **decorrelation** stage followed by outer **selection** stage

Goal: Flexible **trade-off** between **complexity** and **performance** for subsequent receiver stages

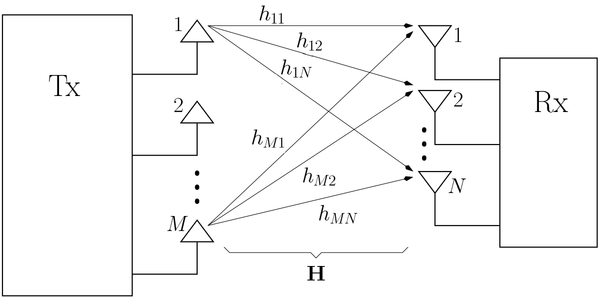
Focus here:

- Duality between transmit and receive diversity scheme
- Performance analysis with focus on complexity-performance trade-off offered by the reduced-dimension receiver
- Combination of reduced-dimension receiver with subsequent space-time decoding/ space-time equalization
- Impact of estimation errors (concerning the transmitter and receiver correlation matrix)

Outline

- System Model
  - Channel and Spatial Correlation Model
  - Structure of Transmit and Receive Diversity Scheme
- Transmit and Receive Diversity Scheme
  - Performance Analysis
  - Impact of estimation errors
- Conclusions

MIMO Channel Model

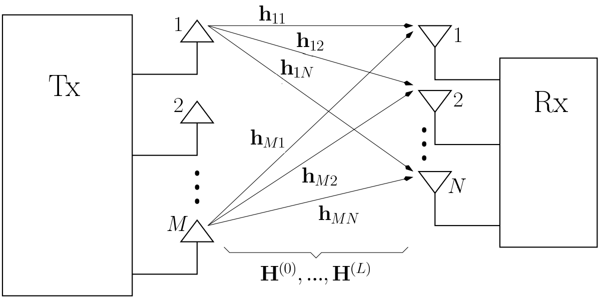


- M Tx antennas, N Rx antennas, quasi-static fading

$$\mathbf{y}[k] = \mathbf{H}\mathbf{x}[k] + \mathbf{n}[k]$$

  - $\mathbf{y}[k]$: Received vector, $\mathbf{x}[k]$: Data vector, $\mathbf{n}[k]$: Noise vector
  - $\mathbf{H}$: Channel matrix, $h_{ij} \sim \mathcal{CN}\{0, \sigma_h^2\}$ (Rayleigh fading)

MIMO Channel Model



- M Tx antennas, N Rx antennas, quasi-static fading

$$\mathbf{y}[k] = \sum_{l=0}^{L} \mathbf{H}^{(l)} \mathbf{x}[k-l] + \mathbf{n}[k]$$

  - $\mathbf{y}[k]$: Received vector, $\mathbf{x}[k]$: Data vector, $\mathbf{n}[k]$: Noise vector
  - $\mathbf{H}^{(l)}$: Channel matrices ($l = 0, \ldots, L$)

Transmit & receive diversity scheme based on **correlation matrices**

- **Flat fading:**

$$\mathbf{R}_{\mathsf{Tx}} = \mathsf{E}\{\mathbf{H}^{\mathsf{H}}\mathbf{H}\}/(N\sigma_h^2), \quad \mathbf{R}_{\mathsf{Rx}} = \mathsf{E}\{\mathbf{H}\mathbf{H}^{\mathsf{H}}\}/(M\sigma_h^2)$$

Example: Kronecker correlation model

$$\mathbf{H} := \mathbf{R}_{\mathsf{Rx}}^{1/2}\,\mathbf{G}\,\mathbf{R}_{\mathsf{Tx}}^{1/2} \qquad g_{ij} \sim \mathcal{CN}\{0, \sigma_h^2\} \text{ i.i.d.}$$

Transmitter and Receiver Correlation Matrix

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- **Frequency-selective fading** (receive diversity scheme):

$$\mathsf{E}\{\mathbf{y}[k]\mathbf{y}^{\mathsf{H}}[k]\} := \sigma^2\,\mathbf{R}_{\mathsf{Rx}} + \sigma_n^2\,\mathbf{I}_N$$

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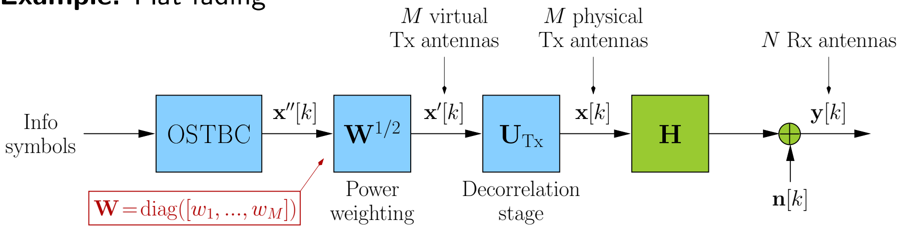
- **Eigenvalue Decompositions:**

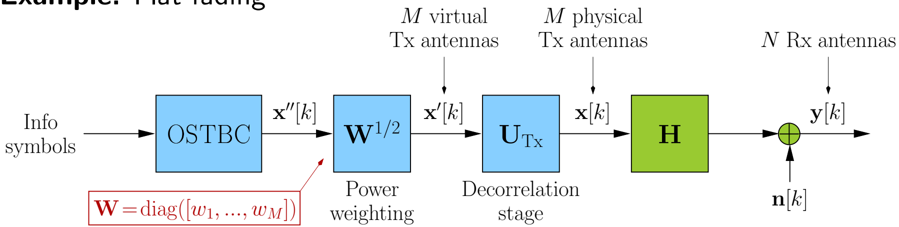
$$\mathbf{R}_{\bullet} := \mathbf{U}_{\bullet}\mathbf{\Lambda}_{\bullet}\mathbf{U}_{\bullet}^{\mathsf{H}}$$

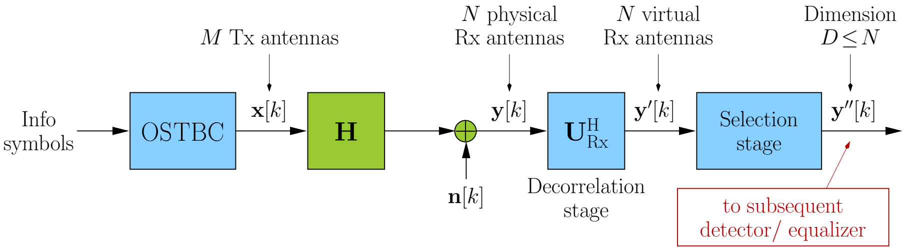
$\mathbf{\Lambda}_{\bullet}$ diagonal with eigenvalues $\lambda_{\mathsf{Tx},i}$ or $\lambda_{\mathsf{Rx},j}$, $\mathbf{U}_{\bullet}$ unitary

Example: Flat fading



Example: Flat fading





Reduced-Dimension Receiver

Selection stage:

- Choice $D = N$ is **optimal**, since decorrelation stage $\mathbf{U}_{\mathsf{Rx}}^{\mathsf{H}}$ does not change performance
- Smallest eigenvalues of $\mathbf{R}_{\mathsf{Rx}}$ can be **discarded** without significant performance loss $\Rightarrow$ **Reduced** complexity for subsequent stages

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Selection criterion adopted from Jelitto & Fettweis (2002):

Choose D as **small as possible**, but such that

Average discarded received power of **desired** signal

$\leq$ Average discarded **noise** power

$$P_{\text{disc}} := \sigma^2 \sum_j \lambda_{\text{Rx},j} \leq (N-D)\,\sigma_n^2$$

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- Closed-form expressions for **BER performance** (flat Rayleigh fading, Kronecker model, BPSK modulation)

No power weighting ($\mathbf{W} = \mathbf{I}_M$), full-dimension receiver ($D = N$)

$$\bar{P}_\mathrm{b} = \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{N} \left(\prod_{\substack{i'=1 \\ i' \neq i}}^{M} \prod_{\substack{j'=1 \\ j' \neq j}}^{N} \frac{\lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j}}{\lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j} - \lambda_{\mathrm{Tx},i'} \lambda_{\mathrm{Rx},j'}} \right)^{M} \times \left(1 - \sqrt{\frac{\sigma^2 \, \lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j}}{M \, \sigma_n^2 + \sigma^2 \, \lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j}}} \right)$$

High-SNR approximation:

$$\bar{P}_\mathrm{b} \approx \left(\frac{M \sigma_n^2}{4 \, \sigma^2} \right)^{MN} \binom{2MN - 1}{MN} \prod_{i=1}^{M} \prod_{j=1}^{N} \frac{1}{\lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j}}$$

- Closed-form expressions for **BER performance** (flat Rayleigh fading, Kronecker model, BPSK modulation)

  No power weighting ($\mathbf{W} = \mathbf{I}_M$), reduced dimension ($D < N$)

$$\bar{P}_\mathrm{b} = \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{D} \left(\prod_{\substack{i'=1 \\ i' \neq i}}^{M} \prod_{\substack{j'=1 \\ j' \neq j}}^{D} \frac{\lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j}}{\lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j} - \lambda_{\mathrm{Tx},i'} \lambda_{\mathrm{Rx},j'}} \right)^M \times \left(1 - \sqrt{\frac{\sigma^2 \, \lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j}}{M \, \sigma_n^2 + \sigma^2 \, \lambda_{\mathrm{Tx},i} \lambda_{\mathrm{Rx},j}}} \right)$$

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Performance Analysis

- Closed-form expressions for **BER performance** (flat Rayleigh fading, Kronecker model, BPSK modulation)

Power weighting ($\mathbf{W} \neq \mathbf{I}_M$), reduced dimension ($D < N$)

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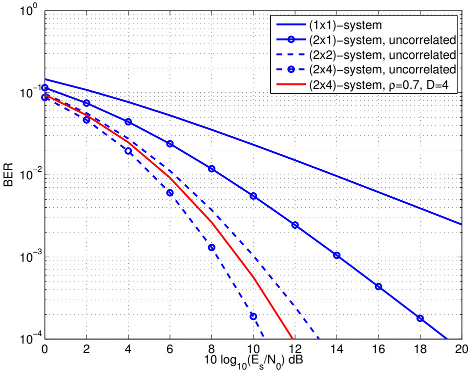
Flat Fading

Example:

- (2x4)-System
- BPSK modulation
- Alamouti STBC & linear decoding
- Transmitter: $\mathbf{R}_{\mathsf{Tx}} = \mathbf{I}_2$, No power weighting
- Receiver: Single-parameter correlation matrix $\mathbf{R}_{N,\rho}$ ($\rho = 0.7$), perfectly known



$D = 4$: Optimal performance (Whole SNR range)

Complexity-Performance Trade-off

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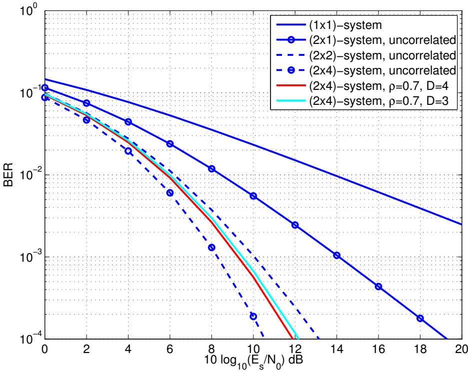

$D = 3$: Close to optimum (Whole SNR range shown)

Flat Fading

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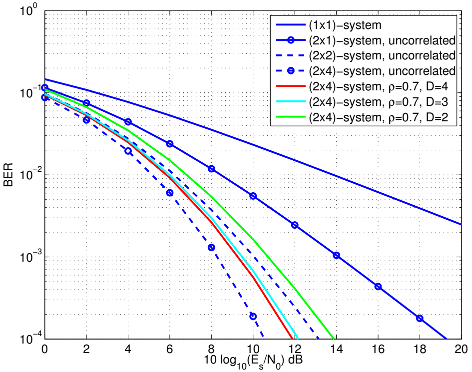

$D = 2$: Same slope as (2x2)-system (Choose only if SNR $\leq$ 4 dB)

Complexity-Performance Trade-off

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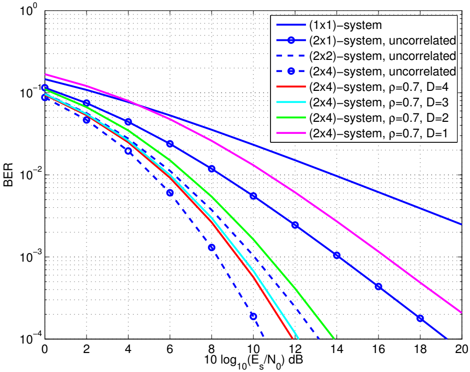

$D = 1$: Same slope as (2x1)-system (Choose only if SNR ≤ 2 dB)

Complexity-Performance Trade-off

Frequency-selective Fading

Example:

- (2x4)-System
- BPSK modulation
- Alamouti STBC & trellis-based equalizer
- Transmitter: $\mathbf{R}_{\mathsf{Tx}} = \mathbf{I}_2$, No power weighting
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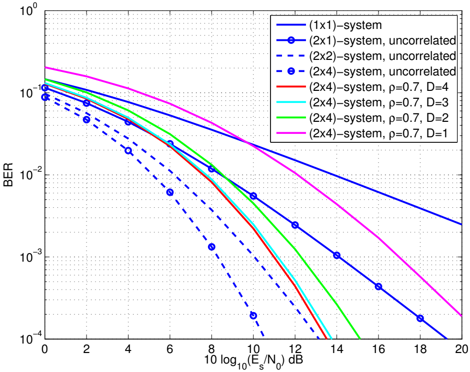

Impact of Estimation Errors

- In practical systems, $\mathbf{R}_\bullet$ has to be **estimated**

$$\Rightarrow \quad \hat{\mathbf{R}}_\bullet := \hat{\mathbf{U}}_\bullet \hat{\mathbf{\Lambda}}_\bullet \hat{\mathbf{U}}_\bullet^{\mathsf{H}}$$

  In general, $\hat{\mathbf{U}}_\bullet \neq \mathbf{U}_\bullet$ and $\hat{\mathbf{\Lambda}}_\bullet \neq \mathbf{\Lambda}_\bullet$ ($\hat{\mathbf{R}}_\bullet$ still Hermitian)

- Reduced-dimension receiver
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  $\hat{\mathbf{\Lambda}}_{\mathsf{Rx}} \Rightarrow$ Selection rule $\hat{P}_{\mathsf{disc}} = \sigma^2 \sum_j \hat{\lambda}_{\mathsf{Rx},j} \leq (N-D)\,\sigma_n^2$

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$\hat{\mathbf{\Lambda}}_{\mathsf{Rx}} \Rightarrow$ Selection rule $\quad \hat{P}_{\mathsf{disc}} = \sigma^2 \sum_j \hat{\lambda}_{\mathsf{Rx},j} \leq (N-D)\,\sigma_n^2$

$\hat{\mathbf{U}}_{\mathsf{Rx}}$: Average power of desired signal **actually** discarded is

$$P_{\mathsf{disc}} := \sigma^2 \sum_j \mathbf{\Xi}_{\mathsf{Rx},j,j} \leq (N-D)\,\sigma_n^2,$$

where $\quad \mathbf{\Xi}_{\mathsf{Rx}} := \hat{\mathbf{U}}_{\mathsf{Rx}}^{\mathsf{H}} \mathbf{U}_{\mathsf{Rx}} \mathbf{\Lambda}_{\mathsf{Rx}} \mathbf{U}_{\mathsf{Rx}}^{\mathsf{H}} \hat{\mathbf{U}}_{\mathsf{Rx}}$

Effect usually **small** as long as $\hat{\mathbf{R}}_{\mathsf{Rx}}$ is not too bad

Mismatched Selection Rule

Example: Single-parameter correlation matrix $\mathbf{R}_{\text{Rx}} = \mathbf{R}_{N,\rho}$ with $N = 4$ and $\rho = 0.7$; direct estimate for ρ available

$\Rightarrow$ Table for the optimal choice of the dimension D

	SNR in dB										
	0	1	2	3	4	5	6	7	8	9	10
$\hat{\rho} = \rho$	1	1	1	2	2	3	3	3	3	3	3
$\hat{\rho} = \rho - 10\%$	1	1	2	2	3	3	3	3	4	4	4
$\hat{\rho} = \rho + 10\%$	1	1	1	1	2	2	2	2	2	3	3

Red: Unnecessarily **high** receiver complexity (marginal gains)
Green: Chosen receiver complexity too **small** (notable losses)

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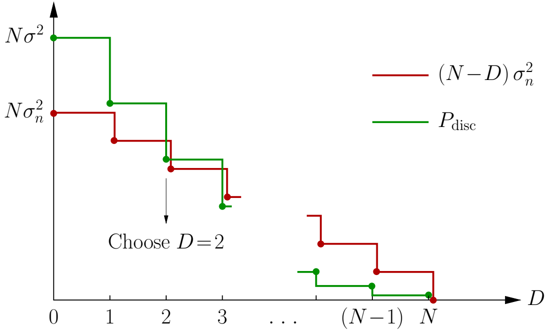
Still: Performance loss/ complexity overhead **limited**, occurs only for certain SNR values $\Rightarrow$ Scheme quite **robust**

Conclusions

Transmit diversity and **receive diversity** scheme for spatially correlated MIMO systems

- Simple structures, based on second-order **channel statistics**
- **Duality** between transmit and receive diversity scheme
- Closed-form expressions for bit-error-rate **performance**
- **Complexity-performance** trade-off of receive diversity scheme (combined with space-time decoding/ equalization)
- Impact of **estimation errors** w.r.t. correlation matrix
  $\Rightarrow$ Transmit and receive diversity scheme quite **robust**

$$P_{\mathsf{disc}} := \sigma^2 \sum_j \lambda_{\mathsf{Rx},j} \;\le\; (N-D)\,\sigma_n^2$$



Improved Selection Rule

Introduce a bias ψ:

$$\hat{P}_{\mathsf{disc}} \leq (N-D)\,\sigma_n^2 + \psi$$

- Some **performance loss** acceptable $\Rightarrow$ Choose $\psi > 0$
- Some **extra complexity** acceptable $\Rightarrow$ Choose $\psi < 0$

The value of ψ has to be optimized numerically, based on the quality of the estimate $\hat{\mathbf{R}}_{\mathsf{Rx}}$