# Prefiltering and Trellis-Based Equalization for Sparse ISI Channels

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$$\mathbf{h} := [\, h_0 \; \overbrace{\underbrace{0\ldots0}_{f_0 \text{ zeros}} \; h_1 \; \underbrace{0\ldots0}_{f_1 \text{ zeros}} \; h_2 \; \ldots \; h_{G-1} \; \underbrace{0\ldots0}_{f_{G-1} \text{ zeros}} \; h_G}^{L} \,]^{\mathsf{T}}$$

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Special case: **Zero-pad channel**

$$f_0 = f_1 = \ldots = f_{G-1} =: f \geq 1, \qquad L = (f+1)G$$

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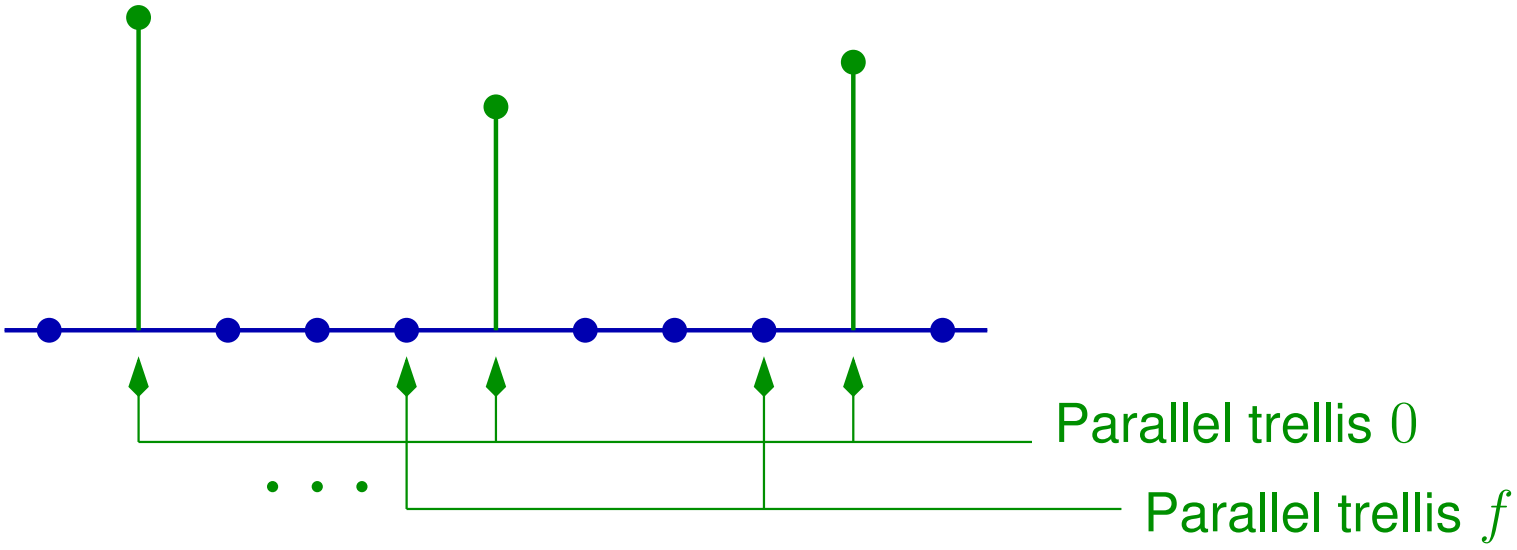
Instead $M^L = M^{(f+1)G}$ states $\Rightarrow$
$(f+1)$ parallel trellises with $M^G$ states each

# P-VA, P-BCJRA

Sparse CIR (zero–pad)




Parallel trellis $i$:

Symbol estimates for time indices $i + (f+1)N$ ($N$ integer)

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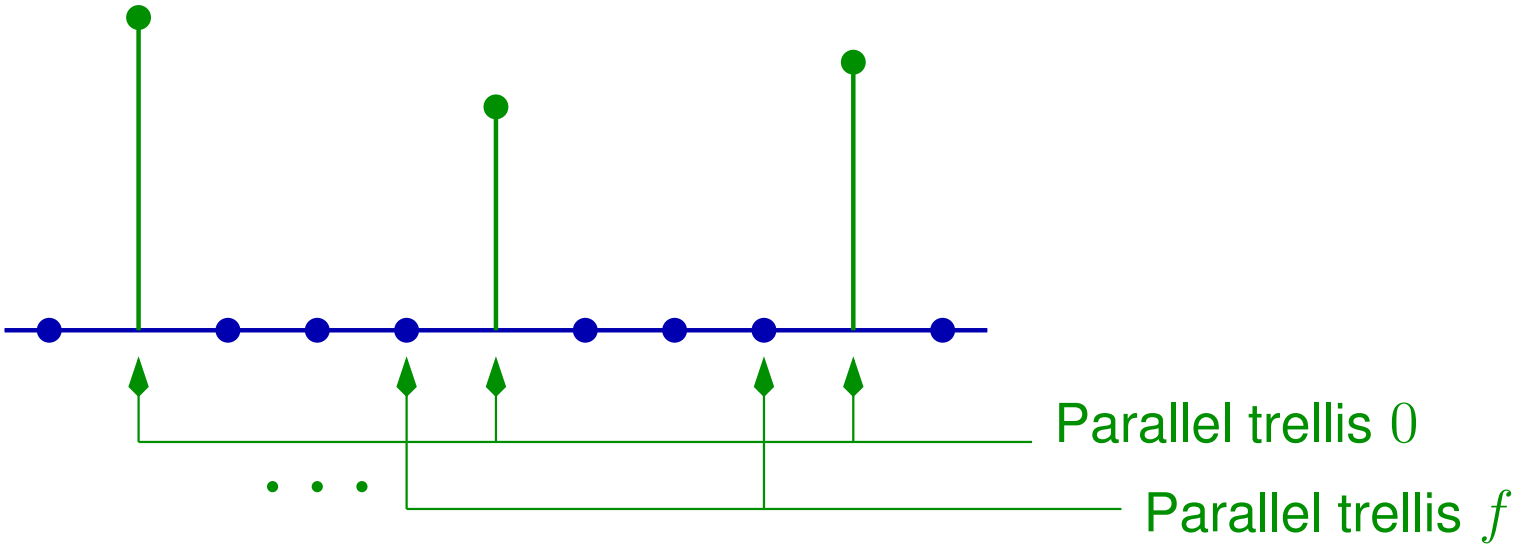


Parallel trellis $i$:

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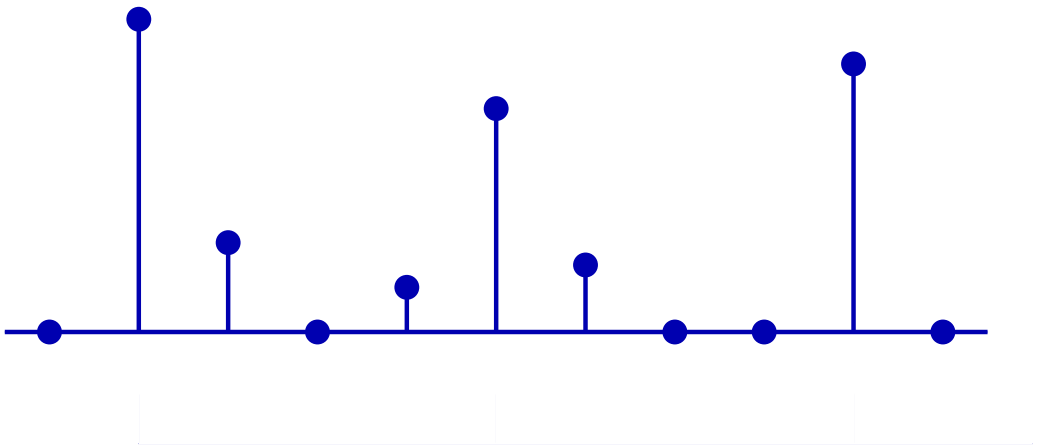
$\Rightarrow$ Still **optimal** in the sense of MLSE

# sub-P-VA, sub-P-BCJRA

Sparse CIR (non zero–pad)



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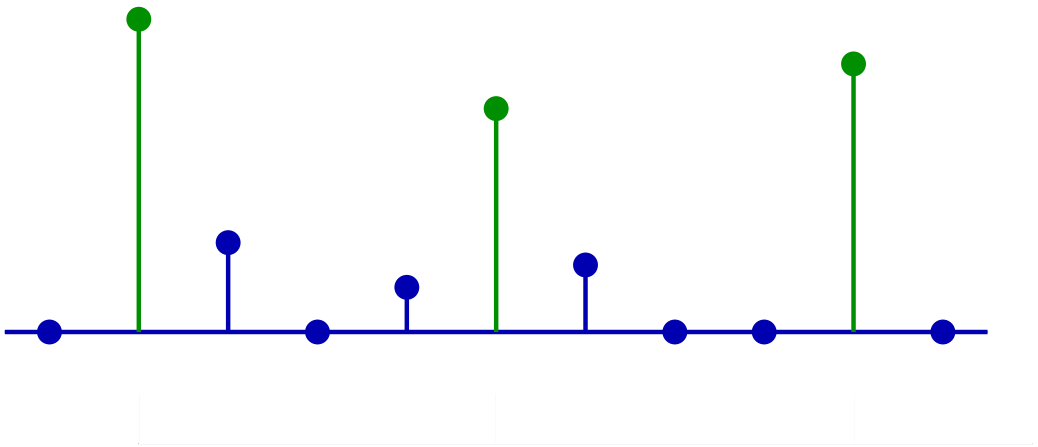

(a) Find an underlying zero-pad CIR similar to the given CIR

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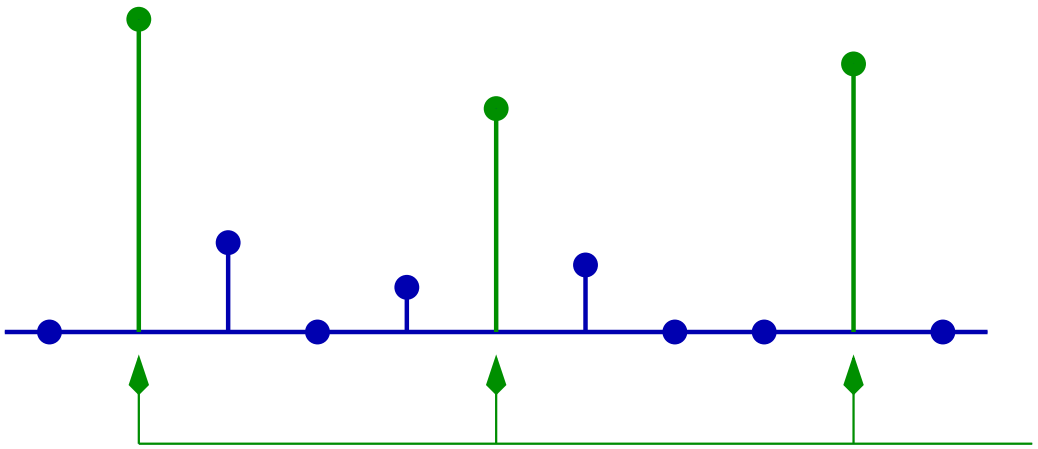

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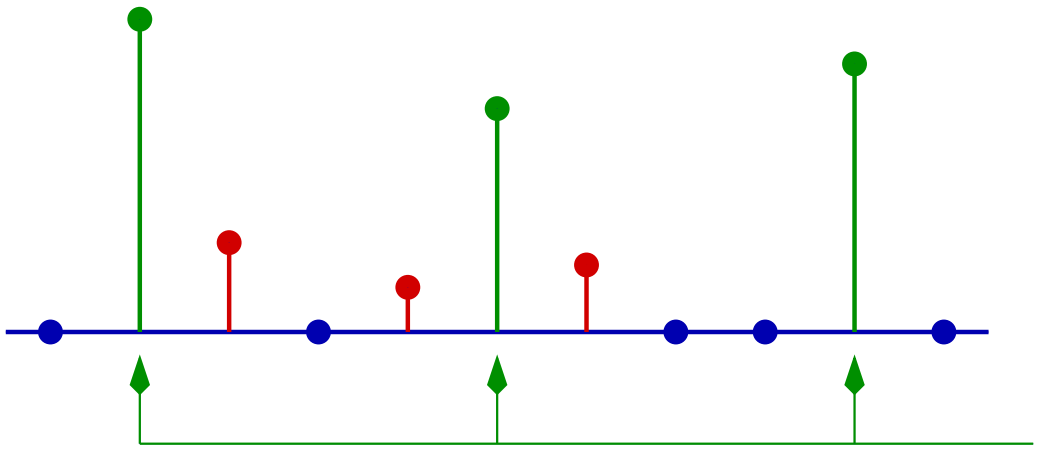

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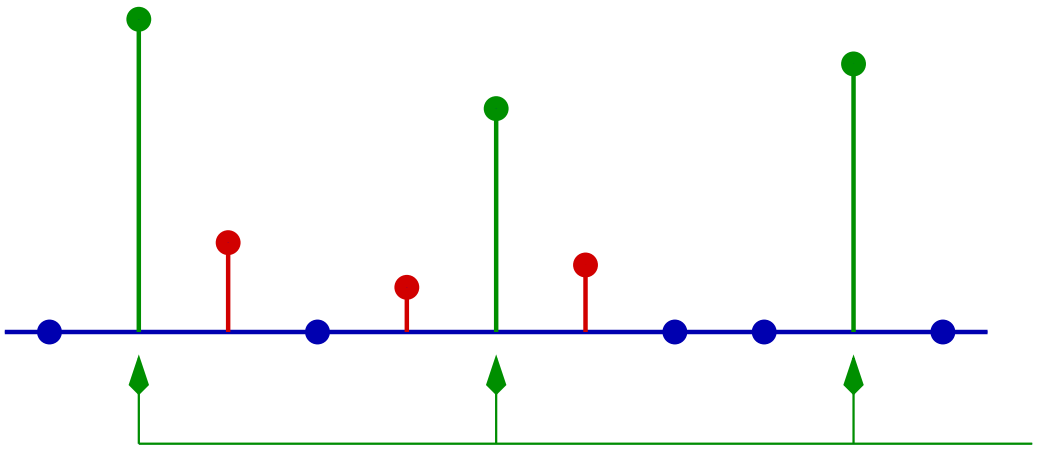

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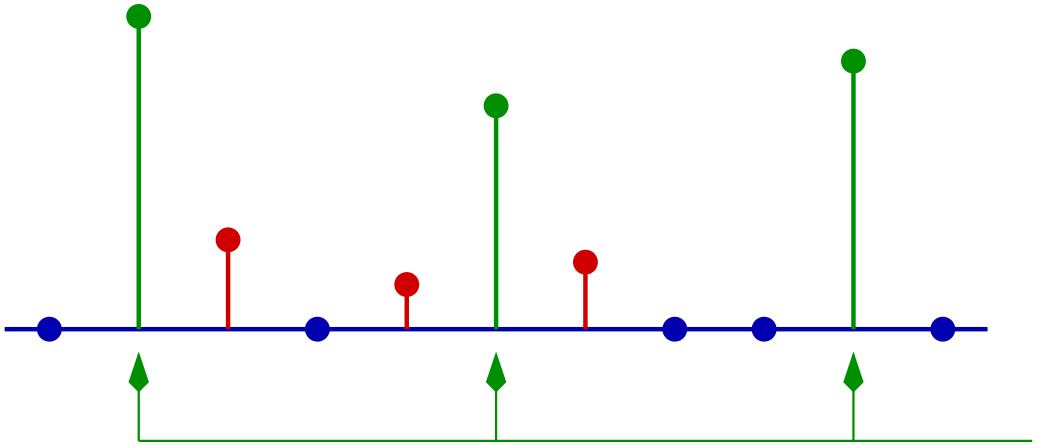
$\Rightarrow$ **suboptimal** parallel-trellis VA/BCJRA

# Fading Channel



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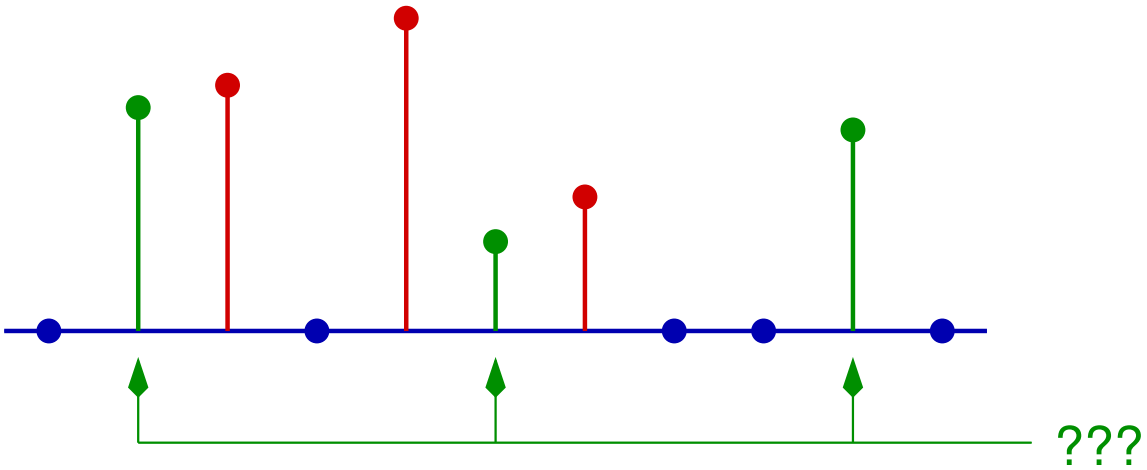


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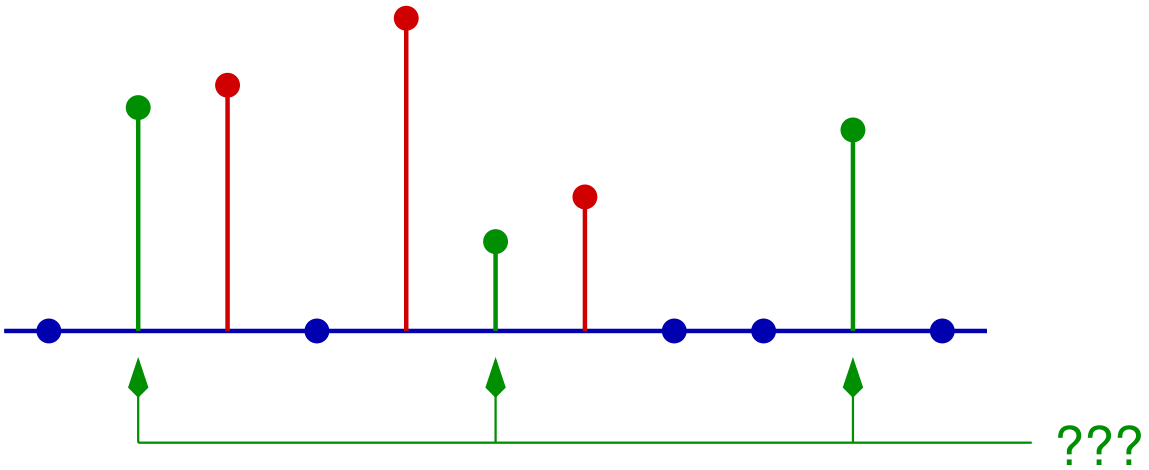


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## Our approach

Use **prefiltering** in conjunction with **standard** reduced-complexity **trellis-based equalizer**

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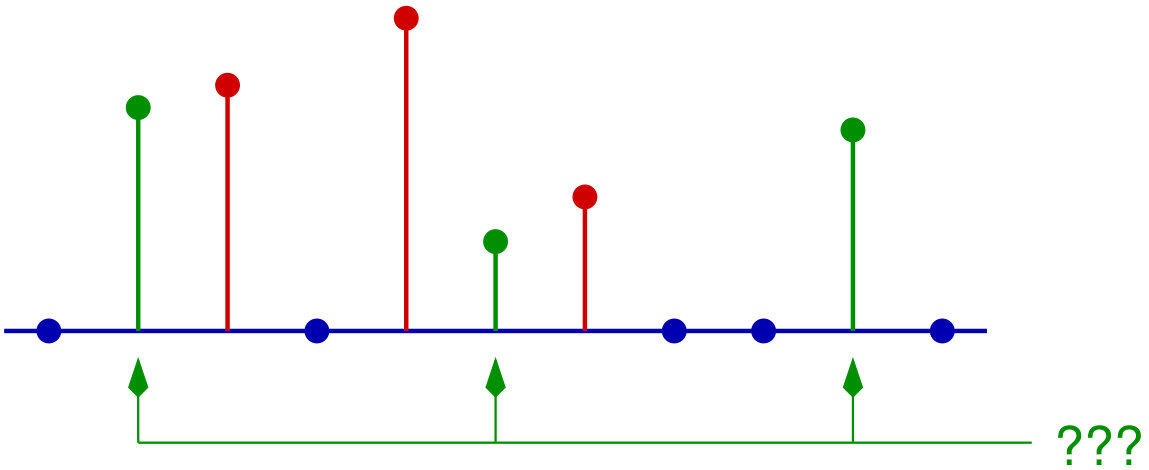


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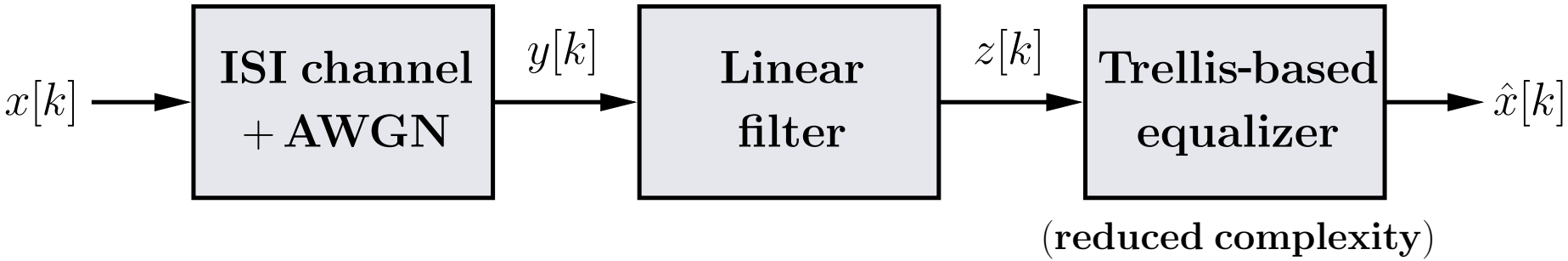
Use **prefiltering** in conjunction with **standard** reduced-complexity **trellis-based equalizer**

$\Rightarrow$ Tackle **general** sparse fading CIRs & provide performance **close** to the matched filter bound (MFB)

# Outline

- Introduction
- Proposed Receiver Structure
  - Linear Prefiltering
  - Reduced-State Trellis-Based Equalization
- Numerical Results
- Conclusions

# Proposed Receiver Structure

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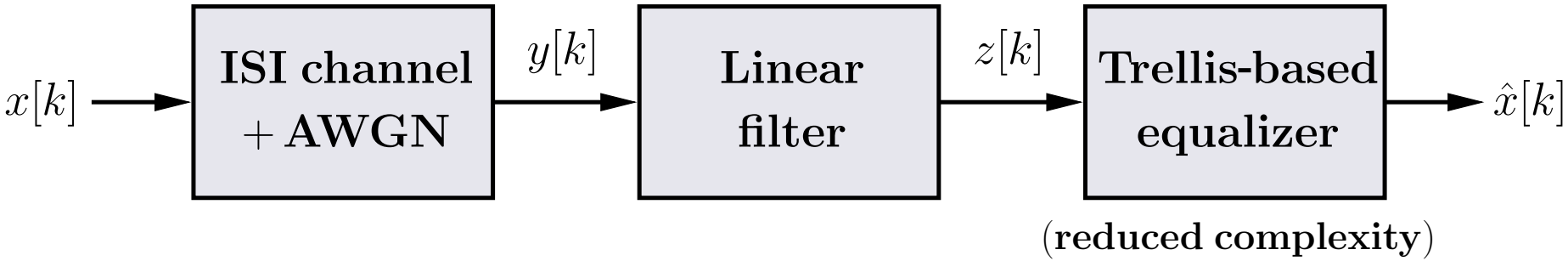


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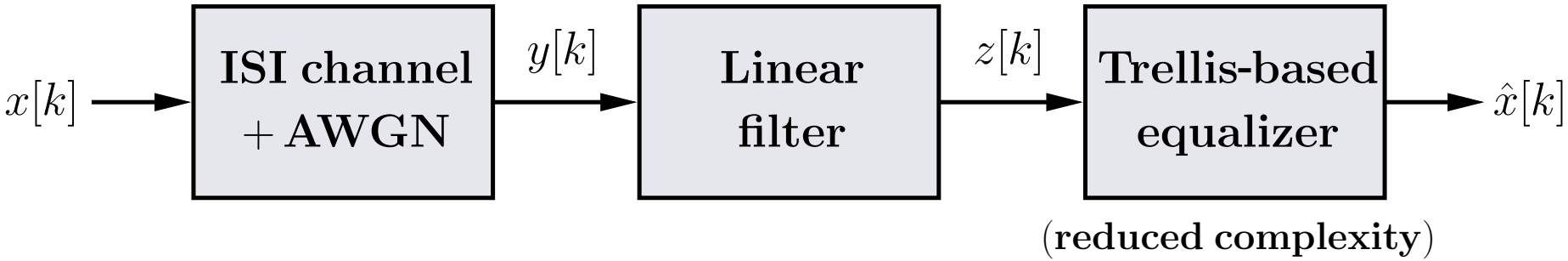


**Linear filter** that can be computed **efficiently**
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(i) Channel shortening filter (CSF)
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(ii) Minimum-phase filter (WMF)
$\Rightarrow$ Energy concentration in the first channel coefficients

# Proposed Receiver Structure

**Standard** reduced-complexity **trellis-based equalizer** (not specifically designed for sparse ISI channels)

Sparse CIR structure is normally **lost** after prefiltering

$\Rightarrow$ Solely the linear filter is adjusted to the current CIR

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(i) Shortened Viterbi detector (SVD)
- $\Rightarrow$ Shortened memory length $L_{\mathsf{s}} \ll L$
- $\Rightarrow$ Use SVD in conjunction with CSF

(ii) Delayed decision-feedback sequence estimator (DDFSE)
- $\Rightarrow$ Parallel decision feedback, memory length $K \ll L$
- $\Rightarrow$ Use DDFSE in conjunction with WMF

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  - Power profiles before and after CSF/WMF
  - Performance results for different channel memory lengths
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# Comparison with sub-P-BCJRA

Static CIR $\mathbf{h} = [h_0\ 0\ 0\ 0\ h_4\ 0\ 0\ h_7\ 0 \ldots 0\ h_{15}]^{\mathsf{T}}$ (no zero-pad)

$h_0 = 0.87,\ h_4 = h_7 = h_{15} = 0.29$

Binary transmission; $L_{\mathsf{F}} = 40$ (WMF), $L_{\mathsf{F}} = 50$ (CSF)

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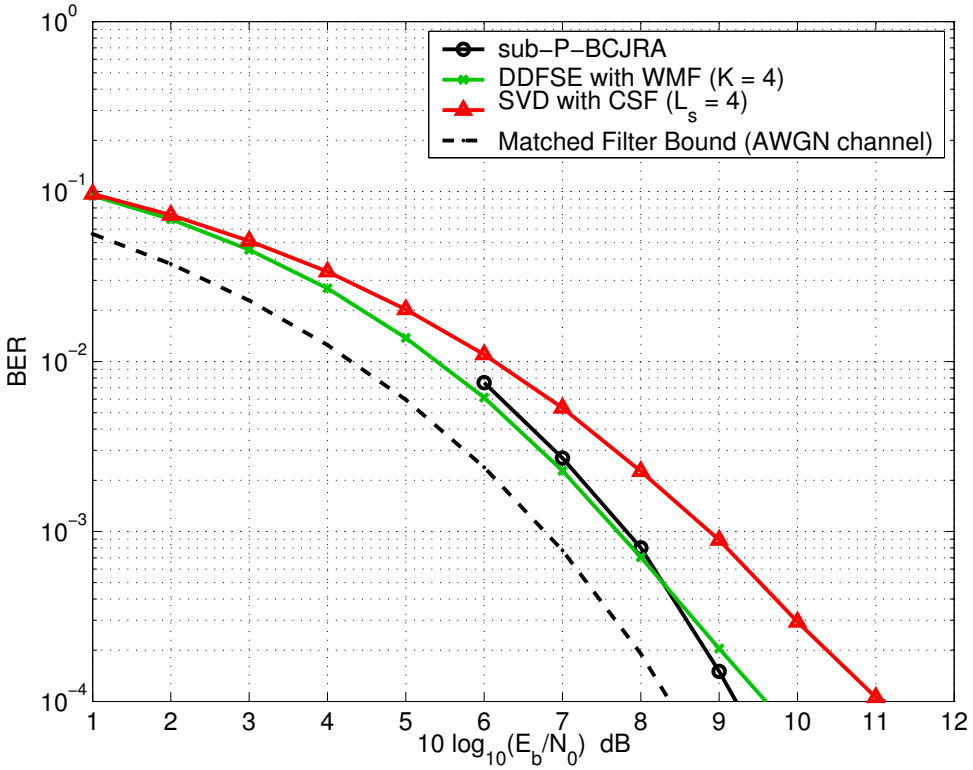


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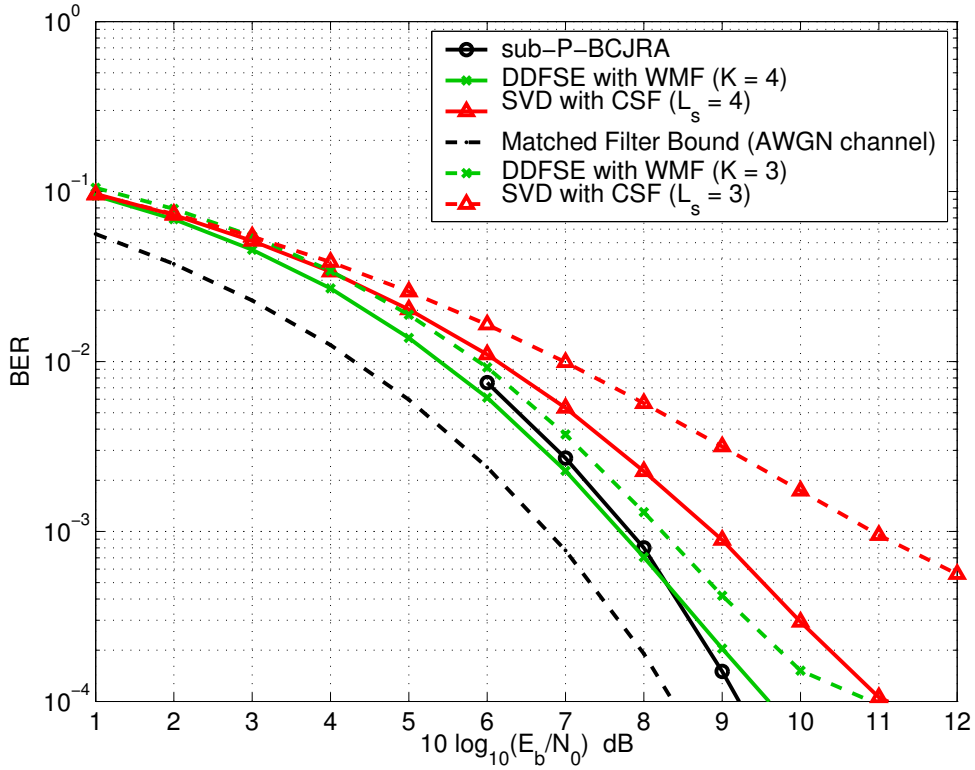


DDFSE ($K = 4$) + WMF: Similar **performance** as sub-P-BCJRA

DDFSE ($K = 3$) + WMF: **Reduced** complexity at expense of **small** loss

# Power Profiles after CSF/WMF

Fading CIR with $h_g \sim \mathcal{CN}(0, \sigma^2_{\mathsf{h},g})$ and power profile

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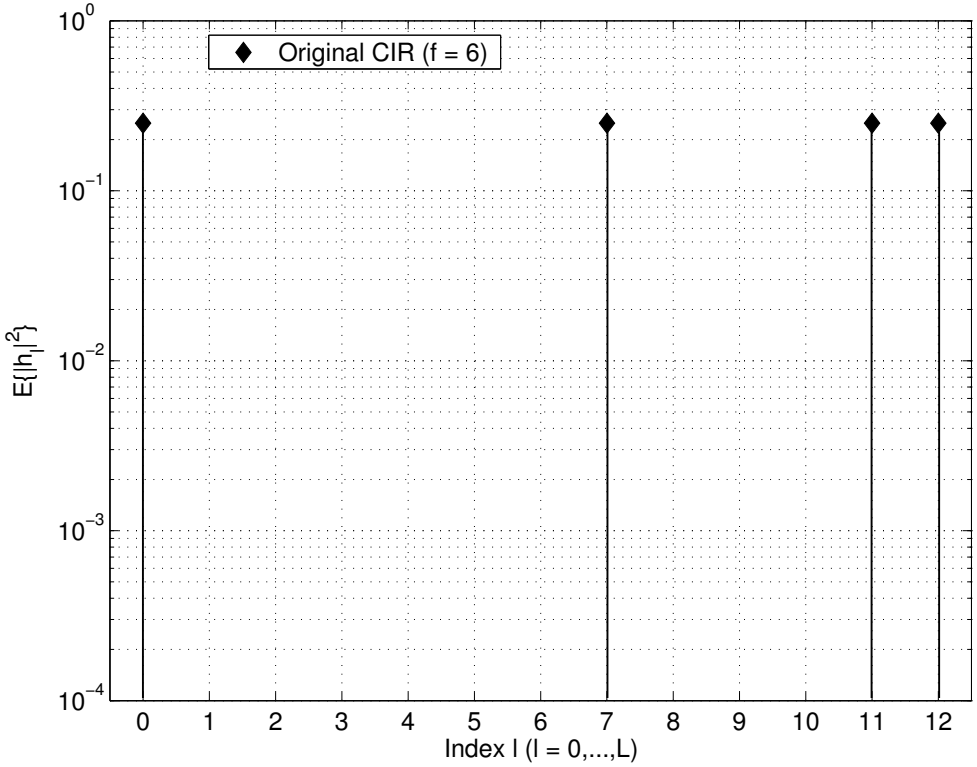

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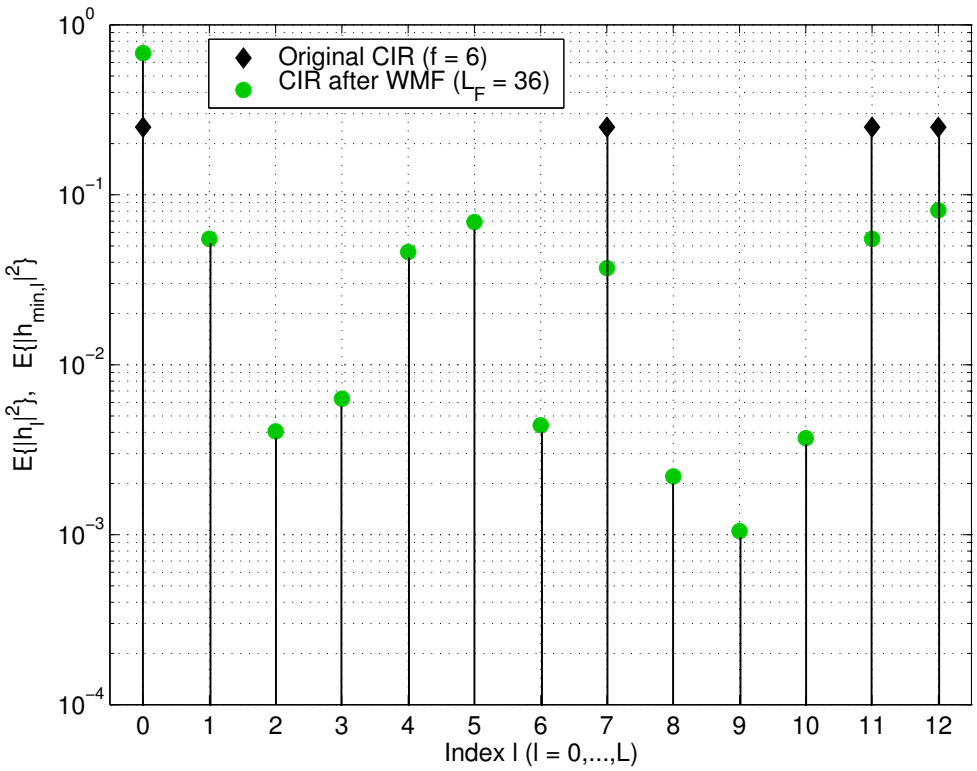


Sparse structure **lost** after prefiltering

**WMF**: Assigns main energy to first channel coefficient

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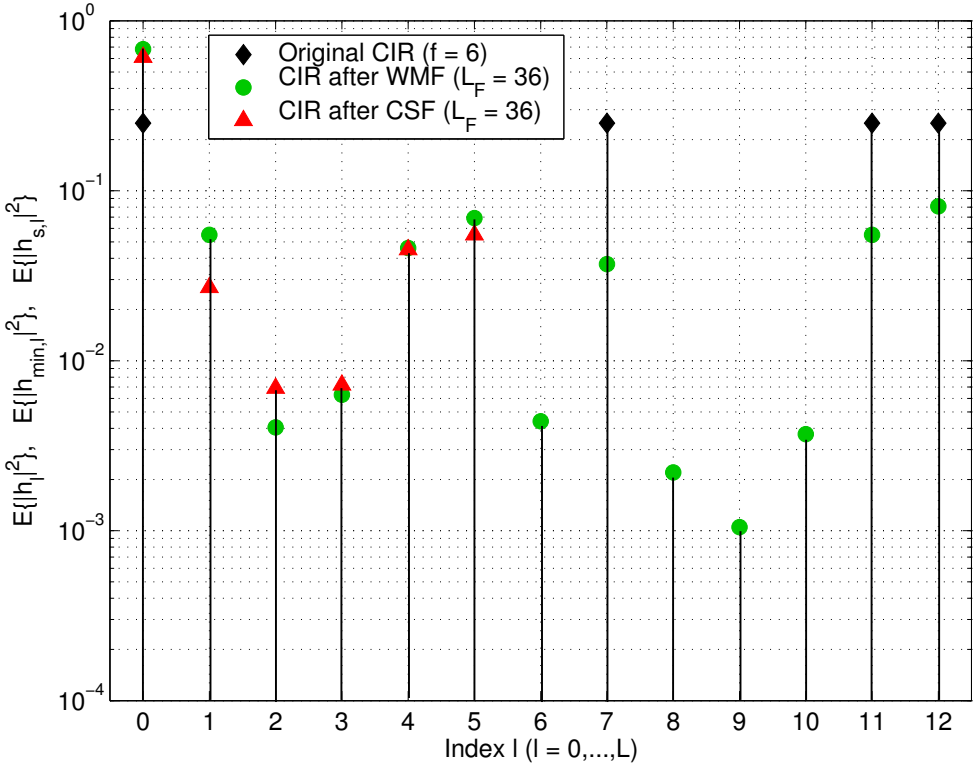


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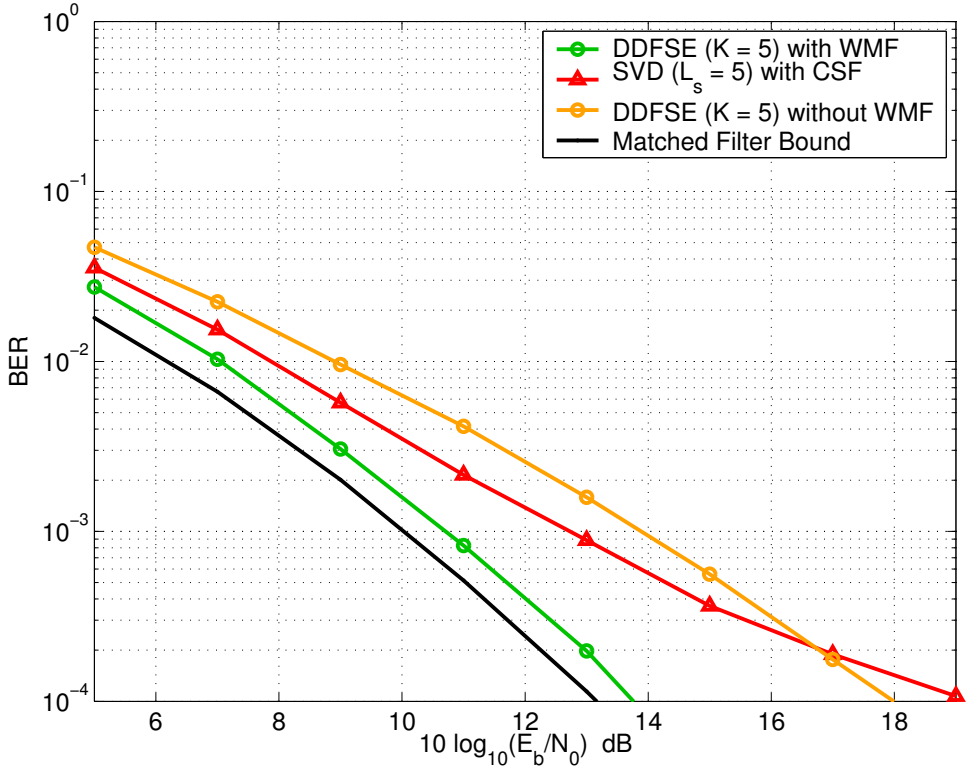
Memory length $L=6 \quad (K, L_s=5)$



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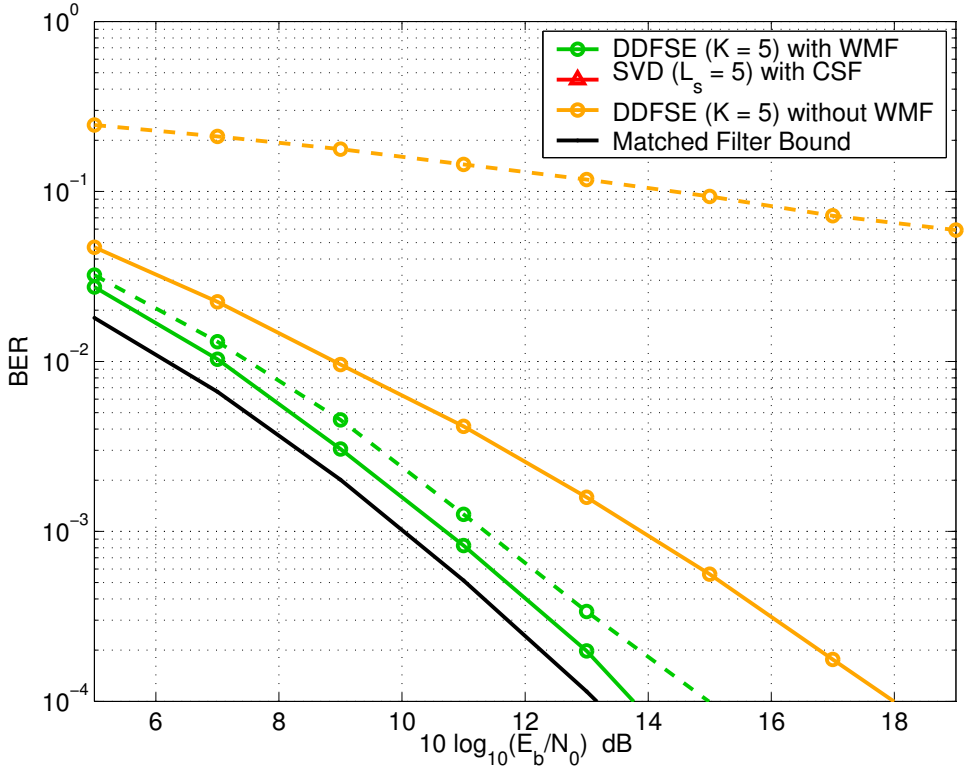
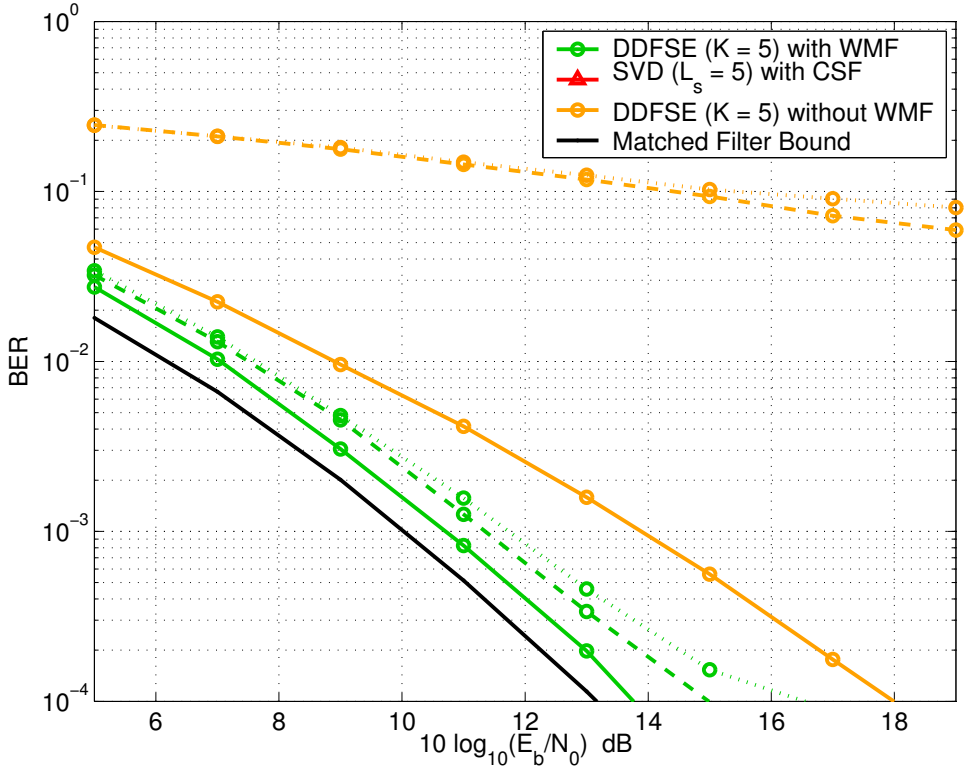
Memory length $L = 12 \quad (K, L_s = 5)$



# Fading CIR, Different Memory Lengths

Memory length $L = 20$ $(K, L_s = 5)$




DDFSE <u>with</u> WMF deviates **only** 1-2 dB from the MFB (at a BER of $10^{-3}$) even for large $L$; WMF makes **huge** difference

# Conclusions

- Efficient equalization of **sparse** ISI channels at reasonable complexity is **demanding task**
- Current **trellis-based** solutions require a certain CIR structure and do not seem practicable for **fading** channels
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  Generic receiver structure consisting of **linear filter** and standard **reduced-complexity** trellis-based equalizer

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$\Rightarrow$ **Alternative:** Tree-based equalizer (LISS alg.) + WMF