On the Duality of Wireless Systems with Multiple Cooperating Transmitters and Wireless Systems with Correlated Antennas

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Spatial Diversity in Wireless Systems

Wireless systems suffer from **fading effects** due to **multipath** signal propagation

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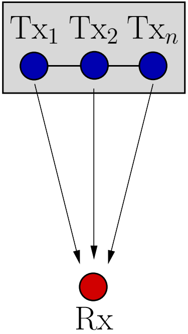
Example

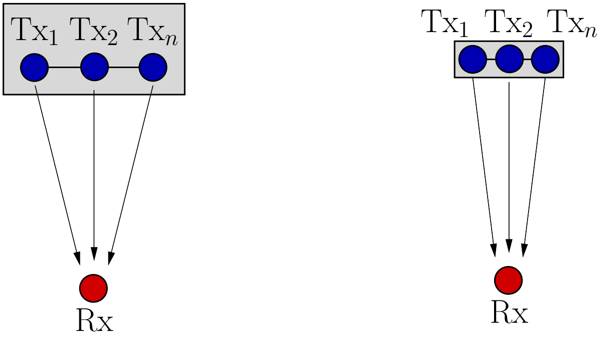
Multiple antennas in conjunction with **space-time coding** or **diversity reception** techniques ⇒ Spatial diversity

Conventionally: Co-located antennas with *sufficient* spacing
$\Rightarrow$ Statistically **independent** links



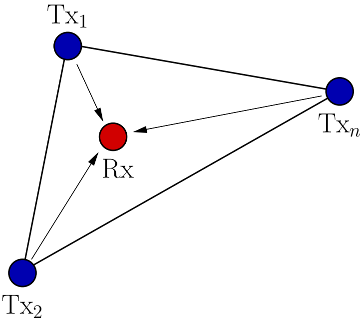
Systems with Multiple Transmitters

Compact system: Densely packed antennas (limited space)
$\Rightarrow$ **Correlated** links, **reduced** diversity gains



Distributed system:

Virtual multiple-antenna system (cooperating network nodes)



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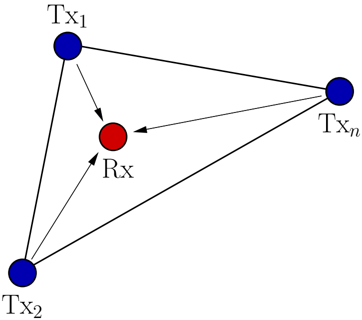

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- Simulcast networks (Broadcast/paging)

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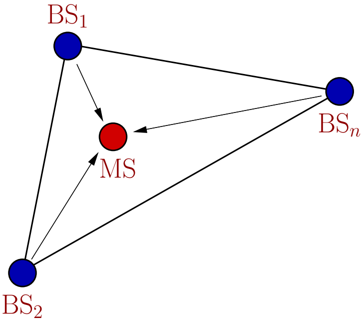

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Example: Simulcast network with cooperating base stations, *distributed* space-time code (e.g., OSTBC)

Distributed System

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⇒ Unequal average link SNRs: $\bar{\gamma}_i = \frac{\Omega_i}{n}\frac{E_\mathsf{s}}{N_0} \quad (i\!=\!1,\ldots,n)$

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We show

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We show

- Unequal SNRs cause **reduced** diversity gains, just as **correlated** links
- Any OSTBC system with **distributed** transmitters can be transformed into *equivalent* **correlated** OSTBC system

  Case of **equal** SNRs corresponds to **uncorrelated** system

Outline

- Introduction
- Error Performance of Distributed OSTBCs
  - Numerical Results
  - High-SNR Analysis
  - Simple Performance Measure
- Duality between Distributed and Correlated Systems
- Conclusions

Error Performance of Distributed OSTBCs

Assumptions:

- Distributed OSTBC, n cooperating transmitters (BSs)
- Single receiver (MS) with fixed position
- All network nodes have one antenna
- Quasi-static flat Rayleigh fading, binary transmission
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Performance analysis

$(n \times 1)$-OSTBC system $\Rightarrow$ Equivalent $(1 \times n)$-**MRC system**:

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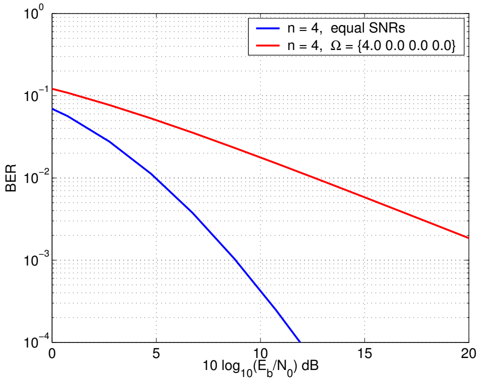
$\mathbf{y}[k]$: Received samples time index k, $\mathbf{h}$: Channel coefficients

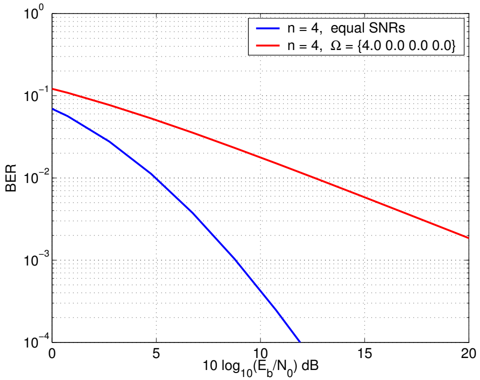
$a[k]$: kth info symbol (before OSTBC), $\mathbf{n}[k]$: AWGN samples

BER Curves



n = 4, equal SNRs
n = 4, Ω = {4.0 0.0 0.0 0.0}
BER
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
5
10
15
20
$10 \log_{10}(E_b/N_0)$ dB

BER Curves



High-SNR Approximation

Equal SNRs: $\bar{P}_\mathrm{b} \approx \left(\frac{n}{4\,E_\mathrm{s}/N_0}\right)^n \binom{2n-1}{n} \propto (E_\mathrm{s}/N_0)^{-n}$

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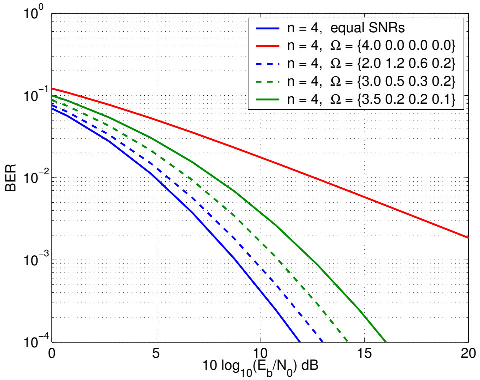

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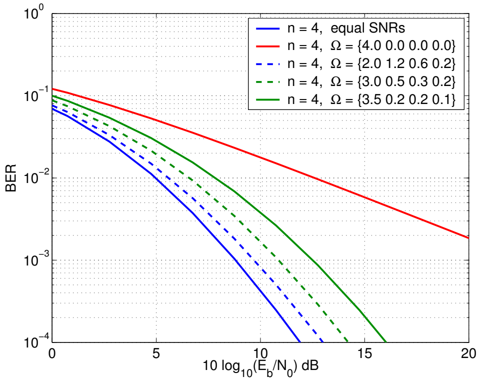

Product term always

$$\prod_{i=1}^{n} \frac{1}{\Omega_i} \geq 1$$

i.e., unequal SNRs cause **up-shift** of the BER curves

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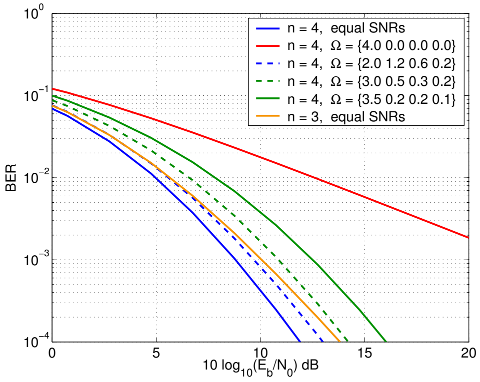

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Simple Performance Measure

Goal: Classify distributed OSTBC systems w.r.t. performance

$\Rightarrow$ Performance measure $\Delta \in [0, 1]$ (SNR unbalance)

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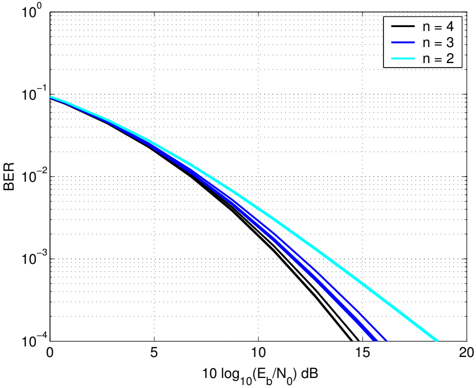
Example:

Different systems with $\Delta \approx 0.7$ and 2, 3, 4 transmitters



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Sketch of Proof

- Via equivalent $(1 \times n)$-MRC system $\mathbf{y}[k] = \mathbf{h}\, a[k] + \mathbf{n}[k]$
- **Unitary transform** does not change statistical properties:

$$\mathbf{y}'[k] := \mathbf{U}\mathbf{y}[k] = \mathbf{U}\mathbf{h}\, a[k] + \mathbf{U}\mathbf{n}[k] =: \mathbf{h}'\, a[k] + \mathbf{n}'[k],$$

$\mathbf{U}$ any unitary matrix $\Rightarrow$ **Equivalent** MRC system

– Spatial correlation properties of **distributed** system:

$$\mathsf{E}\left\{\mathbf{h}\mathbf{h}^{\mathsf{H}}\right\} = \mathsf{diag}\left(\Omega_1, \dots, \Omega_n\right) =: \boldsymbol{\Omega}$$

Spatial correlation properties of **transformed** system:

$$\mathsf{E}\left\{\mathbf{h}'\mathbf{h}'^{\mathsf{H}}\right\} = \mathsf{E}\left\{\mathbf{U}\mathbf{h}\,\mathbf{h}^{\mathsf{H}}\mathbf{U}^{\mathsf{H}}\right\} = \mathbf{U}\boldsymbol{\Omega}\mathbf{U}^{\mathsf{H}}$$

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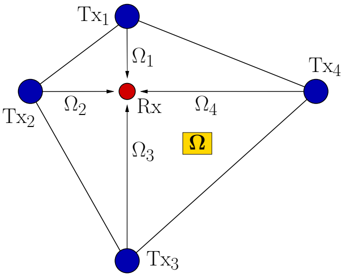
Special cases

$$\mathbf{\Omega} = \mathbf{I}_n \quad \mapsto \quad \mathbf{R} = \mathbf{I}_n \ \text{(uncorrelated)}$$

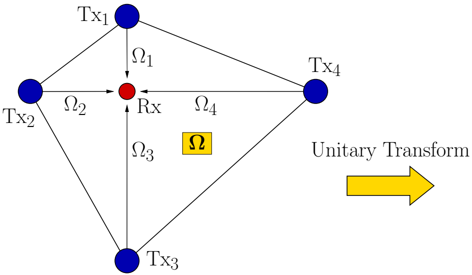
$$\mathbf{\Omega} = \mathrm{diag}\left(n, 0, \ldots, 0\right) \quad \mapsto \quad \mathbf{R} \ \text{with} \ |r_{ij}| = 1$$



Distributed system:

$$\underbrace{\begin{bmatrix} 2.5 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix}}_{\boldsymbol{\Omega}}$$

Numerical Example



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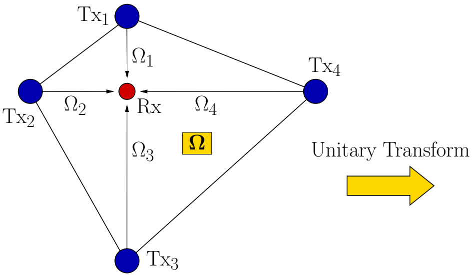

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$$\underbrace{\begin{bmatrix} 1 & 0.4 & 0.75 & 0.35 \\ 0.4 & 1 & 0.35 & 0.75 \\ 0.75 & 0.35 & 1 & 0.4 \\ 0.35 & 0.75 & 0.4 & 1 \end{bmatrix}}_{\mathcal{H}_4 \mathbf{\Omega} \mathcal{H}_4^{\mathsf{H}} = \mathbf{R}}$$

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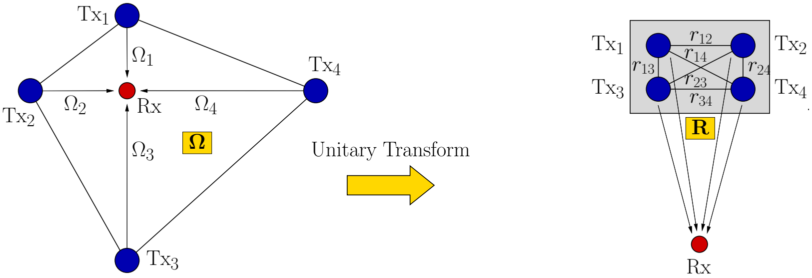

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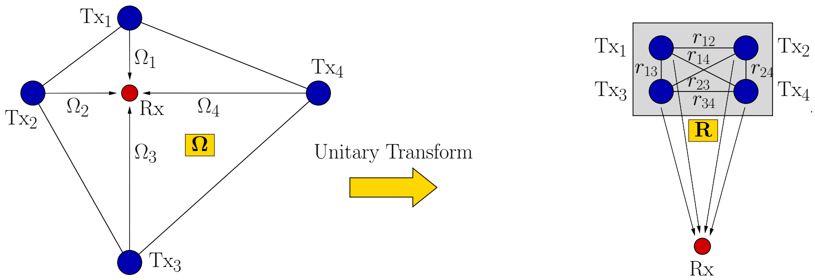

Application Example

Reuse transmit power allocation schemes originally developed for spatially correlated systems

Conclusions

- **Cooperative** networks (e.g., simulcast or relay networks)
  $\Rightarrow$ Virtual antenna arrays
- Performance analysis of **distributed** space-time codes
  $\Rightarrow$ **Unequal** link SNRs can cause significant performance loss
- Simple **performance measure** to classify distributed OSTBC systems
- **Duality** of distributed OSTBC systems and spatially correlated OSTBC systems (unitary matrix transform)