Trellis-Based Equalization for Sparse ISI Channels Revisited

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$$\mathbf{h} := [\, h_0 \; \overbrace{\underbrace{0 \ldots 0}_{f_0 \text{ zeros}} \; h_1 \; \underbrace{0 \ldots 0}_{f_1 \text{ zeros}} \; h_2 \; \ldots \; h_{G-1} \; \underbrace{0 \ldots 0}_{f_{G-1} \text{ zeros}} \; h_G}^{L} \,]^{\mathsf{T}}$$

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Special case: **Zero-pad channel**

$$f_0 = f_1 = \ldots = f_{G-1} =: f \geq 1$$

Equalization for Sparse ISI Channels

Discrete-time channel model

$$y[k] \;=\; h_0\, x[k] \;+\; \sum_{g=1}^{G} h_g\, x[k - d_g] \;+\; n[k]$$

$y[k]$: kth received sample

$x[k]$: kth transmitted data symbol

$n[k]$: kth AWGN sample

d_g: Position of h_g within $\mathbf{h}$

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Due to large channel memory length, **efficient** equalization with reasonable **complexity** is a demanding task

Here: **Trellis-based equalization** (based on VA or BCJRA)

MLSE **prohibitive** $\Rightarrow$ M^L trellis states (M-ary data symbols)

Existing Solutions

Exploiting the sparse channel structure, **reduced-complexity** algorithms can be derived

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Zero-pad channel:

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General sparse channel:

- Benvenuto/Marchesani'96: Multi-trellis VA (M-VA)

$\Rightarrow$ Based on parallel **irregular** (i.e., time-varying) trellises

Outline

- Introduction
- Complexity Reduction Without Loss of Optimality
  - Unified Framework Based on Factor Graphs
  - Recapitulation of the P-VA and the M-VA
  - Drawbacks of the Existing Solutions
- Simple Equalization Scheme for General Sparse ISI Channels
- Conclusions

Framework for Complexity Reduction

Decomposition into multiple parallel trellises

Key question:

Which **symbol decisions** $\hat{x}[k]$, $1 \leq k \leq K_\mathsf{B}$ (K_B block length) are influenced by a certain **symbol hypothesis** $\tilde{x}[k_0]$?

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$\Rightarrow$ $\tilde{x}[k_0]$ and $\tilde{x}[k_1]$ can be accommodated in **separate** trellises **without loss** of optimality

Example 1: $\mathbf{h} := [\, h_0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; h_1 \;\; 0 \;\; h_2 \,]^{\mathsf{T}} \quad (L{=}8,\ G{=}2)$

$\tilde{x}[k_0]$

$x[k_0]$ $x[k_0{+}1]$ $x[k_0{+}2]$ $+3$ $+4$ $+5$ $+6$ $+7$ $+8$ $+9$ $+10$ $+11$ $+12$ $+13$ $+14$ $+15$ $x[k_0{+}16]$

$y[k_0]$ $y[k_0{+}1]$ $y[k_0{+}2]$ $+3$ $+4$ $+5$ $+6$ $+7$ $+8$ $+9$ $+10$ $+11$ $+12$ $+13$ $+14$ $+15$ $y[k_0{+}16]$

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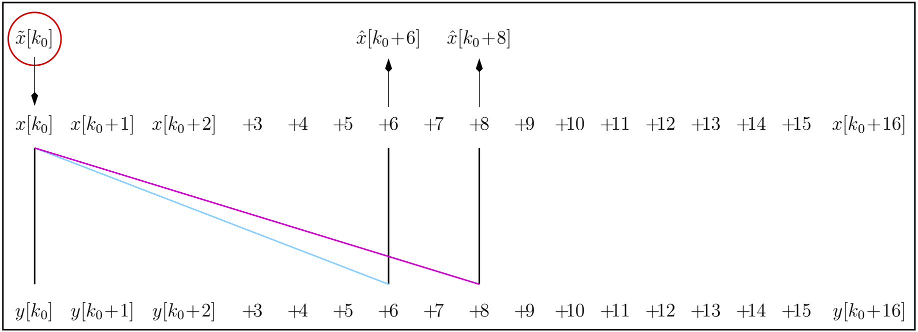

$$
\begin{aligned}
y[k_0] &= h_0\, x[k_0] + h_1\, x[k_0-6] + h_2\, x[k_0-8] \\
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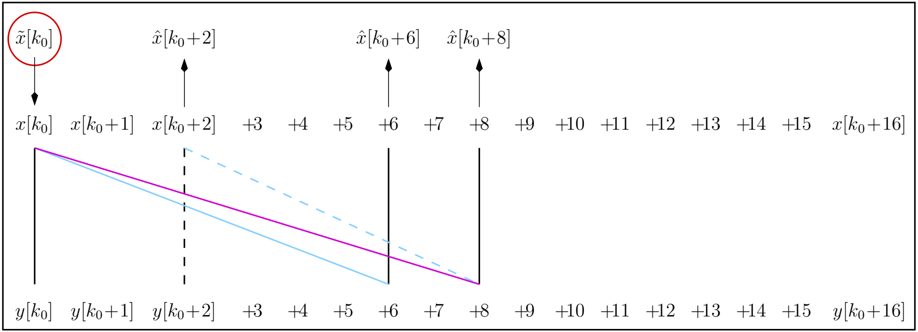

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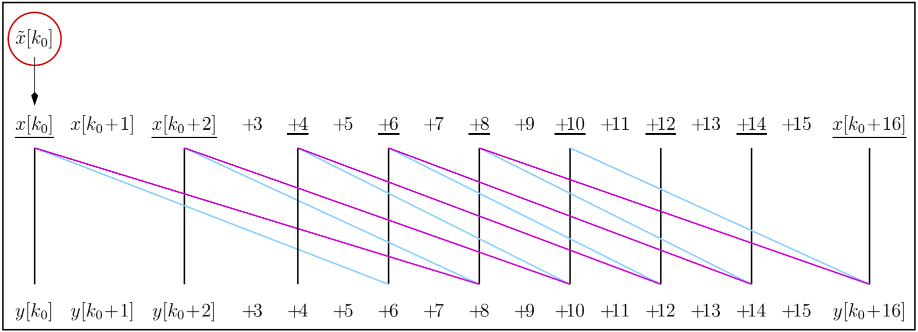

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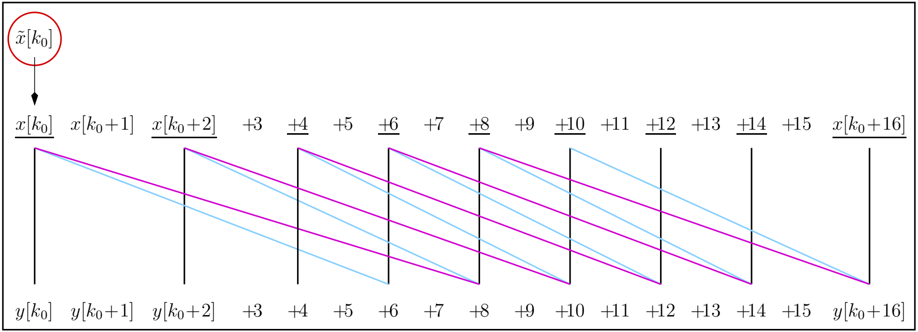

$\Rightarrow$ Two parallel (regular) trellises are still optimal!

$\Rightarrow$ Parallel-trellis VA/BCJRA

Framework for Complexity Reduction

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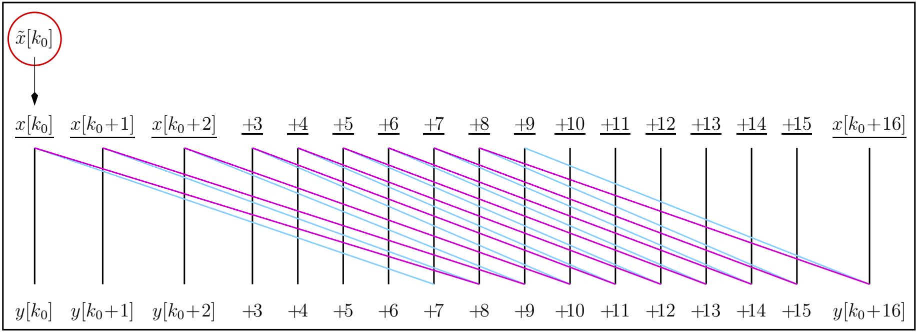

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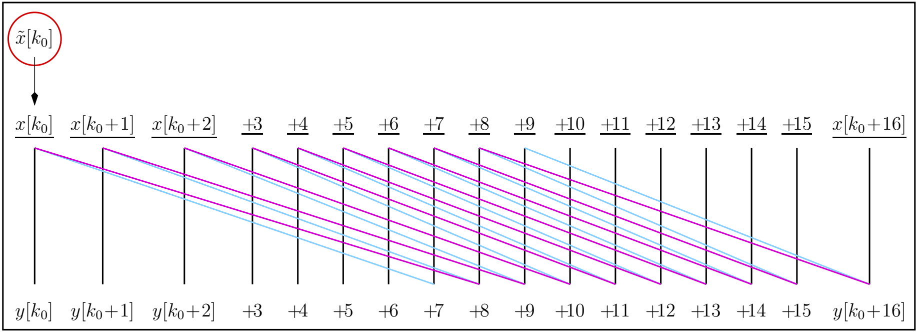

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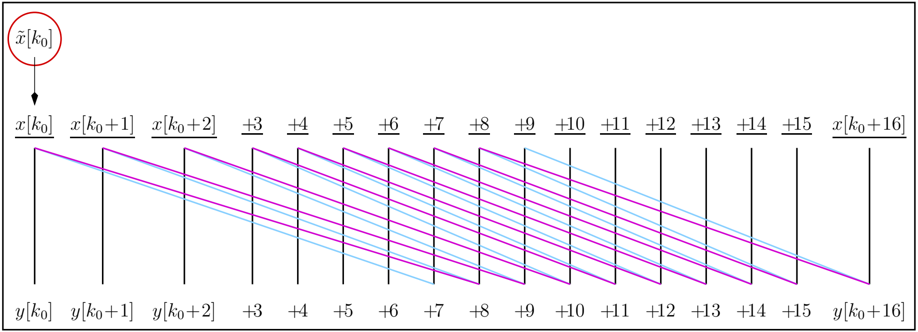

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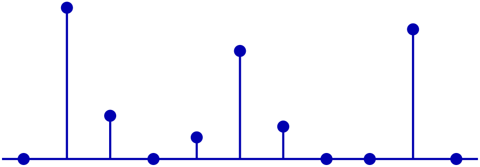
Multi-trellis VA neglects most of the dependencies $\Rightarrow$ suboptimal!

sub-P-VA, sub-P-BCJRA

- Alternative solution for **general sparse channels**:

  Suboptimal parallel-trellis VA/BCJRA
  (McGinty/Kennedy/Hoeher'98, Lee/McLane'02)



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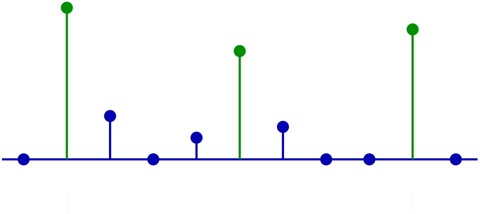

(a) Find an underlying zero-pad CIR similar to the given CIR

sub-P-VA, sub-P-BCJRA

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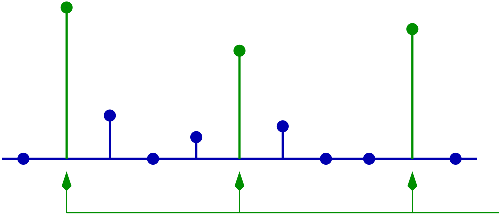

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(b) Define the parallel trellis diagrams

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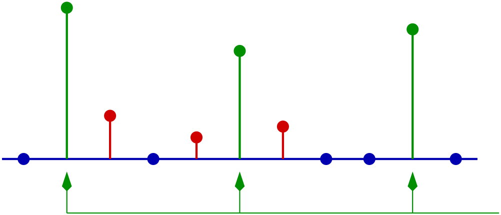
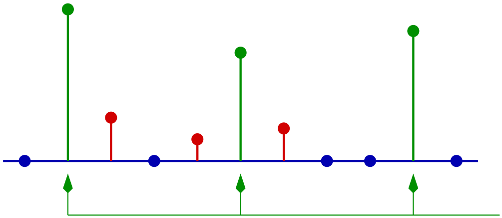


(a) Find an underlying zero-pad CIR similar to the given CIR

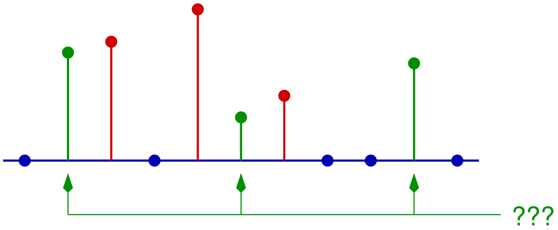
(b) Define the parallel trellis diagrams

(c) Perform decision feedback between the parallel trellises



(a) Fading channel

Drawbacks



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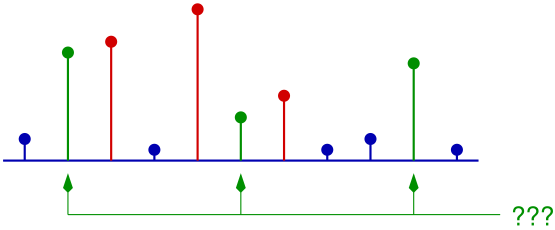

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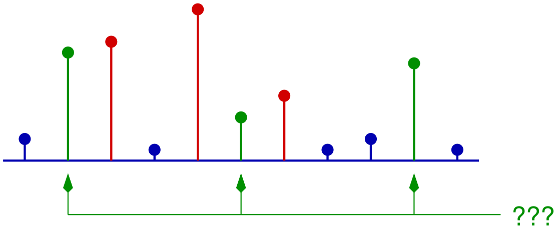

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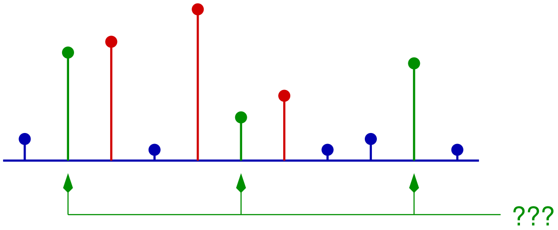
Our approach

It does not seem useful to **explicitly** utilize the **sparse** channel structure

$\Rightarrow$ How good are **standard** suboptimal equalization techniques?

Drawbacks



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Our approach

Use **prefiltering** in conjunction with **standard** reduced-complexity **trellis-based equalizer**

$\Rightarrow$ Tackle **general** sparse fading CIRs & provide performance **close** to the matched filter bound (MFB)

Outline

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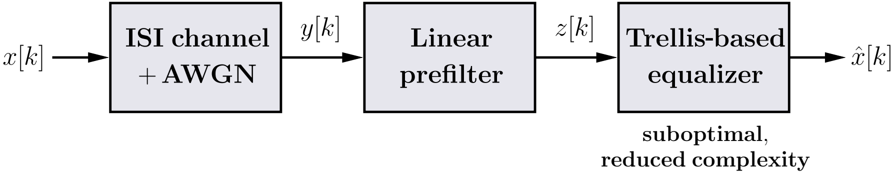
- Complexity Reduction Without Loss of Optimality
- Simple Equalization Scheme for General Sparse ISI Channels
  - Considered Receiver Structure
  - Numerical Results
- Conclusions

Considered Receiver Structure



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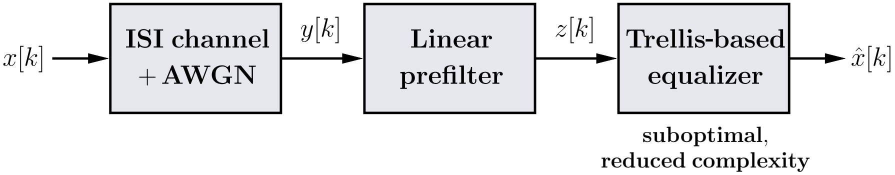

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$\Rightarrow$ Solely the linear prefilter is adjusted to the current CIR

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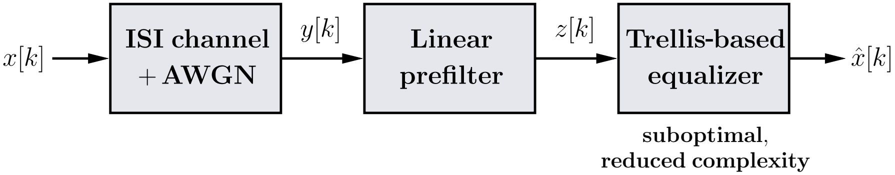

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Example: Minimum-phase filter (WMF) in conjunction with delayed decision-feedback sequence estimator (DDFSE)

Comparison with sub-P-BCJRA

Static CIR $\mathbf{h} = [h_0 \; 0 \; 0 \; 0 \; h_4 \; 0 \; 0 \; h_7 \; 0 \ldots 0 \; h_{15}]^{\mathsf{T}}$ (no zero-pad)

$h_0 = 0.87, \; h_4 = h_7 = h_{15} = 0.29$

Binary transmission; WMF with $L_{\mathsf{F}} = 40$ filter taps;

DDFSE with memory length $K \ll L$

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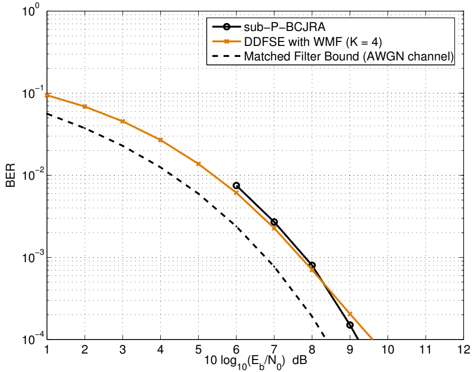

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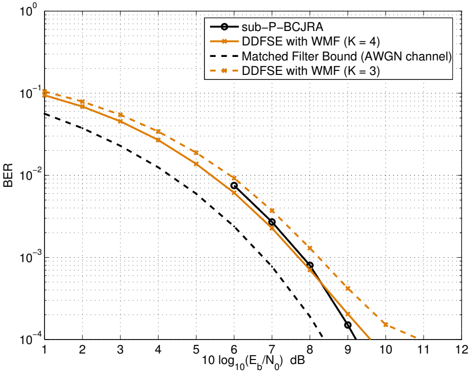

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DDFSE $(K = 3)$ + WMF: **Reduced** complexity at expense of **small** loss

Fading CIR, Different Memory Lengths

Fading CIR with $h_g \sim \mathcal{CN}(0, \sigma^2_{\mathsf{h},g})$ and power profile

$$\mathbf{p} := [\,\sigma^2_{\mathsf{h},0}\ \underbrace{0 \ldots 0}_{f \text{ zeros}}\ \sigma^2_{\mathsf{h},1}\ \ 0\ \ 0\ \ 0\ \ \sigma^2_{\mathsf{h},2}\ \sigma^2_{\mathsf{h},3}\,]^{\mathsf{T}}, \ \ \sigma^2_{\mathsf{h},g} = 0.25$$

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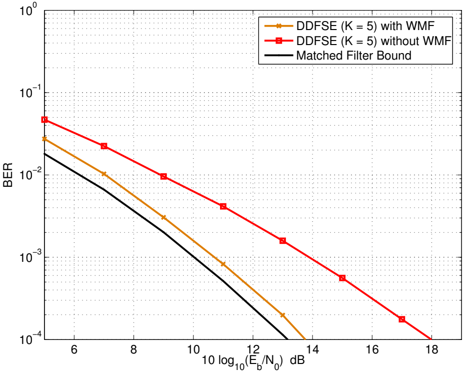
Memory length $L = 6$ $(K = 5)$



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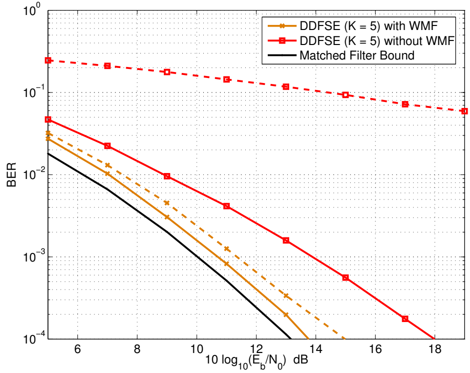
Memory length $L = 12$ $(K = 5)$



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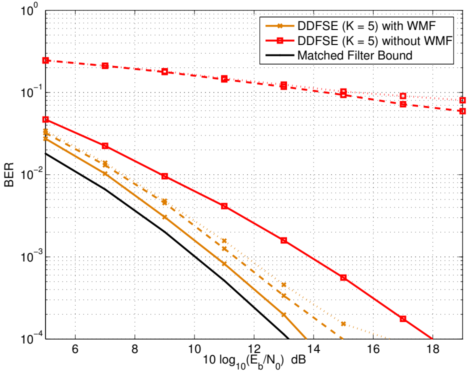
Memory length $L = 20$ $(K = 5)$



Fading CIR with $h_g \sim \mathcal{CN}(0, \sigma^2_{\mathrm{h},g})$ and power profile

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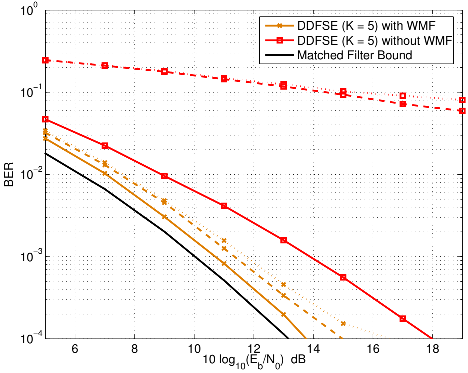

DDFSE with WMF deviates **only** 1-2 dB from the MFB (at BER 10^{-3}) even for a large memory length L

WMF makes **huge** difference

Conclusions

- Efficient equalization of **sparse** ISI channels at reasonable complexity is a **demanding task**
- **Optimal** trellis-based solutions are only applicable for zero-pad channels (factor graph)
- Existing **suboptimal** solutions explicitly exploit the sparse channel structure and seem impracticable for **fading** channels
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- **Our approach:** Use linear prefilter in conjunction with standard reduced-complexity trellis-based equalizer

$\Rightarrow$ **General** sparse ISI channels can be tackled

$\Rightarrow$ Only the linear prefilter is adjusted to the **current CIR**

$\Rightarrow$ Performance **close** to the MFB