Analysis of the Expected Error Performance of Cooperative Wireless Networks Employing Distributed Space-Time Codes

Jan Mietzner[1], Ragnar Thobaben[2], and Peter A. Hoeher[1]

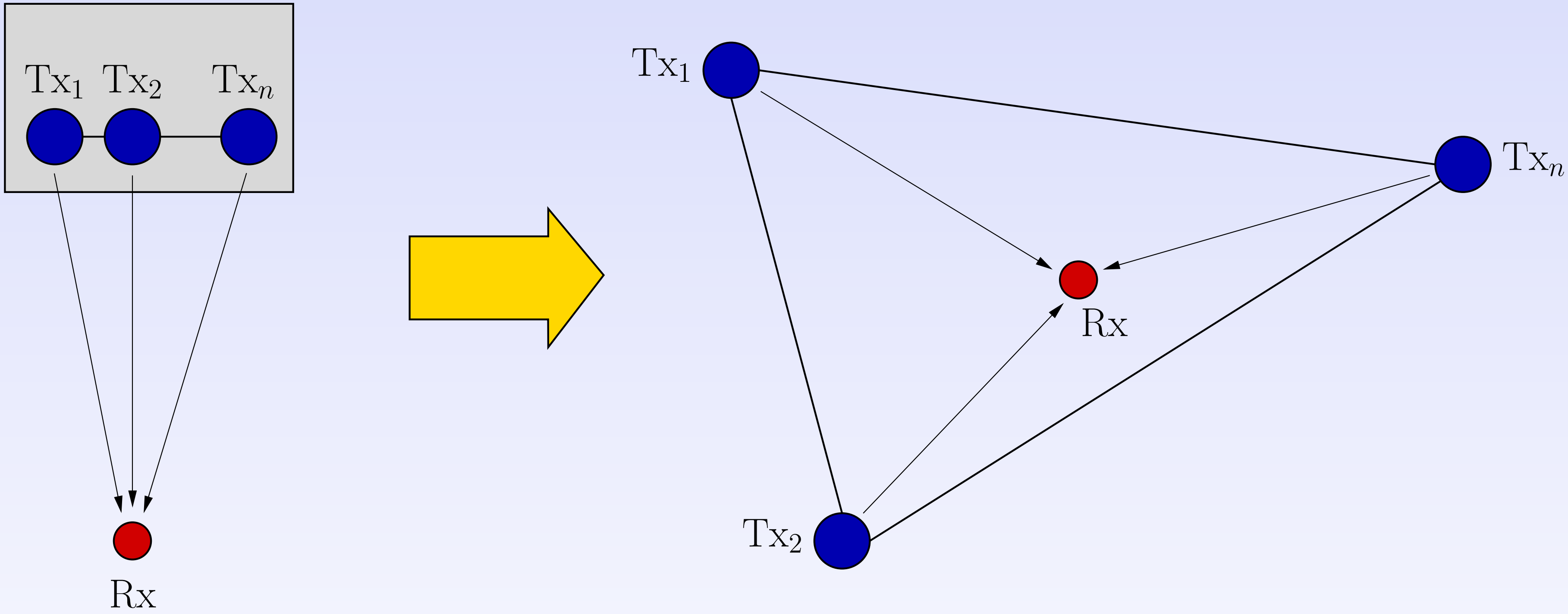
University of Kiel, Germany

[1]Information and Coding Theory Lab (ICT)
[2]Institute for Circuits and System Theory (LNS)

{jm,rat,ph}@tf.uni-kiel.de
http://www-ict.tf.uni-kiel.de

Globecom 2004, Dallas, Texas, USA
November 30, 2004

From Co-located to Distributed Transmitters



Motivation for Distributed Space-Time Codes

- **Benefits** of **multiple antennas** for wireless communication systems:
  - **Performance** of wireless systems often limited by **fading** due to **multipath** signal propagation
  - **System performance** significantly **improved** by exploiting **diversity**

$\Longrightarrow$ Employ **Space-time codes (STCs)** to exploit **spatial diversity**

Motivation for Distributed Space-Time Codes

- **Benefits** of **multiple antennas** for wireless communication systems:
  - **Performance** of wireless systems often limited by **fading** due to **multipath** signal propagation
  - **System performance** significantly **improved** by exploiting **diversity**

$\Longrightarrow$ Employ **Space-time codes (STCs)** to exploit **spatial diversity**

- Concept of multiple antennas can be transferred to **cooperative wireless networks**:
  - **Multiple** (single-antenna) nodes cooperate and perform a **joint transmission strategy**

$\Longrightarrow$ Nodes share their antennas using a **distributed** space-time code

Cooperative Wireless Networks – Examples

- **Simulcast networks** for broadcasting or paging applications:

  Conventionally, all nodes **simultaneously** transmit the **same signal** using the **same carrier frequency** $\Longrightarrow$ Reduced probability of shadowing

Cooperative Wireless Networks – Examples

- **Simulcast networks** for broadcasting or paging applications:

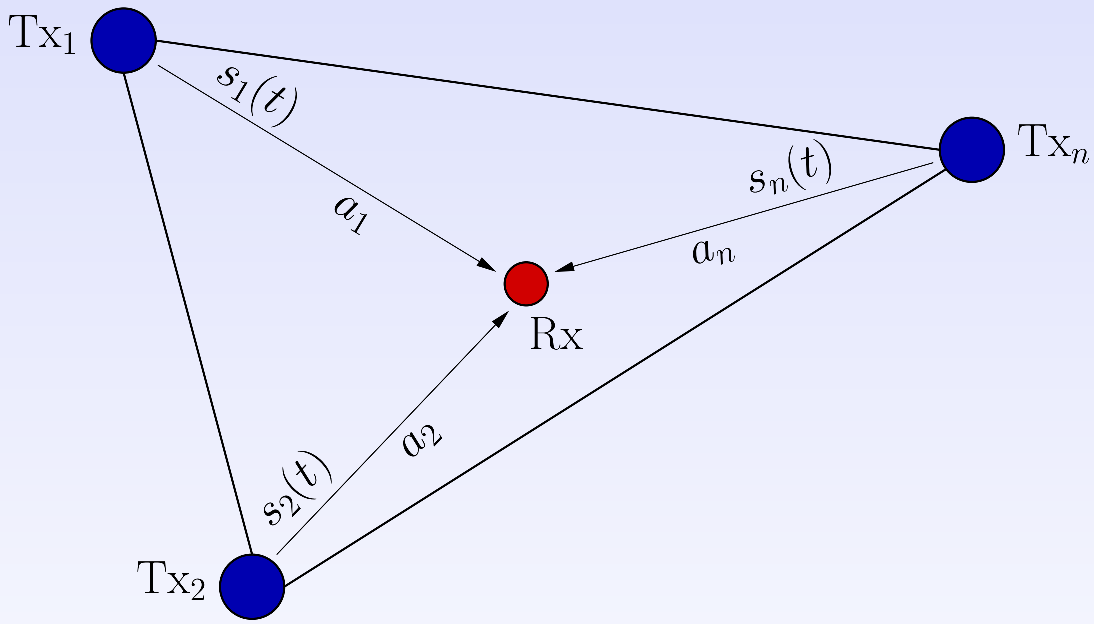
  Conventionally, all nodes **simultaneously** transmit the **same signal** using the **same carrier frequency** $\Longrightarrow$ Reduced probability of shadowing

- **Relay-assisted communication**, e.g., in cellular systems, ad-hoc networks, sensor networks:
  - **Relay nodes** receive signal from a **source node** and forwarded it to a **destination node**
  - **Fixed stations** or other **mobile stations** ('user cooperation diversity')

$\Longrightarrow$ **Distributed STCs** are **suitable** for both types of networks

Cooperative Wireless Networks – General Setting

- n **transmitting** nodes (Tx_1,...,Tx_n), one **receiving** node (Rx); single-antenna nodes



Differences between Co-located and Distributed Transmitters

- **Distributed STCs:**
  - No shadowing: Diversity degree n $\checkmark$
  - **Additionally:** Diversity degree $(n-\nu)$ if **any** subset of ν Tx nodes obstructed $(\checkmark)$
- Higher probability of **line-of-sight (LOS)** component

Differences between Co-located and Distributed Transmitters

- **Distributed STCs:**
  - No shadowing: Diversity degree n ✓
  - **Additionally:** Diversity degree $(n-\nu)$ if **any** subset of ν Tx nodes obstructed (✓)

- Higher probability of **line-of-sight (LOS)** component

- Transmitted signals $s_i(t)$ subject to **different** average **link gains** a_i, due to different distances or shadowing $\Longrightarrow$ **Reduced** degree of diversity

Here: Focus on average link gains a_i and associated diversity loss

Outline

- **Error Performance of Distributed STCs**
  - Basic Assumptions
  - Analytical Results
- **Average Error Performance in a General Uplink Scenario**
- **Average Error Performance in an Uplink Scenario with Additional Constraint**
- **Conclusions**

Basic Assumptions

- Transmitting nodes Tx_1,...,Tx_n perfectly **synchronized** in time and frequency
- All nodes employ a **single antenna**

Basic Assumptions

- Transmitting nodes $\mathrm{Tx}_1,\ldots,\mathrm{Tx}_n$ perfectly **synchronized** in time and frequency
- All nodes employ a **single antenna**
- Frequency-flat **block-fading** channel model (Rayleigh):

Channel coefficients $\boxed{h_i \sim \mathcal{CN}(0, a_i),\ i = 1, \ldots, n}$ **Normalization:** $\boxed{\sum_i a_i := n}$

Basic Assumptions

- Transmitting nodes $\mathrm{Tx}_1,\ldots,\mathrm{Tx}_n$ perfectly **synchronized** in time and frequency
- All nodes employ a **single antenna**
- Frequency-flat **block-fading** channel model (Rayleigh):

  Channel coefficients $h_i \sim \mathcal{CN}(0, a_i), \; i = 1, \ldots, n$ **Normalization:** $\sum_i a_i \; := \; n$
- Same average **transmitter power** P/n for all transmitting nodes Tx_i; no shadowing
- Congenerous antennas at **Tx nodes**, omnidirectional antenna at **Rx node**

Basic Assumptions

- Transmitting nodes $\mathrm{Tx}_1,\ldots,\mathrm{Tx}_n$ perfectly **synchronized** in time and frequency
- All nodes employ a **single antenna**
- Frequency-flat **block-fading** channel model (Rayleigh):

Channel coefficients $h_i \sim \mathcal{CN}(0, a_i),\ i = 1, \ldots, n$ **Normalization:** $\sum_i a_i := n$

- Same average **transmitter power** P/n for all transmitting nodes Tx_i; no shadowing
- Congenerous antennas at **Tx nodes**, omnidirectional antenna at **Rx node**

$$\Longrightarrow \qquad \frac{a_j}{a_i} = \left(\frac{d_i}{d_j}\right)^{\rho} \qquad \text{(according to Friis formula)}$$

d_i: Length of transmission link i, ρ: Path-loss exponent $(2 \le \rho \le 4)$

Analytical Results for the Error Performance

- Average **signal-to-noise ratio (SNR)** for transmission link i: $a_i E_\mathrm{s}/nN_0$
  (E_s/N_0: Overall received SNR)

- In the sequel, **Alamouti's Tx diversity** scheme $(n = 2)$ and **binary transmission**

Analytical Results for the Error Performance

- Average **signal-to-noise ratio (SNR)** for transmission link i: $a_i E_\mathrm{s}/nN_0$
  (E_s/N_0: Overall received SNR)

- In the sequel, **Alamouti's Tx diversity** scheme $(n=2)$ and **binary transmission**

- Using Proakis' theoretical results for **diversity reception**, one obtains the **bit error rate (BER):**

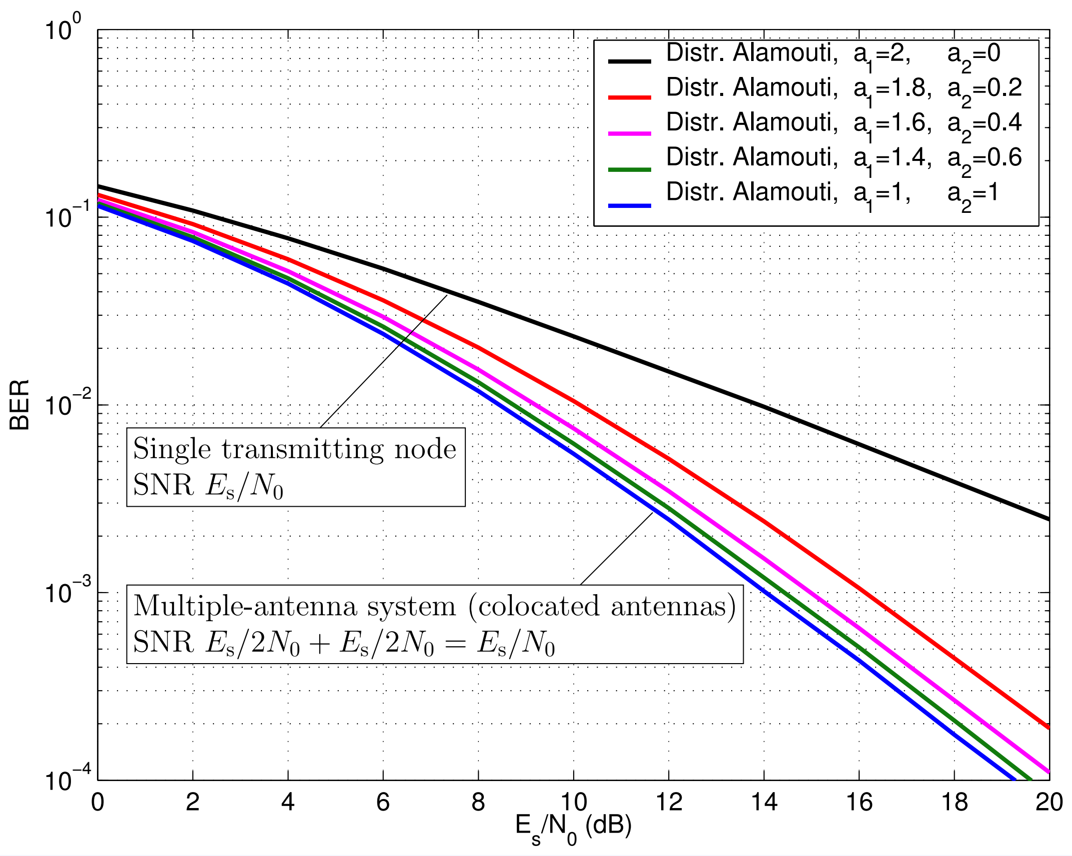
$$P_\mathrm{b}(a_1) = \frac{1}{2}\left[\frac{a_1\,(1-\mu(a_1))}{a_1-a_2} + \frac{a_2\,(1-\mu(a_2))}{a_2-a_1}\right],$$

where $a_1 \in [0,2], \quad a_2 = 2 - a_1$ and $\mu(a_i) = \dfrac{1}{\sqrt{1+\frac{2N_0}{a_i E_\mathrm{s}}}} \qquad (i=1,2)$

Specifically, $P_\mathrm{b}(a_1) = P_\mathrm{b}(2-a_1)$ holds for all a_1

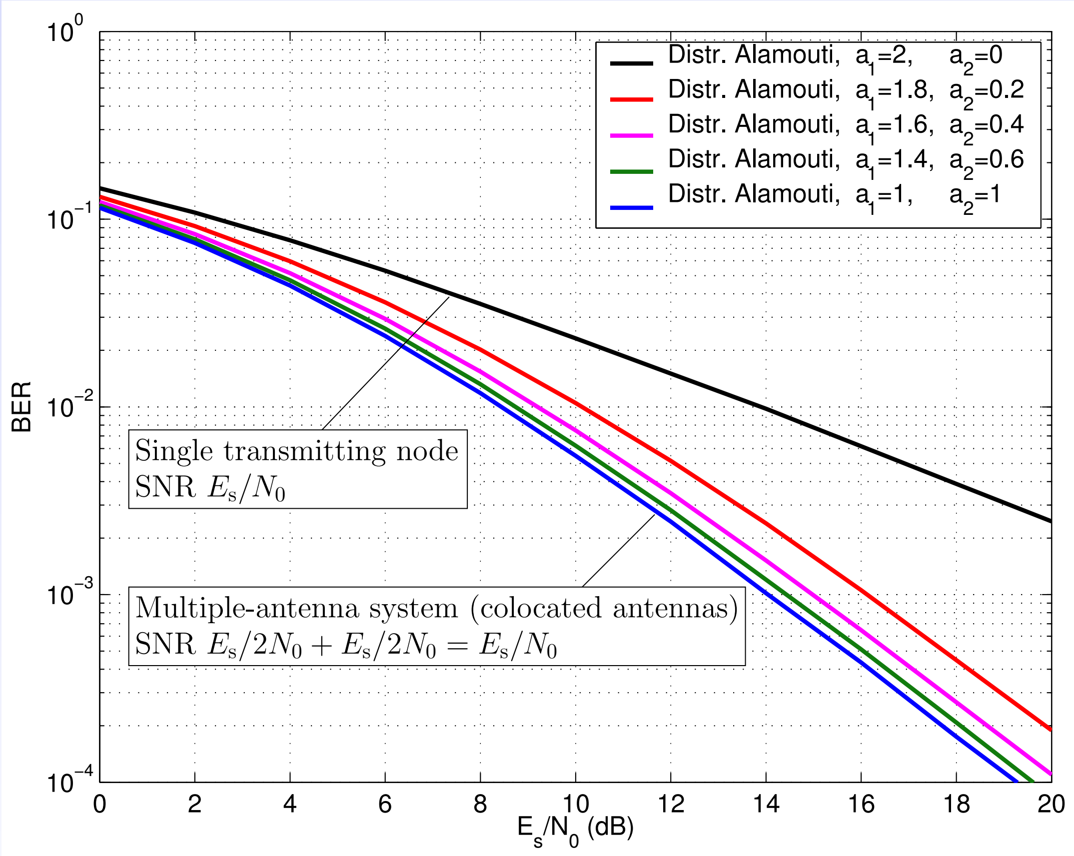
Analytical Results for the Error Performance

$P_{\mathrm{b}}(a_1)$ vs. E_{s}/N_0 in dB



Analytical Results for the Error Performance

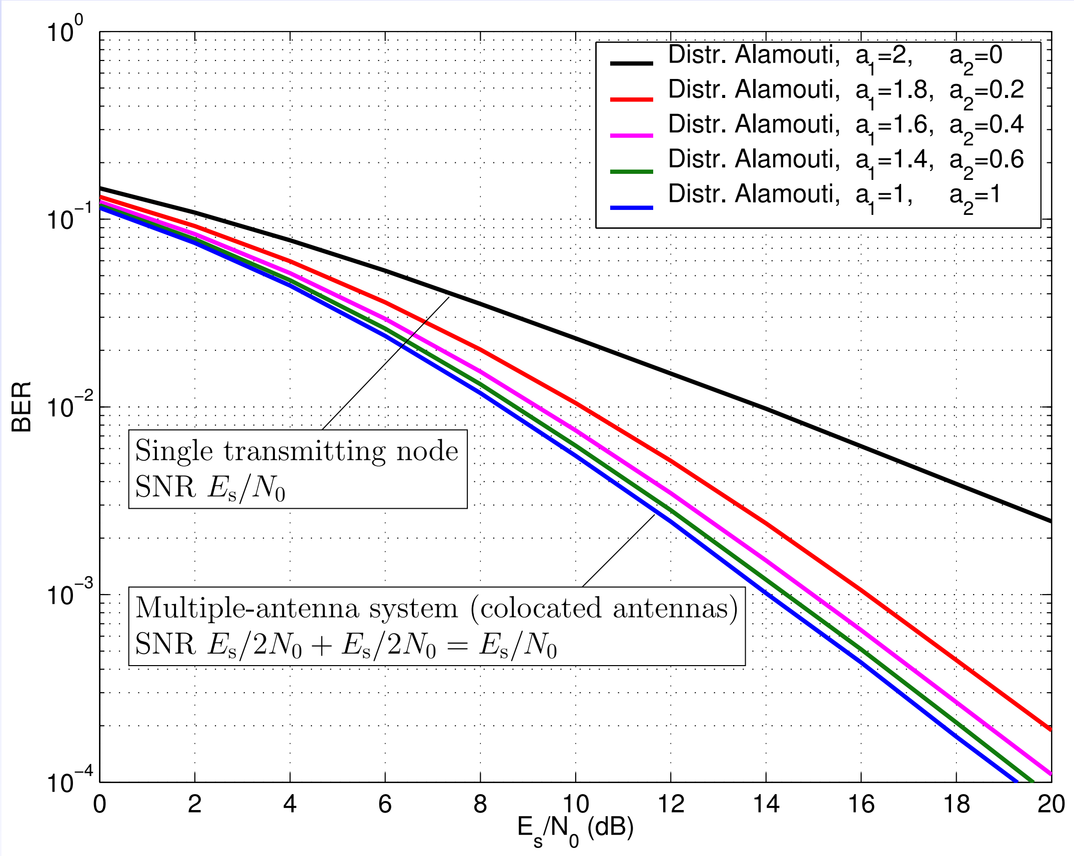
$P_{\mathrm{b}}(a_1)$ vs. E_{s}/N_0 in dB



- Best performance for $a_1 = a_2 = 1$ (diversity degree of two)
- Worst performance for $a_1 = 2$ and $a_2 = 0$ (diversity degree of one)
- Even for large a_1, significant gains w.r.t. single transmission node (diversity degree still close to two)

Analytical Results for the Error Performance

$P_b(a_1)$ vs. E_s/N_0 in dB



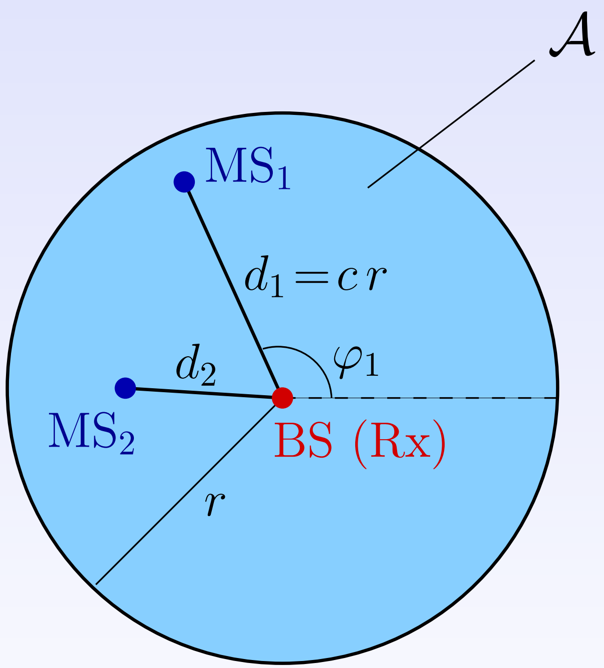
- Best performance for $a_1 = a_2 = 1$ (diversity degree of two)
- Worst performance for $a_1 = 2$ and $a_2 = 0$ (diversity degree of one)
- Even for large a_1, significant gains w.r.t. single transmission node (diversity degree still close to two)
- Results hold approximately also, e.g., for TR-STBCs and delay diversity
- Generalizations are possible:
  - $n > 2$ Tx nodes (e.g., OSTBCs)
  - Rice fading, shadowing

Outline

- **Error Performance of Distributed STCs**
- **Average Error Performance in a General Uplink Scenario**
  - General Uplink Scenario
  - Derivation of the Mean Bit Error Rate
- **Average Error Performance in an Uplink Scenario with Additional Constraint**
- **Conclusions**

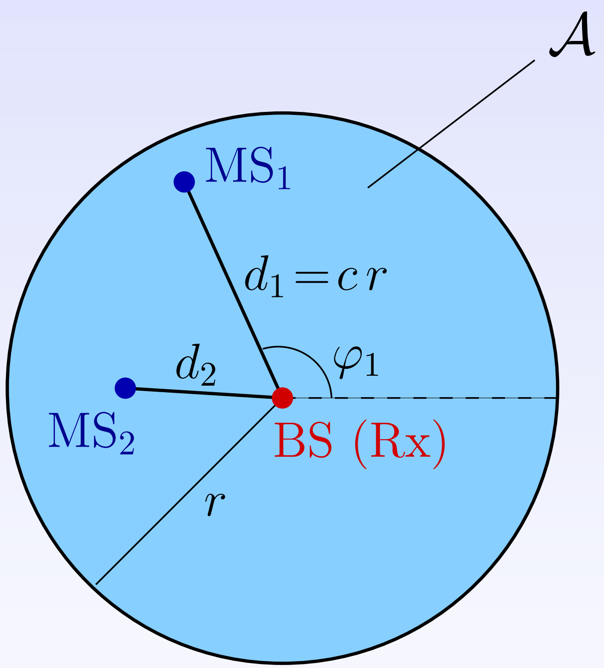
General Uplink Scenario

- **Assumptions:**
  - $n = 2$ **Tx nodes** $(\mathrm{MS}_1,\ \mathrm{MS}_2)$, one **Rx node** (BS), **distributed** Alamouti scheme $(\mathrm{MS}_1$ and MS_2 may be **mobile relays**)
  - **Coverage area** $\mathcal{A}$ of BS is a disk of radius r



General Uplink Scenario

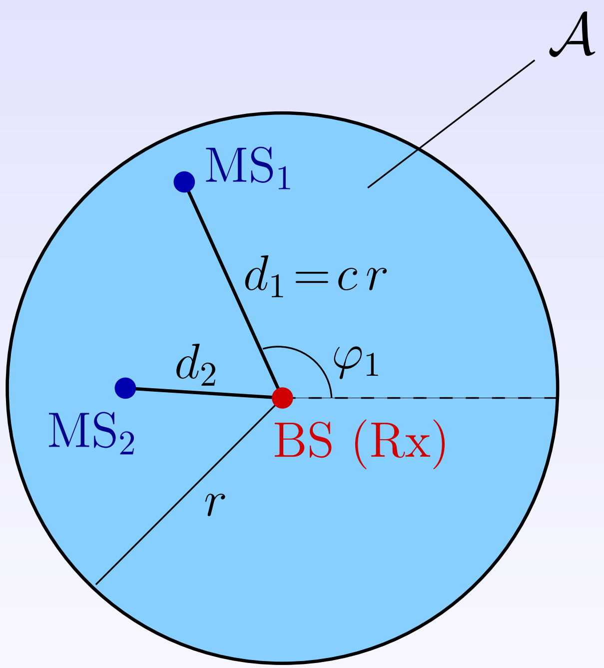
- **Assumptions:**
  - $n = 2$ **Tx nodes** (MS_1, MS_2), one **Rx node** (BS), **distributed** Alamouti scheme (MS_1 and MS_2 may be **mobile relays**)
  - **Coverage area** $\mathcal{A}$ of BS is a disk of radius r



- For MS_1 a fixed distance d_1 to BS is assumed where $\boxed{d_1 := c\,r, \quad c \leq 1}$ (angle φ_1 arbitrary)
- MS_2 is located anywhere within $\mathcal{A}$, according to a **uniform distribution**

General Uplink Scenario

- **Assumptions:**
  - $n = 2$ **Tx nodes** (MS_1, MS_2), one **Rx node** (BS), **distributed** Alamouti scheme (MS_1 and MS_2 may be **mobile relays**)
  - **Coverage area** $\mathcal{A}$ of BS is a disk of radius r



  - For MS_1 a fixed distance d_1 to BS is assumed where $\boxed{d_1 := c\,r, \quad c \le 1}$ (angle φ_1 arbitrary)
  - MS_2 is located anywhere within $\mathcal{A}$, according to a **uniform distribution**

- The **mean BER** can be calculated as

$$\boxed{\bar{P}_\mathrm{b} = \int_0^2 p_{A_1}(a_1)\, P_\mathrm{b}(a_1)\, \mathrm{d}a_1}$$

$\Longrightarrow p_{A_1}(a_1)$ required

Derivation of the Mean Bit Error Rate

- Let $q := d_2/d_1$ (corresponding random variable Q)
- Since d_1 is fixed, the pdf of Q is given by

$$p_Q(q) = d_1 \cdot p_{D_2}(d_1 q) = c\,r \cdot \frac{\partial}{\partial d_2} P(D_2 \leq d_2) \Big|_{d_2 = c\,r\,q}$$

Derivation of the Mean Bit Error Rate

- Let $q := d_2/d_1$ (corresponding random variable Q)
- Since d_1 is fixed, the pdf of Q is given by

$$p_Q(q) = d_1 \cdot p_{D_2}(d_1 q) = c\,r \cdot \frac{\partial}{\partial d_2} P(D_2 \leq d_2)\Big|_{d_2 = c\,r\,q}$$

- With $P(D_2 \leq d_2) = \pi d_2^2 / \pi r^2$ one obtains $p_Q(q) = 2c^2 q$

Derivation of the Mean Bit Error Rate

- Let $q := d_2/d_1$ (corresponding random variable Q)

- Since d_1 is fixed, the pdf of Q is given by

$$p_Q(q) = d_1 \cdot p_{D_2}(d_1 q) = c\,r \cdot \frac{\partial}{\partial d_2} P(D_2 \leq d_2) \Big|_{d_2 = c\,r\,q}$$

- With $P(D_2 \leq d_2) = \pi d_2^2/\pi r^2$ one obtains $p_Q(q) = 2c^2 q$

- Using $a_1/a_2 = (d_2/d_1)^\rho = q^\rho$ and $a_2 = 2 - a_1$

  $\Longrightarrow$ a_1 is a function of q: $a_1 = \dfrac{2\,q^\rho}{1+q^\rho}$

  $\Longrightarrow$ The pdf $p_{A_1}(a_1)$ can be determined using $p_Q(q) = 2c^2 q$

Derivation of the Mean Bit Error Rate

- One obtains

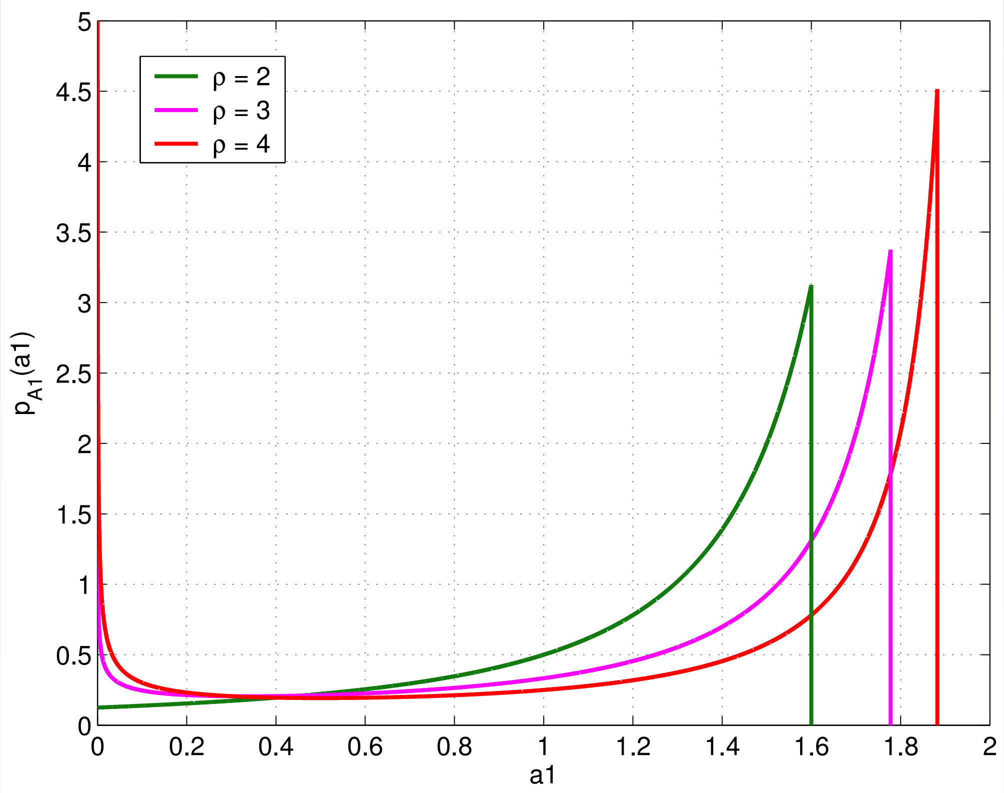
$$
\begin{aligned}
p_{A_1}(a_1) &= \frac{(1+\xi(a_1))^2}{2\rho\ \xi(a_1)^{(\rho-1)/\rho}} \cdot p_Q(\xi(a_1)^{1/\rho}) \\
&= \begin{cases} \dfrac{c^2\,(1+\xi(a_1))^2}{\rho\ \xi(a_1)^{(\rho-2)/\rho}}, & \text{for} \quad a_1 \in [0, a_{1\max}] \\ 0 & \text{else}, \end{cases}
\end{aligned}
$$

where

$$\xi(a_1) := \frac{a_1}{2-a_1}$$ and $$a_{1\max} = a_{1\max}(c,\rho) := \frac{2}{(1+c^\rho)}$$

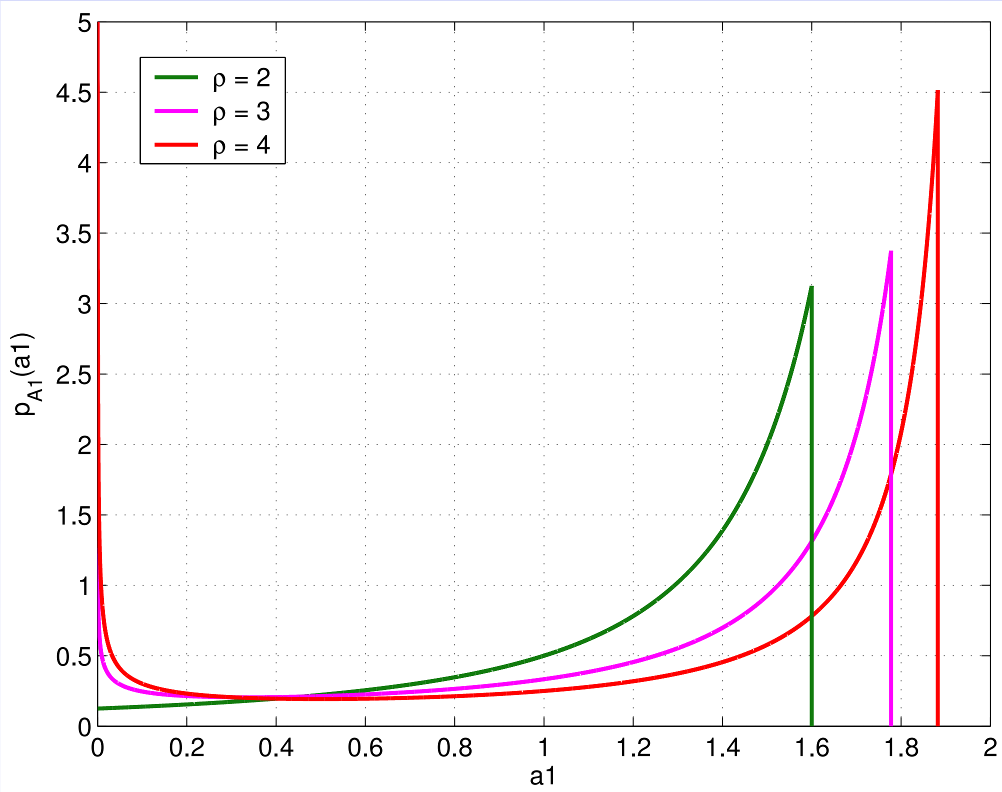
Average Error Performance

$p_{A_1}(a_1)$ vs. a_1 $(\rho = 2, 3, 4;\ c = 0.5)$

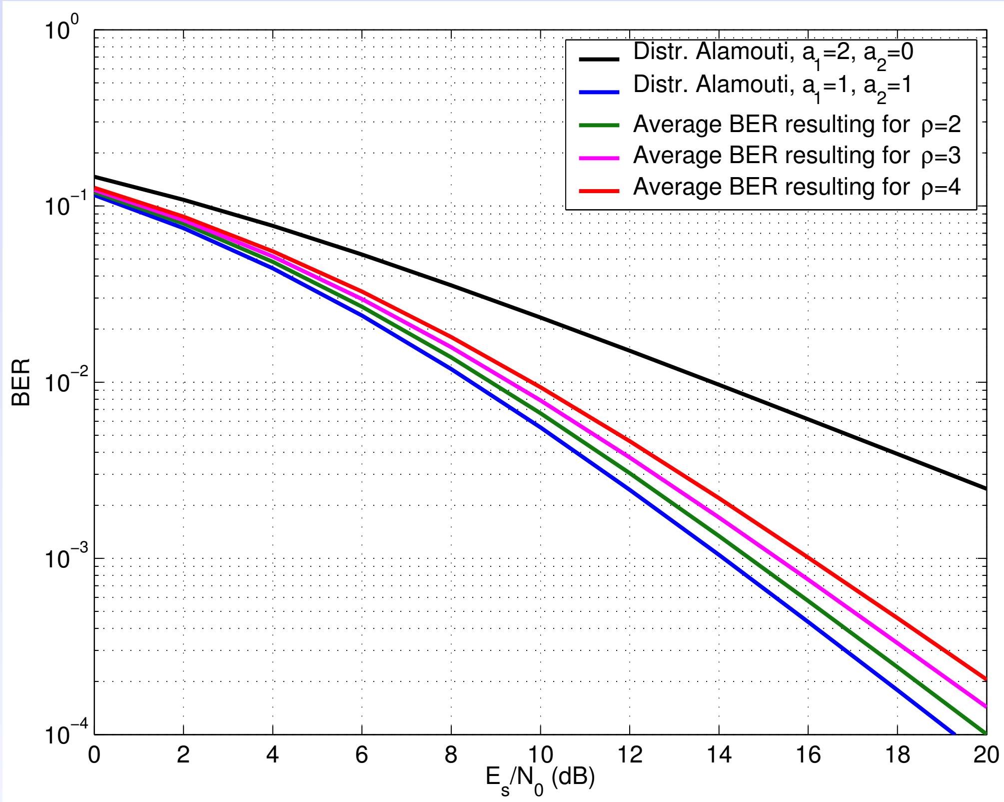


Average Error Performance

$p_{A_1}(a_1)$ vs. a_1 $(\rho = 2, 3, 4;\ \ c = 0.5)$

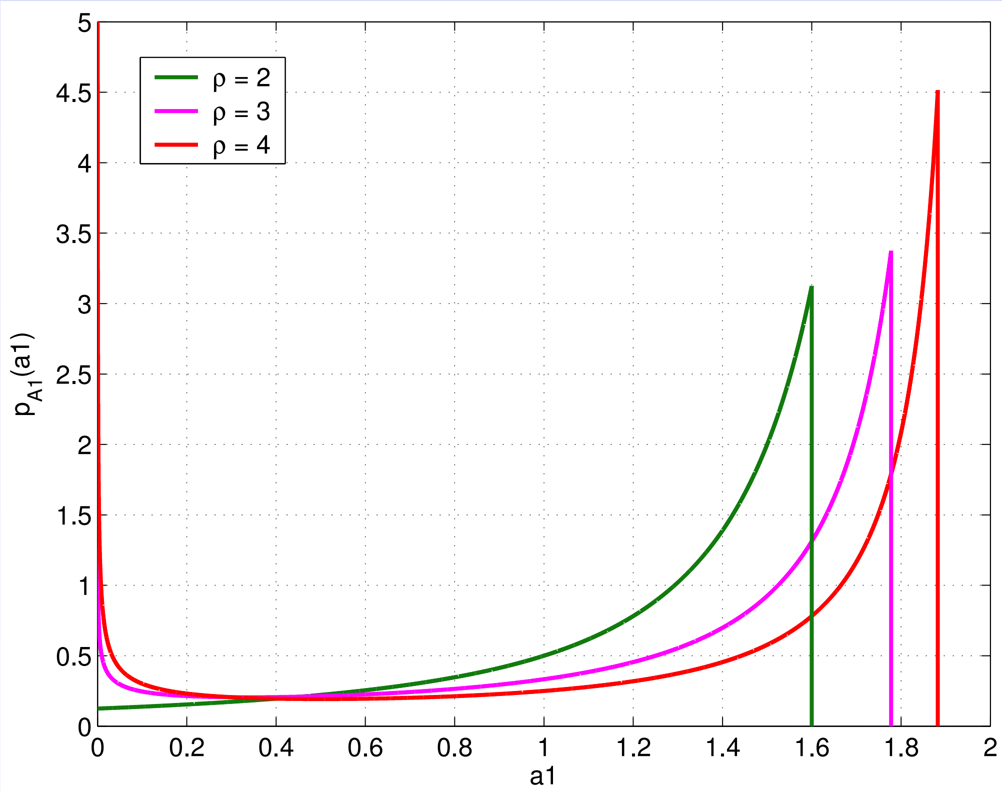


$\bar{P}_\mathrm{b}$ vs. E_s/N_0 in dB

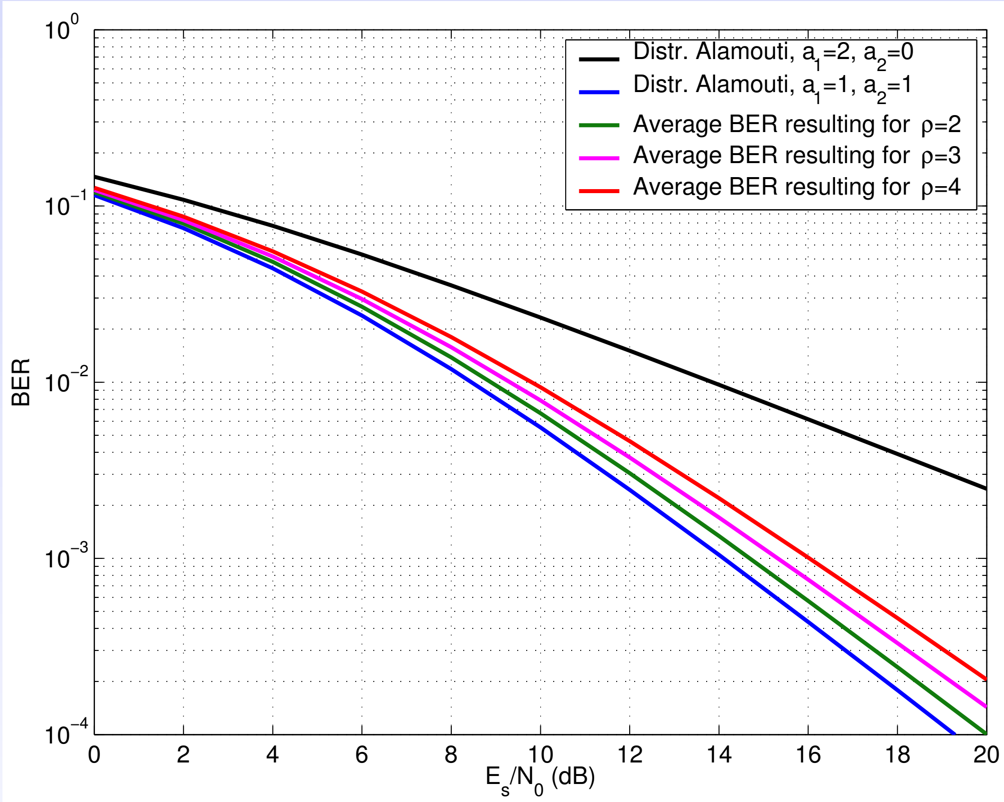


Average Error Performance

$p_{A_1}(a_1)$ vs. a_1 $(\rho = 2, 3, 4;\ c = 0.5)$



$\bar{P}_{\rm b}$ vs. $E_{\rm s}/N_0$ in dB



For **large** path-loss exponent ρ, probability that $a_1 \approx 1$ comparably **small**

$\Longrightarrow$ **Significant** average loss compared to co-located antennas $(a_1 = a_2 = 1)$

Outline

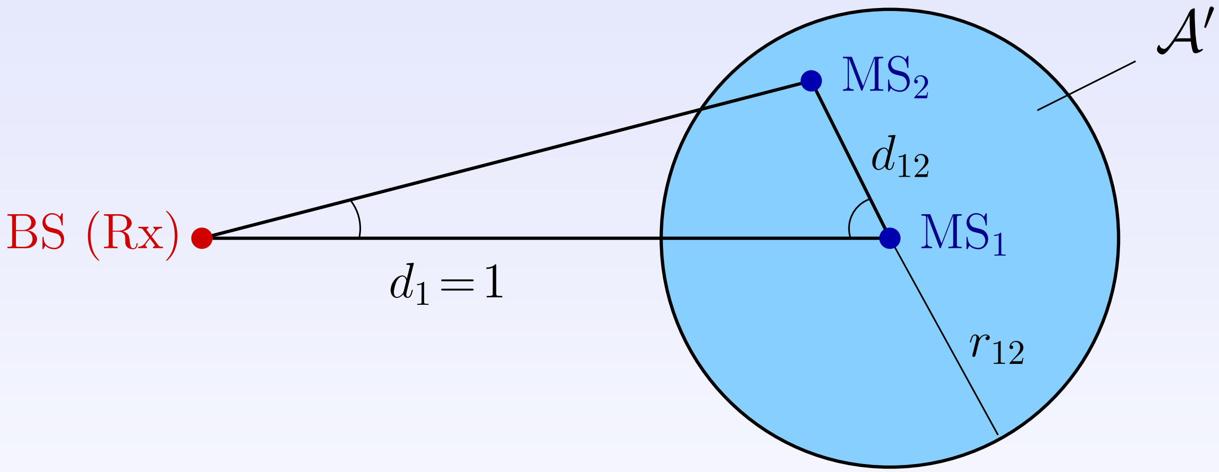
- **Error Performance of Distributed STCs**
- **Average Error Performance in a General Uplink Scenario**
- **Average Error Performance in an Uplink Scenario with Additional Constraint**
  - Uplink Scenario with Additional Constraint
  - Mean Bit Error Rate
- **Conclusions**

Uplink Scenario With Additional Constraint

- **Assumptions:**
  - **Constraint** for MS_2: Distance d_{12} between MS_2 and MS_1 **significantly smaller** than d_1

    $\Longrightarrow$ MS_2 within disk $\mathcal{A}'$ of radius $r_{12} \ll d_1$ around MS_1, according to **uniform distribution**

    $\Longrightarrow$ Constraint **reasonable** when MS_1 and MS_2 act as **mutual relays:** MS_1 and MS_2 only cooperate if $d_{12} \leq r_{12}$, so as to avoid **error propagation**
  - **Distance** d_1 between MS_1 and BS normalized to one

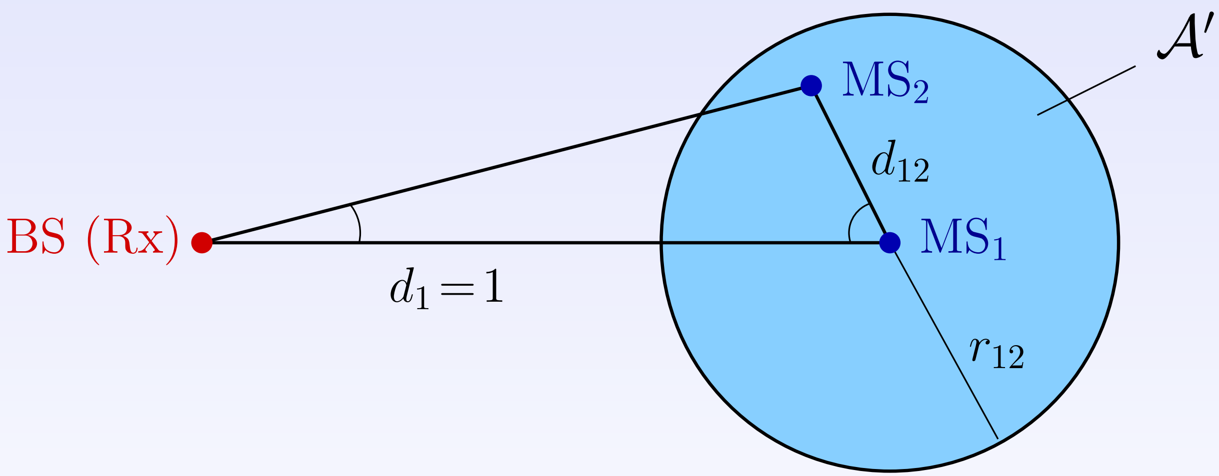


Uplink Scenario With Additional Constraint

- **Assumptions:**
  - **Constraint** for MS_2: Distance d_{12} between MS_2 and MS_1 **significantly smaller** than d_1

    $\Longrightarrow$ MS_2 within disk $\mathcal{A}'$ of radius $r_{12} \ll d_1$ around MS_1, according to **uniform distribution**

    $\Longrightarrow$ Constraint **reasonable** when MS_1 and MS_2 act as **mutual relays:** MS_1 and MS_2 only cooperate if $d_{12} \leq r_{12}$, so as to avoid **error propagation**
  - **Distance** d_1 between MS_1 and BS normalized to one

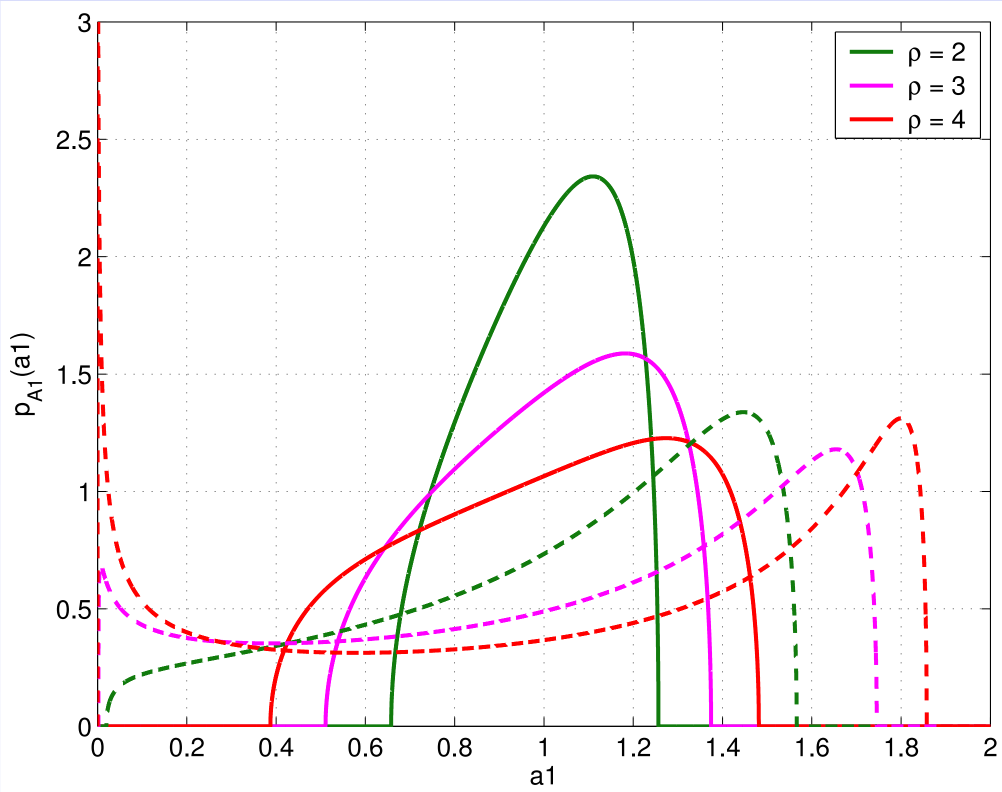


- **Derivation** of $p_{A_1}(a_1)$ and $\bar{P}_b$ as before, via the pdf $p_Q(q)$

  (However, deriving $p_Q(q)$ is more involved)

Average Error Performance

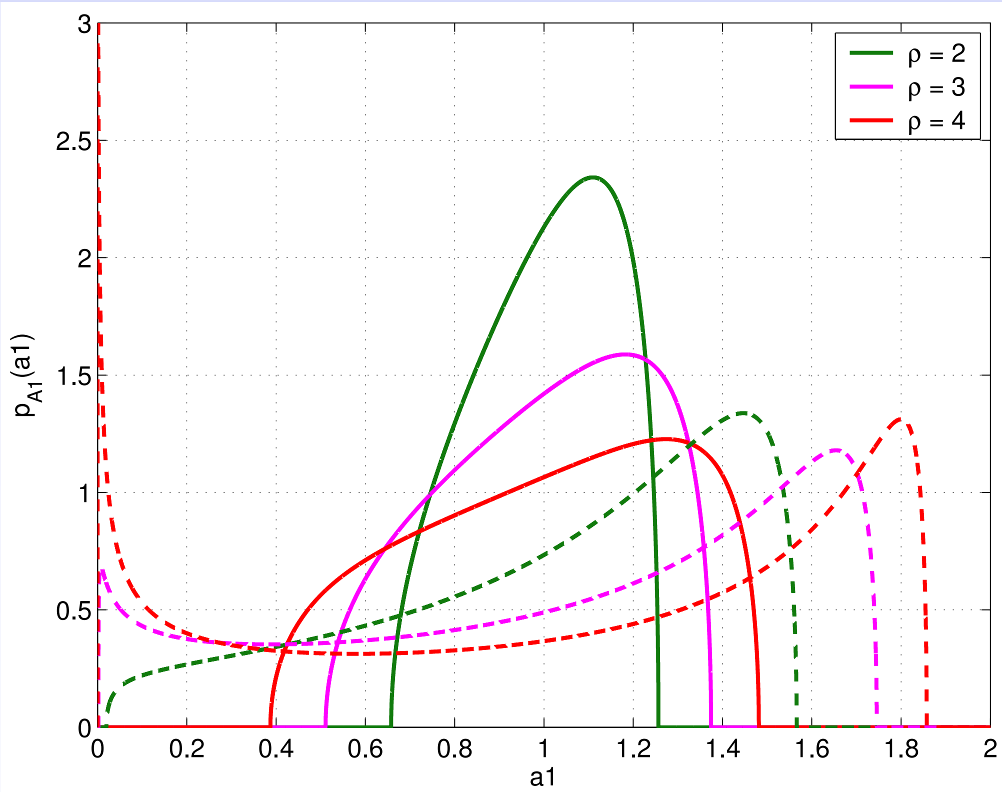
$p_{A_1}(a_1)$ vs. a_1 $(\rho = 2, 3, 4)$



Radius $r_{12} = 0.3$ (solid), $r_{12} = 0.9$ (dashed)

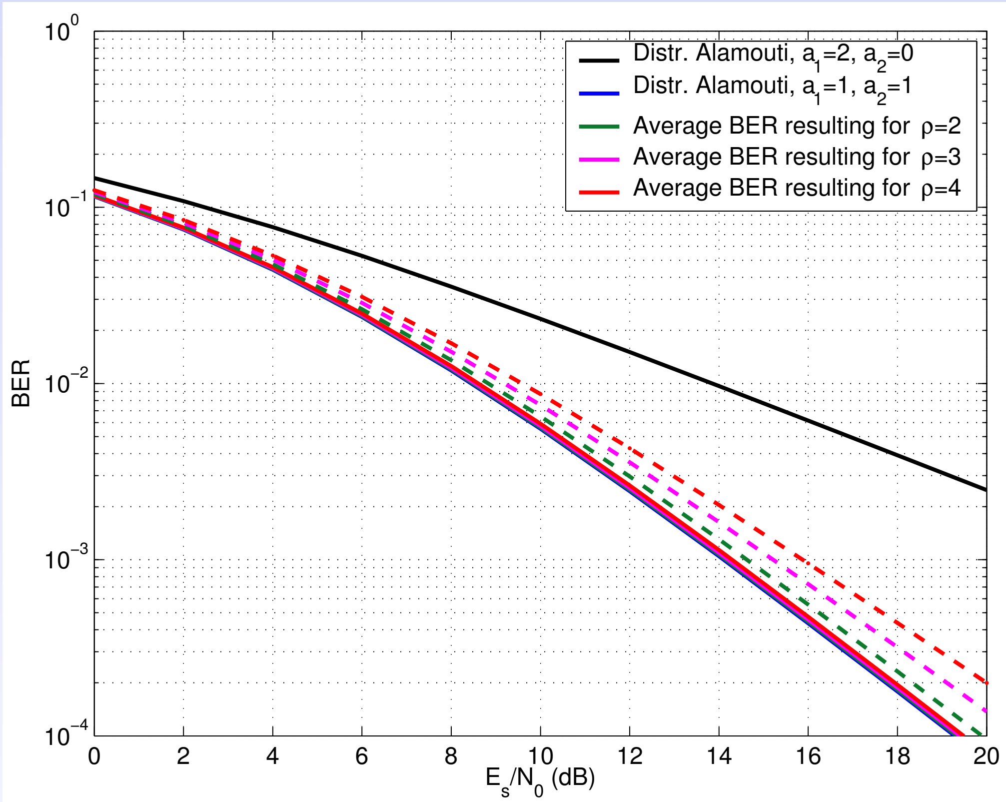
Average Error Performance

$p_{A_1}(a_1)$ vs. a_1 $(\rho = 2, 3, 4)$



Radius $r_{12} = 0.3$ (solid), $r_{12} = 0.9$ (dashed)

$\bar{P}_{\mathrm{b}}$ vs. E_{s}/N_0 in dB



Conclusions

- Wireless systems with **distributed** transmitters: Specific **differences** compared to systems with **co-located** antennas

- **Here:** Focus on different average **link gains** $\Longrightarrow$ **Reduced** diversity degree

- Two typical **uplink scenarios** considered $\Longrightarrow$ Analytical derivation of the **mean BER**

$\Longrightarrow$ In most scenarios **performance loss** < 2 dB at a BER of 10^{-3}

$\Longrightarrow$ Most **significant** performance loss for **large** path-loss exponents (e.g. $\rho = 4$)

Appendix: Expressions for the Uplink Scenario with Additional Constraint

- Probability $P(D_2 \leq d_2)$, where $1 - r_{12} \leq d_2 \leq 1 + r_{12}$:

$$P(D_2 \leq d_2) \;=\; \frac{1}{\pi}\frac{d_2^2}{r_{12}^2}\left(\varphi_{\mathrm{B}}(d_2) - \frac{1}{2}\sin(2\,\varphi_{\mathrm{B}}(d_2))\right) + \frac{1}{\pi}\left(\varphi_{\mathrm{M}}(d_2) - \frac{1}{2}\sin(2\,\varphi_{\mathrm{M}}(d_2))\right),$$

where $\varphi_{\mathrm{B}}(d_2) = \arccos\left(\underbrace{\frac{1 + d_2^2 - r_{12}^2}{2\,d_2}}_{=:\,\psi(d_2)}\right)$ and $\varphi_{\mathrm{M}}(d_2) = \arccos\left(\underbrace{\frac{1 - d_2^2 + r_{12}^2}{2\,r_{12}}}_{=:\,\zeta(d_2)}\right)$

- Pdf $p_Q(q)$, $q = d_2/d_1 = d_2$: ($\longrightarrow$ from $p_Q(q)$ one obtains $p_{A_1}(a_1)$)

$$p_Q(q) \;=\; \frac{\partial}{\partial d_2} P(D_2 \leq d_2)\Big|_{d_2=q} \;=\; \frac{1}{\pi}\frac{2q}{r_{12}^2}\left(\varphi_{\mathrm{B}}(q) - \frac{1}{2}\sin(2\,\varphi_{\mathrm{B}}(q))\right) + \ldots$$

$$\ldots + \frac{1}{\pi}\frac{1 - q^2 - r_{12}^2}{2\,r_{12}^2\sqrt{1 - \psi^2(q)}}\left(1 - \cos(2\,\varphi_{\mathrm{B}}(q))\right) + \frac{1}{\pi}\frac{q}{r_{12}\sqrt{1 - \zeta^2(q)}}\left(1 - \cos(2\,\varphi_{\mathrm{M}}(q))\right)$$