# Equivalence of Spatially Correlated and Distributed MIMO Systems

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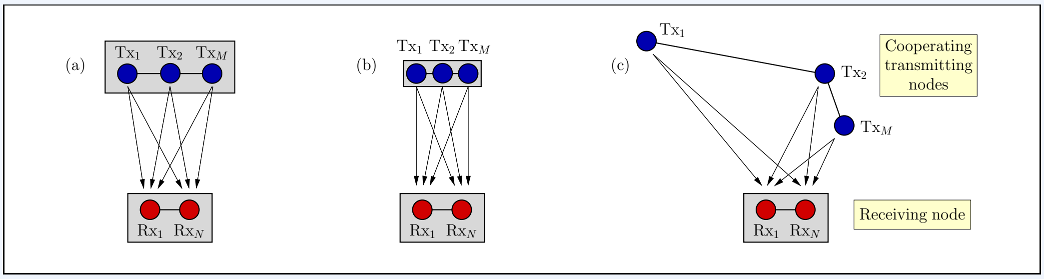
## 1. MIMO Systems with Co-located and Distributed Antennas

- **MIMO systems** for wireless communications offer **huge advantages** over single-antenna systems (capacity, error performance, ...).
- MIMO concept may be transferred to **cooperative wireless networks** $\Rightarrow$ **Distributed MIMO systems**, 'virtual antenna arrays'.
  **Examples:** Simulcast networks, reach-back links for wireless sensors, relay-assisted wireless networks, ...




- In the MIMO literature, however, assumptions concerning **antenna spacings** at transmitter and receiver are typically quite **restrictive**:
  - Antenna elements are normally assumed to be **co-located**, i.e., they belong to some **antenna array**.
  - On the other hand, antenna spacings are often assumed to be **sufficiently large** to justify assumption of **uncorrelated antennas**.
- **Spatial fading correlations** can lead to a **significant loss** in capacity and error performance.
- At the first glance, MIMO systems with **co-located** antennas and MIMO systems with **distributed** antennas have **little in common**.

**We show:** Any **co-located** MIMO system obeying the Kronecker-correlation model can be transformed into an **equivalent distributed** MIMO system, and vice versa.

$\Longrightarrow$ In a **static scenario** (i.e., no macroscopic diversity), distributed antennas can lead to a significant **performance loss**.

$\Longrightarrow$ **Transmit power allocation** schemes for spatially correlated MIMO systems **can be reused** for distributed MIMO systems.

## 2. System and Correlation Model

- **Generic** MIMO transmission model:

$$\mathbf{y}[k] = \mathbf{H}\,\mathbf{x}[k] + \mathbf{n}[k], \quad h_{ji} \sim \mathcal{CN}\{0, \sigma_{ji}^2\}.$$

($M$ transmit antennas, $N$ receive antennas, Rayleigh fading.)

- **Co-located** MIMO system (Kronecker correlation model):

$$\mathbf{H} = \mathbf{R}_{\mathrm{Rx}}^{1/2}\,\mathbf{G}\,\mathbf{R}_{\mathrm{Tx}}^{1/2}, \quad g_{ji} \sim \mathcal{CN}\{0, \sigma^2\} \ \text{ i.i.d.},$$

where $\mathrm{tr}(\mathbf{R}_{\mathrm{Rx}}) = N$ and $\mathrm{tr}(\mathbf{R}_{\mathrm{Tx}}) = M$.

- MIMO system with **distributed** transmit or receive antennas:

$$\mathbf{H} = \mathbf{R}_{\mathrm{Rx}}^{1/2}\,\mathbf{G}\,\mathbf{\Sigma}_{\mathrm{Tx}}^{1/2},$$

where $\mathbf{\Sigma}_{\mathrm{Tx}}$ diagonal with $\mathrm{tr}(\mathbf{\Sigma}_{\mathrm{Tx}}) = M$, or

$$\mathbf{H} = \mathbf{\Sigma}_{\mathrm{Rx}}^{1/2}\,\mathbf{G}\,\mathbf{R}_{\mathrm{Tx}}^{1/2},$$

where $\mathbf{\Sigma}_{\mathrm{Rx}}$ diagonal with $\mathrm{tr}(\mathbf{\Sigma}_{\mathrm{Rx}}) = N$.
(Entries of $\mathbf{\Sigma}_{\mathrm{Tx}}$ and $\mathbf{\Sigma}_{\mathrm{Rx}}$ represent **unequal link SNRs**.)

## 3. Capacity Distribution

- **Capacity distribution** for co-located MIMO systems:

$$p(C(\mathbf{H})) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \mathrm{cf}_C(\mathrm{j}\omega)\,\mathrm{e}^{-\mathrm{j}\omega C(\mathbf{H})}\,\mathrm{d}\omega,$$

where the characteristic function is of form [Park/Shin/Lee'04]

$$\mathrm{cf}_C(\mathrm{j}\omega) = \frac{K\,\varphi(\mathrm{j}\omega)}{\psi(\mathbf{R}_{\mathrm{Tx}}, \mathbf{R}_{\mathrm{Rx}})}\left|\left[\begin{array}{c}\mathbf{V}(\mathbf{R}_{\mathrm{Rx}})\\ \mathbf{M}(\mathbf{R}_{\mathrm{Tx}}, \mathbf{R}_{\mathrm{Rx}}, \mathrm{j}\omega)\end{array}\right]\right|.$$

Expression for $\mathrm{cf}_C(\mathrm{j}\omega)$ depends **only** on the **eigenvalues** of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$, but not on specific entries.

- **Eigenvalue decomposition** of $\mathbf{R}_{\mathrm{Tx}}$ and $\mathbf{R}_{\mathrm{Rx}}$:

$$\mathbf{R}_\bullet := \mathbf{U}_\bullet \mathbf{\Lambda}_\bullet \mathbf{U}_\bullet^{\mathrm{H}}.$$

$\Longrightarrow$ **Equivalent distributed** MIMO system can be found as follows:

$$\mathbf{U}_\bullet^{\mathrm{H}} \mathbf{R}_\bullet \mathbf{U}_\bullet = \mathbf{\Lambda}_\bullet \ =: \ \mathbf{\Sigma}_\bullet\,.$$

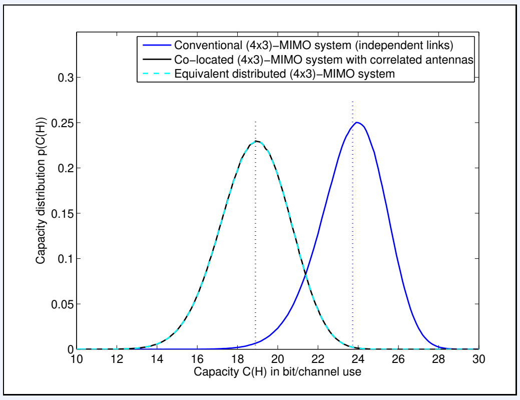
- **Vice versa:**
  Given a distributed MIMO system, the diagonal entries of $\mathbf{\Sigma}_\bullet$ may be interpreted as the **eigenvalues** of some correlation matrix $\mathbf{R}_\bullet$.

$\Longrightarrow$ **Equivalent co-located** MIMO system can be found as follows:

$$\tilde{\mathbf{U}}_\bullet^{\mathrm{H}} \mathbf{\Sigma}_\bullet \tilde{\mathbf{U}}_\bullet \ =: \ \mathbf{R}_\bullet\,.$$

Suitable matrices for $\tilde{\mathbf{U}}_\bullet$ are, e.g., the Hadamard or the Fourier matrix.

- **Numerical example:**
  $M = 4$, $N = 3$, $\rho_{\mathrm{Tx}} = 0.8$, $\rho_{\mathrm{Rx}} = 0.7$, $\mathrm{SNR} = 20$ dB




## 4. Further Contents of the Paper

- (Asymptotic) equivalence with regard to the **pairwise error probability** of space-time codes.
- Optimal **transmit power allocation** based on statistical channel knowledge (maximum ergodic capacity).

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