UWB Transmitted Reference Signaling Schemes - Part II: Narrowband interference Analysis

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***Abstract*— In Part I [1] of this two-part paper, we provide uncoded bit error probability (BEP) analysis of various transmitted-reference (TR) schemes by developing an analytical framework based on the sampling expansion approach. In this paper, the effect of narrowband interference (NBI) is taken into account to derive uncoded BEP expressions of TR and differential TR (DTR) signaling with an autocorrelation receiver (AcR). For simplicity, our NBI is modeled as a single tone interferer with Rayleigh distributed amplitude and an uniformly distributed random phase. We quantify the effect of NBI and channel power dispersion profile (PDP) on the optimum integration interval of AcR. Unlike the NBI-free situation, the optimum integration interval of AcR is not necessarily close to the delay spread, and depends on the signal-to-interference ratio (SIR), the signal-to-noise ratio (SNR) and the channel PDP. Furthermore, our results also allow us to compare the NBI sensitivity of TR and DTR signaling.**

I. Introduction

There is renewed interest in transmitted-reference (TR) signaling due to its simplicity since conventional Rake reception in ultrawide bandwidth (UWB) systems requires a large number of Rake fingers to capture the multipath energy [2]–[5]. Moreover, channel estimation error in Rake reception can deteriorate the bit error probability (BEP) performance. In Part I [1] of this two-part paper, we provide uncoded BEP analysis of various TR schemes for a broad class of fading channels in dense resolvable multipath channels by developing an analytical framework based on the sampling expansion approach [1], [4], [5].

Since UWB systems are required to coexist and contend with a variety of interfering signals, the study of such narrowband interference (NBI) on UWB systems is an important issue [6]–[10]. However, the results available in the literature regarding the impact of NBI on TR signaling schemes are all based on simulations [11], [12], and offers no unifying performance measure for such systems. This motivates us to extend our BEP analysis for both TR and differential TR (DTR) signaling schemes in a single link system to include the impact of NBI [1], [4], [5]. Results in [9] show that NBI can be reasonably well approximated by a tone interference. As in [9], we model the NBI as a single-tone interferer with Rayleigh distributed amplitude and an uniformly distributed random phase. Based on our derived uncoded BEP expressions and numerical results, we quantify the effect of NBI and channel power dispersion profile (PDP) on the optimum integration interval of autocorrelation receiver (AcR). Unlike the NBI-free case, the optimum integration interval of AcR is not necessarily close to the delay spread, and depends on the signal-to-interference ratio (SIR), the signal-to-noise ratio (SNR), and the PDP of the channel. Furthermore, we compare the NBI sensitivity of TR and DTR signaling.

The paper is organized as follows. Section II presents the system model for both TR and DTR signaling schemes of a single link system. In Section III, the impact of NBI on the performance of TR and DTR signaling is studied. To illustrate our proposed methodology, we consider the Nakagami-m fading channels and present numerical results in Section IV. Finally, Section V comprises concluding remarks.

II. System Model

A. Transmitted-Reference

As shown in Fig. 1 of Part I [1], the transmitted signal of TR signaling for a single user is given by

$$s_{\mathrm{TR}}(t) = \sum_i b_{\mathrm{r}}(t - iN_{\mathrm{s}}T_{\mathrm{f}}) + d_i b_{\mathrm{d}}(t - iN_{\mathrm{s}}T_{\mathrm{f}}), \qquad (1)$$

and

$$b_{\mathrm{r}}(t) = \sum_{j=0}^{\frac{N_{\mathrm{s}}}{2}-1} \sqrt{E_{\mathrm{p}}} a_j p(t - j2T_{\mathrm{f}} - c_j T_{\mathrm{p}}),$$
$$b_{\mathrm{d}}(t) = \sum_{j=0}^{\frac{N_{\mathrm{s}}}{2}-1} \sqrt{E_{\mathrm{p}}} a_j p(t - j2T_{\mathrm{f}} - c_j T_{\mathrm{p}} - T_{\mathrm{r}}), \qquad (2)$$

where T_{f} is the average repetition period, $d_i \in \{-1, 1\}$ is the data symbol, symbol duration is $N_{\mathrm{s}}T_{\mathrm{f}}$, and $p(t)$ is the normalized signal pulse with duration T_{p}. The energy of the transmitted pulse is then $E_{\mathrm{p}} = E_{\mathrm{s}}/N_{\mathrm{s}}$, and symbol energy is E_{s}. In direct-sequence (DS) signaling, $\{a_j\}$ is the bipolar pseudo-random sequence (e.g. in [13], [14] Walsh-Hadamard sequences are used). In time-hopping (TH) signaling, $\{c_j\}$ is the pseudo-random TH sequence. The duration of the received UWB pulse is $T_{\mathrm{g}} = T_{\mathrm{p}} + T_{\mathrm{d}}$, where T_{d} is the maximum excess delay of the channel. To preclude inter-symbol interference (ISI) and intra-symbol interference (i.s.i.), we assume that $T_{\mathrm{r}} \geq T_{\mathrm{g}}$ and $N_{\mathrm{h}}T_{\mathrm{p}} + T_{\mathrm{r}} \leq 2T_{\mathrm{f}} - T_{\mathrm{g}}$, where T_{r} is the time separation between each pair of data and reference pulses such that these received pulses will not overlap.

B. Differential Transmitted-Reference

In DTR signaling, the transmitted signal for a single user is given by

$$s_{\mathrm{DTR}}(t) = \sum_i e_i b(t - iN_{\mathrm{s}}T_{\mathrm{f}}), \tag{3}$$

and

$$b(t) = \sum_{j=0}^{N_{\mathrm{s}}-1} \sqrt{E_{\mathrm{p}}} a_j p(t - jT_{\mathrm{f}} - c_j T_{\mathrm{p}}), \tag{4}$$

where the data symbol d_i is now differentially encoded such that $e_i = e_{i-1}d_i$, where $d_i = \pm 1$, $\{a_j\}$ and $\{c_j\}$ are the DS and TH sequences. The length of $\{a_j\}$ is now N_{s}. The TH sequence is pseudo-random with the range $0 \le c_j < N_{\mathrm{h}}$, where N_{h} satisfies $T_{\mathrm{f}} \ge (N_{\mathrm{h}}-1)T_{\mathrm{p}} + T_{\mathrm{g}}$ to preclude ISI and i.s.i. The channel is assumed to be constant over two symbols in order to use differential encoding over every two symbols.

III. Narrowband Interference Analysis

In the presence of NBI, the received signal for TR signaling and similarly for DTR signaling can be written as

$$r(t) = (h * s_{\mathrm{TR}})(t) + J(t) + n(t), \tag{5}$$

where $J(t)$ denotes the NBI, $h(t)$ is the linearly time-invariant impulse response of the channel, and $n(t)$ is zero-mean, white Gaussian noise with two-sided power spectral density $N_0/2$. The channel impulse response can be written as $h(t) = \sum_{l=1}^{L} \alpha_l \delta(t - \tau_l)$ where α_l and τ_l denote respectively the attenuation and delay of l-th path, and L is the number of resolvable multipath components. We can also express $\alpha_l = |\alpha_l| \exp(j\phi_l)$, where $\phi_l = 0$ or π with equal probability. We consider the resolvable multipath channel, i.e., $|\tau_l - \tau_j| \ge T_{\mathrm{p}}, \forall l \ne j$, where $\tau_l = \tau_1 + (l-1)T_{\mathrm{p}}$. Under uncorrelated scattering assumption [15], $\{\alpha_l\}$ are statistically independent random variables (r.v's).

The autocorrelation function of the superposition of the two independent noise processes in (5) is given by

$$R_{n_{\mathrm{T}}}(\tau) = R_{\mathrm{J}}(\tau) + \frac{N_0}{2}\delta(\tau), \tag{6}$$

where $n_{\mathrm{T}}(t) \triangleq J(t) + n(t)$ and $R_{\mathrm{J}}(\tau) = \mathbb{E}\{J(t)J(t+\tau)\}$. The received signal is first passed through an ideal bandpass zonal filter (BPZF) with bandwidth W and center frequency f_{c} to eliminate the out-of-band noise (see Fig. 2 of Part I [1]). Since the bandwidth of a typical NBI is smaller than that of the transmitted pulse, the autocorrelation function of the filtered $n_{\mathrm{T}}(t)$ is then given by

$$R_{\widetilde{n}_{\mathrm{T}}}(\tau) = R_{\mathrm{J}}(\tau) + WN_0 \,\mathrm{sinc}(W\tau)\cos(2\pi f_{\mathrm{c}}\tau), \tag{7}$$

where $\widetilde{n}_{\mathrm{T}}(t)$ is the filtered version of $n_{\mathrm{T}}(t)$ in (6). As in [9], we model the NBI as a single-tone continuous-wave (CW) signal given by

$$J(t) = \alpha_{\mathrm{J}}\sqrt{2J_0}\cos(2\pi f_{\mathrm{J}} t + \theta), \tag{8}$$

where J_0 is the average NBI power, α_{J} is the slowly-varying[1] Rayleigh distributed r.v. with $\mathbb{E}\{\alpha_{\mathrm{J}}^2\} = 1$, f_{J} is the NBI carrier frequency, and θ is the random phase uniformly distributed over $[0, 2\pi)$. Thus, from (6)-(8), $R_{\mathrm{J}}(\tau) = J_0\cos(2\pi f_{\mathrm{J}}\tau)$ and we can see that $n_{\mathrm{T}}(t)$ is now colored, so the samples of $\widetilde{n}_{\mathrm{T}}(t)$ taken at an interval of $1/W$ are expected to be correlated. In the following, we derive the BEP of TR and DTR signaling with AcR, where we define $\mathrm{SIR} = E_{\mathrm{s}}/(N_{\mathrm{s}}T_{\mathrm{f}}J_0)$.

A. Transmitted-Reference

Without loss of generality, we consider the detection of the data symbol at $i = 0$. In addition, we assume perfect synchronization at the AcR. Following [1], [4], [5], we can incorporate the NBI in (8) and the decision statistics generated at the AcR for TR signaling is given by

$$\begin{aligned} Z_{\mathrm{TR}} &= \sum_{j=0}^{\frac{N_{\mathrm{s}}}{2}-1} \int_{j2T_{\mathrm{f}}+T_{\mathrm{r}}+c_jT_{\mathrm{p}}}^{j2T_{\mathrm{f}}+T_{\mathrm{r}}+c_jT_{\mathrm{p}}+T} \widetilde{r}_{\mathrm{TR}}(t)\widetilde{r}_{\mathrm{TR}}(t - T_{\mathrm{r}})dt \\ &= \sum_{j=0}^{\frac{N_{\mathrm{s}}}{2}-1} \int_0^T (w_j(t) + \xi_{1,j}(t) + \eta_{1,j}(t)) \\ &\qquad \times (d_0 w_j(t) + \xi_{2,j}(t) + \eta_{2,j}(t))dt \\ &= \sum_{j=0}^{\frac{N_{\mathrm{s}}}{2}-1} U_j, \end{aligned} \tag{9}$$

where $w_j(t) \triangleq \sqrt{E_{\mathrm{p}}} a_j \sum_{l=1}^{L} \alpha_l p(t - \tau_l)$, $\xi_{1,j}(t) \triangleq J(t + j2T_{\mathrm{f}} + c_jT_{\mathrm{p}})$, $\xi_{2,j}(t) \triangleq J(t + j2T_{\mathrm{f}} + c_jT_{\mathrm{p}} + T_{\mathrm{r}})$, $\eta_{1,j}(t) \triangleq \widetilde{n}(t + j2T_{\mathrm{f}} + c_jT_{\mathrm{p}})$ and $\eta_{2,j}(t) \triangleq \widetilde{n}(t + j2T_{\mathrm{f}} + c_jT_{\mathrm{p}} + T_{\mathrm{r}})$ are defined over the interval $[0, T]$. We can then rewrite U_j in (9) using the sampling approach as

$$\begin{aligned} U_j = \frac{1}{W}\sum_{m=1}^{2WT} \big[& d_0 w_{j,m}^2 + w_{j,m}(\xi_{2,j,m} + \eta_{2,j,m}) \\ & + d_0 w_{j,m}(\xi_{1,j,m} + \eta_{1,j,m}) \\ & + (\xi_{1,j,m} + \eta_{1,j,m})(\xi_{2,j,m} + \eta_{2,j,m}) \big], \end{aligned} \tag{10}$$

where $\xi_{1,j,m}$ and $\xi_{2,j,m}$ are respectively the m-th samples of $\xi_{1,j}(t)$ and $\xi_{2,j}(t)$ in the interval $[0, T]$ given by

$$\begin{aligned} \xi_{1,j,m} &= \alpha_{\mathrm{J}}\sqrt{2J_0}\cos\left[2\pi f_{\mathrm{J}}\left(\frac{m}{W} + j2T_{\mathrm{f}} + c_jT_{\mathrm{p}}\right) + \theta\right], \\ \xi_{2,j,m} &= \alpha_{\mathrm{J}}\sqrt{2J_0}\cos\left[2\pi f_{\mathrm{J}}\left(\frac{m}{W} + j2T_{\mathrm{f}} + c_jT_{\mathrm{p}} + T_{\mathrm{r}}\right) + \theta\right], \end{aligned} \tag{11}$$

and the rest of the terms in (10) are defined similarly as in Part I [1].

By conditioning on d_0, we can rewrite (10) as

$$\begin{aligned} U_{j|d_0=+1} &= \sum_{m=1}^{2WT}\left[\left(\frac{1}{\sqrt{W}}w_{j,m} + \beta_{1,j,m}\right)^2 - \beta_{2,j,m}^2\right], \\ U_{j|d_0=-1} &= \sum_{m=1}^{2WT}\left[-\left(\frac{1}{\sqrt{W}}w_{j,m} - \beta_{2,j,m}\right)^2 + \beta_{1,j,m}^2\right], \end{aligned} \tag{12}$$

[1]The amplitude is assumed to be constant over at least two symbols of TR signaling.

where $\beta_{1,j,m} = \frac{1}{2\sqrt{W}}(\eta_{2,j,m} + \xi_{2,j,m} + \eta_{1,j,m} + \xi_{1,j,m})$ and $\beta_{2,j,m} = \frac{1}{2\sqrt{W}}(\eta_{2,j,m} + \xi_{2,j,m} - \eta_{1,j,m} - \xi_{1,j,m})$. Further conditioning on θ, $\alpha_{\rm J}$ and c_j, the quantities $\xi_{1,j,m}$ and $\xi_{2,j,m}$ in (11) are deterministic and the conditional variance $\sigma^2_{\rm TR}$ of $\beta_{1,j,m}$ and $\beta_{2,j,m}$ is simply $\frac{N_0}{4}$. Thus, from (12), we can observe that the statistical characterization of U_j when conditioned on θ, $\alpha_{\rm J}$, c_j, a_j and the channel is no longer symmetric with respect to d_0 due to the interference term. For example, $U_{j|d_0=+1}$ is simply the difference of two noncentral chi-squared r.v.'s with equal degrees of freedom, but different non-centrality parameters. As a result, we need to calculate separately the conditional BEP with respect to d_0 to obtain the overall BEP.

First, to obtain $\mathbb{P}\{Z_{\rm TR} < 0|d_0 = +1\}$, we calculate the non-centrality parameters of Y_1 and Y_2 (these conditional r.v.'s are defined in (14) of Part I [1]) as follows

$$\begin{aligned}\mu^{\rm (NBI)}_{{\rm TR},Y_1} &\triangleq \frac{1}{2\sigma^2_{\rm TR}} \sum_{j=0}^{\frac{N_{\rm s}}{2}-1} \sum_{m=1}^{2WT} \frac{1}{W}\left(w_{j,m} + \frac{\xi_{1,j,m}+\xi_{2,j,m}}{2}\right)^2 \\ &\approx \frac{E_{\rm s}}{N_0}\sum_{l=1}^{L_{\rm CAP}} \alpha_l^2 + \frac{\alpha_{\rm J}^2 N_{\rm s} J_0 T}{2N_0} + \frac{\alpha_{\rm J}^2 N_{\rm s} J_0 T}{2N_0}\cos(2\pi f_{\rm J} T_{\rm r}),\end{aligned} \tag{13}$$

$$\begin{aligned}\mu^{\rm (NBI)}_{{\rm TR},Y_2} &\triangleq \frac{1}{2\sigma^2_{\rm TR}} \sum_{j=0}^{\frac{N_{\rm s}}{2}-1} \sum_{m=1}^{2WT} \frac{(\xi_{2,j,m}-\xi_{1,j,m})^2}{4W} \\ &\approx \frac{\alpha_{\rm J}^2 N_{\rm s} J_0 T}{2N_0} - \frac{\alpha_{\rm J}^2 N_{\rm s} J_0 T}{2N_0}\cos(2\pi f_{\rm J} T_{\rm r}),\end{aligned} \tag{14}$$

where $L_{\rm CAP} \triangleq \lceil \min\{WT, WT_{\rm g}\}\rceil$ denotes the actual number of multipath components captured by the AcR and the approximation used in (13) and (14) is shown in Appendix I. Note that unlike in (14) of Part I [1], Y_2 is now a noncentral chi-squared r.v. due to the presence of NBI. From (13) and (14), it is interesting to see that the NBI affects the performance by shifting the means and changing the variances of the probability density functions (pdfs) of both r.v.'s Y_1 and Y_2. Since Y_1 and Y_2 are conditionally independent, the pdf of Y' ($Y' = Y_1 - Y_2$) is simply equal to the convolution of the respective pdfs of Y_1 and $-Y_2$. Thus, the conditional probability that $Y' < 0$ is negligible when $\mu^{\rm (NBI)}_{{\rm TR},Y_1} \gg \mu^{\rm (NBI)}_{{\rm TR},Y_2}$. However, this conditional probability becomes significant when $\mu^{\rm (NBI)}_{{\rm TR},Y_1} \ll \mu^{\rm (NBI)}_{{\rm TR},Y_2}$, and this becomes the dominating error of probability. This interchange effect depends particularly on the relationship between $T_{\rm r}$ and $f_{\rm J}$ as these quantities affect the sign of the cosine argument in (13) and (14). The conditional pdfs of Y_1 and Y_2 are then given by

$$f_{Y_1|\{\alpha_l\},\alpha_{\rm J}}(y_1) = f_{\rm NC}(y_1, \mu^{\rm (NBI)}_{{\rm TR},Y_1}, q_{\rm TR}), \tag{15}$$

$$f_{Y_2|\{\alpha_l\},\alpha_{\rm J}}(y_2) = f_{\rm NC}(y_2, \mu^{\rm (NBI)}_{{\rm TR},Y_2}, q_{\rm TR}), \tag{16}$$

where $q_{\rm TR} = \frac{N_{\rm s}WT}{2}$ and the notation $f_{\rm NC}(\cdot)$ is defined in (16) of Part I [1]. Note that we have suppressed the conditional r.v.'s θ, $\{c_j\}$ and $\{a_j\}$ because (13) and (14) do not depend on these r.v.'s. The BEP conditional on $\{\alpha_l\}$, and $\alpha_{\rm J}$ is then given by

$$\begin{aligned}&\mathbb{P}\{Z_{\rm TR} < 0|d_0 = +1, \{\alpha_l\}, \alpha_{\rm J}\} \\ &= \mathbb{P}\{Y_1 - Y_2 < 0|d_0 = +1, \{\alpha_l\}, \alpha_{\rm J}\} \\ &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \left(\frac{1}{1+v^2}\right)^{q_{\rm TR}} \\ &\quad \times \mathfrak{Re}\left[\frac{e^{\left(\frac{-jv\mu^{\rm (NBI)}_{{\rm TR},Y_1}}{1+jv} + \frac{jv\mu^{\rm (NBI)}_{{\rm TR},Y_2}}{1-jv}\right)}}{jv}\right] dv,\end{aligned} \tag{17}$$

where $\mathfrak{Re}(\cdot)$ denotes the real part of $(\cdot)$ and the derivation of (17) can be found by using [16]. By averaging (17) with respect to $\{\alpha_l\}$ and $\alpha_{\rm J}$, the BEP conditioned on $d_0 = +1$ is given by

$$\begin{aligned}&\mathbb{P}\{Z_{\rm TR} < 0|d_0 = +1\} \\ &= \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \left(\frac{1}{1+v^2}\right)^{q_{\rm TR}} \\ &\quad \times \mathfrak{Re}\left[\frac{\psi_{\mu^{\rm (NBI)}_{{\rm TR},Y_1}}\left(\frac{-jv}{1+jv}\right)\psi_{\mu^{\rm (NBI)}_{{\rm TR},Y_2}}\left(\frac{jv}{1-jv}\right)}{jv}\right] dv \\ &\triangleq P_{\rm e}^{\rm (NBI)}\left(\psi_{\mu^{\rm (NBI)}_{{\rm TR},Y_1}}(jv), \psi_{\mu^{\rm (NBI)}_{{\rm TR},Y_2}}(jv).q_{\rm TR}\right),\end{aligned} \tag{18}$$

where $\psi_{\mu^{\rm (NBI)}_{{\rm TR},Y_1}}(jv)$ and $\psi_{\mu^{\rm (NBI)}_{{\rm TR},Y_2}}(jv)$ are respectively the characteristic functions (CFs) of $\mu^{\rm (NBI)}_{{\rm TR},Y_1}$ and $\mu^{\rm (NBI)}_{{\rm TR},Y_2}$, and are given by

$$\begin{aligned}\psi_{\mu^{\rm (NBI)}_{{\rm TR},Y_1}}(jv) &= \psi_{\rm J}\left(jv\left[\frac{N_{\rm s}J_0T}{2N_0} + \frac{N_{\rm s}J_0T}{2N_0}\cos(2\pi f_{\rm J}T_{\rm r})\right]\right) \\ &\quad \times \prod_{l=1}^{L_{\rm CAP}} \psi_l\left(\frac{jvE_{\rm s}}{N_0}\right),\end{aligned} \tag{19}$$

$$\psi_{\mu^{\rm (NBI)}_{{\rm TR},Y_2}}(jv) = \psi_{\rm J}\left(jv\left[\frac{N_{\rm s}J_0T}{2N_0} - \frac{N_{\rm s}J_0T}{2N_0}\cos(2\pi f_{\rm J}T_{\rm r})\right]\right), \tag{20}$$

where $\psi_l(jv)$ and $\psi_{\rm J}(jv)$ are respectively the CFs of α_l^2 and $\alpha_{\rm J}^2$.

Next, from (12), we can observe that conditioned on θ, $\alpha_{\rm J}$, c_j, a_j and the channel, $U_{j|d_0=-1}$ is still the difference of two noncentral chi-squared r.v.s' with equal degrees of freedom, but with non-identical non-centrality parameters. Using Appendix I, the non-centrality parameters of Y_3 and Y_4 defined in (14) of Part I [1] conditioned on θ, $\alpha_{\rm J}$, $\{c_j\}$, $\{a_j\}$ and the channel are given by

$$\begin{aligned}\mu^{\rm (NBI)}_{{\rm TR},Y_3} &\triangleq \frac{1}{2\sigma^2_{\rm TR}} \sum_{j=0}^{\frac{N_{\rm s}}{2}-1} \sum_{m=1}^{2WT} \frac{1}{W}\left(w_{j,m} - \frac{\xi_{1,j,m}-\xi_{2,j,m}}{2}\right)^2 \\ &\approx \frac{E_{\rm s}}{N_0}\sum_{l=1}^{L_{\rm CAP}} \alpha_l^2 + \frac{\alpha_{\rm J}^2 N_{\rm s} J_0 T}{2N_0} - \frac{\alpha_{\rm J}^2 N_{\rm s} J_0 T}{2N_0}\cos(2\pi f_{\rm J} T_{\rm r}),\end{aligned} \tag{21}$$

$$\mu_{\mathrm{TR},Y_4}^{(\mathrm{NBI})} \triangleq \frac{1}{2\sigma_{\mathrm{TR}}^2} \sum_{j=0}^{\frac{N_\mathrm{s}}{2}-1} \sum_{m=1}^{2WT} \frac{\left(\xi_{2,j,m}+\xi_{1,j,m}\right)^2}{4W}$$
$$\approx \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{2N_0} + \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{2N_0} \cos(2\pi f_\mathrm{J} T_\mathrm{r}), \tag{22}$$

and the conditional pdfs of Y_3 and Y_4 are given by

$$f_{Y_3|\{\alpha_l\},\alpha_\mathrm{J}}(y_3) = f_{\mathrm{NC}}(y_3, \mu_{\mathrm{TR},Y_3}^{(\mathrm{NBI})}, q_{\mathrm{TR}}), \tag{23}$$
$$f_{Y_4|\{\alpha_l\},\alpha_\mathrm{J}}(y_4) = f_{\mathrm{NC}}(y_4, \mu_{\mathrm{TR},Y_4}^{(\mathrm{NBI})}, q_{\mathrm{TR}}). \tag{24}$$

Following (17)-(20), the BEP conditioned on $d_0 = -1$ is given by

$$\mathbb{P}\{Z_{\mathrm{TR}} > 0 | d_0 = -1\}$$
$$= P_\mathrm{e}^{(\mathrm{NBI})}\left(\psi_{\mu_{\mathrm{TR},Y_3}^{(\mathrm{NBI})}}(jv), \psi_{\mu_{\mathrm{TR},Y_4}^{(\mathrm{NBI})}}(jv), q_{\mathrm{TR}}\right), \tag{25}$$

where the CFs of $\mu_{\mathrm{TR},Y_3}^{(\mathrm{NBI})}$ and $\mu_{\mathrm{TR},Y_4}^{(\mathrm{NBI})}$ are given by

$$\psi_{\mu_{\mathrm{TR},Y_3}^{(\mathrm{NBI})}}(jv) = \psi_\mathrm{J}\left(jv\left[\frac{N_\mathrm{s} J_0 T}{2N_0} - \frac{N_\mathrm{s} J_0 T}{2N_0}\cos(2\pi f_\mathrm{J} T_\mathrm{r})\right]\right)$$
$$\prod_{l=1}^{L_{\mathrm{CAP}}} \psi_l\left(\frac{jvE_\mathrm{s}}{N_0}\right), \tag{26}$$
$$\psi_{\mu_{\mathrm{TR},Y_4}^{(\mathrm{NBI})}}(jv) = \psi_\mathrm{J}\left(jv\left[\frac{N_\mathrm{s} J_0 T}{2N_0} + \frac{N_\mathrm{s} J_0 T}{2N_0}\cos(2\pi f_\mathrm{J} T_\mathrm{r})\right]\right). \tag{27}$$

Thus, using (18) and (25), the BEP of TR signaling with AcR in the presence of NBI is given by

$$P_{\mathrm{e,TR}}^{(\mathrm{NBI})} = \frac{1}{2}\left[P_\mathrm{e}^{(\mathrm{NBI})}\left(\psi_{\mu_{\mathrm{TR},Y_1}^{(\mathrm{NBI})}}(jv), \psi_{\mu_{\mathrm{TR},Y_2}^{(\mathrm{NBI})}}(jv), q_{\mathrm{TR}}\right)\right.$$
$$\left. + P_\mathrm{e}^{(\mathrm{NBI})}\left(\psi_{\mu_{\mathrm{TR},Y_3}^{(\mathrm{NBI})}}(jv), \psi_{\mu_{\mathrm{TR},Y_4}^{(\mathrm{NBI})}}(jv), q_{\mathrm{TR}}\right)\right]. \tag{28}$$

Note that in the absence of NBI, the form in (18) also gives us an alternative expression for the BEP of TR signaling with AcR, as compared to [1], [4], [5], by substituting $J_0 = 0$ in (13) and (14).

B. Differential Transmitted-Reference

Following the sampling approach and incorporating the NBI in (8), we can rewrite U_j in (21) of Part I [1] as

$$U_j = \frac{1}{W}\sum_{m=1}^{2WT}\left[d_0 w_{j,m}^2 + e_{-1} w_{j,m}(\xi_{2,j,m}+\eta_{2,j,m})\right.$$
$$+ e_0 w_{j,m}(\xi_{1,j,m}+\eta_{1,j,m})$$
$$\left. + (\xi_{1,j,m}+\eta_{1,j,m})(\xi_{2,j,m}+\eta_{2,j,m})\right], \tag{29}$$

where $\xi_{1,j,m}$ and $\xi_{2,j,m}$ are respectively the m-th samples of $\xi_{1,j}(t) \triangleq J(t+jT_\mathrm{f}+c_jT_\mathrm{p}-N_\mathrm{s}T_\mathrm{f})$ and $\xi_{2,j}(t) \triangleq J(t+jT_\mathrm{f}+c_jT_\mathrm{p})$ in the interval $[0,T]$, and the rest of the terms in (29) are defined similarly as in (21) of Part I [1].

Conditioned on d_0, we can rewrite (29) in the form of (12), where $\beta_{1,j,m} = \frac{1}{2\sqrt{W}}(e_{-1}\eta_{2,j,m} + e_{-1}\xi_{2,j,m} + e_0\eta_{1,j,m} + e_0\xi_{1,j,m})$, $\beta_{2,j,m} = \frac{1}{2\sqrt{W}}(e_{-1}\eta_{2,j,m} + e_{-1}\xi_{2,j,m} - e_0\eta_{1,j,m} - e_0\xi_{1,j,m})$. Conditioning on $d_0 = +1$, the conditional variance σ_{DTR}^2 of $\beta_{1,j,m}$ and $\beta_{1,j,m}$ is $\frac{N_0}{4}$. Following Appendix I, the non-centrality parameters of Y_1 and Y_2 in (14) of Part I [1] conditioned on θ, α_J, $\{c_j\}$, $\{a_j\}$ and the channel are given by

$$\mu_{\mathrm{DTR},Y_1}^{(\mathrm{NBI})} \approx \frac{2E_\mathrm{s}}{N_0}\sum_{l=1}^{L_{\mathrm{CAP}}}\alpha_l^2 + \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{N_0}$$
$$+ \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{N_0}\cos(2\pi f_\mathrm{J} N_\mathrm{s} T_\mathrm{f}),$$
$$\mu_{\mathrm{DTR},Y_2}^{(\mathrm{NBI})} \approx \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{N_0} - \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{N_0}\cos(2\pi f_\mathrm{J} N_\mathrm{s} T_\mathrm{f}). \tag{30}$$

Similarly, the non-centrality parameters of Y_3 and Y_4 in (14) of Part I [1] conditioned on θ, α_J, $\{c_j\}$, $\{a_j\}$ and the channel are given by

$$\mu_{\mathrm{DTR},Y_3}^{(\mathrm{NBI})} \approx \frac{2E_\mathrm{s}}{N_0}\sum_{l=1}^{L_{\mathrm{CAP}}}\alpha_l^2 + \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{N_0}$$
$$- \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{N_0}\cos(2\pi f_\mathrm{J} N_\mathrm{s} T_\mathrm{f}),$$
$$\mu_{\mathrm{DTR},Y_4}^{(\mathrm{NBI})} \approx \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{N_0} + \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{N_0}\cos(2\pi f_\mathrm{J} N_\mathrm{s} T_\mathrm{f}), \tag{31}$$

where the above derivations follow straightforwardly from Appendix I. Unlike the non-centrality parameters of TR signaling, the non-centrality parameters of DTR signaling in (30) and (31) depend on f_J, N_s and T_f since the time separation between the two pulses is now $N_\mathrm{s}T_\mathrm{f}$, instead of T_r.

The BEP of DTR signaling with AcR in the presence of NBI is then given by

$$P_{\mathrm{e,DTR}}^{(\mathrm{NBI})} = \frac{1}{2}\left[P_\mathrm{e}^{(\mathrm{NBI})}\left(\psi_{\mu_{\mathrm{DTR},Y_1}^{(\mathrm{NBI})}}(jv), \psi_{\mu_{\mathrm{DTR},Y_2}^{(\mathrm{NBI})}}(jv), q_{\mathrm{DTR}}\right)\right.$$
$$\left. + P_\mathrm{e}^{(\mathrm{NBI})}\left(\psi_{\mu_{\mathrm{DTR},Y_3}^{(\mathrm{NBI})}}(jv), \psi_{\mu_{\mathrm{DTR},Y_4}^{(\mathrm{NBI})}}(jv), q_{\mathrm{DTR}}\right)\right], \tag{32}$$

where $q_{\mathrm{DTR}} = N_\mathrm{s}WT$.

IV. NUMERICAL RESULTS

In this section, we evaluate the numerical results of both TR and DTR signaling with NBI, based on the analytical results developed in previous sections. We consider pulse duration $T_\mathrm{p} = 0.5$ ns, average repetition period $T_\mathrm{f} = 100$ ns, and $N_\mathrm{s} = 16$. We consider a TH sequence of all ones, i.e., $c_j = 1$ for all j. The NBI carrier frequency is $f_\mathrm{J} = 2.45$ GHz. Since the NBI experiences a flat Rayleigh fading, the CF of α_J^2 is then given by $\psi_\mathrm{J}(jv) = 1/(1-jv)$. For UWB channels, we consider a dense resolvable multipath channel, where each multipath gain is Nakagami distributed with fading severity index m_l, and average power $\mathbb{E}\left\{\alpha_l^2\right\}$ [15]. Under uncorrelated scattering assumption, $\{\alpha_l^2\}$ are statistically independent and

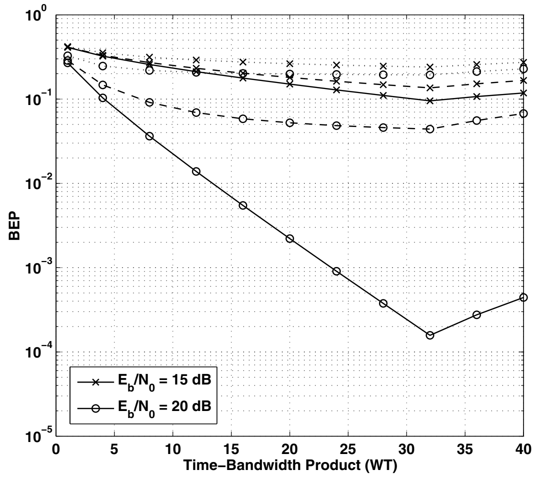

Fig. 1. Effect of integration interval T of AcR on performance of TR signaling when $(L, \epsilon, m) = (32, 0, 3)$. The solid, dashed and dotted lines indicate no NBI, NBI with SIR -5 dB and NBI with SIR -10 dB respectively.

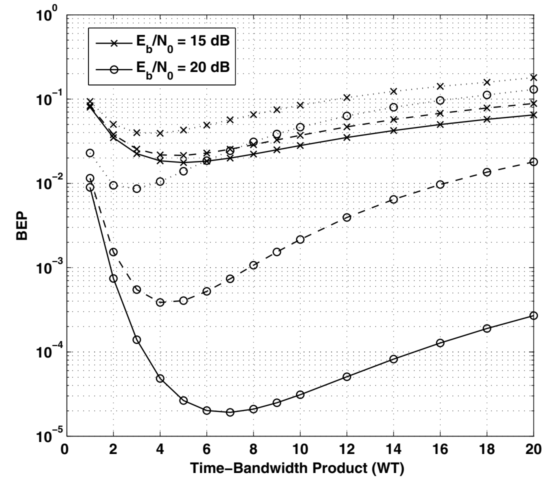

Fig. 2. Effect of integration interval T of AcR on performance of TR signaling when $(L, \epsilon, m) = (32, 0.4, 3)$. The solid, dashed and dotted lines indicate no NBI, NBI with SIR -5 dB and NBI with SIR -10 dB respectively.

the CF of α_l^2 is given by $\psi_l(jv) = 1/(1 - jv\mathbb{E}\left\{\alpha_l^2\right\}/m)^m$. where $\mathbb{E}\left\{\alpha_l^2\right\} = \mathbb{E}\left\{\alpha_1^2\right\}\exp\left[-\epsilon(l-1)\right]$, for $l = 1, \ldots, L$ and $\mathbb{E}\left\{\alpha_l^2\right\}$ is normalized such that $\sum_{l=1}^{L}\mathbb{E}\left\{\alpha_l^2\right\} = 1$. Note that we have assumed that the fading severity index m is identical for all faded paths for simplicity. The average power of the first arriving multipath component is given by $\mathbb{E}\left\{\alpha_1^2\right\}$ and ϵ is the power decay factor. With this model, we consider two set of parameters, $(L, \epsilon, m) = (32, 0, 3)$ for uniform PDP and $(32, 0.4, 3)$ for exponential PDP.

To understand the effects of NBI and channel PDP on the choice of integration interval T of the AcR, we first consider the BEP performance of TR signaling for different values of time-bandwidth product, WT with $(L, \epsilon, m) = (32, 0, 3)$. As shown in Fig. 1, we observe that with this PDP, the optimum T is always equal to $T_{\rm g}$ (i.e. $WT = L = 32$), regardless of the presence of NBI. However, the loss in performance when T is not at optimum point, is much greater at high $E_{\rm b}/N_0$ and NBI free cases. This can be explained by the fact that at high $E_{\rm b}/N_0$ and low SIR values, the loss due to noise and interference accumulation is less than the gain of capturing more multipath energy as WT increases. Note that this trade-off also depends on the PDP of the channel as illustrated in Fig. 2 using $(L, \epsilon, m) = (32, 0.4, 3)$. It can be seen in Fig. 2 that the optimum T is no longer at $T_{\rm g}$, since the gain in collecting more residual multipath energies inherent in the channel with exponential PDP is not sufficient to compensate the noise accumulation beyond the optimum point. Moreover, we observe that the optimum T increases as $E_{\rm b}/N_0$ and SIR increase due to decrease in noise and interference accumulation. Therefore, in general, the optimum T depends on the PDP used, the operating $E_{\rm b}/N_0$, and the SIR.

In Fig. 3, we compare the NBI sensitivity of TR and DTR signaling with AcR for different SIR values when the optimum T is chosen for each $E_{\rm b}/N_0$ and SIR. It is particularly interesting to observe that DTR signaling is less robust against NBI compared to TR signaling. In the absence of NBI, DTR signaling has a gain of about 2 dB compared to TR signaling as expected. However, this gain diminishes as SIR decreases and after certain crossing points, DTR signaling performs worse than TR signaling as indicated by the error floor. This is because interference is more severe in DTR signaling due to presence of more noise and interference terms, although a doubling of the received useful multipath energies is present. Essentially the presence of more interference terms outweigh the gain in signal energy compared to TR signaling as indicated by the crossing points.

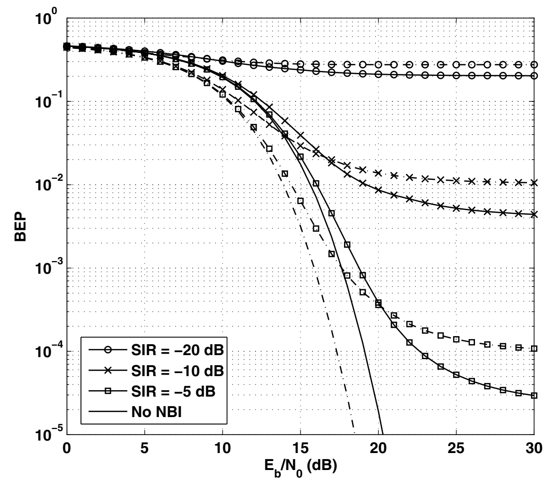

Fig. 3. Effect of NBI on BEP performance of TR and DTR signaling when $(L, \epsilon, m) = (32, 0.4, 3)$ and optimum T for each SNR and SIR is chosen. The solid and dashed lines indicate the TR and DTR signaling respectively.

V. Conclusions

In this paper, we extended our BEP analysis of TR and DTR signaling with AcR to take into account the effect of NBI. We quantified the effect of NBI and channel PDP on the optimum integration interval of AcR. Unlike the NBI-free situation, the optimum integration interval of AcR is not necessarily equal to the delay spread, and depends on the SIR, the SNR and the channel PDP. Numerical results show that DTR signaling performs worse than TR signaling from the NBI robustness point of view.

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Appendix I
Derivation of $\mu_{\mathrm{TR},Y_1}^{(\mathrm{NBI})}$ and $\mu_{\mathrm{TR},Y_2}^{(\mathrm{NBI})}$

In this appendix, the non-centrality parameters in (13) and (14) are derived. First, we begin by deriving $\mu_{\mathrm{TR},Y_1}^{(\mathrm{NBI})}$ as follows

$$\begin{aligned}\mu_{\mathrm{TR},Y_1}^{(\mathrm{NBI})} &= \frac{E_\mathrm{s}}{N_0}\sum_{l=1}^{L_\mathrm{CAP}}\alpha_l^2 + \frac{1}{2N_0}\sum_{j=0}^{\frac{N_\mathrm{s}}{2}-1}\sum_{m=1}^{2WT}\frac{(\xi_{1,j,m}+\xi_{2,j,m})^2}{W}\\ &\quad + \frac{1}{2\sigma_{\mathrm{TR}}^2}\sum_{j=0}^{\frac{N_\mathrm{s}}{2}-1}\int_0^T w_j(t)(\xi_{1,j}(t)+\xi_{2,j}(t))dt\\ &\triangleq \mu_\mathrm{A}+\mu_\mathrm{B}+\mu_\mathrm{C},\end{aligned} \tag{33}$$

and we can simplify μ_B in (33) as

$$\mu_\mathrm{B} \approx \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{2N_0} + \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{2N_0}\cos(2\pi f_\mathrm{J} T_\mathrm{r}), \tag{34}$$

where we have made the above approximations by observing the fact that $T \gg \frac{1}{4\pi f_\mathrm{J}}$ and $|\sin(\phi)| \leq 1$. After some manipulation, we can rewrite μ_C in (33) as

$$\begin{aligned}\mu_\mathrm{C} = &\frac{4\alpha_\mathrm{J}|\widehat{P}(f_\mathrm{J})|\sqrt{2E_\mathrm{p}J_0}\cos(\pi f_\mathrm{J} T_\mathrm{r})}{N_0}\sum_{j=0}^{\frac{N_\mathrm{s}}{2}-1}a_j\sum_{l=1}^{L_\mathrm{CAP}}|\alpha_l|\\ &\times\left[\cos\left(2\pi f_\mathrm{J}\left(\tau_l + j2T_\mathrm{f} + c_j T_\mathrm{p} + (T_\mathrm{r}/2)\right) + \varphi_l\right)\right],\end{aligned} \tag{35}$$

where $\widehat{P}(f_\mathrm{J})$ and $\arg\widehat{P}(f_\mathrm{J})$ are the frequency and phase responses of $p(t)$ at frequency f_J, $\varphi_l \triangleq \arg\widehat{P}(f_\mathrm{J})+\theta-\phi_l$ and is uniformly distributed over $[0,2\pi)$. Following (34), $\mu_{\mathrm{TR},Y_2}^{(\mathrm{NBI})}$ in (14) can also be approximated as

$$\mu_{\mathrm{TR},Y_2}^{(\mathrm{NBI})} \approx \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{2N_0} - \frac{\alpha_\mathrm{J}^2 N_\mathrm{s} J_0 T}{2N_0}\cos(2\pi f_\mathrm{J} T_\mathrm{r}). \tag{36}$$

In order to obtain $\mathbb{P}\{Z_\mathrm{TR} < 0 | d_0 = +1\}$, we can resort to a quasi-analytical approach by substituting (33)-(36) into (17) and then numerically averaging (17) with respect to $\{\alpha_l\}$ and α_J. However, under some cases, we can also obtain analytical BEP expression for $\mathbb{P}\{Z_\mathrm{TR} < 0 | d_0 = +1\}$ as shown in (18) when the last term in (33) is negligible compared to the first two terms. For example, an extensive numerical simulation campaign has been carried out for a typical set of system parameters ($T_\mathrm{f} = 100$ns, $T_\mathrm{p} = 0.5$ns, $T_\mathrm{r} = 40$ns, possible $f_\mathrm{J} = 5.745$ GHz, 1.575 GHz, and 3.5 GHz) by considering $c_j = 1$ for all j, Walsh-Hadamard codes for $\{a_j\}$, and by averaging over random variables α_l, and α_J (a uniform power delay profile with Rayleigh distributed path amplitudes has been considered for the useful signal channel model as a particular "bad" case of the Nakagami distribution). Results have shown that we can assume that $\mu_\mathrm{A} + \mu_\mathrm{B} \gg \mu_\mathrm{C}$ for SIR $\geq$ -20 dB and, thus, we can ignore μ_C in (33) to obtain (13). Note that similar approximations have also been made to obtain (21) and (22).

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