UWB Transmitted Reference Signaling Schemes - Part I: Performance Analysis

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Abstract— **Transmitted-Reference (TR) signaling, in conjunction with an autocorrelation receiver (AcR), offers a low-complexity alternative to Rake reception. Due to its simplicity, there is renewed interest in TR signaling for ultrawide bandwidth (UWB) systems. Different variations of TR signaling have been proposed and investigated, including differential TR (DTR) signaling and noise averaging at the AcR. This paper provides performance analysis of various TR schemes by developing an analytical framework based on the sampling expansion approach. Specifically, we derive uncoded bit error probability (BEP) of different TR signaling schemes such as DTR and noise averaging at the AcR for a broad class of fading channels.**

I. INTRODUCTION

Recently, there has been renewed interest in utilizing ultrawide bandwidth (UWB) spread-spectrum communications for future military, homeland security and commercial applications. UWB systems involve the transmission of a train of extremely narrow pulses by employing either time-hopping (TH) or direct sequence (DS) techniques for multiple access and pulse position modulation (PPM) or pulse amplitude modulation (PAM) for data transmission [1]. The key motivation for using UWB systems is the ability to highly resolve multipath components, as well as the availability of technology to implement and generate UWB signals with relatively low complexity. These fine delay resolution properties make UWB radio a viable candidate for communications in dense multipath environments such as short-range or indoor wireless communications [2]–[4].

This paper considers a signaling scheme referred to as transmitted-reference (TR) signaling [5]–[7]. TR signaling involves the transmission of a reference and data signal pair, separated either in time [6], [7] or in frequency [8]. In order for this pair of separated signals to experience the same channel, either the time separation must be less than the channel coherence time, or the frequency separation must be less than the channel coherence bandwidth. The receiver can simply be an autocorrelation receiver (AcR), which may also be modified to include noise averaging for better performance [9]–[12]. Since TR signaling allocates a significant part of the symbol energy by transmitting reference pulses, differential encoding over consecutive symbols can also be used to alleviate this communication resource wastage problem. This alternative TR signaling is referred to as differential transmitted-reference

(DTR) signaling [10], [13]. The performance analysis of TR and DTR signaling published in literature is based on averaging numerically the conditional bit error probability (BEP) via a quasi-analytical/simulation approach [10] or quasi analytical/experimental approach [9], whose results rely on either simulation parameters or experimental data. Therefore, there is a need to provide an analytical tool to compare TR and DTR signaling schemes. We aim to provide an analytical framework to derive BEP of these schemes in dense resolvable multipath channels. We adopt the sampling expansion approach, which is an extension of our previous work [11], [12]. We derive the BEP for TR and DTR signaling when AcR or modified AcR is used for a broad class of fading channels in dense resolvable multipath channels.

The remainder of the paper is organized as follows. Section II presents the system and channel models for both TR and DTR signaling schemes of a single link system. In Section III, a unified BEP analysis of TR and DTR signaling based on the sampling expansion approach is developed. To illustrate our proposed methodology, we consider Nakagami-m fading channels and present numerical results in Section IV. Finally, Section V comprises concluding remarks.

II. SIGNAL AND CHANNEL MODELS

A. Transmitted-Reference

In TR signaling, the transmitted signal for a single user can be decomposed into a reference signal block $b_\mathrm{r}(t)$ and a data modulated signal block $b_\mathrm{d}(t)$ as given by

$$s_\mathrm{TR}(t) = \sum_i b_\mathrm{r}(t - iN_\mathrm{s}T_\mathrm{f}) + d_i b_\mathrm{d}(t - iN_\mathrm{s}T_\mathrm{f}), \qquad (1)$$

where T_f is the average repetition period, $d_i \in \{-1, 1\}$ is the data symbol, and each block has symbol duration $N_\mathrm{s}T_\mathrm{f}$ [9]–[12] as shown in Fig. 1. Each block, containing $N_\mathrm{s}/2$ transmitted signal pulses, can be written as[1]

$$b_\mathrm{r}(t) = \sum_{j=0}^{\frac{N_\mathrm{s}}{2}-1} \sqrt{E_\mathrm{p}} a_j p(t - j2T_\mathrm{f} - c_j T_\mathrm{p}),$$

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[1]Note that other combination of data and reference pulses is also possible. Here, without loss of generality, we have adopted the conventional TR signaling where the number of reference and data pulses are equal for simplicity [9].
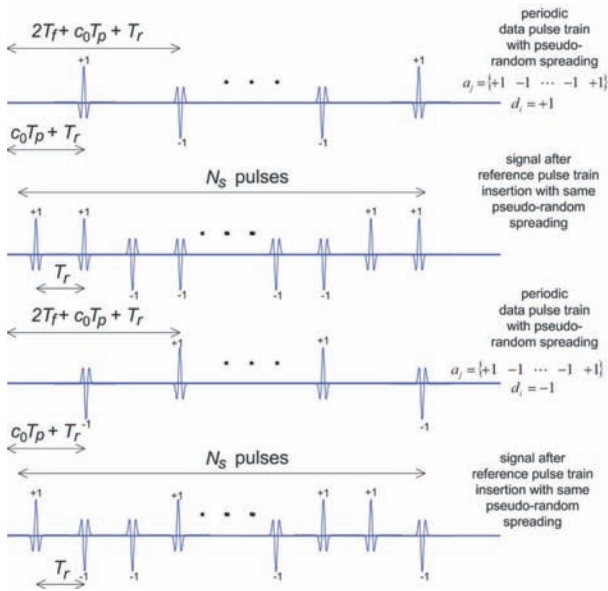
Fig. 1. TR signaling.

$$b_{\mathrm{d}}(t) = \sum_{j=0}^{\frac{N_{\mathrm{s}}}{2}-1} \sqrt{E_{\mathrm{p}}} a_j p(t - j2T_{\mathrm{f}} - c_j T_{\mathrm{p}} - T_{\mathrm{r}}), \qquad (2)$$

where $p(t)$ is the normalized signal pulse with duration T_{p} and $\int_{-\infty}^{+\infty} |p(t)|^2 dt = 1$. The energy of the transmitted pulse is then $E_{\mathrm{p}} = E_{\mathrm{s}}/N_{\mathrm{s}}$, and symbol energy is E_{s}. In our case of binary signaling, $E_{\mathrm{s}} = E_{\mathrm{b}}$, where E_{b} is the energy per bit. To enhance the robustness of TR systems to interference as well as to allow multiple access, DS and/or TH spread spectrum techniques can be used as shown in (2). In DS signaling, $\{a_j\}$ is the bipolar pseudo-random sequence. In TH signaling, $\{c_j\}$ is the pseudo-random TH sequence, where c_j is an integer in the range $0 \leq c_j < N_{\mathrm{h}}$, and N_{h} is the maximum allowable integer shift. The duration of the received UWB pulse is $T_{\mathrm{g}} = T_{\mathrm{p}} + T_{\mathrm{d}}$, where T_{d} is the maximum excess delay of the channel. To preclude inter-symbol interference (ISI) and intra-symbol interference (i.s.i.)[2], we assume that $T_{\mathrm{r}} \geq T_{\mathrm{g}}$ and $N_{\mathrm{h}}T_{\mathrm{p}} + T_{\mathrm{r}} \leq 2T_{\mathrm{f}} - T_{\mathrm{g}}$, where T_{r} is the time separation between each pair of data and reference pulses such that these received pulses will not overlap. Note that the inter-pulse delay between each data-modulated monocycle and its corresponding reference monocycle is given by T_{r}.

B. Differential Transmitted-Reference

In DTR signaling, the transmitted signal for a single user is given by

$$s_{\mathrm{DTR}}(t) = \sum_i e_i b(t - iN_{\mathrm{s}}T_{\mathrm{f}}), \qquad (3)$$

where $b(t)$ is the block-modulated signal with symbol duration $N_{\mathrm{s}}T_{\mathrm{f}}$. The data symbol d_i is now differentially encoded such that $e_i = e_{i-1}d_i$, where $d_i = \pm 1$. Within each $b(t)$-shaped

block, there are N_{s} transmitted signal pulses and it can be written as

$$b(t) = \sum_{j=0}^{N_{\mathrm{s}}-1} \sqrt{E_{\mathrm{p}}} a_j p(t - jT_{\mathrm{f}} - c_j T_{\mathrm{p}}), \qquad (4)$$

where $\{a_j\}$ and $\{c_j\}$ are the DS and TH sequences that provide the multiple access capability of DTR systems. The length of $\{a_j\}$ is now N_{s}. The TH sequence is pseudo-random with the range $0 \leq c_j < N_{\mathrm{h}}$, where N_{h} satisfies $T_{\mathrm{f}} \geq (N_{\mathrm{h}} - 1)T_{\mathrm{p}} + T_{\mathrm{g}}$ to preclude ISI and i.s.i. The channel is assumed to be constant over two symbols in order to use differential encoding over every two symbols.[3]

C. Channel model

The received signal for TR signaling is $r(t) = (h * s_{\mathrm{TR}})(t) + n(t)$, where $h(t)$ is the impulse response of the channel and $n(t)$ is zero-mean, white Gaussian noise with two-sided power spectral density $N_0/2$. Note that similar equation also applies to DTR signaling by replacing $s_{\mathrm{TR}}(t)$ with $s_{\mathrm{DTR}}(t)$. The channel impulse response being modelled as linearly time-invariant can be written as $h(t) = \sum_{l=1}^{L} \alpha_l \delta(t - \tau_l)$ where α_l and τ_l denote respectively the attenuation and delay of l-TH path, and L is the number of resolvable multipath components. We can also express $\alpha_l = |\alpha_l| \exp(j\phi_l)$, where $\phi_l = 0$ or π with equal probability. As in [2], [3], we consider the resolvable channel, i.e., $|\tau_l - \tau_j| \geq T_{\mathrm{p}}, \forall l \neq j$, where $\tau_l = \tau_1 + (l-1)T_{\mathrm{p}}$. However, depending on whether the UWB channel is uncorrelated scattering [3] or correlated scattering [4], $\{\alpha_l\}$ can be statistically independent or correlated random variables (r.v.'s).
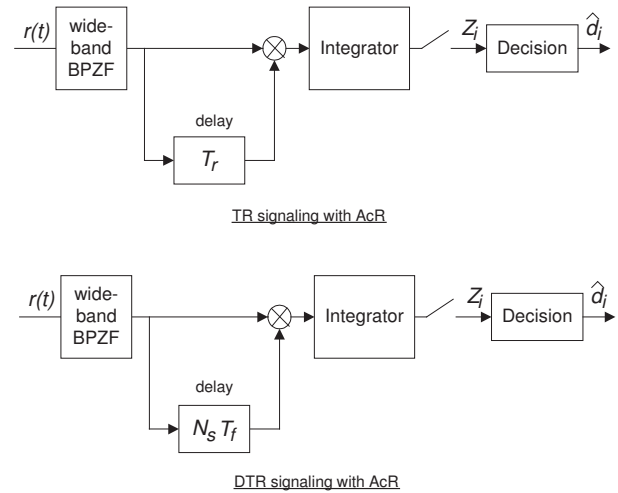

Fig. 2. AcR for TR and DTR signaling.

III. RECEIVER MODEL

A. Autocorrelation Receiver

As shown in Fig. 2, the AcR first passes the received signal through an ideal bandpass zonal filter (BPZF), with

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[2]ISI and i.s.i. may not always be negligible due to constraints on T_{f} and data-rate requirements. In this case, our results will serve as a lower bound.

[3]We can relax this assumption by encoding the data differentially across pairs of pulses.

bandwidth W and center frequency f_c around the signal band to eliminate the out-of-band noise. If W is wide enough, then the signal spectrum will pass undistorted through the BPZF. Consequently, the ISI and i.s.i. caused by filtering will be negligible. The filtered received signal is then passed through a correlator with integration interval T ($T_p \leq T \leq T_g$), as shown in Fig. 2, to collect the received signal energy. The integration interval T determines the number of multipath components (or equivalently, the amount of energy) captured by the receiver as well as the amount of noise and interference accumulation. Without loss of generality, we consider the detection of the data symbol at $i = 0$. In addition, we assume perfect synchronization at the receiver. The decision statistics generated at the AcR for TR and DTR signaling are given respectively by

$$Z_{\mathrm{TR}} = \sum_{j=0}^{\frac{N_s}{2}-1} \int_{j2T_f+T_r+c_jT_p}^{j2T_f+T_r+c_jT_p+T} \widetilde{r}_{\mathrm{TR}}(t)\,\widetilde{r}_{\mathrm{TR}}(t-T_r)dt, \quad (5)$$

and

$$Z_{\mathrm{DTR}} = \sum_{j=0}^{N_s-1} \int_{jT_f+c_jT_p}^{jT_f+c_jT_p+T} \widetilde{r}_{\mathrm{DTR}}(t)\,\widetilde{r}_{\mathrm{DTR}}(t-N_sT_f)dt, \quad (6)$$

where $\widetilde{r}_{\mathrm{TR}}(t) = (h * s_{\mathrm{TR}})(t) + \widetilde{n}(t)$ and $\widetilde{r}_{\mathrm{DTR}}(t) = (h * s_{\mathrm{DTR}})(t) + \widetilde{n}(t)$. Note that $\widetilde{n}(t)$ is a zero-mean, Gaussian random process with autocorrelation function

$$R_{\widetilde{n}}(\tau) = WN_0\,\mathrm{sinc}(W\tau)\cos(2\pi f_c\tau). \quad (7)$$

When $W \gg 1/T_g$, $R_{\widetilde{n}}(t-u)$ in (7) is approximately equal to zero for $|t-u| \geq T_g$. Hence, the noise samples separated by more than T_g or a multiple of $1/W$ can assume to be statistically independent.

B. Modified Autocorrelation Receiver

The AcR performance can be improved for TR and DTR signaling by averaging respectively over $N_s/2$ and N_s received reference pulses from the previous symbol respectively, to be used as an estimate of the channel [9]–[12]. This approach requires that the channel remains constant over two symbols. The decision statistics of this modified AcR for TR and DTR signaling are given respectively by

$$Z_{\mathrm{ATR}} = \sum_{j=0}^{\frac{N_s}{2}-1} a_j \int_{j2T_f+T_r+c_jT_p}^{j2T_f+T_r+c_jT_p+T} \widetilde{r}_{\mathrm{TR}}(t)$$
$$\left(\frac{2}{N_s} \sum_{k=-j}^{\frac{N_s}{2}-1-j} a_{j+k}\widetilde{r}_{\mathrm{TR}}(t-(N_s-2k)T_f) \right.$$
$$\left. - (c_j - c_{j+k})T_p - T_r) \right)dt, \quad (8)$$

and

$$Z_{\mathrm{ADTR}} = \sum_{j=0}^{N_s-1} a_j \int_{jT_f+c_jT_p}^{jT_f+c_jT_p+T} \widetilde{r}_{\mathrm{DTR}}(t)$$
$$\left(\frac{1}{N_s} \sum_{k=-j}^{N_s-1-j} a_{j+k}\widetilde{r}_{\mathrm{DTR}}(t-(N_s-k)T_f) \right.$$
$$\left. - (c_j - c_{j+k})T_p) \right)dt. \quad (9)$$

Next, we develop an analytical framework based on the sampling expansion approach to provide a unified performance analysis of TR and DTR systems in dense multipath channels [11], [12]. Our methodology does not adopt the Gaussian approximation that is commonly used to derive the conditional BEP in terms of the Gaussian Q-function. The methodology still allows us to obtain BEP of TR and DTR systems for a broad class of fading channels.

IV. PERFORMANCE ANALYSIS

A. Transmitted-Reference

It can be shown that Z_{TR} in (5) can be written as [11], [12]

$$Z_{\mathrm{TR}}$$
$$= \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T \left(\breve{b}_r(t+j2T_f+c_jT_p) + \widetilde{n}(t+j2T_f+c_jT_p) \right)$$
$$\times \left(d_0\breve{b}_d(t+j2T_f+c_jT_p+T_r) \right.$$
$$\left. + \widetilde{n}(t+j2T_f+c_jT_p+T_r) \right)dt, \quad (10)$$

where $\breve{b}_r(t) \triangleq (b_r * h * h_{\mathrm{ZF}})(t)$, $\breve{b}_d(t) \triangleq (b_d * h * h_{\mathrm{ZF}})(t)$, and $h_{\mathrm{ZF}}(t)$ is the impulse response of the BPZF. Note that if the symbol duration is less than the coherence time, all pairs of separated pulses will experience the same channel, implying that $\breve{b}_r(t+j2T_f+c_jT_p) = \breve{b}_d(t+j2T_f+c_jT_p+T_r)$ for all $t \in (0, T)$ and c_j. In this case, we can significantly simplify the expression in (10) as follows:

$$Z_{\mathrm{TR}} = \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T (w_j(t) + \eta_{1,j}(t))(d_0w_j(t) + \eta_{2,j}(t))dt$$
$$= \sum_{j=0}^{\frac{N_s}{2}-1} U_j, \quad (11)$$

where we have used $w_j(t) \triangleq \breve{b}_r(t+j2T_f+c_jT_p) = \sqrt{E_p}a_j\sum_{l=1}^L \alpha_l p(t-\tau_l)$, $\eta_{1,j}(t) \triangleq \widetilde{n}(t+j2T_f+c_jT_p)$ and $\eta_{2,j}(t) \triangleq \widetilde{n}(t+j2T_f+c_jT_p+T_r)$ defined over the interval $[0, T]$.

Since the received signal is a real bandpass signal of bandwidth W, one can think of it in terms of its complex baseband equivalent model. In this case, the signal is complex and bandlimited to $W/2$. The Sampling Theorem then states that this must be sampled at a sampling frequency greater than or equal to W. This gives WT complex dimensions or $2WT$

real dimensions [14]. Following this sampling approach [11], [12], we can then represent U_j as

$$U_j = \frac{1}{W} \sum_{m=1}^{2WT} \left(d_0 w_{j,m}^2 + w_{j,m} \eta_{2,j,m} + d_0 w_{j,m} \eta_{1,j,m} \right.$$
$$\left. + \eta_{1,j,m} \eta_{2,j,m} \right), \qquad (12)$$

where the m-th sample of $w_j(t)$, $\eta_{1,j}(t)$ and $\eta_{2,j}(t)$ in (11) are respectively $w_{j,m}$, $\eta_{1,j,m}$, and $\eta_{2,j,m}$ in the interval $[0, T]$. Note that since the noise samples are taken at least T_g apart, they are essentially independent, regardless of c_j. Hence, no further assumption on c_j is required in our analysis. We further observe that U_j is simply the integrator output of the j-th received modulated monocycle.

Due to the statistical symmetry of U_j with respect to d_0, we simply need to calculate the BEP conditioned on $d_0 = +1$. Hence, conditioned on d_0, we can express (12) in the form of a summation of squares

$$U_{j|d_0=+1} = \sum_{m=1}^{2WT} \left[\left(\frac{1}{\sqrt{W}} w_{j,m} + \beta_{1,j,m} \right)^2 - \beta_{2,j,m}^2 \right],$$
$$U_{j|d_0=-1} = \sum_{m=1}^{2WT} \left[-\left(\frac{1}{\sqrt{W}} w_{j,m} - \beta_{2,j,m} \right)^2 + \beta_{1,j,m}^2 \right], \qquad (13)$$

where $\beta_{1,j,m} = \frac{1}{2\sqrt{W}}(\eta_{2,j,m} + \eta_{1,j,m})$, $\beta_{2,j,m} = \frac{1}{2\sqrt{W}}(\eta_{2,j,m} - \eta_{1,j,m})$, and these are statistically independent Gaussian r.v.'s with variance $\sigma_{TR}^2 = \frac{N_0}{4}$. Recall from the definition of $w_j(t)$, the contribution of the sequence $\{a_j\}$ is embedded inside $w_{j,m}$. From (13), we can observe that when conditioned on the channel, $U_{j|d_0=+1}$ has the same probability density functions (pdfs) for $a_j = +1$ and $a_j = -1$. Due to this symmetry and a_j being equally probable, we can assume that $a_j = +1$ in the following analysis without loss of generality. For notational simplicity, we define the normalized r.v.'s Y_1, Y_2, Y_3, and Y_4 as

$$Y_1 \triangleq \frac{1}{2\sigma_{TR}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} (\frac{1}{\sqrt{W}} w_{j,m} + \beta_{1,j,m})^2,$$

$$Y_2 \triangleq \frac{1}{2\sigma_{TR}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \beta_{2,j,m}^2,$$

$$Y_3 \triangleq \frac{1}{2\sigma_{TR}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} (\frac{1}{\sqrt{W}} w_{j,m} - \beta_{2,j,m})^2,$$

$$Y_4 \triangleq \frac{1}{2\sigma_{TR}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \sum_{m=1}^{2WT} \beta_{1,j,m}^2. \qquad (14)$$

Conditioned on the channel, Y_1 and Y_3 are noncentral chi-squared r.v.'s with $N_s WT$ degrees of freedom, whereas Y_2 and Y_4 are central chi-squared r.v.'s with the same degrees of freedom as Y_1 and Y_3. Both Y_1 and Y_3 have the same non-

centrality parameter given by

$$\mu_{TR} = \frac{1}{2\sigma_{TR}^2} \sum_{j=0}^{\frac{N_s}{2}-1} \int_0^T w_j^2(t) dt = \frac{E_s}{N_0} \sum_{l=1}^{L_{CAP}} \alpha_l^2, \qquad (15)$$

where $L_{CAP} \triangleq \lceil \min\{WT, WT_g\} \rceil$ denotes the actual number of multipath components captured by the AcR. Note that $\gamma_{TR} = \mu_{TR}/2$ is the instantaneous received SNR of TR signaling with AcR [11], [12]. The pdfs of Y_1 and Y_2 conditioned on γ_{TR} are given by

$$f_{Y_1|\gamma_{TR}}(y_1) = f_{NC}(y_1, \mu_{TR}, q_{TR}), \qquad (16)$$
$$f_{Y_2|\gamma_{TR}}(y_2) = f_C(y_2, q_{TR}), \qquad (17)$$

where $q_{TR} = \frac{N_s WT}{2}$. We have defined the following pdfs for notational convenience

$$f_{NC}(y, \mu, n) \triangleq e^{-(y+\mu)} \left(\frac{y}{\mu} \right)^{\frac{(n-1)}{2}} I_{n-1}\left(2\sqrt{y\mu} \right), \quad y \geq 0$$

$$f_C(y, n) \triangleq \frac{y^{(n-1)}}{(n-1)!} \exp\left(-y \right), \qquad y \geq 0$$

where $I_{n-1}(\cdot)$ is the $(n-1)$-th order Bessel function of the first kind, and $f_{NC}(y, \mu, n)$ and $f_C(y, n)$ are respectively the pdfs of the noncentral and central chi-squared r.v.'s with $2n$ degrees of freedom and non-centrality parameter μ [15]. Using (16) and (17), the BEP of TR signaling with AcR is given by

$$P_{e,TR} = \mathbb{P}\{Z_{TR} \leq 0 | d_0 = +1\}$$
$$= \mathbb{E}_{\gamma_{TR}}\{\mathbb{P}\{Y_1 < Y_2 | d_0 = +1\}\}$$
$$= \frac{1}{2^{q_{TR}}} \left[\sum_{i=0}^{q_{TR}-1} \left(\frac{(-j)^i}{i!} \right) \frac{d^i}{dv^i} \psi_{\gamma_{TR}}(jv) \Big|_{jv=-1} \right.$$
$$\left. \sum_{k=i}^{q_{TR}-1} \frac{1}{2^k} \frac{(k+q_{TR}-1)!}{(k-i)!(q_{TR}+i-1)!} \right]$$
$$\triangleq P_e(\psi_{\gamma_{TR}}(jv), q_{TR}), \qquad (18)$$

where $\psi_{\gamma_{TR}}(jv) \triangleq \mathbb{E}\{e^{jv\gamma_{TR}}\}$ is the characteristic function (CF) of γ_{TR} and $P_e(\psi_{\gamma_{TR}}(jv), q_{TR})$ is defined for convenience. Under the resolvable multipath and uncorrelated scattering assumption [3], multipath components are statistically independent and $\psi_{\gamma_{TR}}(jv) = \prod_{l=1}^{L_{CAP}} \psi_l(\frac{E_s}{2N_0} jv)$, where $\psi_l(jv)$ is the CF of α_l^2 and it is known in closed form for a wide range of channel fading statistics [15]. When the uncorrelated scattering assumption is no longer valid, multipath components are correlated [4]. In this case, the CF of γ_{TR} can then be found by using eigenvalue decomposition and partial fraction expansion [16], [17]. The detailed derivation of (18) can be found in [11], [12].

Next, we extend the above analysis to derive the BEP of TR signaling with modified AcR [11], [12]. The non-centrality parameter of Y_1 in (14) is given by

$$\mu_{ATR} = \frac{2}{1 + \frac{2}{N_s}} \left(\frac{E_s}{N_0} \sum_{l=1}^{L_{CAP}} \alpha_l^2 \right). \qquad (19)$$

As shown in [11], [12], the BEP of TR signaling with modified AcR becomes

$$P_{e,\text{ATR}} = P_e(\psi_{\gamma_{\text{ATR}}}(jv), q_{\text{TR}}), \tag{20}$$

where the instantaneous received SNR of TR signaling with modified AcR is given by $\gamma_{\text{ATR}} = \mu_{\text{ATR}}/2$.

B. Differential Transmitted-Reference

Following an approach similar to the case of TR signaling, we can represent U_j as

$$U_j = \frac{1}{W} \sum_{m=1}^{2WT} \left(d_0 w_{j,m}^2 + e_{-1} w_{j,m} \eta_{2,j,m} + e_0 w_{j,m} \eta_{1,j,m} \right.$$
$$\left. + \eta_{1,j,m} \eta_{2,j,m} \right), \tag{21}$$

where $w_{j,m}$, $\eta_{1,j,m}$, and $\eta_{2,j,m}$ are the m-th sample of $w_j(t)$, $\eta_{1,j}(t)$ and $\eta_{2,j}(t)$ in the interval $[0,T]$, and $w_j(t) \triangleq (b * h * h_{\text{ZF}})(t + jT_f + c_j T_p) = \sqrt{E_p} a_j \sum_{l=1}^{L} \alpha_l p(t - \tau_l)$, $\eta_{1,j}(t) \triangleq \widetilde{n}(t + jT_f + c_j T_p - N_s T_f)$ and $\eta_{2,j}(t) \triangleq \widetilde{n}(t + jT_f + c_j T_p)$.

As for the case of TR signaling, no assumption on c_j is needed for our analysis. For a_j, we exploit symmetry and consider $a_j = +1$ for all j. Conditioned on $d_0 = +1^4$, in this case, we can express (21) in the form of (13), where $\beta_{1,j,m} = \frac{1}{2\sqrt{W}}(e_{-1}\eta_{2,j,m} + e_0\eta_{1,j,m})$, $\beta_{2,j,m} = \frac{1}{2\sqrt{W}}(e_{-1}\eta_{2,j,m} - e_0\eta_{1,j,m})$, and these are statistically independent Gaussian r.v.'s with variance $\sigma_{\text{DTR}}^2 = \frac{N_0}{4}$. Due to symmetry, we only need to consider Y_1 and Y_2 defined in (14), where the non-centrality parameter of Y_1 is now given by

$$\mu_{\text{DTR}} \triangleq \frac{1}{2\sigma_{\text{DTR}}^2} \sum_{j=0}^{N_s-1} \sum_{m=1}^{2WT} \frac{w_{j,m}^2}{W} = \frac{2E_s}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2, \tag{22}$$

and the pdfs of Y_1 and Y_2 conditioned on γ_{DTR} are given by

$$f_{Y_1|\gamma_{\text{DTR}}}(y_1) = f_{\text{NC}}(y_1, \mu_{\text{DTR}}, q_{\text{DTR}}), \tag{23}$$
$$f_{Y_2|\gamma_{\text{DTR}}}(y_2) = f_C(y_2, q_{\text{DTR}}), \tag{24}$$

where $q_{\text{DTR}} = N_s WT$ and $\gamma_{\text{DTR}} = \mu_{\text{DTR}}/2$. Following (18), the BEP of DTR signaling with AcR is given by

$$P_{e,\text{DTR}} = P_e(\psi_{\gamma_{\text{DTR}}}(jv), q_{\text{DTR}}). \tag{25}$$

Comparing (18) and (25), we can observe that the basic difference between TR and DTR signaling lies not only in a doubled non-centrality parameter, but also in a doubled degree of freedom.

For DTR signaling with modified AcR, the new non-centrality parameter of Y_1 in (14) due to noise averaging in (9) is given by

$$\mu_{\text{ADTR}} \triangleq \frac{1}{2\sigma_{\text{ADTR}}^2} \sum_{j=0}^{N_s-1} \sum_{m=1}^{2WT} \frac{w_{j,m}^2}{W}$$
$$= \frac{4N_s}{(N_s+1)} \left(\frac{N_s E_p}{N_0} \sum_{l=1}^{L_{\text{CAP}}} \alpha_l^2 \right), \tag{26}$$

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[4]When $d_0 = +1$, the pairs of differentially encoded bits are either $(e_{-1}, e_0) = (+1, +1)$ or $(e_{-1}, e_0) = (-1, -1)$ with probability $\frac{1}{2}$ each. By symmetry, we only need to consider $(e_{-1}, e_0) = (+1, +1)$.

where the variance σ_{DATR}^2 of $\beta_{1,j,m}$ and $\beta_{2,j,m}$ is

$$\sigma_{\text{ATR}}^2 = \frac{N_0(N_s+1)}{8N_s}, \tag{27}$$

and the reduced variance of $\eta_{1,j,m}/\sqrt{W}$ in (21) is $N_0/2N_s$. The pdfs of Y_1 and Y_2 conditioned on γ_{ADTR} are now given by

$$f_{Y_1|\gamma_{\text{ADTR}}}(y_1) = f_{\text{NC}}(y_1, \mu_{\text{ADTR}}, q_{\text{DTR}}), \tag{28}$$
$$f_{Y_2|\gamma_{\text{ADTR}}}(y_2) = f_C(y_2, q_{\text{DTR}}), \tag{29}$$

where $\gamma_{\text{ADTR}} = \mu_{\text{ADTR}}/2$. Following (18), the BEP of DTR signaling with modified AcR is given by

$$P_{e,\text{ADTR}} = P_e(\psi_{\gamma_{\text{ADTR}}}(jv), q_{\text{DTR}}). \tag{30}$$

Note that the analytical framework based on sampling approach can also be used to derive the BEP of TH-PPM signaling with energy detector [18].

V. NUMERICAL RESULTS

In this section, we provide some numerical results of both TR and DTR signaling based on our analysis in section III. We consider $N_s = 16$ and $L = 32$. For UWB channels, it has been verified through experimental results that the fading distribution of the multipath gains can be modeled by the Nakagami-m distribution [3]. As a result, we consider a dense resolvable multipath Nakagami-m fading channel. For simplicity, we consider uniform power dispersion profile (PDP), which serves as a benchmark. Moreover, for such a PDP, the optimum integration interval T is shown to be equal to L [11], [12]. Under uncorrelated scattering assumption, $\{\alpha_l^2\}$ are statistically independent and the CF of α_l^2 is given by [15]

$$\psi_l(jv) = \left(\frac{1}{1 - \frac{jv}{mL}} \right)^m, \tag{31}$$

where the fading severity index m is assumed to be identical for all faded paths.

The BEP performance of TR and DTR signaling with different receiver structures in uncorrelated scattering channels are compared in Fig. 3. The solid and dashed lines indicate the results for TR and DTR signaling respectively. The difference between TR and DTR signaling is about 2 dB, slightly less than the 3 dB expected from the doubling of the non-centrality parameter in DTR signaling in (22) compared to that of TR signaling in (15). The loss of 1 dB is associated to the doubled degrees of freedom in DTR signaling (which can be seen by comparing (18) and (25)), which constitutes to more noise accumulation. By comparing the performance between modified AcR and AcR, we can observe that the modified AcR performs better than the AcR by about 3 dB for both signaling schemes [11], [12].
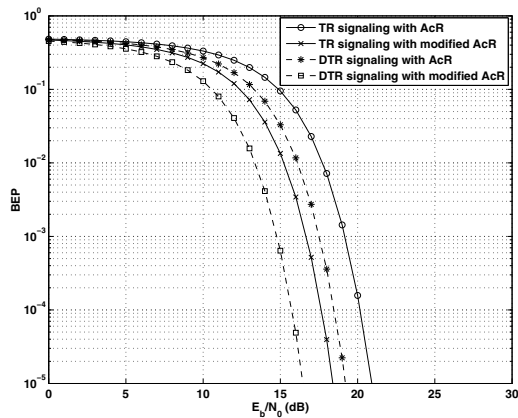
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Fig. 3. BEP performance of TR and DTR signaling with AcR and modified AcR in independent Nakagami-m fading channels with uniform PDP, $m = 3$. The solid and dashed lines indicate the TR and DTR signaling respectively.

VI. Conclusions

In this paper, we developed an analytical framework and provided a unified performance analysis of TR and DTR signaling for both AcR and modified AcR in dense resolvable multipath channels. Specifically, we derived uncoded BEP of TR and DTR signaling schemes with different receiver structures for a broad class of fading channels, including correlated and uncorrelated scattering channels. The analytical framework is based on the sampling expansion approach, without adopting conventional Gaussian approximation.

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