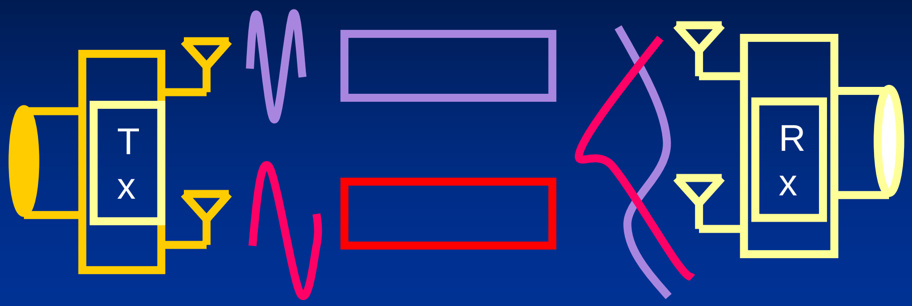
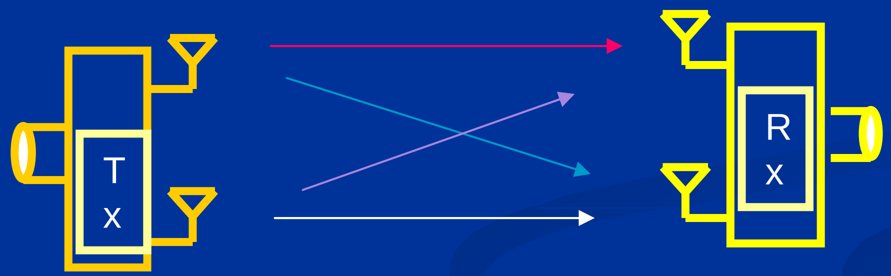
Combining Beamforming and Orthogonal Space-Time block Coding

April 11, 2008

- Full CSIT



- NO CSIT



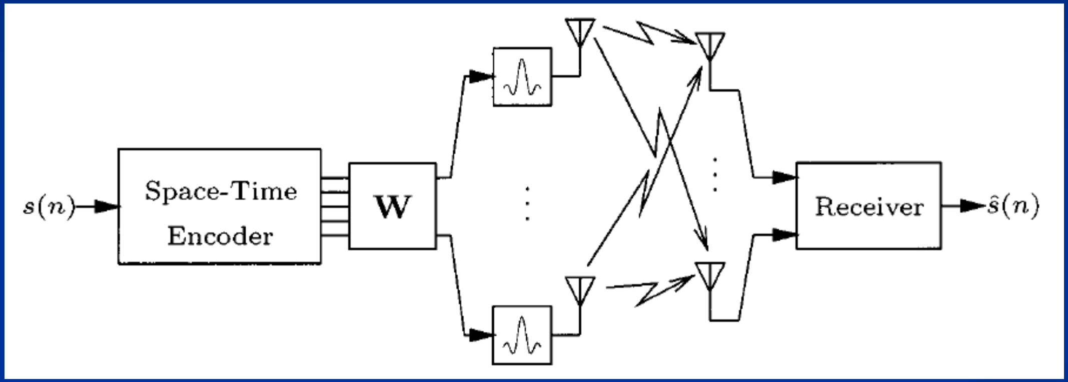
- Partial CSIT

Outline

- System Model and Side Information
- Performance Criterion
- The Transmission Scheme
- Simplified Scenario
- Simulation Results

System Model

- Overview



  - Quasi-static channel with AWGN
  - W: linear transformation matrix
- Receiver

$$\boldsymbol{x}(n) = \boldsymbol{H}^* \boldsymbol{c}(n) + \boldsymbol{e}(n)$$

$$\boldsymbol{c}(n) = \boldsymbol{W}\bar{\boldsymbol{c}}(n)$$

$$\boldsymbol{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N} \\ h_{21} & h_{22} & \cdots & h_{2N} \\ \vdots & \vdots & & \vdots \\ h_{M1} & h_{M2} & \cdots & h_{MN} \end{bmatrix}$$

Side Information at TX

- Assumption:
  - Perfect CSI at Receiver
  - Channel estimates at TX correlated with the true channel

    Outdated channel estimates: correlation determined by the feedback time, similar to Jakes model for mobile users
- Definition:
  - Statistics of side info. $\hat{\boldsymbol{H}}$ $\hat{\boldsymbol{h}}$ $\boldsymbol{m}_{\hat{\boldsymbol{h}}}$
  - Relation to true channel. $\boldsymbol{R}_{\boldsymbol{h}\hat{\boldsymbol{h}}}$ $\boldsymbol{R}_{\boldsymbol{h}\hat{\boldsymbol{h}}}$ $\boldsymbol{R}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}$
    - Perfect side information $\|\boldsymbol{R}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}\| \to 0$
    - No side information $\|\boldsymbol{R}^{-1}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}\| \to 0$

Performance Criterion for STC

- Pairwise Codeword Error Probability (PECP)
  - Received signal corresponding to C $\boldsymbol{X} = \boldsymbol{H}^*\boldsymbol{C} + \boldsymbol{E}$
  - Decoding the codeword according to $\hat{\boldsymbol{C}} = \arg\min_{\boldsymbol{C}\in\mathcal{C}} \|\boldsymbol{X} - \boldsymbol{H}^*\boldsymbol{C}\|_{\mathrm{F}}^2$
  - PCEP conditioned on the side information

$$\Pr\left[\|\boldsymbol{X} - \boldsymbol{H}^*\boldsymbol{C}_k\|_{\mathrm{F}}^2 > \|\boldsymbol{X} - \boldsymbol{H}^*\boldsymbol{C}_l\|_{\mathrm{F}}^2 \,\middle|\, \boldsymbol{C} = \boldsymbol{C}_k, \hat{\boldsymbol{h}}\right]$$

  - C is a function of the channel estimate

$$\Pr\left[\hat{\boldsymbol{C}} \neq \boldsymbol{C}\right] = \int \Pr\left[\hat{\boldsymbol{C}} \neq \boldsymbol{C} \,\middle|\, \hat{\boldsymbol{h}}\right] p_{\hat{\boldsymbol{h}}}\left(\hat{\boldsymbol{h}}\right) d\hat{\boldsymbol{h}}$$

  - Start with the upper bound over true channel

$$P\left(\boldsymbol{C}_k \to \boldsymbol{C}_l \,\middle|\, \boldsymbol{h}, \hat{\boldsymbol{h}}\right) \le \frac{1}{2} e^{-d^2(\boldsymbol{C}_k, \boldsymbol{C}_l)/4\sigma^2} \qquad d^2(\boldsymbol{C}_k, \boldsymbol{C}_l) = \|\boldsymbol{H}^*(\boldsymbol{C}_k - \boldsymbol{C}_l)\|_{\mathrm{F}}^2$$

  - Consider $$p_{\boldsymbol{h}|\hat{\boldsymbol{h}}}\left(\boldsymbol{h} \,\middle|\, \hat{\boldsymbol{h}}\right) = \frac{e^{-(\boldsymbol{h}-\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}})^* \boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}^{-1} (\boldsymbol{h}-\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}})}}{\pi^{MN} \det\left(\boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}\right)}$$

Performance Criterion for STC

$$P\left(\boldsymbol{C}_k \rightarrow \boldsymbol{C}_l \,\middle|\, \hat{\boldsymbol{h}}\right) \leq \int \frac{1}{2} e^{-d^2(\boldsymbol{C}_k, \boldsymbol{C}_l)/4\sigma^2} p_{\boldsymbol{h}|\hat{\boldsymbol{h}}}\left(\boldsymbol{h} \,\middle|\, \hat{\boldsymbol{h}}\right) d\boldsymbol{h}$$

Introduce: $\boldsymbol{\Psi}(\boldsymbol{C}_k, \boldsymbol{C}_l) = (\boldsymbol{I}_N \otimes \boldsymbol{A}(\boldsymbol{C}_k, \boldsymbol{C}_l))/4\sigma^2 + \boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}^{-1}$

Use
$$\int \frac{e^{-(\boldsymbol{h}-\boldsymbol{\Psi}^{-1}\boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}^{-1}\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}})^* \boldsymbol{\Psi}(\boldsymbol{h}-\boldsymbol{\Psi}^{-1}\boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}^{-1}\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}})}}{\pi^{MN} \det(\boldsymbol{\Psi}^{-1})} d\boldsymbol{h} = 1$$

Upper Bound
$$V(\boldsymbol{C}_k, \boldsymbol{C}_l) = \frac{e^{\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}}^* \boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}^{-1}(\boldsymbol{\Psi}(\boldsymbol{C}_k, \boldsymbol{C}_l)^{-1} - \boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}})\boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}^{-1}\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}}}}{2\det\left(\boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}\right)\det(\boldsymbol{\Psi}(\boldsymbol{C}_k, \boldsymbol{C}_l))}$$

$$\ell(\boldsymbol{C}_k, \boldsymbol{C}_l) = \underline{\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}}^* \boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}^{-1} \Psi(\boldsymbol{C}_k, \boldsymbol{C}_l)^{-1} \boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}^{-1} \boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}}} \underline{- \log\det(\boldsymbol{\Psi}(\boldsymbol{C}_k, \boldsymbol{C}_l))}$$

Channel knowledge

Open-loop system

Transmission Scheme

- i. To design a STC using the criterion x
- ii. To improve a predetermined STC √

$$\boldsymbol{C}_k = \boldsymbol{W}\overline{\boldsymbol{C}}_k \qquad \boldsymbol{A}(\boldsymbol{C}_k, \boldsymbol{C}_l) = (\boldsymbol{C}_k - \boldsymbol{C}_l)(\boldsymbol{C}_k - \boldsymbol{C}_l)^* = \mu_{kl}\boldsymbol{I}_M$$

$$\ell(\boldsymbol{W}\boldsymbol{W}^*, \mu_{kl}) = \boldsymbol{m}^*_{\boldsymbol{h}|\hat{\boldsymbol{h}}}\boldsymbol{R}^{-1}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}\Psi(\boldsymbol{W}\boldsymbol{W}^*, \mu_{kl})^{-1}\boldsymbol{R}^{-1}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}} - \log\det(\Psi(\boldsymbol{W}\boldsymbol{W}^*, \mu_{kl}))$$

$$\Psi(\boldsymbol{W}\boldsymbol{W}^*, \mu_{kl}) = (\boldsymbol{I}_N \otimes \boldsymbol{W}\boldsymbol{W}^*)\mu_{kl}/4\sigma^2 + \boldsymbol{R}^{-1}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}$$

Define: $\boldsymbol{Z} = \boldsymbol{W}\boldsymbol{W}^*$

Optimization Problem: $\boldsymbol{Z}_{\text{opt}} = \arg \min_{\substack{\boldsymbol{Z} \\ \boldsymbol{Z}=\boldsymbol{Z}^*\succeq 0 \\ \text{tr}(\boldsymbol{Z})=1}} \ell(\boldsymbol{Z})$

$$\ell(\boldsymbol{Z}) = \boldsymbol{m}^*_{\boldsymbol{h}|\hat{\boldsymbol{h}}}\boldsymbol{R}^{-1}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}\left((\boldsymbol{I}_N \otimes \boldsymbol{Z})\eta + \boldsymbol{R}^{-1}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}\right)^{-1}\boldsymbol{R}^{-1}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}} - \log\det\left((\boldsymbol{I}_N \otimes \boldsymbol{Z})\eta + \boldsymbol{R}^{-1}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}\right)$$

Convexity of the Criterion Function

$$\underbrace{m^*_{h|\hat{h}} R^{-1}_{hh|\hat{h}} \Psi^{-1} R^{-1}_{hh|\hat{h}} m_{h|\hat{h}}}_{1} + \log \underbrace{\det(\Psi^{-1})}_{2}$$

- 1st term: A function is convex over X if it is convex when restricted to any line that intersects X

$$\Psi = \lambda \Psi_1 + (1-\lambda)\Psi_2 \quad 0 \le \lambda \le 1$$

The second-order derivative of 1 with respect to λ :

$$2m^*_{h|\hat{h}} R^{-1}_{hh|\hat{h}} \Psi^{-1} (\Psi_1 - \Psi_2) \Psi^{-1} (\Psi_1 - \Psi_2) \Psi^{-1} R^{-1}_{hh|\hat{h}} m_{h|\hat{h}}$$

- 2nd term: convex

Criterion function is **Convex** !

Special Cases

- 1) No CSI → conventional OSTBC

$$\|\boldsymbol{R}_{\boldsymbol{hh}|\hat{\boldsymbol{h}}}^{-1}\| \rightarrow 0 \qquad \boldsymbol{Z}_{\text{as}} = \boldsymbol{I}_M / M$$

$$\boldsymbol{W}_{\text{as}} = \boldsymbol{I}_M / \sqrt{M}$$

- 2) Infinite SNR → conventional OSTBC

$$\eta = \mu_{\min}/4\sigma^2 \rightarrow \infty$$

$$\boldsymbol{W}_{\text{as}} = \boldsymbol{I}_M / \sqrt{M}$$

Special Cases

- Perfect CSI $\|\boldsymbol{R}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}}\| \to 0$

  $\boldsymbol{\Omega}_k^{(m)}$: k[th] block on the diagonal of $\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}}\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}}^*$

  $$\boldsymbol{\Theta} = \sum_{k=1}^{N} \boldsymbol{\Omega}_k^{(m)} \rightarrow \boldsymbol{W}_{\text{as}} = [\boldsymbol{v}_M \quad \boldsymbol{0} \quad \cdots \quad \boldsymbol{0}]$$

  $\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}} \to \boldsymbol{h}$ $\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}}\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}}^* \to \boldsymbol{h}\boldsymbol{h}^*$ $\boldsymbol{\Theta} \to \boldsymbol{H}\boldsymbol{H}^*$

- SNR=0, $\eta \to 0$

  $$\boldsymbol{\Theta} = \sum_{k=1}^{N} \boldsymbol{\Omega}_k^{(m)} + \boldsymbol{\Omega}_k^{(R)} \rightarrow \boldsymbol{W}_{\text{as}} = [\boldsymbol{v}_M \quad \boldsymbol{0} \quad \cdots \quad \boldsymbol{0}]$$

An Algorithm for a Simplified Scenario

- Scenario: $R_{hh|\hat{h}} = \alpha I_{MN}$

Introduce $\hat{\Upsilon} = \frac{1}{\alpha}\sum_{k=1}^{N} \Omega_k^{(m)}$

$$\ell(Z) = \frac{1}{\alpha} m_{h|\hat{h}}^* ((I_N \otimes Z\alpha\eta) + I_{MN})^{-1} m_{h|\hat{h}} - \log\det((I_N \otimes Z\alpha\eta) + I_{MN})$$

$$= \frac{1}{\alpha} \mathrm{tr}\left((I_N \otimes (Z\alpha\eta + I_M))^{-1} m_{h|\hat{h}} m_{h|\hat{h}}^* \right) - \log\det((I_N \otimes Z\alpha\eta) + I_{MN})$$

$$= \mathrm{tr}\left((Z\alpha\eta + I_M)^{-1} \hat{\Upsilon} \right) - \log\det((I_N \otimes Z\alpha\eta) + I_{MN})$$

Let $Z = V\Lambda V^*$ $\hat{\Upsilon} = \hat{V}\hat{\Lambda}\hat{V}^*$

$$\ell(\{\lambda_i\}_{i=1}^{M}, V) = \mathrm{tr}\left((\Lambda\alpha\eta + I_M)^{-1} V^* \hat{V}\hat{\Lambda}\hat{V}^* V \right) - N \log\det(\Lambda\alpha\eta + I_M)$$

$$\sum_{k=1}^{M} \lambda_k = 1$$

$$\lambda_i \geq 0, \qquad i = 1, \ldots, M$$

$$\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_M.$$

A Simplified Scenario

- h_{ij} i.i.d. zero-mean complex Gaussian

Introduce $\rho = E[h_{ij}\hat{h}^*_{ij}]/\sigma_h^2$

$$\boldsymbol{R}_{\boldsymbol{h}\boldsymbol{h}} = \sigma_h^2 \boldsymbol{I}_{MN} \quad \boldsymbol{R}_{\boldsymbol{h}\hat{\boldsymbol{h}}} = \sigma_h^2 \rho \boldsymbol{I}_{MN} \quad \boldsymbol{R}_{\hat{\boldsymbol{h}}\hat{\boldsymbol{h}}} = \sigma_h^2 \boldsymbol{I}_{MN}$$

$$\boldsymbol{m}_{\boldsymbol{h}} = \boldsymbol{m}_{\hat{\boldsymbol{h}}} = \boldsymbol{0}$$

Conditional channel distribution:

$$\boldsymbol{m}_{\boldsymbol{h}|\hat{\boldsymbol{h}}} = \rho \hat{\boldsymbol{h}} \quad \boldsymbol{R}_{\boldsymbol{h}\boldsymbol{h}|\hat{\boldsymbol{h}}} = \sigma_h^2 (1 - |\rho|^2) \boldsymbol{I}_{MN}$$