The Effect upon Channel Capacity in Wireless Communications of Perfect and Imperfect Knowledge of the Channel

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Reading Group Discussion

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Intro ./ Overview

- Time Variation, ISI and multiple access dealt with an error in channel est. perspective.
- The general idea is to establish bounds for the asymptotic performance with imperfections in CSI.
- Look at Mutual Info. Use of Gaussian codebooks.. hence looking at capacity ?
- Use perturbation to model error in CSI - Error with known variance.
- Will focus on the uplink case . . . and start by looking at the mutual info for a known channel.
- General Trick: Upper bound the mutual info. with that of an equal variance Gaussian process

Perfectly Known Channel

Channel Model

- Imp. Resp. at $t' - t$:
  $g(t',t) = \sum_m g_m(t',t) = \sum_m a^m(t')\delta(\tau^m(t') - t)$
  where $\tau^m(t) = \tau^m + (B^m/f_o)t$.
- Discrete Time Model: $y[k] = \sum_{\text{all paths}} \sum_l s[k-l]g^m[k,l] + n[k]$

Max. Mutual Info.– Complex Transmission (Single User)

- Mutual Info. given by $I(\underline{Y}_k;\underline{S}_k) = h(\underline{Y}_k) - h(\underline{N}_k)$
- Eqn.(12) - approx. assumption of time and BW limiting - ???
- Using sampling rate assumptions construct $\underline{f}_{2k'}^{2k}$.
- Using W.filling Max MI $= \frac{1}{2T}\sum_{i=1}^{2k'} \ln\left(1 + \frac{u_i\lambda_i}{WN_0/2}\right)$

Multiple Users- Non-cooperating case

Mutual Info

- The case where the antennas cooperate $\rightarrow$ antenna array (development similar to single user).
- Maximise $I(\{\underline{S}_{i_{2k}},\}_{i=1}^{K}; \underline{Y}_{2k}) = h(\underline{Y}_{2k}) - h(\underline{N}_{2k})$
(each source has individual power constraints - sum of all M.I / time)
- $h(\underline{Y}_{2k})$ is maximized by using Gaussian inputs.

2-user case

- Maximize (Typo in eqn. 23)
$\max \ln \left(|\underline{f}_1^{2k} \underline{\Lambda}_{\underline{S}_1} \underline{f}_1^{2k^T} + \underline{f}_2^{2k} \underline{\Lambda}_{\underline{S}_2} \underline{f}_2^{2k^T} + \underline{\Lambda}_{\underline{N}_{2k}}| \right)$
- The above region is proved to be convex.. hence maximization $\rightarrow$ global max.

Effect of Channel Error of Known Variance

Assumptions on channel

- Unknown at Tx.
- Partially known at Rx.
- Idea: Look at *asymptotic* variations of the channel.

Mutual Info.

- $\underline{Y} = \underline{FS} + \underline{N}$.
-
$$I(\underline{Y};\underline{S}) = h(\underline{S}) - h(\underline{S}|\underline{Y}) \tag{1}$$
- Known Channel:
$$I(\underline{Y};\underline{S}|\underline{F}) = h(\underline{S}|\underline{F}) - h(\underline{S}|\underline{Y},\underline{F}) \tag{2}$$
- Eqn.2 - Eqn.1 gives (typo in Eqn.34)
$$I(\underline{Y};\underline{S}|\underline{F}) - I(\underline{Y};\underline{S}) = I(\underline{S};\underline{F}|\underline{Y}) \tag{3}$$

Single User - cont'd (single symbol)

- Channel known within some MSE.
- Therefore $F = \bar{F} + \tilde{F} \dots$ where $\tilde{F}$ accounts for the meas. error.
- Recall $I(Y;S) = h(Y) - h(Y|S)$ and from $Y = S\bar{F} + S\tilde{F} + N$ we have

$$h(Y|S = s) = h(s\tilde{F} + N) \tag{4}$$

- Integrating over dist. of $\tilde{F}$

Lower Bound on MI using the worst case dist. under a cov. constraint

- Max. $I(Y;S) = h(S) - h(S|Y) \dots$ Use Gaussian dist.S - might not be max.
- Max $h(S|Y)$... thus yielding lower bound on I.

$$h(S|Y) \leq \frac{1}{2}\ln(2\pi e \mathrm{Var}(S - \alpha Y)) \tag{5}$$

.. and this is the part I dont understand very well

lower bound

- $\alpha = \frac{E[SY]}{E[Y^2]} = \frac{\bar{F}\sigma_s^2}{\bar{F}^2\sigma_s^2 + \sigma_F^2\sigma_s^2 + \sigma_N^2}$
- Use above to get $\text{Var}(S - \alpha Y)$ and hence lower bound on $I(S;Y)$

Upper bound

- For upper bound use the fact $I(S;Y) \leq I(S;Y|F)$ and then Jensen's ineq. to get Eqn.(49).
- Loss in mutual info. due to uncertainty about the channel

$$I(S;Y|F) - I(S;Y) \leq \frac{1}{2}\ln\left(1 + \frac{\sigma_F^2\sigma_S^2}{\sigma_N^2}\right) \qquad (6)$$

Interpretation

- Upper bound : Meas. noise = Extra Tx power
- Lower bound : Not useful at all
- Not discussing the extension to multiple symbols but (Interpretations are the same) ..
  Ques. why does a matrix have to be invertible if its white (pg. 940).

Multiple Access Case

Channel Assumptions and idea

- Channels of different users are mutually independent.
- If user k can be decoded $\rightarrow$ interference from this user be cancelled with error owing to channel est. errors.

Mutual Info. (2 user case)

- Assumption: All Rx components part of signal ... Therefore you get upper bound on mutual info.
- $I(Y;S_1|S_2) \leq I(Y;S_1|S_2,F_1,F_2) = h(Y|S_2,F_1,F_2) - h(N)$
- Again upper bounded using Gaussian dist.

$$\begin{aligned} I(Y;S_1|S_2) &\leq \frac{1}{2}E_{F_1}[\ln(\sigma_N^2 + \sigma_{S_1}^2 F_1^2)] - \frac{1}{2}\ln(\sigma_N^2) \\ &\leq \frac{1}{2}ln\left(1 + \frac{\sigma_{S_1}^2 F_1^2}{\sigma_N^2}\right) \end{aligned}$$

Lower bound

- $$\begin{aligned} I(Y;S_1|S_2) &= h(S_1|S_2) - h(S_1|Y,S_2) \\ &= h(S_1) - h(S_1|Y,S_2) \\ &= h(S_1) - h(S_1|(Y-\bar{F}_2 S_2),S_2) \\ &\geq h(S_1) - h(S_1|(Y-\bar{F}_2 S_2)) \end{aligned}$$
- . . . and then use the LMMSE estimate

Interpretation

- Main Result : Uncertainty of channel of other users while doing Interference cancellation - lower bounded by more AWGN.
- Fig. 6 reinstates the widely known result that interference cancellation is better than treating the MAC as an interference channel.