Capacity and Mutual Information of Wideband Multipath Fading Channels

by I.E. Telatar and D.N.C. Tse (IEEE Trans. IT, July 2000)

proposed by Jan

CTG Reading Group

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Outline

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1. **Capacity Bounds for Wideband Multipath Fading Channels**
   - Basic Assumptions
   - Upper Bound on Capacity
   - Lower Bound on Capacity

2. **Mutual Information Achieved by Spread-Spectrum Signaling**
   - Motivation and Assumptions
   - Bounds on Mutual Information
   - Practical Implications

Capacity Bounds for Wideband Fading Channels I

Basic Assumptions

- Wideband multipath fading channel, bandwidth W
  - Wideband: received power spread out over large bandwidth
  - Still narrowband in the sense $W \ll f_c$ (f_c: carrier frequency)
- Channel model:

$$y(t) = \sum_{l=1}^{L} a_l(t)\, x(t - d_l(t)) + z(t)$$

  - $y(t)$: received waveform, $x(t)$: transmitted waveform, $z(t)$: AWGN
  - L: number of physical multipaths
  - $a_l(t)$: path amplitudes, constant during coherence time T_c, unknown at receiver
  - $d_l(t)$: path delays, slowly time-varying, perfectly known at receiver
- Capacity derivation:
  - Constraint on average received power $P \Rightarrow$ SNR $= P/N_0$

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Capacity Bounds for Wideband Fading Channels II

Upper Bound on Capacity

- Capacity of infinite bandwidth fading channel with SNR P/N_0 and *perfect* channel state information (CSI) at receiver:

$$C^* = \frac{P}{N_0}$$

- Wideband multipath fading channel: path amplitudes $a_l(t)$ *unknown* at receiver $\Rightarrow$

$$C \leq C^* = \frac{P}{N_0}$$

- C^* corresponds to capacity of infinite bandwidth AWGN channel (non-fading, perfect CSI at receiver):

$$\lim_{W \to \infty} W \log\left(1 + \frac{P}{N_0 W}\right) \approx \lim_{W \to \infty} W \frac{P}{N_0 W} = \frac{P}{N_0} =: C_{AWGN}$$

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Capacity Bounds for Wideband Fading Channels III

Lower Bound on Capacity I

Design efficient signaling scheme and assess mutual information (MI)

- Choose symbol duration T_s such that $2T_d \leq T_s \leq T_c$ (T_d: delay spread, $T_d \ll T_c$ assumed)
- Convey message $m \in \{1, ..., M\}$ using signal

$$x_m(t) = \begin{cases} \sqrt{\lambda}\exp(j2\pi f_m t) & 0 \leq t \leq T_s \\ 0 & else \end{cases}$$

$\Rightarrow$ Single sinusoid at frequency f_m ($\hat{=}$ FSK scheme)

- Receiver correlates received signal against all possible $x_m(t)$, $m \in \{1, ..., M\} \Rightarrow$ *non-coherent* detection
- Choose frequencies as $f_m := n/(T_s - 2T_d)$ (n integer) to obtain *orthogonal* scheme
- Repeat transmission of $x_m(t)$ on N disjoint time intervals $\Rightarrow$ receiver can average over fading

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Capacity Bounds for Wideband Fading Channels IV

Lower Bound on Capacity II

- Using *low duty cycle* above scheme achieves MI

$$I(x;y|d_l) = \left(1 - 2\frac{T_d}{T_c}\right)\frac{P}{N_0}$$

  Due to *average* power constraint we have $\lambda := P/\theta \gg P \quad (\theta \to 0)$

- Altogether:

$$\left(1 - 2\frac{T_d}{T_c}\right) C_{AWGN} \leq C \leq C_{AWGN}$$

  $(C_{AWGN} = P/N_0)$

  - Since $T_d \ll T_c$, lower and upper bound approximately *coincide*
  - Capacity-achieving signaling is "peaky" in time and frequency domain

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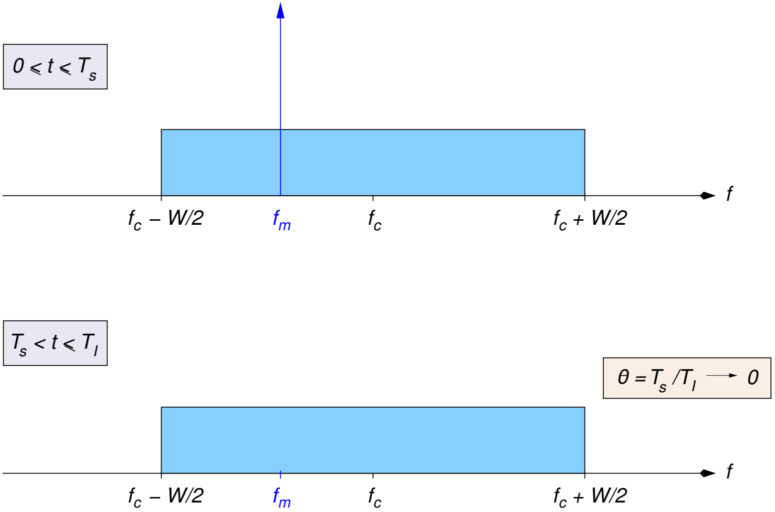

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MI bounds for Spread-Spectrum Signaling I

Spread-spectrum (SS) schemes (DS-CDMA, code-spread CDMA, ...) commonly used for communication over wideband channels

Key result

- Capacity-achieving signaling for wideband multipath fading channels *maximal different* from SS signaling

  $\Rightarrow$ SS signals are "white-like" and non-peaky in time

Question

- How good is SS signaling for wideband multipath fading channels?

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MI bounds for Spread-Spectrum Signaling II

Assumptions

- Discrete-time channel model:

$$Y_i = \sqrt{\frac{\mathcal{E}}{K_c}} \sum_{l=1}^{\tilde{L}} G_l \, X_{(i-D_l)} + Z_i$$

  - Y_i: received sample, X_i: transmitted symbol, Z_i: AWGN sample
  - $\tilde{L}$: number of *resolvable* multipaths at system bandwidth W ($\tilde{L} \leq L$)
  - G_l, D_l: amplitudes/delays of resolvable multipaths
  - $\mathcal{E} := PT_c/N_0$, K_c normalization factor

- Two different notions of “white-like” signals
  - info symbols modulated on pseudo-random spreading sequences with near-perfect auto-correlation ($\triangleq$ DS-CDMA)
  - info symbols spread onto wide bandwidth using low-rate FEC ($\triangleq$ code-spread CDMA)

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MI bounds for Spread-Spectrum Signaling III

Bounds on MI

- Upper bound on MI per unit time (holds for large W and large $\tilde{L}$; equal average path energies assumed):

$$I(X;Y|D_I) \leq \frac{\mathcal{E}^2}{T_c^2 \tilde{L}}$$

- Lower bound on MI per unit time (holds for large W and *any* $\tilde{L}$):

$$I(X;Y|D_I) \geq \frac{\mathcal{E}}{T_c} - \frac{\tilde{L}}{T_c} \log\left(1 + \frac{\mathcal{E}}{\tilde{L}}\right)$$

  - If $\tilde{L} \ll \mathcal{E}$, lower bound close to $\mathcal{E}/T_c = P/N_0 = C_{AWGN}$, i.e., SS signaling *near-optimal*
  - If $\tilde{L} \gg \mathcal{E}$, upper bound holds and is close to zero, i.e., SS signaling *highly suboptimal* (!)

$\Rightarrow$ $\mathcal{E} =: \tilde{L}_{crit}$ critical system parameter indicating *overspreading*

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MI bounds for Spread-Spectrum Signaling IV

Critical Parameter $\tilde{L}_{crit}$

- Critical parameter also plays *key role* for detection error probability of (specific) binary orthogonal modulation schemes ($W \rightarrow \infty$)
- Interpretation of case $\tilde{L} \gg \tilde{L}_{crit}$:
  $\Rightarrow$ energies of resolvable paths very small
  $\Rightarrow$ poor estimates of complex gains
  $\Rightarrow$ effective multipath combining at the receiver *difficult*

Question

- How good are DS-UWB systems?

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