MATLAB in Digital Signal Processing and Communications

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MATLAB Tutorial

October 15, 2008

Objective and Focus

- Learn how MATLAB can be used efficiently in order to perform tasks in digital signal processing and digital communications
- Learn something about state-of-the-art digital communications systems and how to simulate/analyze their performance

Focus

- Wireless multi-carrier transmission system based on Orthogonal Frequency-Division Multiplexing (OFDM) including
  - a simple channel coding scheme for error correction
  - interleaving across subcarriers for increased frequency diversity
- OFDM is extremely popular and is used in e.g.
  - Wireless LAN air interfaces (Wi-Fi standard IEEE 802.11a/b/g, HIPERLAN/2)
  - Fixed broadband wireless access systems (WiMAX standard IEEE 802.16d/e)
  - Wireless Personal Area Networks (WiMedia UWB standard, Bluetooth)
  - Digital radio and digital TV systems (DAB, DRM, DVB-T, DVB-H)
  - Long-term evolution (LTE) of third-generation (3G) cellular systems
  - Cable broadband access (ADSL/VDSL), power line communications

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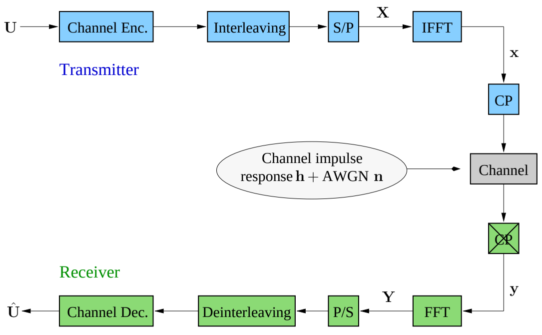
System Overview



- N_c: number of orthogonal carriers ($N_c := 2^n$); corresponds to (I)FFT size
- R: code rate of employed channel code ($R := 1/2^m \leq 1$)
- $\mathbf{U}$: vector of info symbols (length RN_c), $\hat{\mathbf{U}}$: corresponding estimated vector
- $\mathbf{X}$: transmitted OFDM symbol (length N_c), $\mathbf{Y}$: received OFDM symbol

⇒ We will consider each block in detail, especially their realization in MATLAB

Info Vector $\mathbf{U}$

- Info symbols U_k carry the actual information to be transmitted (e.g., data files or digitized voice)
- Info symbols U_k typically regarded as independent and identically distributed (i.i.d.) random variables with realizations , e.g., in $\{0, 1\}$ (equiprobable)
- We use antipodal representation $\{-1, +1\}$ of bits as common in digital communications

MATLAB realization

- Generate vector $\mathbf{U}$ of length RN_c with i.i.d. random entries $U_k \in \{-1, +1\}$

```
U = 2*round(rand(1,R*Nc))-1;
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Channel Encoding

- Channel coding adds redundancy to info symbols in a structured fashion
- Redundancy can then be utilized at receiver to correct transmission errors (channel decoding)
- For each info symbol U_k the channel encoder computes N code symbols $X_{k,1}, \ldots, X_{k,N}$ according to pre-defined mapping rule $\Rightarrow$ Code rate $R := 1/N$
- Design of powerful channel codes is a research discipline on its own

- We focus on simple repetition code of rate R, i.e.,

$$\mathbf{U} = [\ldots, U_k, U_{k+1}, \ldots] \mapsto \mathbf{X} = [\ldots, U_k, U_k, \ldots, U_{k+1}, U_{k+1}, \ldots]$$

- Example: $U_k = +1$, $R = 1/4$, received code symbols $+0.9$, $+1.1$, -0.1, $+0.5$ $\Rightarrow$ High probability that $U_k = +1$ can be recovered

MATLAB realization

- Apply repetition code of rate R to info vector $\mathbf{U}$ $\Rightarrow$ Vector $\mathbf{X}$ of length N_c

```
X = kron(U,ones(1,1/R));
```

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Interleaving

- Code symbols in vector $\mathbf{X}$ (length N_c) will be transmitted in parallel over the N_c orthogonal subcarriers (via IFFT operation)
- Each code symbol 'sees' frequency response of underlying channel on particular subcarrier
- Channel impulse response (CIR) is typically considered random in wireless communications (see slide 'Channel Model')
- Channel frequency response of neighboring subcarriers usually correlated; correlation between two subcarriers with large spacing typically low
- Idea: Spread code symbols $X_{k,1}, ..., X_{k,N}$ associated with info symbol U_k across entire system bandwidth instead of using N subsequent subcarriers
- We use maximum distance pattern for interleaving
- Example: $N_c = 128$ subcarriers, code rate $R = 1/4$
  $\Rightarrow$ Use subcarriers $\#k$, $\#(k+32)$, $\#(k+64)$, and $\#(k+96)$
  for code symbols associated with info symbol U_k $(k = 1, ..., 32)$

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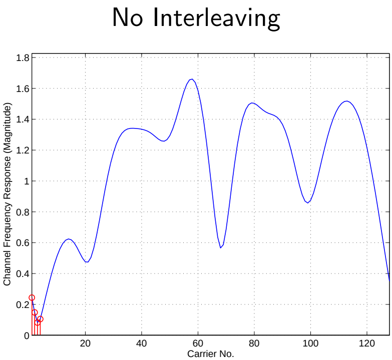
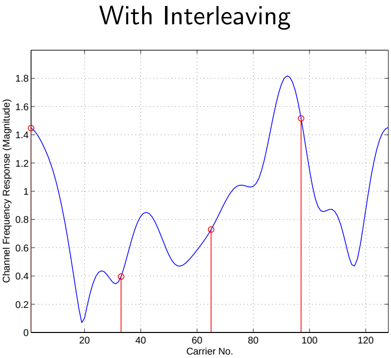
Interleaving





MATLAB realization

- Interleave vector $\mathbf{X}$ according to maximum distance pattern ($N_c = 128$, $R = 1/4$)

```
index = [1 33 65 97 2 34 66 98 ... 32 64 96 128];
X(index) = X;
```

OFDM Modulation

- OFDM symbol $\mathbf{X}$ ($\hat{=}$ frequency domain) converted to time domain via IFFT operation $\Rightarrow$ Vector $\mathbf{x}$ (length N_c)
- Assume CIR $\mathbf{h}$ of length N_ch $\Rightarrow$ To avoid interference between subsequent OFDM symbols, guard interval of length $N_\mathrm{ch}-1$ required
- Often cyclic prefix (CP) is used, i.e., last $N_\mathrm{ch}-1$ symbols of vector $\mathbf{x}$ are appended to $\mathbf{x}$ as a prefix $\Rightarrow$ Vector of length $N_\mathrm{c}+N_\mathrm{ch}-1$
- Details can be found in

  Z. Wang and G. B. Giannakis, "Wireless multicarrier communications – Where Fourier meets Shannon," *IEEE Signal Processing Mag.*, May 2000.

MATLAB realization

- Perform IFFT of vector $\mathbf{X}$ and add CP of length $N_\mathrm{ch}-1$

```
x = ifft(X)*sqrt(Nc);
x = [ x(end-Nch+2:end) x ];
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Channel Model

- OFDM typically employed for communication systems with large bandwidth $\Rightarrow$ Underlying channel is frequency-selective, i.e., CIR $\mathbf{h}$ has length $N_{\text{ch}} > 1$
- In wireless scenarios channel coefficients $h_0, ..., h_{N_{\text{ch}}-1}$ considered random
- We consider baseband transmission model, i.e., channel coefficients h_l are complex-valued (equivalent passband model involves real-valued quantities)
- In rich-scattering environments, Rayleigh-fading channel model has proven useful, i.e., channel coefficients h_l are complex Gaussian random variables:

$$\text{Re}\{h_l\}, \text{Im}\{h_l\} \sim \mathcal{N}(0, \sigma_l^2/2) \quad \Rightarrow \quad h_l \sim \mathcal{CN}(0, \sigma_l^2)$$

- We assume exponentially decaying channel power profile, i.e.,

$$\frac{\sigma_l^2}{\sigma_0^2} := \exp(-l/c_{\text{att}}), \quad l = 0, .., N_{\text{ch}} - 1$$

- We assume block-fading model, i.e., CIR $\mathbf{h}$ stays constant during entire OFDM symbol and changes randomly from one OFDM symbol to the next
- Noiseless received vector given by convolution of vector $\mathbf{x}$ with CIR $\mathbf{h}$
- Noise samples are i.i.d. complex Gaussian random variables $\sim \mathcal{CN}(0, \sigma_{\text{n}}^2)$

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Channel Model

MATLAB realization

- Generate exponentially decaying channel power profile $\Rightarrow$ Variances σ_l^2

```
var_ch = exp(-[0:Nch-1]/c_att);
```

- Normalize channel power profile such that overall average channel energy is 1

```
var_ch = var_ch/sum(var_ch);
```

- Generate random CIR realization with independent complex Gaussian entries and specified channel power profile

```
h = sqrt(0.5)*(randn(1,Nch)+j*randn(1,Nch)) .* sqrt(var_ch);
```

- Calculate noiseless received vector via convolution of vector $\mathbf{x}$ with CIR $\mathbf{h}$

```
y = conv(x,h);
```

- Add additive white Gaussian noise (AWGN) samples with variance σ_n^2

```
n = sqrt(0.5)*( randn(1,length(y))+j*randn(1,length(y)) );
y = y + n * sqrt(sigma2_n);
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OFDM Demodulation

- Received vector resulting from convolution of transmitted vector $\mathbf{x}$ with CIR $\mathbf{h}$ has length $(N_c + N_{ch} - 1) + N_{ch} - 1$

  $\Rightarrow$ Received vector is truncated to same length $N_c + N_{ch} - 1$ as vector $\mathbf{x}$
- Then CP is removed to obtain received vector $\mathbf{y}$ of length N_c
- Finally, FFT is performed to convert received vector $\mathbf{y}$ back to frequency domain $\Rightarrow$ received OFDM symbol $\mathbf{Y}$ of length N_c

MATLAB realization

- Truncate vector $\mathbf{y}$ by removing last $N_{ch} - 1$ entries, remove CP (first $N_{ch} - 1$ entries), and perform FFT to obtain received OFDM symbol $\mathbf{Y}$

```
y(end-Nch+2:end) = [];
y(1:Nch-1) = [];
Y = fft(y)/sqrt(Nc);
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OFDM Demodulation

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Deinterleaving and Channel Decoding

- For coherent detection of the info symbols U_k, the channel phases associated with the entries of the received OFDM symbol $\mathbf{Y}$ have to be derotated

  $\Rightarrow$ We need to calculate the channel frequency response via FFT of CIR $\mathbf{h}$
- Perform deinterleaving based on employed interleaver pattern
- If repetition code is used, all entries of $\mathbf{Y}$ that are associated with same info symbols U_k are optimally combined using maximum ratio combining (MRC)
- Finally, estimates $\hat{U}_k$ are formed based on the RN_c output symbols $Z_{\mathrm{mrc},k}$ of the MRC step
- In simulation, determine the number of bit errors in current OFDM symbol by comparing $\hat{\mathbf{U}}$ with $\mathbf{U}$
- Update error counter and finally determine average bit error rate (BER) by dividing overall number of errors by overall number of transmitted info bits

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Deinterleaving and Channel Decoding

MATLAB realization

- Calculate channel frequency response via FFT of zero-padded CIR $\mathbf{h}$

```
h_zp = [h zeros(1,Nc-Nch)];
H = fft(h_zp);
```

- Derotate channel phases associated with entries of received OFDM symbol $\mathbf{Y}$

```
Z = conj(H) .* Y;
```

- Perform deinterleaving (N_c=128, R=1/4)

```
index_matrix = [1 33 65 97 ; 2 34 66 98 ; ... 32 64 96 128];
matrix_help = Z(index_matrix);
```

- Perform MRC $\Rightarrow$ Vector $\mathbf{Z}_{\mathrm{mrc}}$ of length RN_c

```
Z_mrc = sum(matrix_help,2);
```

- Form estimates $\hat{U}_k$ based on vector $\mathbf{Z}_{\mathrm{mrc}}$

```
Uhat = sign(real(Z_mrc))';
```

Deinterleaving and Channel Decoding

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```

Deinterleaving and Channel Decoding

MATLAB realization (cont'd)

- Count bit errors in current OFDM symbol and update error counter

```
err_count = err_count + sum(abs(Uhat-U))/2;
```

- After transmission of N_{real} OFDM symbols calculate final BER

```
ber = err_count/(R*Nc*Nreal);
```

Simulation Results

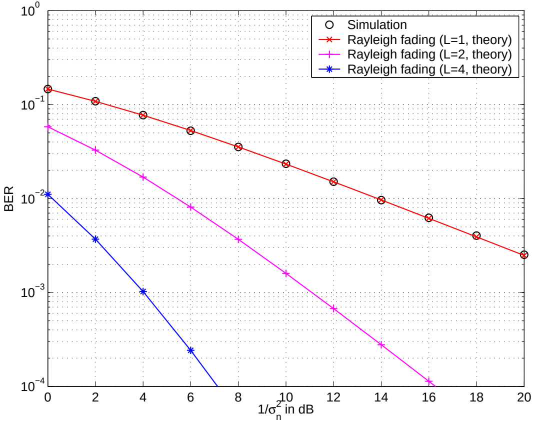
Uncoded transmission

$N_{\mathrm{real}} = 10,000$ OFDM symbols

$N_{\mathrm{c}} = 128$ subcarriers

$N_{\mathrm{ch}} = 10$ channel coefficients

$c_{\mathrm{att}} = 2$ for channel profile



Comparison with analytical results for $L = 1$ Rayleigh fading branch (see Appendix) validates simulated BER curve

Simulation Results

Repetition code (rate 1/2)
No interleaving

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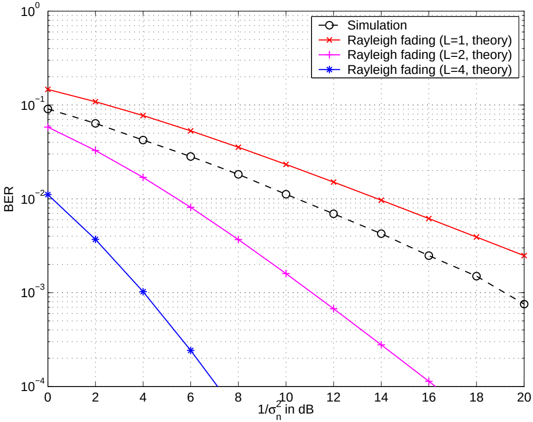

Repetition code yields significant gain, mostly due to increased received power per info bit; hardly any diversity gain as no interleaver is used

Simulation Results

Repetition code (rate 1/2)
With interleaving

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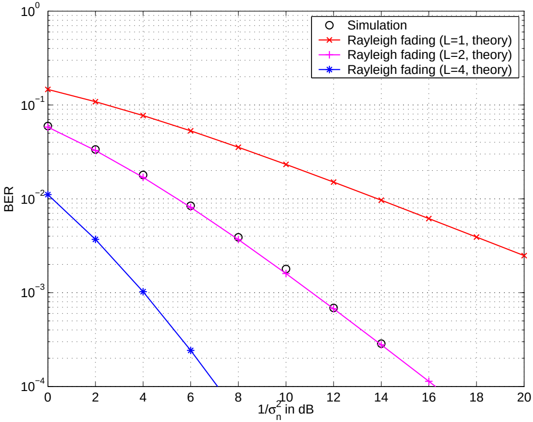
$N_{\mathrm{c}} = 128$ subcarriers

$N_{\mathrm{ch}} = 10$ channel coefficients

$c_{\mathrm{att}} = 2$ for channel profile



Comparison with analytical results for diversity reception over $L = 2$ i.i.d. Rayleigh fading branches (see Appendix) validates simulated BER curve

Simulation Results

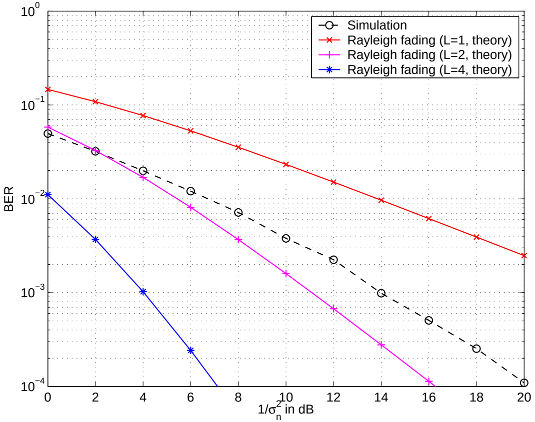
Repetition code (rate 1/4)
No interleaving

$N_{\text{real}} = 10,000$ OFDM symbols

$N_{\text{c}} = 128$ subcarriers

$N_{\text{ch}} = 10$ channel coefficients

$c_{\text{att}} = 2$ for channel profile



Repetition code yields significant gain, mostly due to increased received power per info bit; slight diversity gain visible even without interleaver

Simulation Results

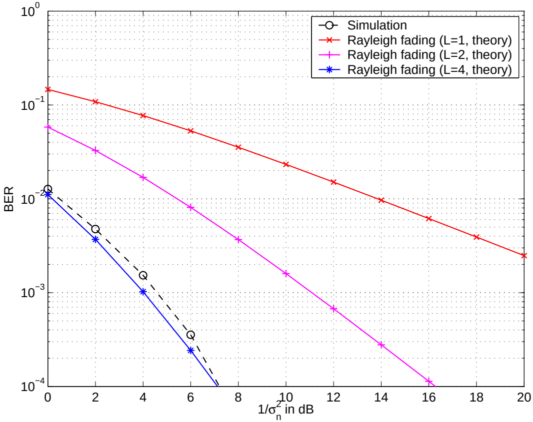
Repetition code (rate 1/4)
With interleaving

$N_{\mathrm{real}} = 10,000$ OFDM symbols

$N_{\mathrm{c}} = 128$ subcarriers

$N_{\mathrm{ch}} = 10$ channel coefficients

$c_{\mathrm{att}} = 2$ for channel profile



Comparison with analytical results for diversity reception over $L = 4$ i.i.d. Rayleigh fading branches shows that channel does not quite offer diversity order of 4

Appendix

Analytical results

- Analytical BER performance of binary antipodal transmission over $L \geq 1$ i.i.d. Rayleigh fading branches with MRC at receiver was calculated according to

$$P_\mathrm{b} = \frac{1}{2^L}\left(1-\sqrt{\frac{1}{1+\sigma_\mathrm{n}^2}}\right)^L \sum_{l=0}^{L-1}\binom{L-1+l}{l}\frac{1}{2^l}\left(1+\sqrt{\frac{1}{1+\sigma_\mathrm{n}^2}}\right)^l$$

  using MATLAB function `proakis_equalSNRs.m`

- Details can be found in Chapter 14 of

  J. G. Proakis, *Digital Communications*, 4th ed., McGraw-Hill, 2001.

MATLAB code

- The MATLAB code and these slides can be downloaded from my homepage

  `www.ece.ubc.ca/~janm/`

  (see 'Teaching')