Multiple-Antenna Systems

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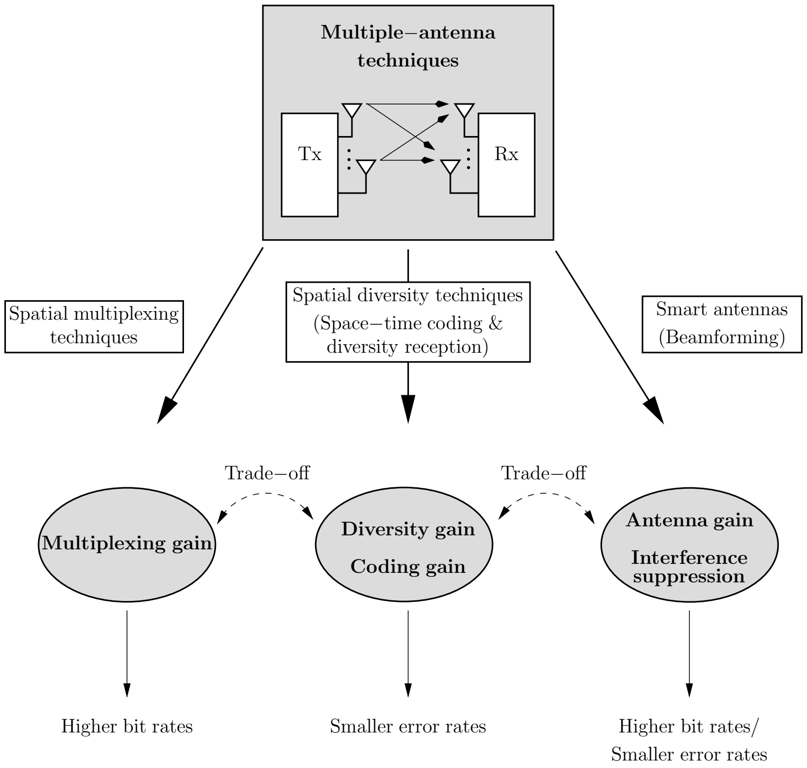
1. Introduction

- How is it possible to build (digital) wireless communication systems offering **high data rates** and **small error rates** ?
- **Trade-off** between spectral efficiency (high data rates) and power efficiency (small error rates), given fixed bandwidth & transmission power
- **Example:**
  Increase cardinality of modulation scheme $\Rightarrow$ Data rate $\uparrow$, error rate $\uparrow$
  Decrease rate of channel code $\Rightarrow$ Error rate $\downarrow$, data rate $\downarrow$
- Conventional transmitter & receiver techniques operate in **time domain** and/ or in **frequency domain**
- **Idea:**
  Utilize **multiple antennas** at the transmitter and/ or the receiver
  - Multiple-input multiple-output (MIMO) system
  - Single-input multiple-output (SIMO) system
  - Multiple-input single-output (MISO) system

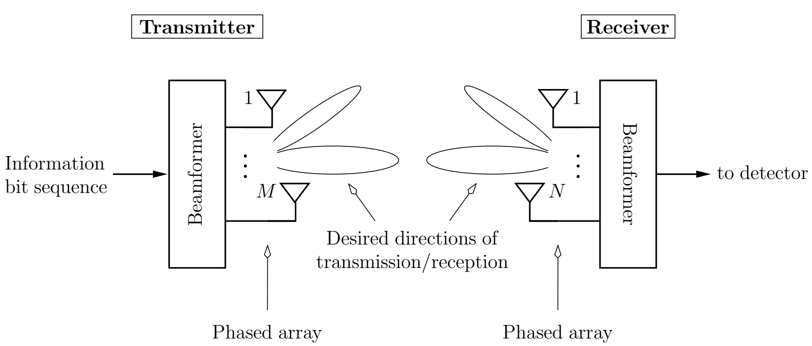
  $\Rightarrow$ Exploit **spatial domain** (in addition to time/ frequency domain)
  $\Rightarrow$ **Better** trade-off between spectral efficiency and power efficiency
- **Benefits of multiple antennas:**
  - Increased data rates by means of **spatial multiplexing techniques**
  - Decreased error rates by means of **spatial diversity techniques**
  - Improved signal-to-noise ratios (SNRs)/ signal-to-interference-plus-noise ratios (SINRs) by means of **beamforming techniques**



2. Basic Principles

2.1 Beamforming Techniques

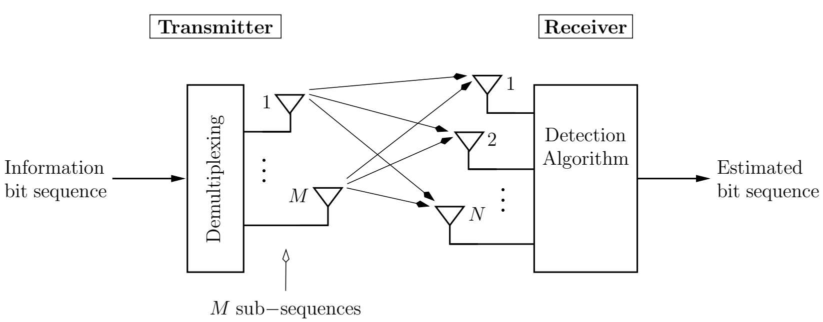
- **Goal:** Improved SNRs or SINRs in multiuser scenarios
- Beamforming can be interpreted as **linear filtering** in the spatial domain
- Consider **antenna array** with N elements and directional antenna pattern receiving a radio-frequency (RF) signal from a certain direction
- Due to antenna array geometry, impinging RF signal reaches antenna elements at **different** times (underlying baseband signal does not change)
  $\Rightarrow$ Adjust **phases** of RF signals to achieve constructive superposition
  $\Rightarrow$ Corresponds to **steering** of antenna pattern towards desired direction
  $\Rightarrow$ Additional **weighting** of RF signals can shape antenna pattern
  ($N-1$ degrees of freedom for placing maxima or nulls)
- Principle can also be utilized at the **transmitter** (reciprocity)



- **Improved SNRs:**
  Focus antenna patterns on desired angles of reception/ transmission, e.g., towards line-of-sight (LoS) or significant scatterers $\Rightarrow$ **Antenna gain**
- **Improved SINRs:**
  Steer nulls towards co-channel users $\Rightarrow$ **Interference suppression**
- Beamforming/ smart antenna techniques thus enable **space-division multiple access (SDMA)**, as an alternative to time-division or frequency-division multiple access (TDMA/ FDMA)
- SNR/ SINR gains can be utilized to **decrease error rates** or to **increase data rates** (by switching to a higher-order modulation scheme)
- In practical systems directions of significant scatterers must be **estimated** (e.g., MUSIC or ESPRIT algorithm); SINR can also be optimized **without** knowing the directions of all co-channel users (Capon beamformer)
- Beamforming techniques are **well established** since the 1960's (origins are in the field of radar technology); however, intensive research for **wireless communication** systems started only in the 1990's
- **Literature:** An exhaustive **overview** on smart antenna techniques for wireless communications can be found in [Godara'97]
- **Final remark:**
  Beamforming can also be performed in baseband domain, if channel is known at transmitter and receiver (**eigen-beamforming**)

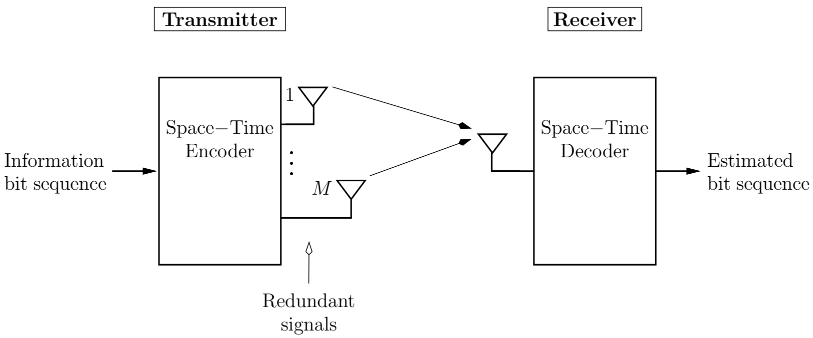
2.2 Spatial Multiplexing Techniques

- **Goal:** Increased data rates compared to single-antenna system
- Capacity of MIMO systems grows **linearly** with $\min\{M, N\}$
- At the **transmitter**, the data sequence is split into M sub-sequences that are transmitted simultaneously using the same frequency band
  $\Rightarrow$ Data rate increased by factor M (**multiplexing gain**)
- At the **receiver**, the sub-sequences are separated by means of **interference-cancellation** algorithm, e.g., linear zero-forcing (ZF)/ minimum-mean-squared-error (MMSE) detector, maximum-likelihood (ML) detector, successive interference cancellation (SIC) detector, ...
- Typically, channel knowledge required **solely** at the receiver
- For a good error **performance**, typically $N \geq M$ required
- Intensive **research** started at the end of the 1990's
- **Literature:** [Foschini'96]
  (Tutorials can be found in [Gesbert et al.'03], [Paulraj et al.'04])



2.3 Spatial Diversity Techniques

- **Goal:** Decreased error rates compared to single-antenna system
- Send/ receive multiple **redundant** versions of the same data sequence and perform appropriate **combining** (in baseband domain)
  $\Rightarrow$ If the redundant signals undergo statistically **independent** fading, it is unlikely that all signals simultaneously experience a deep fade
  $\Rightarrow$ Spatial **diversity gain** (typically, small antenna spacings sufficient)
- **Receive diversity:** SIMO system with N receive antennas and linear combining of the received signals
  - Various **combining strategies**, e.g., equal-gain combining (EGC), selection combining (SC), maximum-ratio combining (MRC), ...
  - Well-established since the 1950's, see [Brennan'59]
- **Transmit diversity:** MISO system with M transmit antennas
  - Appropriate **pre-processing** of transmitted redundant signals to enable **coherent** combining at receiver (space-time encoder/ decoder)
  - **Optionally**, $N > 1$ receive antennas for enhanced performance
  - Typically, channel knowledge required **solely** at the receiver
  - Intensive **research** started at the end of the 1990's
  - Well-known techniques are **Alamouti's scheme** for $M = 2$ transmit antennas [Alamouti'98], **space-time trellis codes** [Tarokh et al.'98], and **orthogonal space-time block codes** [Tarokh et al.'99]
  - An **abundance** of transmitter/ receiver structures has been proposed (some offer additional **coding gain**)
- **Literature:** An exhaustive **overview** of the benefits of spatial diversity in wireless communication systems can be found in [Diggavi et al.'04]



3. Mathematical Details

3.1 System Model

- Consider a **MIMO system** with M transmit and N receive antennas
- **Assumptions:**
  - **Frequency non-selective fading** & square-root Nyquist filters at transmitter and receiver (pulse energy $E_g := 1$)
    $\Rightarrow$ No intersymbol interference (ISI)
  - **Rayleigh fading** (no LoS component), i.e., channel gains are zero-mean complex Gaussian random variables
  - **Block fading**, i.e., channel gains are invariant over complete data block and change randomly from one block to the next
- **Discrete-time channel model:**
  - k: Discrete time index ($1 \le k \le N_\mathrm{B}$, N_B block length)
  - μ: Transmit antenna index ($1 \le \mu \le M$)
  - ν: Receive antenna index ($1 \le \nu \le N$)

- **Discrete-time channel model (cont'd):**
  - $x_\mu[k]$: Transmitted symbol of transmit antenna μ, time index k,

$$\mathsf{E}\{x_\mu[k]\} = 0, \quad \mathsf{E}\{|x_\mu[k]|^2\} =: \sigma_{x_\mu}^2$$

    (Underlying information symbols are denoted as $a[k]$)
  - $h_{\nu,\mu}$: Channel gain between μth transmit & νth receive antenna,

$$h_{\nu,\mu} \sim \mathcal{CN}(0, \sigma_h^2) \quad \text{(i.i.d)}$$

    (Amplitude $|h_{\nu,\mu}|$ is **Rayleigh** distributed)
  - $n_\nu[k]$: Additive white Gaussian noise (AWGN) sample at receive antenna ν, time index k,

$$n_\nu[k] \sim \mathcal{CN}(0, \sigma_n^2) \quad \text{(i.i.d)}$$

  - $y_\nu[k]$: Received symbol at receive antenna ν, time index k

- **Matrix-vector model**
  - Transmitted vector: $\mathbf{x}[k] := [\, x_1[k], \ldots, x_M[k] \,]^\mathrm{T}$
  - Noise vector: $\mathbf{n}[k] := [\, n_1[k], \ldots, n_N[k] \,]^\mathrm{T}$
  - Received vector: $\mathbf{y}[k] := [\, y_1[k], \ldots, y_N[k] \,]^\mathrm{T}$
  - Channel matrix:

$$\mathbf{H} := \begin{bmatrix} h_{1,1} & \cdots & h_{1,M} \\ \vdots & \ddots & \vdots \\ h_{N,1} & \cdots & h_{N,M} \end{bmatrix}$$

$\Rightarrow$ **System model:**

$$\mathbf{y}[k] = \mathbf{H}\,\mathbf{x}[k] + \mathbf{n}[k] \qquad (1)$$

3.2 Eigen-Beamforming

- Consider a **quadratic** MIMO system with $M = N > 1$ antennas
- Assume that the instantaneous realization of the channel matrix is **perfectly known** both at the transmitter **and** at the receiver
- **Eigenvalue decomposition** of $\mathbf{H}$:

$$\mathbf{H} := \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^{\mathrm{H}} \qquad (2)$$

$\mathbf{U}$: **Unitary** $(N \times N)$-matrix, i.e., $\mathbf{U}^{\mathrm{H}}\mathbf{U} = \mathbf{I}_N$

$\boldsymbol{\Lambda}$: **Diagonal** $(N \times N)$-matrix containing eigenvalues $\lambda_1, \ldots, \lambda_N$ of $\mathbf{H}$:

$$\boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \ldots, \lambda_N) = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_N \end{bmatrix}$$

- Since $\mathbf{H}$ is perfectly known, transmitter and receiver can **calculate** the matrix $\mathbf{U}$ (e.g., using the Jacobian algorithm [Golub et al.'96, Ch. 8.4])
- **Eigen-beamforming:**
  - Instead of $\mathbf{x}[k]$, transmitter sends **pre-processed** vector $\mathbf{x}'[k] := \mathbf{U}\mathbf{x}[k]$
  - The received vector $\mathbf{y}'[k]$ is **post-processed** as $\mathbf{U}^{\mathrm{H}}\mathbf{y}'[k] =: \mathbf{y}[k]$

$$\Rightarrow \quad \mathbf{y}[k] = \mathbf{U}^{\mathrm{H}}\mathbf{y}'[k] = \mathbf{U}^{\mathrm{H}}(\mathbf{H}\mathbf{x}'[k] + \mathbf{n}[k]) = \mathbf{U}^{\mathrm{H}}\mathbf{H}\mathbf{U}\mathbf{x}[k] + \underbrace{\mathbf{U}^{\mathrm{H}}\mathbf{n}[k]}_{=:\,\bar{\mathbf{n}}[k]}$$
$$= \mathbf{U}^{\mathrm{H}}\mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^{\mathrm{H}}\mathbf{U}\mathbf{x}[k] + \bar{\mathbf{n}}[k] = \boldsymbol{\Lambda}\mathbf{x}[k] + \bar{\mathbf{n}}[k]$$

$$\Rightarrow \quad y_\nu[k] = \lambda_\nu x_\mu[k] + \bar{n}_\nu[k] \quad \text{for all} \quad \mu, \nu = 1, \ldots, N \qquad (3)$$

  - Thus, assuming full rank ($\lambda_1 \neq 0, \ldots, \lambda_N \neq 0$) we have N parallel scalar channels **without** spatial interference (i.e., data rate enhanced by factor N compared to single-antenna system)
  - Noise samples $\bar{n}_\nu[k]$ are **still** i.i.d. $\sim \mathcal{CN}(0, \sigma_n^2)$, due to unitarity of $\mathbf{U}$

- **Transmit power allocation:**
  In addition, the transmit power allocated to the parallel channels can be **optimized**, based on the instantaneous SNRs $\frac{|\lambda_\nu|^2 \sigma_{x_\mu}^2}{\sigma_n^2}$ ($\nu = 1, \ldots, N$) and a certain optimization criterion

3.3 Spatial Multiplexing

- Consider a **MIMO system** with $N \geq M > 1$ antennas
  (For $N < M$, the system is inherently rank-deficient)
- Assume that the instantaneous realization of the channel matrix is **known solely** at the receiver
- **Linear ZF detection:** Received vector $\mathbf{y}[k]$ is **post-processed** as

$$\mathbf{z}_{\mathrm{ZF}}[k] := (\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{y}[k] =: \mathbf{H}^{+}\mathbf{y}[k] \qquad (4)$$

($\mathbf{H}^{+}$: Left-hand pseudo-inverse of $\mathbf{H}$; for $M = N$ and full rank use $\mathbf{H}^{-1}$)

$$\Rightarrow \qquad \mathbf{z}_{\mathrm{ZF}}[k] = \mathbf{H}^{+}\mathbf{y}[k] = \mathbf{H}^{+}(\mathbf{H}\mathbf{x}[k] + \mathbf{n}[k]) = \mathbf{x}[k] + \mathbf{H}^{+}\mathbf{n}[k],$$

i.e., spatial interference **completely removed**; however, variance of the resulting noise samples may be significantly **enhanced**

- **Linear MMSE detection:** (assume $\sigma_{x_1}^2 = \ldots = \sigma_{x_M}^2 =: \sigma_x^2$)
  Received vector $\mathbf{y}[k]$ is **post-processed** as

$$\mathbf{z}_{\mathrm{MMSE}}[k] := (\mathbf{H}^{\mathrm{H}}\mathbf{H} + \sigma_n^2/\sigma_x^2 \cdot \mathbf{I}_M)^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{y}[k] \qquad (5)$$

  - Usually **better** performance than ZF detection, since better **trade-off** between spatial interference mitigation & noise enhancement
  - For high SNR values ($\sigma_n^2 \to 0$), both detectors become **equivalent**
- Performance of ZF/ MMSE detection often quite **poor**, unless $N \gg M$

- **ML detection:**

$$\hat{\mathbf{x}}_{\mathrm{ML}}[k] := \mathrm{argmin}_{\tilde{\mathbf{x}}[k]} \; ||\mathbf{y}[k] - \mathbf{H}\tilde{\mathbf{x}}[k]||^2 \qquad (6)$$

  – For example, **brute-force** search over all possible hypotheses $\tilde{\mathbf{x}}[k]$ for the transmitted vector $\mathbf{x}[k]$

  ⇒ For Q-ary modulation scheme, there are Q^M possibilities

  ⇒ **Optimal** detection strategy (w.r.t. ML criterion), but very **complex**

- **SIC detection:**
  – **Good trade-off** between complexity and performance
  – Originally proposed in [Foschini'96] for the well-known **BLAST scheme** ('Bell-Labs Layered Space-Time Architecture')
  – **QR decomposition** of $\mathbf{H}$: (assume $N = M$)

$$\mathbf{H} := \mathbf{Q}\mathbf{R} \qquad (7)$$

  $\mathbf{Q}$: **Unitary** $(N{\times}N)$-matrix, i.e., $\mathbf{Q}^{\mathrm{H}}\mathbf{Q} = \mathbf{I}_N$

  $\mathbf{R}$: Upper **triangular** $(N{\times}N)$-matrix:

$$\mathbf{R} = \begin{bmatrix} r_{1,1} & \cdots & r_{1,N} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & r_{N,N} \end{bmatrix}$$

  (There are various algorithms for calculating the QR decomposition)

  – Received vector $\mathbf{y}[k]$ is first **post-processed** as $\mathbf{z}_{\mathrm{SIC}}[k] := \mathbf{Q}^{\mathrm{H}}\mathbf{y}[k]$

$$\Rightarrow \; \mathbf{z}_{\mathrm{SIC}}[k] := \mathbf{Q}^{\mathrm{H}}\mathbf{y}[k] = \mathbf{Q}^{\mathrm{H}}(\mathbf{H}\mathbf{x}[k]+\mathbf{n}[k]) = \mathbf{R}\mathbf{x}[k]+\bar{\mathbf{n}}[k] \qquad (8)$$

  – Symbol $x_N[k]$ is not affected by spatial interference and can **directly** be detected
  – Assuming that the detection of $x_N[k]$ was correct, the influence of $\hat{x}_N[k]$ can be **subtracted** from the $(N-1)$th row of (8); then symbol $x_{N-1}[k]$ can **directly** be detected, and so on ...

3.4 Receive Diversity

- Consider a **SIMO system** with N receive antennas

- Assume that the instantaneous realization of the $(N{\times}1)$**-channel matrix** is **perfectly known** at the receiver

- **Received sample** at receive antenna ν, time index k:

$$y_\nu[k] = h_{\nu,1}\, x_1[k] + n_\nu[k]$$

  – $h_{\nu,1} \sim \mathcal{CN}(0,\sigma_h^2) \;\Rightarrow\;$ Amplitude $|h_{\nu,1}| =: \alpha_\nu$ **Rayleigh** distributed

$$p(\alpha_\nu) = \frac{2\alpha_\nu}{\sigma_h^2}\exp\left(-\frac{\alpha_\nu^2}{\sigma_h^2}\right) \qquad (\alpha_\nu \geq 0), \qquad (9)$$

  – Instantaneous SNR $\frac{|h_{\nu,1}|^2\sigma_{x_1}^2}{\sigma_n^2} =: \gamma_\nu$ **Chi-square** (χ^2) distributed

$$p(\gamma_\nu) = \frac{1}{\bar{\gamma}}\exp\left(-\frac{\gamma_\nu}{\bar{\gamma}}\right) \qquad (\gamma_\nu \geq 0), \qquad (10)$$

  where $\bar{\gamma} := \frac{\sigma_h^2\sigma_{x_1}^2}{\sigma_n^2} \Rightarrow$ **Large** probability of **small** instantaneous SNRs

- **Idea:** Combine received samples $y_1[k], ..., y_N[k]$ to obtain more favorable SNR distribution at combiner output (γ_{comb})
  – **Equal-gain combining (EGC):** Add up all samples

$$z_{\mathrm{comb}}[k] := \sum_{\nu=1}^{N} y_\nu[k] = \underbrace{\left(\sum_{\nu=1}^{N} h_{\nu,1}\right)}_{=:\,h_{\mathrm{comb}}} x_1[k] + \underbrace{\sum_{\nu=1}^{N} n_\nu[k]}_{=:\,n_{\mathrm{comb}}[k]}$$

  ⇒ $h_{\mathrm{comb}} \sim \mathcal{CN}(0, N\sigma_h^2)$, $n_{\mathrm{comb}}[k] \sim \mathcal{CN}(0, N\sigma_n^2)$, i.e., **no gain!**

  ⇒ Do it **coherently** $\;(h_{\nu,1} := \alpha_\nu \mathrm{e}^{\mathrm{j}\phi_\nu})$

$$z'_{\mathrm{comb}}[k] := \sum_{\nu=1}^{N} \mathrm{e}^{-\mathrm{j}\phi_\nu} y_\nu[k] = \underbrace{\left(\sum_{\nu=1}^{N} \alpha_\nu\right)}_{=:\,h'_{\mathrm{comb}}} x_1[k] + \underbrace{\sum_{\nu=1}^{N} \mathrm{e}^{-\mathrm{j}\phi_\nu} n_\nu[k]}_{=:\,n'_{\mathrm{comb}}[k]}$$

  Combiner-output SNR: $\gamma_{\mathrm{comb}} = (\Sigma_\nu\, \alpha_\nu)^2 \sigma_{x_1}^2 / (N\sigma_n^2)$

– **Selection combining (SC):** Select branch with largest instant. SNR
Combiner-output SNR: $\gamma_{\text{comb}} = \max_\nu\{\alpha_\nu^2\}\sigma_{x_1}^2/\sigma_n^2 = \max_\nu\{\gamma_\nu\}$

– **Maximum-ratio combining (MRC):**

$$z_{\text{comb}}[k] := \sum_{\nu=1}^{N} h_{\nu,1}^*\, y_\nu[k] = \underbrace{\left(\sum_{\nu=1}^{N} |h_{\nu,1}|^2\right)}_{=:\, h_{\text{comb}}} x_1[k] + \underbrace{\sum_{\nu=1}^{N} h_{\nu,1}^*\, n_\nu[k]}_{=:\, n_{\text{comb}}[k]}$$

Combiner-output SNR: $\gamma_{\text{comb}} = (\Sigma_\nu\, |h_{\nu,1}|^2)\sigma_{x_1}^2/\sigma_n^2 = \Sigma_\nu\, \gamma_\nu$

$\Rightarrow$ **Maximizes** combiner-output SNR; **optimal** w.r.t. ML criterion

- **Symbol error rates (SERs) with MRC:** (without derivation ;-))
  $\bar{\gamma}$: Average SNR **per** receive branch

  – Binary Phase-Shift Keying (BPSK) [Proakis'01, Ch. 14]

$$\text{SER}(\bar{\gamma}) = \frac{1}{2^N}\left(1-\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)^N \sum_{i=0}^{N-1}\binom{N-1+i}{i}\frac{1}{2^i}\left(1+\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)^i \quad (11)$$

  – Q-ary Phase-Shift Keying (Q-PSK) [Simon et al.'00]

$$\text{SER}(\bar{\gamma}) = \frac{1}{\pi}\int_0^{\frac{(Q-1)\pi}{Q}} \left(\frac{\sin^2\varphi}{\sin^2\varphi + \bar{\gamma}\sin^2(\pi/Q)}\right)^N \mathrm{d}\varphi \quad (12)$$

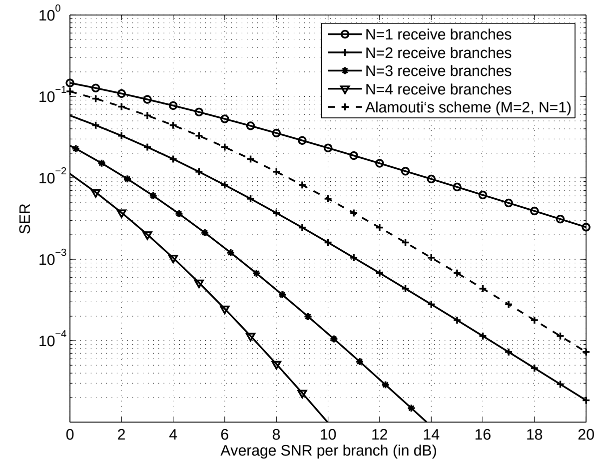
  – Q-ary Amplitude-Shift Keying (Q-ASK) [Simon et al.'00]

$$\text{SER}(\bar{\gamma}) = \frac{2(Q-1)}{Q\pi}\int_0^{\frac{\pi}{2}} \left(\frac{(Q^2-1)\sin^2\varphi}{(Q^2-1)\sin^2\varphi + 3\bar{\gamma}}\right)^N \mathrm{d}\varphi \quad (13)$$

  – Q-ary Quadrature-Amplitude Modulation (Q-QAM) [Simon et al.'00]

$$\text{SER}(\bar{\gamma}) = \frac{4}{\pi}\left(1-\frac{1}{\sqrt{Q}}\right)\int_0^{\frac{\pi}{2}} \left(\frac{2(Q-1)\sin^2\varphi}{2(Q-1)\sin^2\varphi + 3\bar{\gamma}}\right)^N \mathrm{d}\varphi - \frac{4}{\pi}\left(1-\frac{1}{\sqrt{Q}}\right)^2\int_0^{\frac{\pi}{4}} \left(\frac{2(Q-1)\sin^2\varphi}{2(Q-1)\sin^2\varphi + 3\bar{\gamma}}\right)^N \mathrm{d}\varphi \quad (14)$$

Example: BPSK, $N = 1, \ldots, 4$ receive branches



Asymptotic slope (i.e., $\bar{\gamma} \to \infty$) of the curves is $-N$ ('diversity order N')

3.5 Transmit Diversity

- Consider a **MISO system** with M transmit antennas
- Assume that the instantaneous realization of the ($1{\times}M$)-channel matrix is **perfectly known** at the receiver, but **not** at the transmitter
- **Transmit Diversity:** Suitable **pre-processing** of transmitted data sequence required to allow for **coherent** combining at the receiver
  – **Example:** Send **identical** signals over all transmit antennas
  $\Rightarrow$ **No diversity gain!** (corresponds to EGC without co-phasing)
  – **Instead:** Perform appropriate two-dimensional mapping/ encoding in **time** and **space** (i.e., over the transmit antennas)

- **Example:** Alamouti's scheme for $M=2$ transmit antennas
  ($N=1$ receive antennas considered; can be extended to $N>1$)
  - **Space-time mapping:** Information symbols to be transmitted are processed in pairs $[\,a[k], a[k+1]\,]$; at time index k, symbol $a[k]$ is transmitted via the first antenna and symbol $a[k+1]$ via the second antenna; at time index $k+1$, symbol $-a^*[k+1]$ is transmitted via the first antenna and symbol $a^*[k]$ via the second antenna

$$\mathbf{A} = \begin{bmatrix} a[k] & a[k+1] \\ -a^*[k+1] & a^*[k] \end{bmatrix} \begin{matrix} \longleftarrow \text{time index } k \\ \longleftarrow \text{time index } k+1 \end{matrix}$$
$$\begin{matrix} \uparrow & \uparrow \\ \text{antenna 1} & \text{antenna 2} \end{matrix} \qquad (15)$$

  (In terms of prior system model: $\mathbf{A} =: [\,\mathbf{x}^{\mathrm{T}}[k], \mathbf{x}^{\mathrm{T}}[k+1]\,]^{\mathrm{T}}$)
  - **Received samples** (time index $k, k+1$):

$$y_1[k] = h_{1,1}\, a[k] + h_{1,2}\, a[k+1] + n_1[k]$$
$$y_1[k+1] = -h_{1,1}\, a^*[k+1] + h_{1,2}\, a^*[k] + n_1[k+1]$$

  - **Equivalent matrix-vector model** (by taking the $(.)^*$ of $y_1[k+1]$)

$$\underbrace{\begin{bmatrix} y_1[k] \\ y_1^*[k+1] \end{bmatrix}}_{=:\, \mathbf{y}_{\mathrm{eq}}[k]} = \underbrace{\begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2}^* & -h_{1,1}^* \end{bmatrix}}_{=:\, \mathbf{H}_{\mathrm{eq}}} \underbrace{\begin{bmatrix} a[k] \\ a[k+1] \end{bmatrix}}_{=:\, \mathbf{a}[k]} + \underbrace{\begin{bmatrix} n_1[k] \\ n_1^*[k+1] \end{bmatrix}}_{=:\, \mathbf{n}_{\mathrm{eq}}[k]}$$

  - **Detection step at the receiver:**
    $\mathbf{H}_{\mathrm{eq}}$ is always **orthogonal** (!), while $\mathbf{H}_{\mathrm{eq}}^{\mathrm{H}}\mathbf{H}_{\mathrm{eq}} = (|h_{1,1}|^2 + |h_{1,2}|^2)\,\mathbf{I}_2$

$$\Rightarrow \quad \mathbf{z}_{\mathrm{comb}}[k] := \mathbf{H}_{\mathrm{eq}}^{\mathrm{H}}\,\mathbf{y}_{\mathrm{eq}}[k] = \mathbf{H}_{\mathrm{eq}}^{\mathrm{H}}\mathbf{H}_{\mathrm{eq}}\mathbf{a}[k] + \underbrace{\mathbf{H}_{\mathrm{eq}}^{\mathrm{H}}\mathbf{n}_{\mathrm{eq}}[k]}_{=:\, \mathbf{n}'_{\mathrm{eq}}[k]}$$
$$= (|h_{1,1}|^2 + |h_{1,2}|^2)\,\mathbf{a}[k] + \mathbf{n}'_{\mathrm{eq}}[k]$$

$\Rightarrow$ Two **parallel** scalar channels for the symbols $a[k]$ and $a[k+1]$ (no spatial interference)

$\Rightarrow$ Corresponds to **MRC** with $M=1$ transmit and $N=2$ receive antennas; however, using the same average transmit power, Alamouti's scheme exhibits a **3 dB loss** compared to MRC

4. Literature

4.1 Cited References

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